

Chaotic Fluctuations in a Universal Set of Transmon Qubit Gates

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Transmon qubits arise from the quantization of nonlinear resonators, systems that are prone to the buildup of strong, possibly chaotic, fluctuations. Such instabilities will likely affect fast gate operations which involve the transient population of higher excited states outside the computational subspace. Here, we extend the statistical analysis from the spectrum of the generator to that of dynamics itself and show that the instantaneous eigenphases of the time evolution operator, in particular of their curvatures, allows for identifying the subspace most affected by instabilities. Our analysis shows that fast entangling gates, operating at speeds close to the so-called quantum speed limit, contain transient regimes where the dynamics indeed becomes partially chaotic for just two transmons. Surprisingly, this does not reflect in gate reliability, rather, slower operations lead to relatively enhanced errors.

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Introduction—In coupled superconducting circuits, one of the leading platforms in quantum information science [1,2], the nonlinearity of Josephson junctions serves to separate an energetically low-lying qubit subspace from a higher-lying state space whose presence has become a focus of attention [3–6]. The restriction to the computational subspace rests on the assumption that the energy stored in the initial state remains decentralized, and higher excited states of individual “qubits” are not accessed. However, residual qubit-qubit couplings can be made algebraically, but never exponentially, small in the detuning from the resonator frequencies. Indeed, the presence of the higher-lying state space may be detrimental for stationary states relevant to storage of information, in particular so for fixed-frequency architectures [3]. While, for information storage, tunable couplers present one route to alleviate signatures of nonintegrability, the issue of instabilities due to the nonlinearity becomes all the more pressing for the implementation of gates.

Indeed, the population of energy levels outside of the computational subspace is rather common when implementing fast gates [7] or resetting superconducting qubits [8]. Fast device operation is desirable in view of optimal clock speeds but also to combat decoherence. The shortest possible time in which a given task can be carried out is referred to as the quantum speed limit (QSL) [9]. The QSL

for quantum gates is fundamentally determined by the size of the effective time-dependent qubit-qubit couplings which are needed to generate entanglement—or, in the case of local rotation gates, to undo undesired entanglement due to residual couplings. Operation at the QSL is enabled by suitably tailored pulses which bring the population of higher lying levels during the gate operation back to the computational subspace at the end of the gate [10]. The corresponding time evolution, under the perspective of (classical) nonlinear dynamics, takes, however, place in a twilight zone between integrable dynamics in the limit of weak intertransmon coupling and possibly chaotic fluctuations at larger couplings [5,6]. Quantum mechanically, this ambiguity shows in the structure of transmon many-body spectra which generically show statistical signatures of both integrability (evidenced by Poisson statistics) and chaos (Wigner-Dyson statistics) [3]. A transmon gate is then a driven protocol transiently exciting a subsystem of qubits close to the ones targeted by the gate operation. The faster a gate operation, the higher the chances that it connects to semiclassically unstable sectors of the state space [4,5]. For gates transiently populating part of the higher-lying state space, instabilities may thus become relevant already in architectures where the computational subspace itself is operated in a stability island [3]. It is thus imperative to clarify the role of the nonlinearity for the dynamics of coupled transmons. Because optimal control is central to scaling up superconducting architectures, identifying how nonlinearity shapes the dynamical structure of $U(t)$ is essential for understanding—and ultimately mitigating—control bottlenecks in fast-gate operation.

Here, we show how to analyze and detect signatures of nonintegrability in a universal set of transmon qubit gates.

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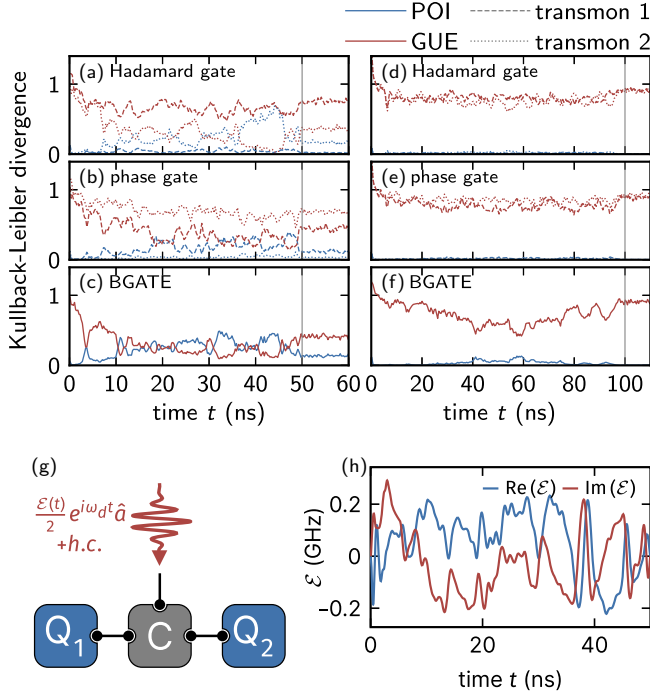


FIG. 1. Spectral analysis of chaotic fluctuations in single-qubit and two-qubit gates. (a)–(f) The statistics of the relative eigenphase differences of the time evolution are compared to Poisson (POI), respectively, Gaussian (GUE) statistics using the Kullback-Leibler (KL) divergence for gate durations of $T = 50$ ns (left) and $T = 100$ ns (right). The single-qubit gates are identified by their line style. Deviations from Poisson statistics are observed in panels (a)–(c) and to a lesser extent in panel (f). (g) Pictorial view of two transmon qubits coupled via a joint transmission line cavity. Gates are implemented by applying a fine-tuned microwave field driving the cavity [10], shown in (h) for the fast entangling BGATE (c).

We adapt methodology from quantum nonlinear dynamics and many-body localization theory to the unitary operators $U(t)$ implementing single-qubit and two-qubit gates. The time evolutions are realized by time-dependent external fields, designed with optimal control [11] as a map on the computational state space, but transiently acting in the larger space of high-lying states. To be specific, we consider the smallest possible structure comprising two transmon qubits coupled via an intermediate cavity subject to a microwave drive [10], see Fig. 1. Analyzing the statistical properties of $U(t)$ in terms of its instantaneous eigenvalues, we find that while traces of nonlinear and even chaotic dynamics are visible in the operators implementing fast gates, they present no principal obstruction to high gate fidelities. The reason is the finite dimensionality of the effective Hilbert space subject to control. While classical chaotic time evolution is unpredictable by definition, projection onto a finite dimensional quantum Hilbert space leads to more manageable dynamics, which can be optimized towards a desired output. Moreover, the presence of partially chaotic dynamics does not seem to impact gate

reliability, a finding that we rationalize in terms of Landau-Zener transitions.

Model and control protocols—We analyze two universal sets of gates, with Hadamard and local phase gates as representative single-qubit operations and the BGATE as entangling operation. The latter yields the smallest decomposition of an arbitrary two-qubit operation into gates of a universal set [12,13]. For full operability of the quantum hardware, it is important to inspect a complete universal set, as this requires the ability to both create and destroy entanglement. The Hamiltonian for two transmon qubits with a joint transmission line cavity subject to a microwave drive in a frame rotating with the drive frequency ω_d reads [10]

$$H(t) = \sum_{q=1}^2 \left[\delta_q b_q^\dagger b_q + \frac{\alpha_q}{2} b_q^\dagger b_q^\dagger b_q b_q + g(b_q^\dagger a + b_q a^\dagger) \right] + \delta_c a^\dagger a + \frac{1}{2} \mathcal{E}(t) a + \frac{1}{2} \mathcal{E}^*(t) a^\dagger, \quad (1)$$

where b_q and a are the annihilation operators for the q th transmon and the cavity, respectively, $\delta_q = \omega_q - \omega_d$ and $\delta_c = \omega_c - \omega_d$ are the qubit and cavity detunings from ω_d , α_q the qubit anharmonicities, and g is the qubit-cavity coupling strength, see Supplemental Material [13] for their values. $\mathcal{E}(t)$ denotes the amplitude of the cavity drive. For all gates analyzed below, its time dependency was derived via gate optimization [10]. Equation (1) is reminiscent of tunable couplers save for the transversal drive. The quantum gates we analyze operate in the quasidispersive straddling qutrits (QuaDiSQ) regime where transmon levels outside the computational subspace are exploited for fast gate operation [10] (with $\dim \mathcal{H} = 150$ [13]). The fastest possible universal set of gates uses gate durations of $T = 50$ ns [10]. Fast gate operations come at the expense of complex and spectrally broad pulse shapes $\mathcal{E}(t)$, see Fig. 1(h) for an example, whereas longer durations significantly reduce both temporal and spectral complexity of the pulses [10].

Spectral analysis—The time evolution operator $U(t)$ comprises all information about the system dynamics, making its eigenstates and eigenphases the natural objects for the analysis. Whereas the eigenstates reflect the occupation of Hilbert space during the gate operation, the eigenphases witness departures from integrability and the degree of adiabaticity of the drive. As a first probe of correlations in the spectrum, we consider the distribution of nearest neighbor eigenphase separations, P_{rel} , compared to the Poisson distribution P_{POI} of integrable, and the Wigner-Dyson distributions P_{GUE} of fully chaotic dynamics, respectively. The statistical differences between these distributions are quantified by the Kullback-Leibler (KL) divergence [20]. Figures 1(a)–1(f) show the two KL divergences D_{KL} (see SM [13] for computational details): While $D_{\text{KL}}(P_{\text{rel}}|P_{\text{POI}})$ vanishes almost everywhere for the

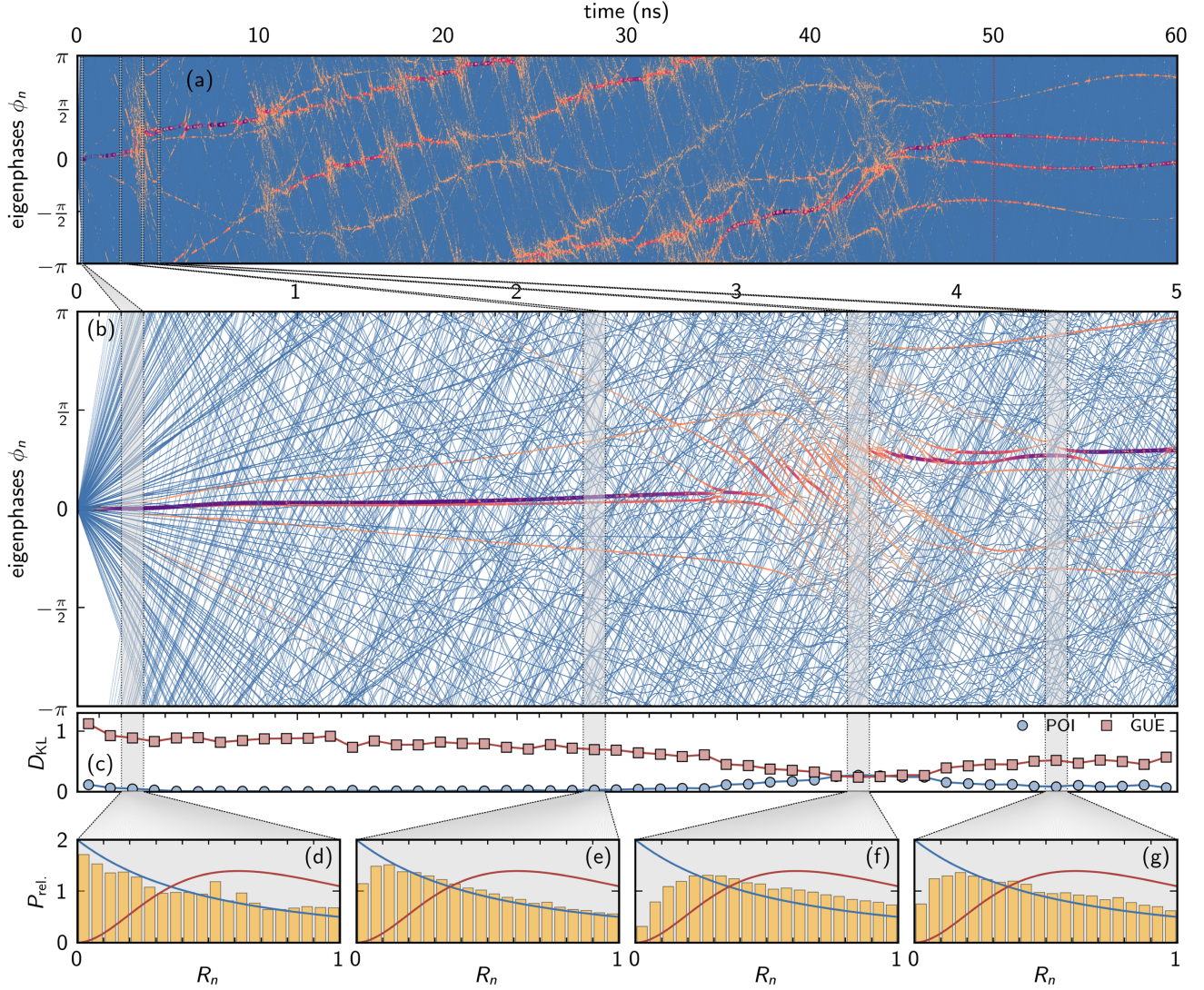


FIG. 2. Detailed analysis of the fast entangling BGATE. (a),(b) Instantaneous eigenphases $\{\phi_n(t)\}$ of $U(t)$ (blue lines) with the instantaneous eigenstate population indicated by size and color of the dots [smaller and lighter (larger and darker) dots imply smaller (larger) population]. (c) KL divergences as in Fig. 1(c) (red dotted line: end of pulse). (d)–(g) Probability distributions of the relative eigenphases in the time intervals highlighted in gray in (b), compared to ideal Poisson (blue) and Gaussian (red) distributions.

slower set of gates [Figs. 1(d) and 1(f)], the gates at the quantum speed limit [Figs. 1(a)–1(c)] have a smaller $D_{KL}(P_{rel}|P_{GUE})$, heralding chaotic fluctuations.

Figure 2 examines the BGATE, more precisely, a two-qubit gate from the BGATE’s local equivalence class and one of the fast gates with chaotic fluctuations, in detail. Panel (a) displays the eigenphases $\phi_n(t)$ of $U(t)$ as blue lines, appearing as blue areas due to their high density, and the color code showing exemplarily the evolution of the computational basis state $|00\rangle$ [where larger and darker (smaller and brighter) dots indicate a larger (smaller) overlap of the time-evolved state $|\Psi_{00}(t)\rangle = U(t)|00\rangle$ with the eigenstate $|\Phi_n(t)\rangle$ to which the blue line corresponds]. Figure 2(b) provides an enlargement of the first 5 ns of the dynamics. This time frame is interesting because it shows the transition from straight lines and line crossings for all

$\phi_n(t)$ into a regime where avoided crossings between lines start to become more frequent. The emerging many-body nonlinear behavior is witnessed by the KL divergence of the eigenphases $\phi_n(t)$ in Fig. 2(c). While up to $t \approx 3$ ns, the KL divergence shows an almost perfect match with an ideal POI statistics, evidencing integrable behavior, it diverges from POI statistics and shows a larger match with GUE statistics, signaling emergent chaotic behavior, between 3 and 4 ns. Figures 2(d)–2(g) show the actual distributions at four representative times, with an almost perfect match with ideal POI statistics before avoided crossings play a role (d). Similarities of P_{rel} to POI statistics also dominate just before (e) and after (g) the chaotic regime, whereas a larger (but far from perfect) match with GUE statistics is observed in (f). Interestingly, after 4 ns the dynamics becomes more regular again. Such complex behavior changing between

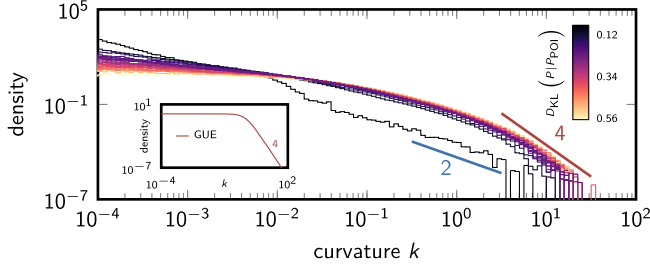


FIG. 3. Curvature statistics for the time evolution implementing the BGATE. The color of the curvature histogram encodes the KL divergence of the eigenphase spacing. For large curvatures, solid lines indicate polynomial fits to the data, consistent with GUE-like behavior (shown in the inset) of a power-law decay with exponent $\tilde{\beta} = 4$ for all but the very first time interval which is consistent with Poisson-like behavior and $\tilde{\beta} = 2$.

integrable, mixed, and chaotic dynamics is also observed for the other fast gates in Fig. 1. Remarkably, the (re) emergence of chaotic behavior does not have any negative impact on the gate errors—these are identically small for both fast and slow protocols in Fig. 1. The optimized control field seems to navigate the computational basis states safely through regions of instability.

Curvature distributions—Beyond the statistics of eigenphases, the structure of avoided crossings can be another indicator of nonlinear dynamics. Correlations lead to effective phase repulsion and to an overall more “curvy” pattern of phase evolutions in Fig. 2(b). To quantify this structure, we consider the curvature κ_n of the eigenphases, $\kappa_n(t) = d^2\phi_n(t)/dt^2$. In the limiting cases of localization and chaos, they follow distinct distributions [21–23],

$$P_{\text{POI}}(k) = \frac{\mathcal{N}_{\text{POI}}}{(1+k^2)}, \quad P_{\text{GUE}}(k) = \frac{\mathcal{N}_{\text{GUE}}}{(1+k^2)^2}, \quad (2)$$

where $\mathcal{N}_{\text{POI/GUE}}$ is a normalization constant and k the curvature κ , rescaled by the average spacing Δ and the variance of the velocity distribution [22,23], $k_n = \kappa_n \Delta(\phi_n)/[2\pi \text{Var}(\partial_t \phi_n)]$. For large k , Eq. (2) gives a power-law behavior, $P(k \rightarrow \infty) \propto \|k\|^{-\tilde{\beta}}$ with $\tilde{\beta} = 2(4)$ for the Poisson (GUE) case [21,23,24]. Analysis of the eigenphase curvatures in Figs. 2(h)–2(k) corroborates the occurrence of chaotic fluctuations indicated by the statistical analysis of the eigenphase differences: noting the double-logarithmic scale, it is clear that small curvatures are prevalent for regular dynamics in Figs. 2(h), 2(i), and, to a lesser extent (k), whereas larger curvatures have significantly more weight in the region of nonlinear behavior, cf. Fig. 2(j). To make the correspondence between the eigenphase statistics and the type of dynamics more quantitative, we calculate the curvature of the eigenphases in each of 30 equally spaced t segments of length 0.5 ns. The resulting distributions of the curvatures are shown in Fig. 3, where the color code corresponds to the KL

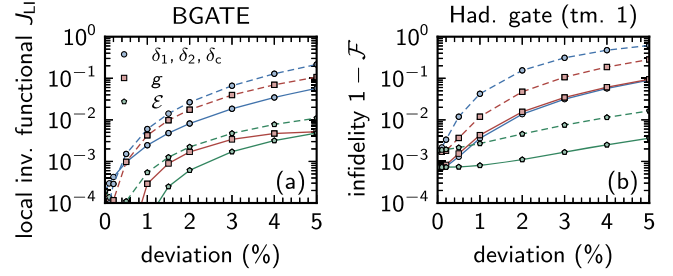


FIG. 4. Gate error for BGATE and Hadamard gate on transmon 1 with gate durations $T = 50$ ns (solid lines) and $T = 100$ ns (dashed lines) vs deviation in detunings, coupling strength and field amplitude, all quantified relative to their original values.

divergence $D(P|P_{\text{POI}})$ of the eigenphase spacing, calculated for the same time interval. Importantly, from the perspective of curvatures, there are traces of nonlinear GUE-like dynamics for all but the initial time intervals. Despite the fact that our spectrum exhibits only 150 levels, the curvature statistics show general similarity to pure GUE behavior (see inset of Fig. 3), exhibiting a plateau of constant probabilities for small curvatures $k \lesssim 1$ followed by a crossover to polynomial decay of probabilities in the high curvature regime. While our data does not exhibit a perfectly flat plateau, it does show the more defining power-law decay for large curvatures, reproducing the GUE exponent of $\tilde{\beta} = 4$ for all but the initial time interval. The latter instead follows a POI-like behavior with exponent $\tilde{\beta} = 2$. This change of curvature profiles reflects the finite time (a many-body scrambling time) required to build up the characteristics of a strongly correlated many-body system.

We thus find that statistical analysis of the curvatures reflects the presence of avoided crossings in a quantitative way. In particular, it allows for the isolation of a subset of states which can clearly be classified according to random matrix theory [25]. Remarkably, this subset of states can comprise only a small portion of Hilbert space. In such cases, the KL divergence has difficulty indicating the onset of chaotic fluctuations, due to the low weight of the affected states in the sum over all states. This difficulty affects also wave function statistics and out-of-time-ordered correlators which are not sufficiently sensitive as we show in the SM [13]. In contrast, “sorting” the evolutions according to their curvature allows for identifying the avoided crossings in highly complex dynamics.

Robustness—How will the signatures discussed above reflect in gate performance, specifically in their sensitivity to inevitable system parameter uncertainties? Figure 4 shows data for gate errors caused by parametric variations in circuit frequencies [shown as detunings and varied identically, cf. Eq. (1)], coupling strength, and driving field amplitudes for four qualitatively different representative classes: fast (50 ns, solid lines) and slow (100 ns, dashed lines), and partially chaotic (left), and integrable

(right) gates. Perhaps unexpectedly, we find that the presence of partially chaotic dynamics does *not* seem to have a detrimental effect on gate reliability. Furthermore, for all gates investigated in Figs. 1(a)–1(f), *slower* operation leads to relatively enhanced errors.

While we have no first principles explanation why the 50 ns gates are more robust to small parameter variations than the 100 ns gates, we can at least formulate a hypothesis: the level occupancies shown color coded in Figs. 2(a) and 2(b) suggest an interpretation of the protocol as a succession of adiabatic, diabatic, and hybridizing Landau-Zener transitions. A time-dependent Landau-Zener transition between two levels is characterized by two parameters, their rate of approach, v , i.e., the slope of the levels in Fig. 2(b), and their separation at the point of maximal proximity, Δ . The probability for a diabatic transition depends on the dimensionless ratio Δ^2/v as $P_{LZ} = \exp(-2\pi\Delta^2/v)$. A given transition probability may thus be achieved by an avoided crossing of levels with shallow slope approaching each other closely or, conversely, by levels with steep slope maintaining a larger separation. Differentiating P_{LZ} with respect to Δ and v , at a fixed ratio Δ^2/v , we find that its sensitivity to variations in these two parameters diminishes for large v and Δ . This implies that steep slope encounters maintaining a large separation are more robust than shallow slope close encounters. The relatively larger representation of the former type in fast partially chaotic gates might therefore positively reflect in their robustness. However, the full understanding of this phenomenon certainly requires systematic investigation beyond the scope of this Letter.

Conclusions—Extending statistical analysis to time-dependent protocols, we have found that fast transmon gates display clear signatures of emergent nonintegrability in both local and entangling operations. While conventional measures of eigenphase or eigenstate statistics are hampered by an indiscriminate average over state space, the eigenphase curvatures cross-correlate eigenphases with level occupations. Analyzing their statistics therefore allows for identifying the dynamically relevant part of the dense level manifold. The performance of the gates under small parameter variations seems to depend more on the nonadiabaticity of curve crossings (which are witnessed by the eigenphase curvatures) than on the (non)integrability of the dynamics. Future work should clarify how robust gate operations can be made with respect to parameter but also model uncertainties, such as the impact of higher harmonics [26]. Our findings are a first step toward elucidating the complex role of the transmon nonlinearity in gate operations, the understanding of which will be essential for scaling up and improving superconducting qubit architectures.

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Data availability—The details of the optimized gate protocols and the numerical data shown in the figures are available on Zenodo [27].

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- [1] P. Krantz, M. Kjaergaard, F. Yan, T.P. Orlando, S. Gustavsson, and W.D. Oliver, A quantum engineer's guide to superconducting qubits, *Appl. Phys. Rev.* **6**, 021318 (2019).
 - [2] J.J. García-Ripoll, *Quantum Information and Quantum Optics with Superconducting Circuits* (Cambridge University Press, Cambridge, England, 2022).
 - [3] C. Berke, E. Varvelis, S. Trebst, A. Altland, and D.P. DiVincenzo, Transmon platform for quantum computing challenged by chaotic fluctuations, *Nat. Commun.* **13**, 2495 (2022).
 - [4] R. Shillito, A. Petrescu, J. Cohen, J. Beall, M. Hauru, M. Ganahl, A.G. Lewis, G. Vidal, and A. Blais, Dynamics of transmon ionization, *Phys. Rev. Appl.* **18**, 034031 (2022).
 - [5] J. Cohen, A. Petrescu, R. Shillito, and A. Blais, Reminiscence of classical chaos in driven transmons, *PRX Quantum* **4**, 020312 (2023).
 - [6] S.-D. Börner, C. Berke, D.P. DiVincenzo, S. Trebst, and A. Altland, Classical chaos in quantum computers, *Phys. Rev. Res.* **6**, 033128 (2024).
 - [7] F.W. Strauch, P.R. Johnson, A.J. Dragt, C.J. Lobb, J.R. Anderson, and F.C. Wellstood, Quantum logic gates for coupled superconducting phase qubits, *Phys. Rev. Lett.* **91**, 167005 (2003).
 - [8] P. Magnard, P. Kurpiers, B. Royer, T. Walter, J.-C. Besse, S. Gasparinetti, M. Pechal, J. Heinsoo, S. Storz, A. Blais, and A. Wallraff, Fast and unconditional all-microwave reset of a superconducting qubit, *Phys. Rev. Lett.* **121**, 060502 (2018).
 - [9] S. Deffner and S. Campbell, Quantum speed limits: From Heisenberg's uncertainty principle to optimal quantum control, *J. Phys. A* **50**, 453001 (2017).
 - [10] M.H. Goerz, F. Motzoi, K.B. Whaley, and C.P. Koch, Charting the circuit QED design landscape using optimal control theory, *npj Quantum Inf.* **3**, 37 (2017).
 - [11] C.P. Koch, U. Boscain, T. Calarco, G. Dirr, S. Filipp, S.J. Glaser, R. Kosloff, S. Montangero, T. Schulte-Herbrüggen, D. Sugny, and F.K. Wilhelm, Quantum optimal control in quantum technologies. Strategic report on current status, visions and goals for research in Europe, *Eur. Phys. J. Quantum Technol.* **9**, 19 (2022).
 - [12] J. Zhang, J. Vala, S. Sastry, and K.B. Whaley, Minimum construction of two-qubit quantum operations, *Phys. Rev. Lett.* **93**, 020502 (2004).
 - [13] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/8nch-ld18> for a brief recap and further details of the analysis, which includes Refs. [14–19].
 - [14] Y. Makhlin, Nonlocal properties of two-qubit gates and mixed states, and the optimization of quantum computations, *Quantum Inf. Process.* **1**, 243 (2002).

- [15] J. Zhang, J. Vala, S. Sastry, and K. B. Whaley, Geometric theory of nonlocal two-qubit operations, *Phys. Rev. A* **67**, 042313 (2003).
- [16] V. Oganesyan and D. A. Huse, Localization of interacting fermions at high temperature, *Phys. Rev. B* **75**, 155111 (2007).
- [17] T. Guhr, A. Müller-Groeling, and H. A. Weidenmüller, Random-matrix theories in quantum physics: Common concepts, *Phys. Rep.* **299**, 189 (1998).
- [18] Y. Y. Atas, E. Bogomolny, O. Giraud, and G. Roux, Distribution of the ratio of consecutive level spacings in random matrix ensembles, *Phys. Rev. Lett.* **110**, 084101 (2013).
- [19] B. Swingle, Unscrambling the physics of out-of-time-order correlators, *Nat. Phys.* **14**, 988 (2018).
- [20] S. Kullback and R. A. Leibler, On information and sufficiency, *Ann. Math. Stat.* **22**, 79 (1951).
- [21] M. Filippone, P. W. Brouwer, J. Eisert, and F. von Oppen, Drude weight fluctuations in many-body localized systems, *Phys. Rev. B* **94**, 201112(R) (2016).
- [22] F. von Oppen, Exact distribution of eigenvalue curvatures of chaotic quantum systems, *Phys. Rev. Lett.* **73**, 798 (1994).
- [23] A. Maksymov, P. Sierant, and J. Zakrzewski, Energy level dynamics across the many-body localization transition, *Phys. Rev. B* **99**, 224202 (2019).
- [24] S. J. Garratt, S. Roy, and J. T. Chalker, Local resonances and parametric level dynamics in the many-body localized phase, *Phys. Rev. B* **104**, 184203 (2021).
- [25] T. Guhr, A. Müller-Groeling, and H. A. Weidenmüller, Random-matrix theories in quantum physics: Common concepts, *Phys. Rep.* **299**, 189 (1998).
- [26] D. Willsch *et al.*, Observation of Josephson harmonics in tunnel junctions, *Nat. Phys.* **20**, 815 (2024).
- [27] D. Basilewitsch, S.-D. Börner, C. Berke, A. Altland, S. Trebst, and C. P. Koch, Data underpinning “Chaotic fluctuations in a universal set of transmon qubit gates,” 2026, [10.5281/zenodo.10160591](https://zenodo.org/record/10160591).