Conformal Defects and Goldstone Bosons in Anti–de Sitter Space

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We study local quantum field theories in anti–de Sitter (AdS) space, with boundary conditions that break some of the bulk isometries. Specifically, we focus on conformal defects and we prove that their spectrum supports a displacement operator of protected dimension, despite the nonlocal nature of the conformal theory living at the boundary of AdS. If the defect breaks a global symmetry, a tilt operator is also present. The existence of a displacement was conjectured in Gabai *et al.* [J. High Energy Phys. 06 (2026) 029] for Wilson loops in Yang-Mills theories in AdS. Our proof is valid in general and applies, in particular, to defects in long-range models, as we discuss in various examples. In the bulk, the modes sourced by the protected operators have Compton wavelength of order of the AdS radius: they constitute the AdS analogue of the Goldstone bosons for the spontaneous breaking of the corresponding symmetries.

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Introduction—The study of quantum field theory (QFT) in hyperbolic space has seen a remarkable development, since the dream of using curvature as an infrared (IR) regulator to study strongly coupled physics [1] was married to the conformal bootstrap [2] and large-N technologies [3]. When the anti–de Sitter (AdS) [4] radius R is larger than any other scale, physics in AdS smoothly connects to flat space dynamics, where some of the most interesting questions have to do with low energy properties of asymptotically free theories: spontaneous symmetry breaking (SSB) or symmetry restoration [5], confinement, and chiral symmetry breaking [6]. Many of these phases are distinguished by the presence (or absence) of massless excitations: the Goldstone bosons [7] associated with the breaking of global or spacetime symmetries. In AdS, on the other hand, the radius regulates the IR and gaps the dynamics. Furthermore, the boundary conditions do not decouple. Vacua, where the order parameter has a vacuum expectation value (VEV), correspond to boundary conditions which break the symmetry explicitly, and one may wonder what feature of SSB persists as the radius is varied.

In the case of continuous global symmetries, this question was clarified thanks to the connection between QFT in AdS and defect conformal field theory (dCFT) [8,9]. When a conformal defect breaks a continuous symmetry of the CFT, a protected *tilt* operator exists on the defect [10]. Correspondingly, the boundary of AdS_{d+1} supports a tilt operator with dimension $\Delta = d$, which, in the large R limit, sources a single Goldstone boson [11]. At finite radius, the Goldstone bosons are strongly interacting, and the existence of the tilt and the identities obeyed by its correlation functions [12–18] provide the nonperturbative hallmark of SSB.

Confining theories arguably represent the most attractive challenge for QFT in AdS. In flat space, Wilson loops exhibit area law and source a string whose transverse fluctuations are Goldstone bosons for the breaking of isometries [19]. In AdS, the area law fails to diagnose confinement [1], but the flux tube sourced by a Wilson loop anchored at the boundary exists at all radii due to the gravitational potential, opening the enticing possibility that perturbative computations might capture flat space spectra and Wilson coefficients [20,21]. In [20], it was conjectured that a protected *displacement* operator exists on the Wilson line, based on two complementary perspectives: at small R, the explicit definition of the line as the holonomy of the gauge field; at large R, the assumption that a two-dimensional description of the flux tube becomes valid.

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The displacement operator has $\Delta = 2$ [22,23], and sources a Goldstone boson which propagates on the worldsheet in the large radius limit. At intermediate radii, the flux tube spreads over a size R, and a worldsheet description is not available at the scale of the mass of the boson. Conversely, the standard argument for the existence of the displacement [9] does not apply because the conformal theory at the boundary of AdS lacks a stress tensor.

Nevertheless, the main purpose of this Letter is to prove that the displacement exists, at any value of R. The proof is similar in spirit to the flat space one [24], based on the spectral density of the two-point function of the order parameter and the current. It is valid for any conformal defect on the boundary of AdS, and, in particular, it does not assume a Lagrangian, nor any particular description of the field configuration in AdS. With minor modifications, our argument also applies to the breaking of continuous global symmetries—which imply the existence of a tilt operator on the defect—and of higher spin symmetries.

The rest of the Letter is organized as follows. Our main result is derived in Sec. II, followed by some examples in Sec. III and conclusions in Sec. IV. The Appendixes A–C, are dedicated to useful technical details. In the Supplemental Material [25], we describe the consequences of SSB of global symmetries (Sec. S.2), more details on the examples mentioned in the main text (Sec. S.3), and, for completeness, some consequences of the conservation of the stress tensor which go beyond what is needed in the main text (Sec. S.1).

Several consequences of the main statement are explored in the conclusions, including its validity for defects in generic non-local CFTs and its implications on the properties of defect conformal manifolds. The role of the displacement as an order parameter for confinement is also further discussed.

Conformal charges in AdS and the Goldstone theorem—Consider a *local* QFT—by which we mean a theory endowed with a stress tensor—on a fixed $(d+1)$-dimensional AdS background. The d-dimensional CFT at the AdS boundary is non-local. A conformal defect, located on a flat p-dimensional surface $\mathcal{D}$ on the boundary ∂AdS, breaks the AdS isometries to the defect symmetry group $\mathrm{SO}(p+1,1)\times\mathrm{SO}(q)$, with $p+q=d$. In a *local* d-dimensional CFT, a standard argument [9] shows the existence of a *displacement* operator $\hat{D}_i$—$i=1,\ldots q$—with dimension $\hat{\Delta}=p+1$, transforming as a vector under the transverse rotation group $\mathrm{SO}(q)$.

To show that the same happens on ∂AdS, we will use the fact that the conformal charges have a local expression in terms of the bulk stress tensor. The action of the charge Q_ξ, associated to the conformal Killing vector ξ, on boundary operators enclosed by the surface $\hat{\Sigma}$, is

$$Q_\xi = \int_\Sigma d\Sigma_\mu \xi_\nu T^{\mu\nu}, \tag{1}$$

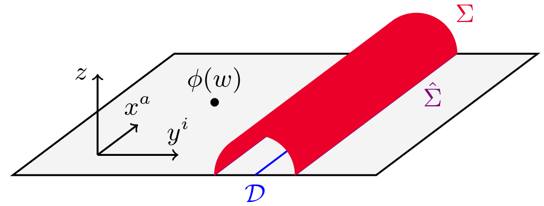

FIG. 1. The charge Q_ξ is integrated over Σ surrounding the defect. The operator ϕ lies on the AdS boundary: for brevity, we use the notation $\phi(w)$ with the understanding that in this case $w^\mu = (x^a, y^i, 0)$.

where Σ is an open codimension-one surface in the AdS interior, whose boundary $\hat{\Sigma}$ lies in ∂AdS—see Fig. 1. As in the flat space story [24], it is useful to consider the spectral decomposition of the two-point function $\langle \phi T_{\mu\nu}\rangle$ of the current with an order parameter, i.e., an operator ϕ whose expectation value in the presence of the defect $\langle\phi\rangle$ does not vanish (hereafter, the presence of the defect will always be understood in correlators). By choosing the quantization scheme appropriately, the spectrum in AdS is labeled by scaling operators on the defect. The action of the charges (1) on local operators constrains the spectral density, and the constraints can only be satisfied if the defect spectrum includes a displacement multiplet. The displacement obeys the usual integrated constraints [9], which are encapsulated in the following Ward identity

$$\sqrt{g}\nabla^\mu T_{\mu i}(w) = \sqrt{\mathcal{C}_{\hat{D}}}\,\delta^q(y)\delta(z)\hat{D}_i(x), \tag{2}$$

where $\mathcal{C}_{\hat{D}}$ is a physical constant, provided that the two-point function of $\hat{D}_i$ is unit normalized, as we will assume of all operators except the stress tensor, unless otherwise stated. For clarity, we chose Poincaré coordinates $w^\mu = (x^a, y^i, z)$ and split them among parallel to $\mathcal{D}$, perpendicular to it on ∂AdS, and orthogonal to ∂AdS.

The proof relies on unitarity, and crucially, on the topological dependence of the charges on the surface Σ, including its boundary $\hat{\Sigma}$. In the bulk, the condition is violated by defects which extend in the AdS interior, like nondynamical branes [31], thus breaking the isometries explicitly. On ∂AdS, the independence of the charges from the local shape of $\hat{\Sigma}$ is violated, for instance, by turning on background field configurations which break $\mathrm{SO}(d+1,1)$ to $\mathrm{SO}(p+1,1)\times\mathrm{SO}(q)$ away from the defect [32]. As we describe in Sec. III, in both cases the displacement is absent.

In the rest of the section, we describe the proof in detail.

The stress tensor and the displacement operator—It will be useful to distinguish two ways of slicing AdS. In Poincaré coordinates, we can pick a constant slice to be a half-sphere centered on a point away from the defect, or on a point belonging to it (see Fig. 2). Quantizing on the first kind of slices, one concludes that a product of

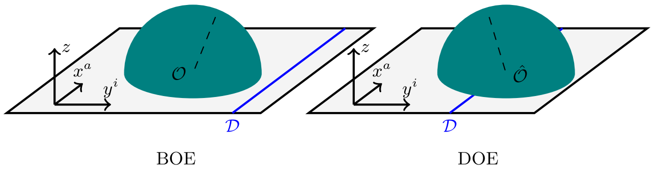

FIG. 2. The two quantization schemes we consider.

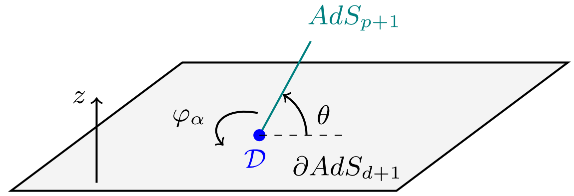

FIG. 3. A representation of the AdS_{p+1} slicing coordinate system.

boundary operators can be replaced with a sum over scaling boundary operators [operator product expansion (OPE)], and a bulk operator can be replaced with a sum of boundary operators [boundary operator expansion (BOE)]. The second kind of slicing proves the convergence of the defect operator expansion (DOE), which replaces a bulk or boundary operator with a sum of excitations of the defect.

Now, let ϕ be a scaling operator, other than the identity, on the boundary of AdS, such that $\langle\phi\rangle \neq 0$. At least one such operator must exist, for otherwise the defect would be disconnected from any correlators of AdS or ∂AdS operators, as one can conclude by repeated application of the BOE and of the OPE. Then, consider $\langle Q_\xi \phi\rangle$, where the charge (1) is defined over the surface Σ depicted in Fig. 1. Its boundary can be chosen as $\hat{\Sigma} = S^{q-1} \times \mathbb{R}^p$. By deforming the integration region outward, one concludes $\langle Q_\xi \phi\rangle = \partial_\xi \langle\phi\rangle \neq 0$, for an AdS Killing vector belonging to $\mathrm{SO}(d+1,1)/\mathrm{SO}(q) \times \mathrm{SO}(p+1,1)$.

One can also evaluate $\langle Q_\xi \phi\rangle$ using the DOE of the stress tensor, i.e., compute the spectral decomposition of $\langle \phi T_{\mu\nu}\rangle$. It is important that the DOE commutes with the integral in (1), a fact that follows from convergence of the DOE and BOE and that we prove in Appendix B. Defect operators are labeled by representations of $\mathrm{SO}(p) \times \mathrm{SO}(q)$ and we focus on the ones that can contribute to the charge. It is sufficient to pick $\xi^\mu = \delta^\mu_i$, a broken translation, to identify the relevant operators, so from now on we consider the equation

$$\langle \partial_i \phi(w)\rangle = \langle \phi(w) \int_\Sigma d\Sigma_\mu T^\mu_i \rangle. \tag{3}$$

$d\Sigma_\mu T^\mu_i$ is in the $(\mathbf{1},\mathbf{q})$ under $\mathrm{SO}(p) \times \mathrm{SO}(q)$, so no defect operator with parallel indices appears in the DOE. While representations of $\mathrm{SO}(q)$ different from the vector appear, they contribute schematically as $d\Sigma_\mu T^\mu_i \sim (\alpha \delta^{j_1}_i + \beta y_i y^{j_1}) y^{j_2} \ldots y^{j_s} \hat{O}_{j_1\ldots j_s}$. Integrating over the S^{q-1} component of Σ, one sees that only operators with $s = 1$ survive.

We are led to consider the contribution of the conformal family of a primary $\hat{O}_i$, with scaling dimension $\hat{\Delta}_{\hat{O}}$, to the DOE of the stress tensor. It is convenient to move to the following AdS_{p+1}-slicing coordinates for AdS_{d+1}

$$\rho = \sqrt{z^2 + |y|^2}, \qquad \theta = \arctan\left(\frac{z}{|y|}\right), \qquad \varphi_\alpha, \qquad x_a,$$

$$ds^2 = \frac{d\rho^2 + dx^2}{\rho^2 \sin^2\theta} + \frac{d\theta^2}{\sin^2\theta} + \cot^2\theta \gamma_{\alpha\beta} d\varphi^\alpha d\varphi^\beta. \tag{4}$$

Here, $|y|^2 = \delta_{ij} y^i y^j$. The AdS_{p+1} slices, fibered over the interval $\theta \in [0, \pi/2]$, share their boundaries, at $\rho = 0$, where the defect lies. The boundary of AdS_{d+1} is also reached at $\theta = 0$, and $\gamma_{\alpha\beta}$ is the metric of the unit $(q-1)$-sphere whose normal vectors are $\hat{y}^i = y^i/|y|$. The coordinate system is illustrated in Fig. 3. The correlator $\langle T_{\mu\nu} \hat{O}_i\rangle$ is determined by the isometries up to seven functions of θ, which appear in the DOE. The counting can be performed using a conformal frame [33]. The components of the DOE appearing in $d\Sigma_\mu T^\mu_i$ are

$$T_{\rho\rho} \sim \hat{y}^i (\mathcal{N}_{2,0}(\theta) + \mathcal{N}^{(1)}_{0,0}(\theta)) \left(\rho^{\hat{\Delta}_{\hat{O}}-2} \hat{O}_i(x) + \ldots\right), \tag{5a}$$

$$T_{\theta\rho} \sim \hat{y}^i \mathcal{N}_{1,0}(\theta) \left(\rho^{\hat{\Delta}_{\hat{O}}-1} \hat{O}_i(x) + \ldots\right), \tag{5b}$$

$$T_{\alpha\rho} \sim \partial_\alpha \hat{y}^i \mathcal{N}_{1,1}(\theta) \left(\rho^{\hat{\Delta}_{\hat{O}}-1} \hat{O}_i(x) + \ldots\right). \tag{5c}$$

Here, the dots hide the descendants, whose contribution is fixed by the isometries: no function of θ appears in their coefficient relative to the primary. The notation for the undetermined functions $\mathcal{N}_{a,b}(\theta)$ is related to the spins of the components of $T_{\mu\nu}$ under the little group of a point in the bulk, $\mathrm{SO}(p+1) \times \mathrm{SO}(q)$. Their dependence on the exchanged operator is not explicit but is understood. The functions $\mathcal{N}_{a,b}(\theta)$ have power-law asymptotic as $\theta \to 0$, fixed by the BOE of the stress tensor. These limits are reported in Appendix A, together with the leading term in the DOE of all the components of the stress tensor. It can be checked that, in the absence of relevant or marginal scalars in the DOE, the unitarity bounds guarantee that the integral (1) converges. A neutral scalar with $\Delta < d$ must be fine-tuned, and depending on the regularization choice, this leads to extra boundary terms in the definition of the charges Q_ξ. These contributions precisely subtract the divergence, and one can show that this can be done preserving the topological nature of the charges [34]. The net result is simply that one should regulate the integral, subtract the divergence and keep the finite part. In the marginal case, the logarithmic divergence cannot be fine tuned in general [35,36]: for simplicity, we assume that no marginal operator appears in the BOE of $T_{\mu\nu}$ [37].

All in all, by plugging Eqs. (5) in (3), one gets

$$\langle \phi(w) \int_\Sigma d\Sigma_\mu T^\mu_i \rangle \Big|_{\hat{O}} = \sqrt{\mathcal{C}_{\hat{O}}} \rho^{\hat{\Delta}_{\hat{O}}-p-1} \int d^p x \langle \hat{O}_i(x) \phi(w) \rangle, \tag{6}$$

where the correlator is restricted to the contribution of one conformal family—but all descendants integrate to zero—and

$$\sqrt{\mathcal{C}_{\hat{O}}} = \frac{\Omega_q}{q} \int_0^{\pi/2} d\theta \frac{\cos^q\theta}{\sin^{d-1}\theta} \tilde{\mathcal{N}}_{\hat{O}}(\theta),$$
$$\tilde{\mathcal{N}}_{\hat{O}}(\theta) = \Big(\mathcal{N}_{2,0}(\theta) + \mathcal{N}^{(1)}_{0,0}(\theta) - \tan\theta \mathcal{N}_{1,0}(\theta) + (q-1)\mathcal{N}_{1,1}(\theta)/\cos^2(\theta) \Big). \tag{7}$$

We performed the integral over S^{q-1} which yields the factor $\Omega_q = 2\pi^{q/2}/\Gamma(q/2)$. Minor modifications are needed when $q = 1$, since an interface may separate two different boundary CFTs: we address this case in Appendix C. Now, the conservation equation $\nabla^\mu T_{\mu\nu} = 0$ imposes three differential constraints on the seven functions $\mathcal{N}_{a,b}(\theta)$, reported in (S1–S3) in the Supplemental Material [25]. The following linear combination is especially important for us:

$$(\hat{\Delta}_{\hat{O}} - p - 1) \frac{\cos^q\theta}{\sin^{d-1}\theta} \tilde{\mathcal{N}}_{\hat{O}}(\theta) = \frac{d}{d\theta} \left(\frac{\cos^q\theta}{\sin^{d-1}\theta} \left[-\mathcal{N}_{1,0}(\theta) - (q-1) \frac{\mathcal{N}_{0,1}(\theta)}{\cos^2\theta} + \tan\theta \mathcal{N}^{(2)}_{0,0}(\theta) \right] \right), \tag{8}$$

where $\mathcal{N}_{0,1}, \mathcal{N}^{(2)}_{0,0}$ are defined in (A1). For generic $\hat{\Delta}_{\hat{O}}$, therefore, the integrand in (7) is a total derivative. Under the assumptions of continuity in AdS and conformality of the boundary theory, the total derivative vanishes at $\theta = 0$, $\pi/2$—see Appendixes A and C for the precise statement. Hence, $\mathcal{C}_{\hat{O}} = 0$ if $\hat{\Delta}_{\hat{O}} \neq p + 1$, consistent with the fact that the right-hand side of (3) is independent of ρ. On the other hand, when $\hat{\Delta}_{\hat{O}} = p + 1$, (8) does not constrain $\tilde{\mathcal{N}}_{\hat{O}}$. In fact, we know from (3) that the result cannot vanish. We conclude that a primary defect operator $\hat{D}_i$, with the quantum numbers of the displacement, exists, and satisfies

$$\int_\Sigma d\Sigma_\mu \xi_\nu T^{\mu\nu} = \sqrt{\mathcal{C}_{\hat{D}}} \int_{\mathcal{D}} \xi^i \hat{D}_i, \tag{9}$$

as an operator equation. We recall that $\mathcal{D}$ is the defect surface at $\rho = 0$ and $\mathcal{C}_{\hat{D}}$ is defined by (5), (7), with $\hat{O}$ chosen to be the displacement. The fact that (9) is valid for any Killing vector can be argued in various ways, for instance, noticing that the contribution of the displacement to $\nabla^\mu T_{\mu i}$ provides a representation of a delta function—see Eq. (2)—as we derive in Sec. S.1 of the Supplemental Material [25]. Alternatively, one can of course repeat the computation in the general case, or simply take the $\rho \to 0$ limit in (3), which suppresses the contribution of descendants [38].

Examples—One can verify the presence of the displacement in many examples. The case of a Wilson loop W in a gauge theory is treated in detail in [20]. Wilson lines can be pushed to the boundary if the gauge field is allowed to fluctuate on ∂AdS. In particular, at large N, one-point functions of boundary operators normalized by $\langle W \rangle$ scale like N^{-1}, while $\mathcal{C}_{\hat{D}}$ scales like N^0.

The double-trace interface described in [39] provides another instructive example. On the two sides of the interface, the same free scalar has either of the two boundary conditions allowed by the bulk equations of motion. The resulting defect has a displacement operator.

In Sec. S.3 of the Supplemental Material [25], we discuss four other examples. In two of them, the displacement operator exists: (i) A defect obtained by coupling a free massive boson to a p-dimensional generalized free field on the boundary of AdS, following [40]; (ii) the pinning field defect [41,42] in the long-range $O(N)$ model [40], at leading order in $d = 4 - \varepsilon$ expansion. The tilt operator is present as well [43].

Two other defects for a free massive boson in AdS have no displacement operator and each highlights the optimality of one of the assumptions of the theorem: (iii) A defect that extends in the AdS interior, modeled by a source for the free scalar localized on an AdS_{p+1} slice. In this case, the conservation of the stress tensor is violated in the bulk; (iv) a modification of the theory on the boundary of AdS by a nonconstant source, following [32]. The modification preserves $\mathrm{SO}(p+1,1) \times \mathrm{SO}(q)$, but extends over the whole ∂AdS. In this case, the BOE of $T_{\mu\nu}$ is modified away from the defect, and Q_ξ depends on $\hat{\Sigma}$ nontopologically.

Conclusions—Let us first comment on the implications of our result for the physics of confinement in AdS. On one hand, the existence of the displacement operator puts the study of string excitations in AdS via the corresponding Ward identities on solid ground. On the other hand, our proof shows that the displacement is generic, and in this sense it is not an order parameter for confinement. Any QFT in AdS with a local defect on the boundary supports this excitation, including, for instance, Abelian gauge theories. Only screened defects [44]—i.e., defects invisible to local bulk and boundary operators—have no displacement operator. This implies, for instance, that string breaking in gauge theories with appropriate matter is realized as screening in the boundary CFT. One way to interpret the generic presence of the displacement is that the flux tube always exists at the scale of the curvature. This suggests that better order parameters might be hidden in the spacetime dependence of expectation values or in the scattering amplitudes of the Goldstone bosons, which should exhibit two dimensional dynamics for a confining theory up to scales parametrically larger than the AdS scale.

It is perhaps useful to comment explicitly on the codimension one case. An interface on ∂AdS which is not attached to a brane of course obeys the theorem. Such a defect can never be factorizing: the displacement which obeys (9) with the unique bulk stress tensor appears in the boundary OPE of operators with a VEV on both sides of the interface, whose connected correlator, therefore, does not vanish. Vice versa, boundary conditions are always associated with explicit breaking of the bulk isometries, typically via an end-of-the-world brane [31,45]. At the other extreme, an interface which can be tuned to be topological can only exist in a theory with a marginal operator, or extend in the bulk nontrivially [46] [47].

Turning our attention away from AdS, the proof presented here, with minimal modifications, shows the existence of a displacement operator on conformal defects embedded in other conformal defects in generic CFTs. Under the same assumptions as reference [46], this implies that continuous families of defects in a fixed CFT are generated by marginal defect operators, unless the defect is attached to (topological) surfaces. In the latter case, the interface between two defects in the same family can extend in the bulk as a (nontopological) defect on the topological surface: this happens, for instance, in the case of a monodromy defect [23,48].

Our proof applies to defects in long-range CFTs reached in the IR from a local relevant deformation of generalized free fields [40,49,50]. Indeed, these theories are equivalent to free fields in AdS with boundary couplings. One may ask whether protected operators associated to broken symmetries exist for defects in nonlocal CFTs without a local higher-dimensional embedding. Intuitively, these operators reflect the ability to couple the defect to the appropriate background fields in a local way, i.e., such that their variation inserts the corresponding local operator in the path integral [8,9]. The role of the higher-dimensional theory with its conserved currents and its local DOE is to give a sharp criterion for this possibility.

Finally, the fact that the displacement multiplet alone accounts for the breaking of all relevant isometries is the rigorous AdS counterpart to the inverse Higgs constraint in flat space [51–55]. It would be interesting to explore whether, in the flat space limit, our construction can provide an equally rigorous proof of the existence of Goldstone modes that propagate in $p+1$ dimensions, without assuming the presence of a $p+1-$ dimensional classical surface as in the standard CCWZ treatment [56,57].

Note added—While finalizing this manuscript, the authors became aware of closely related work by Jiaxin Qiao, who independently obtained the same results with similar arguments in [58]. Following mutual communication, we coordinated the submission so that the two papers appear on the same date.

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End Matter

Appendix A: The DOE of the stress tensor—The leading contribution of a defect vector primary to the components of the stress tensor $T_{\mu\nu}$ is

$$
\begin{aligned}
&T_{\theta a} \sim \hat{y}_i \mathcal{N}_{1,0}(\theta)\left(\frac{\rho^{\hat{\Delta}_{\hat{O}}}}{\hat{\Delta}_{\hat{O}}}\partial_a \hat{O}^i(x) + \dots\right),\\
&T_{\theta\rho} \sim \hat{y}_i \mathcal{N}_{1,0}(\theta)\left(\rho^{\hat{\Delta}_{\hat{O}}-1}\hat{O}^i(x) + \dots\right),\\
&T_{\theta\theta} \sim \hat{y}_i \mathcal{N}^{(2)}_{0,0}(\theta)\left(\rho^{\hat{\Delta}_{\hat{O}}}\hat{O}^i(x) + \dots\right),\\
&T_{a\rho} \sim \hat{y}_i \left(\frac{p+1}{p}\mathcal{N}_{2,0}(\theta)\right)\left(\frac{\rho^{\hat{\Delta}_{\hat{O}}-1}}{\hat{\Delta}_{\hat{O}}}\partial_a \hat{O}^i(x) + \dots\right),\\
&T_{ab} \sim \hat{y}_i \delta_{ab}\left(\mathcal{N}^{(1)}_{0,0}(\theta) - \frac{1}{p}\mathcal{N}_{2,0}(\theta)\right)\left(\rho^{\hat{\Delta}_{\hat{O}}-2}\hat{O}^i(x) + \dots\right),\\
&T_{\rho\rho} \sim \hat{y}_i (\mathcal{N}_{2,0}(\theta) + \mathcal{N}^{(1)}_{0,0}(\theta))\left(\rho^{\hat{\Delta}_{\hat{O}}-2}\hat{O}^i(x) + \dots\right),\\
&T_{\alpha\theta} \sim \partial_\alpha \hat{y}_i \mathcal{N}_{0,1}(\theta)\left(\rho^{\hat{\Delta}_{\hat{O}}}\hat{O}^i(x) + \dots\right),\\
&T_{\alpha\beta} \sim \gamma_{\alpha\beta}\hat{y}_i \mathcal{N}_{0,2}(\theta)\left(\rho^{\hat{\Delta}_{\hat{O}}}\hat{O}^i(x) + \dots\right),\\
&T_{\alpha a} \sim \partial_\alpha \hat{y}_i \mathcal{N}_{1,1}(\theta)\left(\frac{\rho^{\hat{\Delta}_{\hat{O}}}}{\hat{\Delta}_{\hat{O}}}\partial_a \hat{O}^i(x) + \dots\right),\\
&T_{\alpha\rho} \sim \partial_\alpha \hat{y}_i \mathcal{N}_{1,1}(\theta)\left(\rho^{\hat{\Delta}_{\hat{O}}-1}\hat{O}^i(x) + \dots\right).
\end{aligned}
\tag{A1}
$$

These expansions depend on θ only through the seven functions $\mathcal{N}_{a,b}(\theta)$. The ratios of the coefficients of descendants and primary are fixed and independent of θ, as the DOE (A1) is a standard BOE on each AdS_{p+1} slice. The combination of stress-tensor components relevant to the Ward identity (6) for orthogonal translations is $T_{i\rho}$ in mixed components, which is explicitly

$$
T_{i\rho} = \hat{y}_i\left(\cos\theta T_{\rho\rho} - \frac{\sin\theta}{\rho}T_{\theta\rho}\right) + \frac{\partial^\alpha \hat{y}_i}{\rho\cos\theta}T_{\alpha\rho}. \tag{A2}
$$

We now derive asymptotic estimates for the functions $\mathcal{N}_{a,b}(\theta)$ as $\theta \to 0$, using consistency between the $\theta \to 0$ limit at finite ρ (equivalently, $z \to 0$ at finite $|y|$) and the BOE away from the defect. The BOE of $T_{\mu\nu}$ away from the defect can exchange boundary scalars ($\mathcal{O}$) and rank two symmetric traceless primaries ($\mathcal{S}$) of any dimension, or a boundary vector of dimension $\Delta = d+1$. The latter is absent if there is no energy flux across $\partial\mathrm{AdS}$, and indeed the existence of conformal charges (1) requires it. We assume $\Delta_{\mathcal{O}} > d$—see comments in the main text for the opposite case. As for $\mathcal{S}$, it can be checked that, if it is at the unitarity bound, it does not appear in the BOE. All in all,

$$
\begin{aligned}
&T_{zz} = \mathrm{O}(z^{d-2+\varepsilon}),\ T_{az} = \mathrm{O}(z^{d-1+\varepsilon}),\ T_{iz} = \mathrm{O}(z^{d-1+\varepsilon}),\\
&T_{ab} = \mathrm{O}(z^{d-2+\varepsilon}),\ T_{ij} = \mathrm{O}(z^{d-2+\varepsilon}),\ T_{ai} = \mathrm{O}(z^{d-2+\varepsilon}),
\end{aligned}
\tag{A3}
$$

for some $\varepsilon > 0$. In turn, this implies the following asymptotic estimates for the functions $\mathcal{N}_{a,b}(\theta)$ as $\theta \to 0$:

$$
\begin{aligned}
&\mathcal{N}_{1,0}(\theta) = \mathrm{O}(\theta^{d-1+\varepsilon}), \qquad \mathcal{N}_{2,0}(\theta) = \mathrm{O}(\theta^{d-2+\varepsilon}),\\
&\mathcal{N}_{0,1}(\theta) = \mathrm{O}(\theta^{d-1+\varepsilon}), \qquad \mathcal{N}_{0,2}(\theta) = \mathrm{O}(\theta^{d-2+\varepsilon}),\\
&\mathcal{N}^{(1)}_{0,0}(\theta) = \mathrm{O}(\theta^{d-2+\varepsilon}), \qquad \mathcal{N}^{(2)}_{0,0}(\theta) = \mathrm{O}(\theta^{d-2+\varepsilon}),\\
&\mathcal{N}_{1,1}(\theta) = \mathrm{O}(\theta^{d-2+\varepsilon}).
\end{aligned}
\tag{A4}
$$

The estimates (A4) guarantee the convergence of the θ integral in (7) and the vanishing of the boundary term in (8) as $\theta \to 0$. It is also possible to check that, at $\theta = \pi/2$, there are no convergence problems in (7), and that the boundary term in (8) vanishes, by assuming that $\langle \phi T_{\mu\nu}\rangle$ expressed in Cartesian coordinates is finite at $|y|=0$ and $z\neq 0$ and continuous everywhere else. This condition is strengthened for $q=1$, as discussed in Appendix C.

Appendix B: Convergence of the DOE of Q_ξ—Here, we show that the correlator $\langle \phi Q_\xi\rangle$ can be computed by integrating the DOE of the stress tensor term by term. While the DOE converges absolutely in the region of integration, this is not sufficient in order to exchange the ρ expansion and the θ integral: a sufficient condition is met if the integral of the series of absolute values converges (Fubini-Tonelli theorem). More directly, we will use the dominated convergence theorem to bound the partial sums. The integral along the parallel directions x_a is harmless and we discuss it last. The required property can be shown to hold by means of the Cauchy-Schwarz inequality. Since the details of the SO (d) representation of $T_{\mu\nu}$ are irrelevant to the argument and clutter notation, we will replace the stress tensor with a bulk scalar operator Φ. Hence, consider the following correlator and its DOE:

$$\langle \phi(w')\Phi(x,\hat{y},\rho,\theta)\rangle = \sum_{\hat{O}_n}\mathcal{N}_{\hat{\Delta}_n}(\theta)\rho^{\hat{\Delta}_n}\langle \phi(w')\hat{O}_n(x,\hat{y})\rangle,$$
$$\hat{O}_n(x,\hat{y})\equiv \hat{y}^{i_1}\ldots\hat{y}^{i_{s_n}}\hat{O}_{n,i_1\ldots i_{s_n}}(x), \tag{B1}$$

where $\rho<1$, $\hat{O}_n$ are defect scaling operators and ϕ is a spectator that lies at $\rho'>1$. With an abuse of notation, the argument of Φ is written in the coordinate system (4) while w' denotes Poincaré coordinates as in the main text. As usual, the terms in the ρ expansion are in one-to-one correspondence with coefficients in the expansion of the state $|\Phi(w)\rangle$ on the surface at $\rho=1$ in a complete orthonormal set. The partial sums are, therefore, all bounded by the product of the norms as follows:

$$|S_N|\equiv\left|\sum_{\substack{\hat{O}_n\\ n<N}}\mathcal{N}_{\hat{\Delta}_n}(\theta)\rho^{\hat{\Delta}_n}\langle \phi(w')\hat{O}_n(x,\hat{y})\rangle\right|$$
$$\leq (\langle \Phi(x,\hat{y},1/\rho,\theta)\Phi(x,\hat{y},\rho,\theta)\rangle\langle \phi(w')\phi(\mathcal{I}w')\rangle)^{1/2}, \tag{B2}$$

where $\mathcal{I}w'$ is the result of an inversion centered at the point $w_0=(x^a,y^i=0,z=0)$, i.e., $(\mathcal{I}w')^\mu=(w'^\mu-w_0^\mu)/(w'-w_0)^2+w_0^\mu$. In particular, $(\mathcal{I}w'-w_0)^2=(\rho')^{-2}$ lies inside the unit sphere. Since the right-hand side of (B2) is N independent, the hypotheses of the dominated convergence theorem are satisfied if the two-point function $\langle \Phi(x,\hat{y},1/\rho,\theta)\Phi(x,\hat{y},\rho,\theta)\rangle$ is integrable in θ with the appropriate measure.

One then notices that the small θ limit of such correlator is

$$\langle \Phi(x,\hat{y},1/\rho,\theta)\Phi(x,\hat{y},\rho,\theta)\rangle \sim \theta^{2\Delta}, \tag{B3}$$

where Δ is the scaling dimension of the leading boundary operator in the BOE of Φ. We conclude that integrability of the right-hand side of (B2) is guaranteed if the original correlator (B1) is integrable. In particular, we know this to be the case for the stress tensor (or for an improved version, following [34], when $\Delta<d$). Convergence of the integral over the transverse sphere S^{q-1} is obvious.

Regarding the integral over the directions parallel to the defect, we notice that the dependence of $\langle \phi(w')\Phi(w)\rangle$ on the AdS_{p+1} coordinates is restricted by isometries to the combination

$$\zeta=\frac{\rho\rho'}{(x-x')^2+\rho^2+\rho'^2}. \tag{B4}$$

Therefore, the ρ expansion converges well at large $(x-x')^{-2}$. To prove that it is legal to integrate term by term, we simply bound the tail of the series and show that its integral vanishes in the limit. One can integrate in $\theta,\hat{y},x$ in any order, so we show the swapping property directly for the x integral of the correlator:

$$\left|\int d^px\langle\phi\Phi\rangle-\int d^pxS_N\right|$$
$$\leq K(\theta,\hat{y})\rho^{\hat{\Delta}_N}\left|\int d^px(x^2+\rho'^2)^{-(\hat{\Delta}_N+\kappa)/2}\right|$$
$$=K'(\theta,\hat{y})\frac{(\rho')^{p-\hat{\Delta}_N-\kappa}}{(\hat{\Delta}_N+\kappa)^{p/2}}\left(1+\mathrm{O}(\hat{\Delta}_N^{-1})\right)\xrightarrow{N\to\infty}0. \tag{B5}$$

Here, K,K',κ are positive. To obtain the inequality, we used that the DOE converges absolutely to bound the tail by the leading term $\propto\rho^{\hat{\Delta}_N}$. The function K can be taken independent of ρ and $(x-x')$, by bounding the tail with the sum of absolute values evaluated at $x=x'$ and at a fixed ρ_0, $1>\rho_0>\rho$. The presence of the positive constant κ can be deduced from the cross ratio (B4) or from a direct analysis of the DOE.

Of course, one should still check that each term integrates to a finite value, and this fails if there are defect operators of dimension $\hat{\Delta}\leq p/2$. Notice that this issue concerns finitely many contributions to the DOE, and is separate from swapping integration and ρ series. When applied to the two-point function of interest $\langle\phi T_{\rho i}\rangle$, a divergence would mean that the charge Q_ξ is not defined. That this does not happen can be seen as follows. First, descendants in the DOE of $T_{\rho i}$ have $\hat{\Delta}>p/2+1$ by unitarity [59], so their leading contribution to the series is $O(x^{-p-\epsilon})$ and can be integrated. For the primaries, we first cutoff the integral to a compact region in $\mathbb{R}^p$. As explained in the main text, the integral over θ of the contribution of primaries with dimension $\hat{\Delta} \neq p+1$ vanishes. Therefore, less irrelevant primaries do not contribute to the flux on any surface Σ of the kind in Fig. 1, cutoff along $\mathbb{R}^p$. Removing the cut-off, we see that the charge is finite [60].

Appendix C: Comments on the codimension one case—The case of an interface, $q=1$, is degenerate, as the sphere S^{q-1} collapses to a set of two distinct points $\{1,-1\}$. However, the analysis of the previous sections still holds with minor modifications. First of all, the stress tensor no longer has components in the SO(q)-charged indices α, and the corresponding functions $\mathcal{N}_{a,b}(\theta)$ with $b \neq 0$ must be set to zero. The integral formula (7) is still valid, provided that one removes the prefactor Ω_q/q and extends the domain of integration to $[0,\pi]$; note that, in the integrand, the SO(q)-charged degrees of freedom decouple automatically. Equation (8) is also valid, as also there SO(q)-charged degrees of freedom decouple. In this case, to discard the boundary term in Eq. (8) we need $\langle \phi T_{\mu\nu} \rangle$ to be continuous also at $\theta = (\pi/2)$ rather than just finite, reflecting the fact that a discontinuity on a codimension one surface precisely leads to a delta function contribution in $\nabla^\mu T_{\mu\nu}$. Finally, if there is reflection symmetry $x \to -x$ (or $\theta \to \pi - \theta$), then the formula (7) holds as it stands, provided that $\Omega_q/q \to 2$. Note that in this case $\mathcal{N}^{(2)}_{0,0}(\theta)$ must be odd under reflection and therefore vanish at $\theta = \pi/2$, so that it does not contribute to the boundary term in (8).