

Proton's Isovector Parton Distribution Function with Updated Analysis of Large-Momentum Lattice Data

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The proton's unpolarized $u(x) - d(x)$ parton distribution function (PDF) has been studied by a number of lattice QCD groups through large momentum expansion. However, due to lattice artifacts (excited state contaminations, unphysical pion masses, and discretization effects) and less-advanced theoretical analysis (renormalizations, large-distance extrapolations, and large-log resummations), the resulting PDFs cannot be compared strictly with experimental data. By using the state-of-the-art theoretical tools and mitigating the lattice artifacts empirically, we reanalyze the available datasets in the literature and find that the new PDF for the physical-limit-extrapolated dataset (MSULat20) is consistent with global fits within $\sim 1\sigma$, while others are broadly compatible. This provides evidence that large momentum expansion is capable of accurately predicting the x dependence of the PDFs when ideal lattice data become available.

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Introduction—The proton's unpolarized isovector parton distribution function (PDF), $u(x) - d(x)$, is among the best determined from more than 50 years of high-energy experimental data, with uncertainties at a few percent level in the moderate- x region [1–7]. In recent years, this quantity has been explored from first principles lattice quantum chromodynamics (QCD) [8] within the framework of large momentum expansion (LaMET) [9–13], which supposedly has the prediction power of the x dependence. The isovector PDF is significantly easier to compute on lattice than the flavor-singlet ones because it avoids “quark-line-disconnected diagrams.” Therefore, it is an ideal benchmark observable for the quantitative performance of lattice QCD and large momentum expansion.

A number of lattice QCD groups have published results on $u(x) - d(x)$ using different lattice configurations with assorted lattice spacings and pion masses [14–28]. After performing LaMET expansion, the resulting x dependences exhibit noticeable variations and mostly differ from phenomenological determinations. These discrepancies may originate from both lattice artifacts and less-sophisticated theoretical analysis. On the lattice side, excited-state contaminations, unphysical pion masses, and discretization effects remain important sources of systematic uncertainties. Compared with the first-moment calculations [29,30]

where the lattice artifacts have important impacts on the final results, these effects in PDFs are expected to be strong. In fact, they could be even stronger because large momenta result in smaller energy gaps between the ground and excited states and additional suppressions of the signal-to-background ratios. On the theoretical side, much theoretical development for large momentum expansion has become available only in recent years [31–41]. Most existing calculations predate these developments and therefore have potential systematic errors from incomplete theoretical analysis.

In this Letter, we reanalyze the available $u - d$ PDF lattice datasets in Refs. [20,24,25,27] using updated theoretical tools, including hybrid and self-renormalization [31–33], large distance extrapolation [35], next-to-next-to-leading order (NNLO) matching [36,37], and perturbative large-log resummations [34,38–41]. To mitigate the majority of lattice artifacts, we subject the data to the constraint of the first moment $\langle x \rangle$ through an empirical procedure. This is because the first moment is very sensitive to various lattice artifacts and has been very challenging to obtain accurately in moment calculations [29,30]. Therefore, a fair test of the large-momentum expansion at present is made by assuming that the first moment can already be reproduced in a standard lattice calculation. We compare the resulting PDFs with one another and with global fits [4,42], and find that the new results are largely consistent with them in physical limits. This provides evidence that large momentum expansion can potentially predict the x dependence of the PDFs when ideal lattice data become available.

Lattice QCD datasets—The starting point of our analysis is the $u - d$ unpolarized quasi-PDF matrix elements in a

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large-momentum proton,

$$\tilde{h}^B(z, P^z, a) = \langle N(P) | [\bar{u}(z) U(z, 0) \gamma^t u(0) - \bar{d}(z) U(z, 0) \gamma^t d(0)] | N(P) \rangle_B, \quad (1)$$

where various notations are standard [$|N(P)\rangle$ is a nucleon state of momentum P^μ , u , and d are quark fields, γ^μ the Dirac matrices, U is the Wilson line, and $x^\mu = (t, x, y, z)$ are spacetime coordinates] and “ B ” denotes the bare matrix elements before renormalization. Previous calculations of this quantity used for large momentum expansion analysis include Refs. [14–21, 23–27].

The excited-state controls in the above works are significantly less sophisticated than the first-moment calculation in Ref. [29], which demonstrates the level of difficulty in the nucleon matrix elements. Moreover, only one existing work [25] performs both physical pion mass and continuum extrapolations for the isovector PDF. Two papers [26, 27] utilize the more advanced hybrid renormalization technique [31]. All calculations precede the recent theoretical developments regarding the latest long-distance expansion formulas for spatial correlations [35] and complete resummation formulas [34, 38–41]. Therefore, these calculations predicted various different x dependences, and many of them deviate from the experimental fittings.

To test if large-momentum expansion can realize the promise of predicting the x dependence of PDFs, we present a new analysis of the lattice data with a better control of the above effects. Considering large momentum (at least ~ 2 GeV, corresponding to $v \sim 0.9c$), statistical precision, and data accessibility, we select four characteristic datasets in Refs. [20, 24, 25, 27] for the present analysis, as summarized in Table I. In particular, the MSULat20 dataset [25] allows extrapolations to the continuum and physical pion mass limits, with the coordinate and momentum space statistical uncertainties extracted from Figs. 4 and 5 of the reference, respectively. The other datasets [20, 24, 27] are bootstrap samples obtained through private communications. These matrix elements are normalized with respect to the real part at $z = 0$ for charge conservation, while the imaginary part at $z = 0$ is set to zero according to the Hermiticity of the correlator.

In addition to the construction in Eq. (1), the Coulomb gauge method is under exploration [28], and the results are consistent with the global fit PDFs. However, this method requires further theoretical developments, such as next-to-

next-to-leading-order matching and threshold log resummation. Therefore, it is not incorporated into this Letter’s analysis framework.

New analysis—The bare matrix elements in Eq. (1) contain the linear and logarithmic divergences [43–48] regularized by finite lattice spacing a , which are absent in renormalized theory. To eliminate these ultraviolet (UV) divergences, early papers proposed or adopted various approaches [43, 47, 49–52], which, however, suffer from additional infrared (IR) effects at large z that are not perturbatively controllable [31, 32]. To avoid this issue, the hybrid renormalization [31, 33] implements an almost minimal UV subtraction while mitigating short-distance discrepancies between lattice and continuum theories. The renormalized matrix elements take the form,

$$\tilde{h}^R(z, P^z) = \frac{\tilde{h}^B(z, P^z, a)}{\tilde{h}^B(z, 0, a)} \theta[z_s - |z|] + \frac{\tilde{h}^B(z, P^z, a) Z_R(z_s, a, \mu)}{Z_R(z, a, \mu) \tilde{h}^B(z_s, 0, a)} \theta[|z| - z_s]. \quad (2)$$

The cutoff z_s is chosen within the window of $a < z_s \ll 1/\Lambda_{\text{QCD}}$. For $|z| < z_s$, the hybrid scheme reduces to the ratio method [50, 53, 54], which removes the UV divergences together with part of the discretization effects. For $|z| > z_s$, it is the same as the $\overline{\text{MS}}$ scheme up to an overall constant, which subtracts only the UV divergences without introducing extra nonperturbative effects. The factor $Z_R(z, a, \mu)$, converting lattice data to the $\overline{\text{MS}}$ scheme, is extracted using self-renormalization [32]. In particular, Z_R contains a finite renormalization factor $\sim e^{-m_0|z|}$, with a new parameter m_0 . This additional scheme dependence shall be matched, at small z , to an infrared-renormalon resummation scheme of the divergent perturbative QCD series in $\alpha_s(z)$, which has exactly the same uncertainty of order $\mathcal{O}(z\Lambda_{\text{QCD}})$ [34, 52, 55]. More explicitly, we fix the m_0 by fitting the renormalized lattice zero momentum matrix element $\tilde{h}^B(z, 0, a)/Z_R(z, a, \mu)$ with the perturbative coefficient $C_0(z^2\mu^2)$ [50] at NNLO [36, 37] RG resummation (RGR) [38] and leading renormalon resummation (LRR) [34], within the window $a < z \ll 1/\Lambda_{\text{QCD}}$. Therefore, perturbative uncertainties in C_0 are propagated into the systematic uncertainty of m_0 .

Lattice artifacts, such as excited-state contaminations, unphysical pion masses, and discretization effects, could

TABLE I. Selected lattice QCD datasets for the unpolarized $u - d$ proton quasi-PDF matrix elements. Their information, including lattice spacing a , pion mass m_π , and hadron momentum P^z , is listed in the table.

Works	MSULat20 [25]	ETMC20 [24]	LPC18 [20]	CLQCD24 [27]
a (fm)	0.12, 0.09, 0.06	0.0820	0.0854	0.105
m_π (MeV)	310, 230, 138	373	356	293
P^z (GeV)	2.2	1.89	2.27	1.85

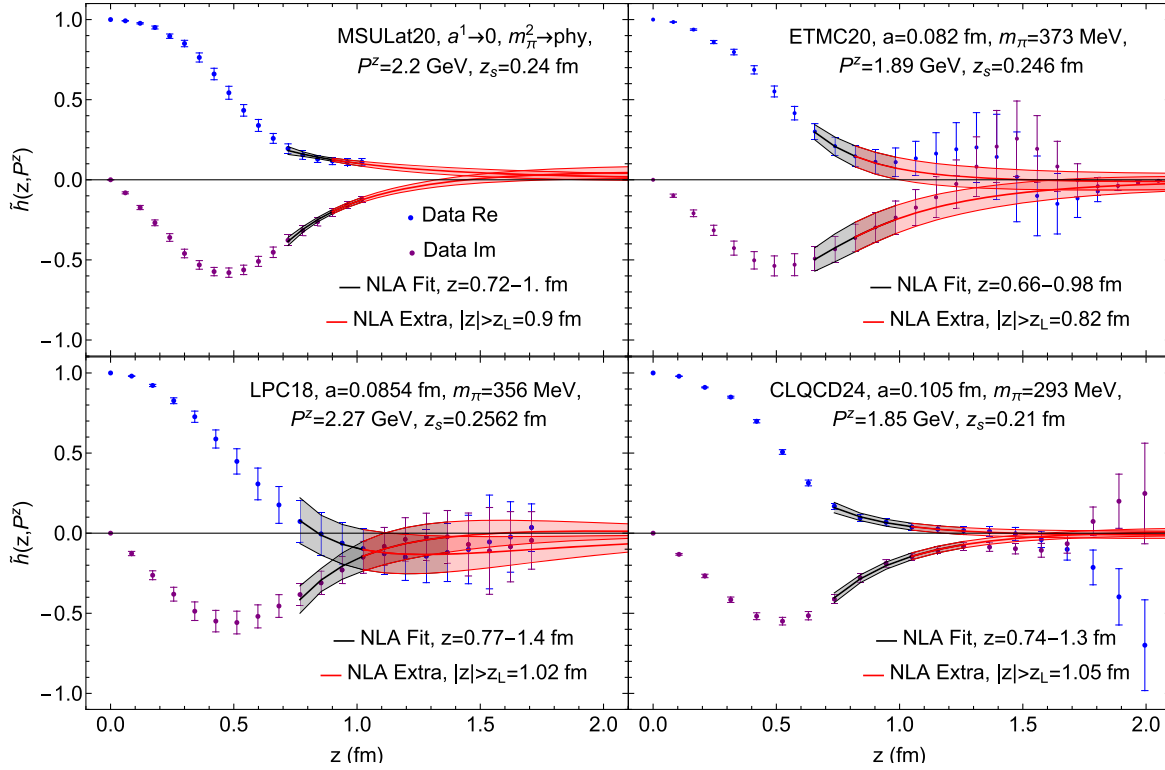


FIG. 1. Large distance asymptotic analysis. The blue and purple points denote the real and imaginary parts of the lattice data, respectively. The black bands cover the fit regions in the asymptotic analysis, while the red bands show the extrapolated large-distance behavior.

distort the global behavior of the lattice PDF, causing a larger first moment $\langle x \rangle$ than the phenomenological value [24,29,30]. To mitigate these effects, we decide to enforce the first moment of the PDF to the state-of-the-art lattice-determined physical value from Ref. [30], which is consistent with other recent lattice calculations [56,57] and phenomenological results [4,42]. In coordinate space, it is the slope of the imaginary part of the correlator near $z = 0$, namely the coefficient of izP^z according to the operator product expansion (OPE). Since the correction factor $1 + i\delta y z P^z$ at large z has an unrealistically large imaginary part, we unitarize it to an exponential. We thus use the following *Ansatz*,

$$\tilde{h}(z, P^z) = \tilde{h}^R(z, P^z) e^{izP^z \delta y}, \quad (3)$$

to remove a large part of the lattice artifacts. The parameter δy is determined by fitting the imaginary part of $\tilde{h}(z, P^z)$ to the short distance OPE [50,53,58],

$$-izP^z \langle x \rangle(\mu) \frac{C_1(z^2 \mu^2)}{C_0(z^2 \mu^2)} + \dots \quad \text{for } a < z \ll 1/\Lambda_{\text{QCD}}, \quad (4)$$

where the first moment is fixed to the Mainz24 value [30], $\langle x \rangle(\mu = 2 \text{ GeV}) = 0.153(17)$. The Wilson coefficients $C_i(z^2 \mu^2)$ [50] are evaluated at NNLO + RGR + LRR

accuracy. We find $\delta y = 0.104(16)$, $0.094(16)$, $0.068(16)$, and $0.085(17)$ for the four datasets, respectively, where the uncertainty originates from the first moment. The coordinate-space results $\tilde{h}(z, P^z)$ are shown as the blue and purple points in Fig. 1.

The next step is to Fourier transform (FT) the spatial correlator into momentum space. Formally, the FT requires the correlator over the entire z range, including the large-distance region (e.g., $\gtrsim 1 \text{ fm}$). Because of confinement, the correlator decays exponentially at large distances, so that the long-distance tail contributes only weakly to the FT. In practice, however, lattice data at large z suffer from exponentially deteriorating signal-to-noise ratios and cannot be directly used. This issue has been investigated through asymptotic analyses in Refs. [31,59,60]. A recent work [35] derives the large-distance asymptotic forms for various quasi-correlators, which serve as physical constraints on the FT. For the unpolarized u - d proton quasi-PDF matrix element, the leading asymptotic (LA) and next-to-leading asymptotic (NLA) forms in the $z \rightarrow \infty$ expansion are

$$\tilde{h}^{\text{LA}}(z, P^z) = A e^{i\phi \text{sign}(z)} e^{-\Lambda|z|}, \quad (5)$$

$$\tilde{h}^{\text{NLA}}(z, P^z) = \left[A e^{i\phi \text{sign}(z)} + \frac{A' e^{i\phi' \text{sign}(z)}}{|z|} \right] e^{-\Lambda|z|}, \quad (6)$$

where A , ϕ , Λ , A' , and ϕ' are real fit parameters. The mass gap Λ corresponds to the binding energy of an intermediate state within the quasi-PDF correlator, which is a vector heavy-light meson. Its value is estimated as 400–600 MeV, and we impose a loose constraint on it during the fit to ensure stability. We fit the asymptotic form $\tilde{h}_{\text{asym}}(z, P^z)$ ($\tilde{h}_{\text{asym}}$ denotes either $\tilde{h}^{\text{LA}}$ or $\tilde{h}^{\text{NLA}}$) to the lattice data $\tilde{h}(z, P^z)$ at distances larger than $\sim 1/\Lambda$ (e.g., 0.7–1.4 fm), with representative NLA fits shown as the black bands in Fig. 1. Uncertainties in the fit parameters will be propagated into the statistical uncertainty. The differences between the LA and NLA analyses, which are at the few-percent level, will be included in the systematic uncertainty. The quasi PDF is then obtained through the FT,

$$\tilde{f}(y, P^z) = P^z \int_{-\infty}^{+\infty} \frac{dz}{2\pi} e^{izP^z y} [\tilde{h}(z, P^z) \theta(z_L - |z|) + \tilde{h}_{\text{asym}}(z, P^z) \theta(|z| - z_L)], \quad (7)$$

where the switching point z_L is chosen around 0.8–1 fm.

The PDF f and quasi-PDF $\tilde{f}$ share the same IR physics but differ in UV physics [50,61–66]. They are connected through a LaMET expansion [13],

$$f(x, \mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z) + O\left(\frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{(1-x)^2 P_z^2}\right), \quad (8)$$

where the perturbative matching kernel $C(x/y, \mu/yP^z)$ is at NNLO accuracy [36,37], supplemented by DGLAP log resummation (RGR) [38], leading-renormalon resummation (LRR) [34], and threshold log resummation (TR) [39–41]. The RGR and TR primarily control the precision at small and large x , respectively. The LRR improves perturbative convergence. Moreover, the renormalon-regularization scheme of the matching kernel is chosen consistently with the constant m_0 in Z_R of Eq. (2), ensuring the cancellation of the linear power correction [34]. Systematics arising from omitted higher-order perturbative corrections are estimated by varying the physical scales that enter the resummation procedures.

Results—The LaMET results for the proton u - d unpolarized PDF are shown as the red bands in Fig. 2. For ETMC20, LPC18, and CLQCD24 datasets, the uncertainties involve statistical fluctuations, the renormalization parameter m_0 , first-moment calibration, Fourier transform, and perturbative matching. Particularly, the uncertainty associated with finite hadron momentum P^z is estimated

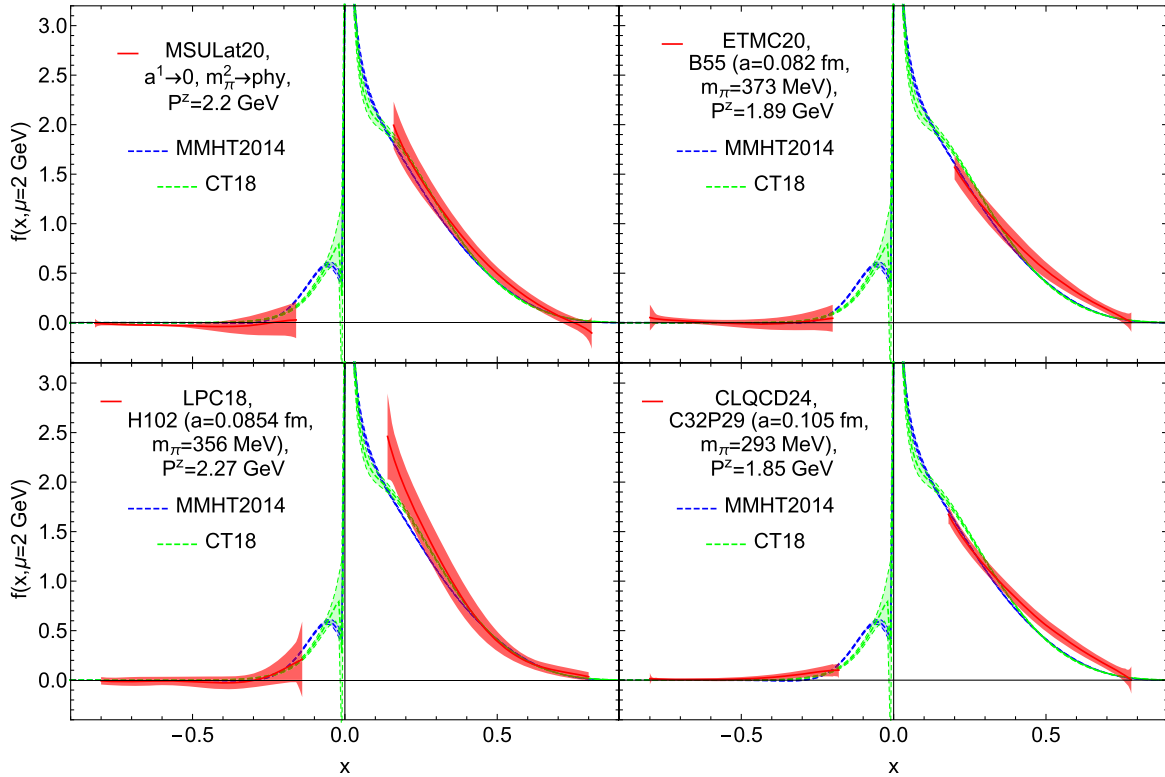


FIG. 2. The proton isovector unpolarized PDFs. The red bands denote the LaMET results in the region $\Lambda_{\text{QCD}}/2P^z \lesssim x \lesssim 1 - \Lambda_{\text{QCD}}/2P^z$, where the perturbative matching is reliable. The blue and green bands correspond to the phenomenological PDFs extracted from global analyses by MMHT2014 [42] and CT18 [4], respectively.

through omitted higher-order terms in the perturbative matching rather than through higher-twist corrections. For the MSULat20 dataset, an additional systematic uncertainty from the continuum and physical pion-mass extrapolations is estimated from the spread among various *Ansätze* in Fig. 4 of Ref. [25]. The detailed explanations of the uncertainty estimate and propagation are included in Supplemental Material [67].

After applying a common state-of-the-art framework to all four lattice datasets, the spread among the resulting PDFs is reduced compared with the original analyses, as shown in Supplemental Material [67]. Since phenomenological PDFs [1–7] are nearly identical in the moderate- x region, only two representative results are shown in Fig. 2 for comparison. The continuum- and physical-mass-extrapolated MSULat20 result agrees with the phenomenological PDFs within approximately 1σ in the accessible x region. The remaining datasets exhibit broadly compatible x dependence, with small deviations that may arise from residual discretization effects or unphysical pion masses. Additional lattice data will help to confirm this expectation. According to our numerical tests, the MSULat20 results for each ensemble before the extrapolations are similar to those of ETMC20 and CLQCD24. The ratios of the LaMET and global fit PDFs are shown in Supplemental Material [67] to further illustrate the above observations.

Summary—We present a LaMET calculation of the proton isovector unpolarized PDF using existing lattice QCD datasets, along with theoretical precision-control techniques and an empirical method to remove a large part of lattice artifacts. The disagreement among earlier lattice PDF calculations appears to be largely attributable to analysis methodology and lattice artifacts. The resulting PDF for the physical-limit-extrapolated dataset, MSULat20, agrees with phenomenological determinations, while other datasets show broad alignment, supporting LaMET’s potential to achieve quantitatively reliable determinations of the x dependence of parton distributions. In particular, hadron momenta of order $P^z \sim 2$ GeV may already be sufficient for precision studies when combined with the latest analysis techniques. This work is an important step toward precision LaMET calculations, which will offer essential inputs for high-energy experiments at the Large Hadron Collider (LHC), Jefferson Laboratory (JLab), and Electron-Ion Colliders (EICs).

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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