

Asymptotic Exceptional Steady States in Dissipative Dynamics

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Spectral degeneracies in Liouvillian generators of dissipative dynamics generically occur as exceptional points, where the corresponding non-Hermitian operator becomes nondiagonalizable. Steady states, i.e., zero modes of Liouvillians, are considered a fundamental exception to this rule since a no-go theorem excludes nondiagonalizable degeneracies there. Here, we demonstrate that the crucial issue of diverging timescales in dissipative state preparation is largely tantamount to an asymptotic approach toward the forbidden scenario of an exceptional steady state in the thermodynamic limit. With case studies ranging from NP-complete satisfiability problems encoded in a quantum master equation to the dissipative preparation of a symmetry protected topological phase, we reveal the close relation between the computational complexity of the problem at hand, and the finite size scaling toward the exceptional steady state, exemplifying both exponential and polynomial scaling. Formally treating the weight W of quantum jumps in the Lindblad master equation as a parameter, we show that exceptional steady states at the physical value $W = 1$ may be understood as a critical point hallmarking the onset of dynamical instability.

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Quantum state preparation remains a formidable challenge due to diverging timescales for large systems, with implications reaching far beyond its natural habitat of quantum science. In particular, the solution to hard problems of general interest such as NP-complete satisfiability (SAT) problems [1–3] and their quantum counterparts known as QMA-complete problems [4–6] may be readily viewed as quantum state preparation tasks, e.g., solved by preparing the ground state of a suitable local parent Hamiltonian in a quantum simulator [7–10]. For the common approach of adiabatic quantum computing [11–13], the central obstruction is a (discontinuous) quantum phase transition along the path to nontrivial ground states, manifesting as a (exponentially) divergent preparation time, known as critical slowdown [14–20].

Dissipative state preparation aims at addressing this challenge by considering incoherent processes as a resource [21–34]. In this way, a targeted many-body state is approached independently of initial conditions as a steady state of dissipative dynamics. There, a central equation of

motion is the Lindblad master equation [35,36]

$$\frac{d}{dt}\rho = \mathcal{L}_W[\rho] = -i(H_{\text{NH}}\rho - \rho H_{\text{NH}}^\dagger) + W \sum_{\mu} L_{\mu}\rho L_{\mu}^\dagger, \quad (1)$$

governing the dynamics of the density matrix ρ of a system weakly coupled (Born) to a bath with negligible memory (Markov). Incoherent processes reflecting the influence of the bath are described by quantum jump operators L_{μ} , while $H_{\text{NH}} = H - (i/2) \sum_{\mu} L_{\mu}^\dagger L_{\mu}$ is the effective non-Hermitian (NH) Hamiltonian combining the Hermitian system Hamiltonian H with the anti-Hermitian damping enacted by the bath. We emphasize that the weight W of the quantum jump term is physically fixed to $W = 1$, but for our subsequent analysis, it will be fruitful to formally consider W as a parameter [37–42].

Importantly, the aforementioned critical slowdown persists in the dissipative context, namely as an asymptotic degeneracy at eigenvalue zero of the Liouvillian $\mathcal{L}_{W=1}$ [cf. Fig. 1(a)], raising the question of the nature of this degeneracy. On one hand, it is known that NH degeneracies generically occur in the form of nondiagonalizable (or exceptional) points (EP) [43–48], such as Liouvillian exceptional points [49–57], while diagonalizable degeneracies would require additional fine-tuning [58]. On the other hand, a rigorous no-go theorem precludes degenerate steady states (i.e., Liouvillian zero modes) from forming an EP [59].

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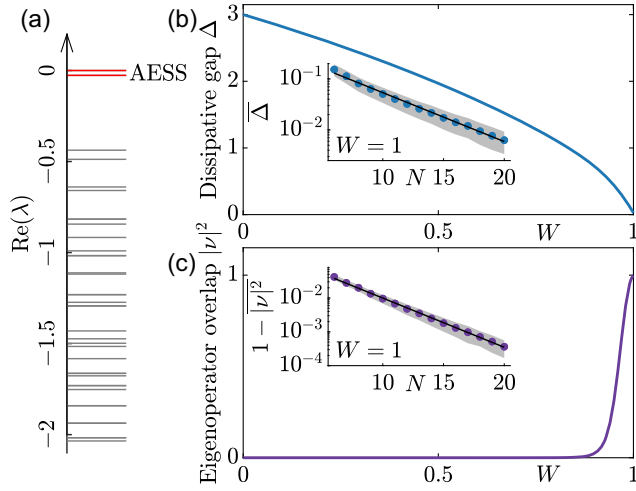


FIG. 1. (a) Liouvillian spectrum [$W = 1$ in Eq. (1)] for a satisfiable 3SAT instance with a unique solution, $N = 14$ variables, and $M = \text{round}(\alpha_c N)$ clauses, where the satisfiability threshold $\alpha_c \approx 4.267$. The two red states form the asymptotic exceptional steady state (AESS). (b),(c) Liouvillian gap Δ and eigenoperator overlap $|\nu|^2$ as a function of W [other parameters as in (a)]. The insets in (b),(c) show the mean values of $\bar{\Delta}$ and $\overline{|\nu|^2}$ at $W = 1$ change with the number of variables N , averaging over 10^3 random instances for each N with $M = \text{round}(\alpha_c N)$. All instances have a unique solution. Shadow regions indicate the standard deviation among instances.

Here, we report on the discovery of *asymptotic exceptional steady states* (AESS) as the generic form of an asymptotic degeneracy associated with critical slowdown in dissipative dynamics. In essence, the forbidden scenario of a steady state EP is approached in the thermodynamic limit (see Fig. 1 for a paradigmatic example), while the exact EP is pushed (infinitesimally) away from the physical value $W = 1$ [cf. Eq. (1)] to $W_c > 1$ in any finite system (see Fig. 2). Furthermore, we reveal that the complexity of a state preparation task at hand naturally determines the finite size scaling properties of AESS, as we exemplify by two concrete systems of distinct scientific interest. First, for the 3SAT problem (cf. Fig. 1), the AESS is approached exponentially in system size N , reflecting the NP-completeness of the problem. Second, for a Lindbladian with a symmetry-protected topological phase as a steady state, we find that an AESS is approached with polynomial scaling, reflecting the continuous topological quantum phase transition separating the steady state from a trivial initial state in symmetry-preserving dynamics [cf. Fig. 3(c) below].

AESS in dissipative state preparation—The central idea of the dissipative state preparation paradigm is to target a pure state $\rho_t = |\psi_t\rangle\langle\psi_t|$ by encoding it into the steady-state subspace of a physical open quantum system, i.e., $\mathcal{L}_1[\rho_t] = 0$. In our present approach, we first set $H = 0$ for simplicity and consider jump operators of the form $L_\mu = O_\mu P_\mu$. The case with $H \neq 0$ will be discussed further

below. Here, the projector P_μ satisfies $P_\mu|\psi_t\rangle = 0$ and the operator O_μ maps unwanted states into the subspace containing $|\psi_t\rangle$. In other words, $|\psi_t\rangle$ lies in the decoherence-free subspace [60] while jump operators dissipate undesirable states.

These jump operators lead to an anti-Hermitian effective Hamiltonian,

$$H_{\text{NH}} = -(i/2) \sum_\mu P_\mu O_\mu^\dagger O_\mu P_\mu = -iH', \quad (2)$$

such that $|\psi_t\rangle$ is a frustration-free ground state of the Hermitian H' . Neglecting the quantum jump terms (i.e., $W = 0$), $\mathcal{L}_0[\rho] = -\{H', \rho\}$ generates the deterministic imaginary time evolution of H' [61,62]. While the finite spectrum gap at $W = 0$ [see Fig. 1(b)] suggests convergence to the solution in a time scaling at most linearly in system size [63,64], we emphasize that a direct experimental implementation of $W = 0$ requires postselection of processes (quantum trajectories) without quantum jump events, the probability of which is attenuated exponentially over time.

As for the physical Liouvillian $\mathcal{L}_{W=1}$, a key quantity to characterize critical slowdown is the relaxation time τ , which is determined by the Liouvillian spectrum. A (diagonalizable) Liouvillian $\mathcal{L}_W$ satisfies $\mathcal{L}_W[r_i] = \lambda_i r_i$ and $\mathcal{L}_W^\dagger[l_i] = \lambda_i^* l_i$, respectively, [65]. Here, λ_i is the i th eigenvalue, ordered by their real parts $\text{Re}(\lambda_0) \geq \text{Re}(\lambda_1) \geq \text{Re}(\lambda_2) \geq \dots \geq \text{Re}(\lambda_{D-1})$, where D is the Hilbert space dimension, and r_i (l_i) are the right (left) eigenoperators whose Hilbert-Schmidt inner product satisfies the biorthonormal relation $\text{Tr}[l_i^\dagger r_j] = \delta_{ij}$. At the physical point ($W = 1$), the eigenmodes with $\lambda_i = 0$ correspond to the steady-state subspace. To clarify the concept of AESS, we assume a unique steady state of $\mathcal{L}_1$ throughout the Letter, which is the most common case in dissipative state preparation. Then, $\lambda_0 = 0$ and $\text{Re}(\lambda_1) < 0$ at $W = 1$. Consequently, it is the Liouvillian gap $\Delta = |\text{Re}(\lambda_0 - \lambda_1)|$ between the steady state r_0 and the slowest relaxation mode r_1 that reveals the relaxation time $\tau \sim \Delta^{-1}$. For generalizations in which the AESS may coexist with multiple steady states, see Supplemental Material (SM) [42].

For nontrivial dissipative preparation tasks, including the preparation of topological states and solution states to computationally hard problems, a divergent timescale in the thermodynamic limit is inherent, thus demanding asymptotic gap closing with increasing system size. This generically occurring critical slowdown is simply the manifestation of many-body complexity in dissipative state preparation. Nevertheless, r_0 and r_1 in a gapless Liouvillian are never strictly degenerate because of the finite-size gap that is typically present in all physically accessible systems. While a no-go theorem precludes EPs for exactly degenerate steady states at $W = 1$ [59], the asymptotic (avoided) degeneracy in gapless Liouvillians does not fall into this

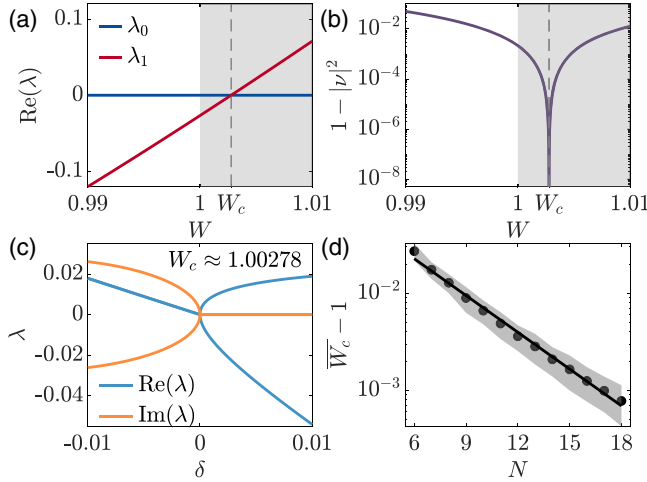


FIG. 2. Eigenvalues (a) of steady and metastable states and their eigenoperator overlap (b) over W [same instance as in Fig. 1]. The shaded region represents $W > 1$, where $\text{Re}(\lambda_1) > 0$ renders the system dynamically unstable in the thermodynamic limit. We observe an EP at $W_c \approx 1.00278$. (c) Perturbed eigenvalues for $\mathcal{L}_{W_c} + \mathcal{L}_{W_c}^{\text{pert}}$, where δ is the perturbation strength. (d) The mean W_c decays with N . Each point is averaged over 10^3 instances with one solution and the shaded region marks the standard deviation between instances.

category. As EPs are the most generic form of NH spectral degeneracy [58], we argue that without fine tuning or additional symmetry, any asymptotic steady-state degeneracy must also lie close to an exact EP. This EP can be revealed by considering an extended (and thus mathematically generic) parameter space (as is explicated below by considering $W \neq 1$; see Fig. 2), where the no-go theorem no longer applies. Thus, the AESS where both eigenvalues and eigenstates between the steady state and slowest relaxation mode asymptotically coalesce with increasing system size indeed represents the typical manifestation of critical slowdown in dissipative state preparation.

To quantitatively describe the properties of AESS, we define the eigenoperator overlap between r_0 and r_1 :

$$\nu = \text{Tr}[\hat{r}_0^\dagger \hat{r}_1], \quad (3)$$

where we define the normalized eigenoperators $\hat{r}_i = r_i / \|r_i\|$ with the Hilbert-Schmidt norm $\|\rho\| = \sqrt{\text{Tr}[\rho^\dagger \rho]}$. $|\nu|^2 = 0$ ($|\nu|^2 = 1$) reveals that these two states are orthogonal (parallel) to each other. In this sense, an AESS is characterized by $\Delta \rightarrow 0$ and $|\nu|^2 \rightarrow 1$ in a physical system (at $W = 1$) being simultaneously approached in the thermodynamic limit.

Whenever $|\nu|^2 \neq 1$, an orthonormal operator basis between r_0 and r_1 can be constructed: $\tilde{r}_0 = \hat{r}_0$ and $\tilde{r}_1 = (\hat{r}_1 - \nu \hat{r}_0) / (1 - |\nu|^2)^{1/2}$, in which the Liouvillian superoperator $\mathcal{L}_W$ becomes [42]

$$\mathcal{L}_W^{\text{eff}} = \begin{pmatrix} 0 & \lambda_1 \frac{\nu}{\sqrt{1-|\nu|^2}} \\ 0 & \lambda_1 \end{pmatrix}. \quad (4)$$

Here, we always have $\lambda_0 = 0$ for generic W since $r_0 = |\psi_t\rangle\langle\psi_t|$ by assumption lies in the decoherence-free subspace of the preparation protocol. As an effective description of the AESS, $\lambda_1 \rightarrow 0$ and $|\nu|^2 \rightarrow 1$ are simultaneously approached by either taking the thermodynamic limit [Figs. 1(b) and 1(c)] or slightly tuning W away from the physical value $W = 1$ [Figs. 2(a) and 2(b)]. In this sense, the effective matrix $\mathcal{L}_W^{\text{eff}}$ asymptotically approaches the generic Jordan-block structure of an EP [66].

Having introduced the general notion of AESS in dissipative dynamics, we now demonstrate the characteristic dependence of its scaling behavior on the class of state preparation problem with paradigmatic examples.

Dissipative 3SAT solver—We first consider the NP-complete 3SAT problem. With N Boolean variables $x_1, x_2, \dots, x_N$, we define M disjunction clauses, each containing three variables or their negations (e.g., $C_m = x_{m_1} \vee \neg x_{m_2} \vee x_{m_3}$). The 3SAT problem asks whether a conjunction of clauses $C_1 \wedge C_2 \wedge \dots \wedge C_M$ can be satisfied by assigning TRUE (1) or FALSE (0) to each binary variable. By mapping TRUE and FALSE of a variable x_n to the states $|1\rangle$ and $|0\rangle$ of a qubit σ_n , the 3SAT problem is converted into finding the ground state of a 3-local Ising-like Hamiltonian $H_{3\text{SAT}} = \sum_{m=1}^M P_m$ [67]. The projector P_m acts on three qubits involved in clause C_m and assigns a unit energy penalty to unsatisfied clauses. For example, $C_m = x_{m_1} \vee \neg x_{m_2} \vee x_{m_3}$ corresponds to $P_m = (1 - \sigma_{m_1}^z)(1 + \sigma_{m_2}^z)(1 - \sigma_{m_3}^z)/8$, which punishes the configuration $|010\rangle$ of the three participating variables. Here, $\sigma_n^{x,y,z}$ are Pauli operators for the n th qubit, with $\sigma_n^z|1\rangle = |1\rangle$ and $\sigma_n^z|0\rangle = -|0\rangle$. The solution to a satisfiable 3SAT instance is encoded in a zero-energy ground state $|\psi_{\text{sol}}\rangle$ of $H_{3\text{SAT}}$, where $P_m|\psi_{\text{sol}}\rangle = 0$ and $|\psi_{\text{sol}}\rangle$ represents a bit string. This maps the problem to the standard state preparation task of preparing the ground states of a target Hamiltonian $H_{3\text{SAT}}$. Yet, the NP-complete nature of 3SAT indicates the generic hardness of completing this task.

To harness the dissipative state preparation framework, we design dissipative processes for each clause C_m through three jump operators:

$$L_{m,\alpha} = \sigma_{m_\alpha}^x P_m, \quad \alpha = 1, 2, 3. \quad (5)$$

These $L_{m,\alpha}$ dissipate the unsatisfiable configuration of clause C_m and rotate qubit m_α into its satisfiable subspace. Therefore, the solution to a satisfiable 3SAT instance is given by the steady state $r_0 = |\psi_{\text{sol}}\rangle\langle\psi_{\text{sol}}|$ of the corresponding $\mathcal{L}_1$. Remarkably, Eq. (5) leads to $H' = 3H_{3\text{SAT}}/2$ [cf. Eq. (2)], indicating that the imaginary time evolution of $H_{3\text{SAT}}$ is achieved by $\mathcal{L}_{W=0}$. While the evolution of $\mathcal{L}_0$ approaches $|\psi_{\text{sol}}\rangle$ at a timescale that is at most linear

in N due to a finite (and constant with N) gap of $H_{3\text{SAT}}$ [Fig. 1(b)], the hardness of the 3SAT problem hides in postselecting an exponentially small subset of quantum trajectories to effectively realize $W = 0$. An interesting question relating to our present discussion of AESS is how does the complexity of 3SAT problem manifest in the spectrum of $\mathcal{L}_W$ as a function of W , in particular at the natural physical value $W = 1$?

To examine the spectrum of $\mathcal{L}_W$, we consider satisfiable 3SAT instances with a unique solution. Similar results with multiple solutions are presented in SM [42,68]. Hard satisfiable instances are generated using the method from Ref. [69]. The clause-to-variable ratio M/N is set to the satisfiability threshold $\alpha_c \approx 4.267$, below (above) which a generic instance is satisfiable (unsatisfiable) [3]. In the computational basis with $H = 0$, the structure of Eq. (5) allows us to focus on the classical dynamics in the diagonal part of ρ , thus reducing the effort in diagonalizing $\mathcal{L}_W$ [42].

The spectrum of $\mathcal{L}_1$ for a typical satisfiable instance is presented in Fig. 1(a). Besides the steady state $r_0 = |\psi_{\text{sol}}\rangle\langle\psi_{\text{sol}}|$, we observe a metastable state r_1 that is well separated from other damping modes, with eigenvalue close to zero. As shown in Fig. 1(b), this state has a small Liouvillian gap at $W = 1$, in contrast to the finite gap at the imaginary time evolution point $W = 0$. Importantly, the mean Liouvillian gap $\bar{\Delta}$ at $W = 1$, averaged over 10^3 satisfiable instances, exhibits an exponential decay as N increases, indicating the exponential runtime of a dissipative 3SAT solver.

The most interesting result appears in the eigenoperator overlap ν defined in Eq. (3). As shown in Fig. 1(c), we find that $|\nu|^2 = 0$ at $W = 0$, as the two eigenoperators correspond to eigenstates of a Hermitian Hamiltonian $H_{3\text{SAT}}$. However, $|\nu|^2$ grows to a value that is extremely close to 1 at $W = 1$. The inset of Fig. 1(c) further demonstrates that the averaged quantity $1 - |\nu|^2$ at $W = 1$ decays exponentially as N increases. These numerical findings imply that the metastable and steady states asymptotically approach an EP in the thermodynamic limit ($N \rightarrow +\infty$), leading to the appearance of AESS.

In a finite system, an exponentially small finite Liouvillian gap at $W = 1$ prevents two nearly degenerate states from becoming an exact EP. However, a slight increase of W to a critical value $W_c > 1$ makes $\Delta = 0$ and $|\nu|^2 = 1$, indicating that two states coalesce to an exact EP in an extended parameter space [Figs. 2(a) and 2(b)]. This shows that the effective Liouvillian in Eq. (4) approaches a Jordan block as $W \rightarrow W_c$. Notably, the EP at $W = W_c$ signifies a transition into a dynamically unstable region where $\lambda_1 > 0$ and the steady state becomes ill-defined [70]. Moreover, the critical value W_c converges to 1 in the thermodynamic limit, whose mean deviation $\bar{W}_c - 1$ is compatible with an exponential decay with system size [Fig. 2(d)]. These results demonstrate that the physical AESS at $W = 1$ is extremely close to an exact EP.

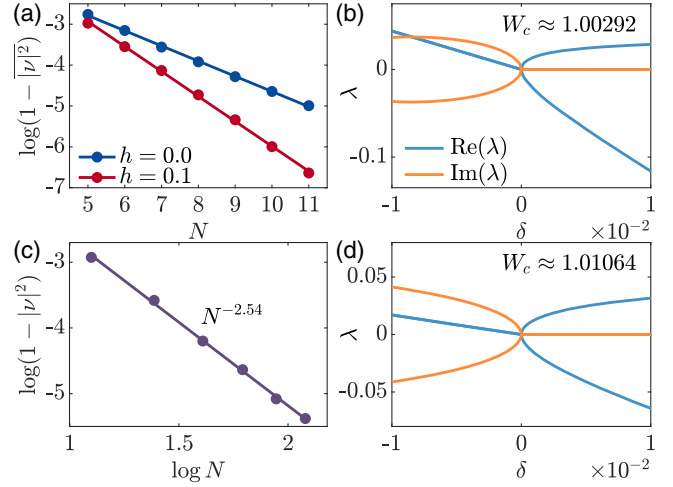


FIG. 3. AESS in quantum systems. (a) The case of including a modified transverse field [Eq. (6)] in dissipative 3SAT solver. We set $M = \text{round}(\alpha_c N)$ and $W = 1$. All points are obtained by averaging over 10^3 satisfiable instances with a unique solution. (b) The perturbed eigenvalues of $\mathcal{L}_{W_c} + \mathcal{L}_{W_c}^{\text{pert}}$ for a uniquely solvable 3SAT instance with $N = 11$. (c) Eigenoperator overlap of $\mathcal{L}_1$ for preparing the AKLT state [Eq. (7)]. (d) The perturbed eigenvalues of $\mathcal{L}_{W_c} + \mathcal{L}_{W_c}^{\text{pert}}$ for AKLT state preparation with $N = 7$.

To further demonstrate that the AESS at $W = 1$ is near an exact EP at $W = W_c$, we perturb the spectrum of $\mathcal{L}_W$ by adding a perturbative Liouvillian $\mathcal{L}_W^{\text{pert}}[\rho] = \delta \sum_{n=1}^N (W L_n \rho L_n^\dagger - \frac{1}{2} \{L_n^\dagger L_n, \rho\})$ with $L_n = \sigma_n^-$, where δ is the perturbation strength. Since a solution $|\psi_{\text{sol}}\rangle$ to a random satisfiable instance is unlikely to be a dark state for all $L_n = \sigma_n^-$, this perturbation erases the solution in the steady state. While a physical dissipative dynamics requires $\delta > 0$ and $W = 1$, we mathematically consider both $\delta > 0$ and $\delta < 0$ at $W = W_c > 1$. As shown in Fig. 2(c), the perturbed eigenvalues are purely real when $\delta > 0$ and form complex-conjugated pairs when $\delta < 0$, undergoing a parity-time symmetry breaking at $\delta = 0$ [71–74]. This is a clear signature of an exact EP at $\delta = 0$ and $W = W_c$.

Quantum AESS—While the above dissipative 3SAT solver is governed by classical dynamics, our arguments for the occurrence of AESS also apply to genuine quantum dynamics. To exemplify this case, we consider two quantum models: a modified dissipative 3SAT solver with $H \neq 0$, and a dissipative preparation of the Affleck-Kennedy-Lieb-Tasaki (AKLT) state [75].

As a modified 3SAT solver, we add a quantum Hamiltonian H_X to Eq. (5):

$$H_X = h \sum_{m \neq m'}^M \sum_{n \in \text{Ind}_{m,m'}} P_{m,m'} \sigma_n^x P_{m,m'}, \quad (6)$$

where $P_{m,m'} = I - (I - P_m)(I - P_{m'})$, h is the interaction strength, and $\text{Ind}_{m,m'}$ contains the qubit indices involved in

clauses C_m and $C_{m'}$. H_X describes a transverse field that only generates dynamics in the subspace orthogonal to $|\psi_{\text{sol}}\rangle$. Therefore, $|\psi_{\text{sol}}\rangle\langle\psi_{\text{sol}}|$ is still the steady state at $W = 1$. Figure 3(a) indicates that the AEES still exists in quantum dynamics, where both the eigenvalues and eigenoperators of steady and metastable states for $\mathcal{L}_1$ coalesce in the thermodynamic limit. Additionally, an exact EP appears at $W = W_c > 1$, which is confirmed by the perturbed eigenvalues [Fig. 3(b)] at $W = W_c$ after adding a perturbation $\mathcal{L}_{W_c}^{\text{pert}}[\rho] = \delta \sum_{n=1}^N (W_c L_n \rho L_n^\dagger - \frac{1}{2} \{L_n^\dagger L_n, \rho\})$ with $L_n = \sigma_n^x$.

Both 3SAT solvers eventually stabilize a classical steady state. We now demonstrate that an AEES can also occur as a nontrivial quantum state, such as the symmetry-protected topological AKLT state [21,31,76]. We consider a one-dimensional $S = 1$ spin system ($H = 0$) with four types of jump operators:

$$L_{n,\alpha} = S_{n,n+1}^\alpha P_{n,n+1}^{(2)}, \quad L'_{n,\alpha} = S_{n+1}^\alpha P_{n,n+1}^{(2)}, \quad (7)$$

where $\alpha = x, y$. $S_n^{x,y,z}$ are spin-1 operators. $P_{n,n+1}^{(2)}$ projects two neighboring $S = 1$ spins into the $S = 2$ subspace [75]. We assume that there are N spins and take periodic boundary conditions $S_1 = S_{N+1}$. By construction, the AKLT state $|\psi_{\text{AKLT}}\rangle$ lives in the decoherence-free subspace since $P_{n,n+1}^{(2)}|\psi_{\text{AKLT}}\rangle = 0$ for $n = 1, \dots, N$, thus giving the steady state $|\psi_{\text{AKLT}}\rangle\langle\psi_{\text{AKLT}}|$ of $\mathcal{L}_1$. We find that the AEES still exists at $W = 1$ [Fig. 3(c)] and is close to an exact EP at a slightly larger $W = W_c > 1$. We also perturb the exact EP by adding a perturbative Liouvillian $\mathcal{L}_{W_c}^{\text{pert}}[\rho] = \delta \sum_{n=1}^N (W_c L_n \rho L_n^\dagger - \frac{1}{2} \{L_n^\dagger L_n, \rho\})$ with $L_n = S_n^z$. The perturbed eigenvalues [Fig. 3(d)] elucidate the spectral EP structure.

Interestingly, the eigenoperator overlap $1 - |\nu|^2$ in Fig. 3(c) decays as a power law with increasing system size, distinct from the exponential decay observed in Figs. 1 and 3(a) for 3SAT. These numerical findings further corroborate the close relation between the finite-size scaling of the AEES and the complexity of preparing target states. Preparing a symmetry-protected topological state in a symmetry-preserving fashion requires a polynomial time reflecting the obstruction by a continuous topological quantum phase transition, while an exponentially long time is expected to be required to solve the NP-complete 3SAT problem.

Dynamical consequences of AEES—We now return to a more general discussion of the physical Liouvillian at $W = 1$. Although a finite-size gap prevents a direct perturbative analysis, the AEES still plays a crucial role in the complexity of approaching the physical steady state. With the biorthonormal relation $\text{Tr}[l_i^\dagger r_j] = \delta_{ij}$ for $\mathcal{L}_1$, an initial density matrix ρ_{ini} follows the time evolution $\rho(t) = r_0 + \sum_{i \geq 1} e^{\lambda_i t} c_i r_i$, where $c_i = \text{Tr}[l_i^\dagger \rho_{\text{ini}}]$. In the

longtime regime, the evolution effectively becomes $\rho(t) = r_0 + e^{\lambda_1 t} c_1 r_1 + o(e^{\lambda_2 t})$.

The eigenoperators r_1 and l_1 of AEES have the form $r_1 = r_0 + \delta_r$ and $l_1 = \kappa(I - r_0) + \delta_l$, where $||\delta_r||/||r_1||$ and $||\delta_l||/||l_1||$ decay to 0 and $\kappa \equiv \text{Tr}[l_1]/(2^N - 1)$ approaches -1 as N increases [42]. Since l_1 is close to the projector $I - r_0$, a random initial state without prior knowledge of $r_0 = |\psi_{\text{sol}}\rangle\langle\psi_{\text{sol}}|$ will typically lead to $c_1 = \text{Tr}[l_1^\dagger \rho_{\text{ini}}] = -1 + \epsilon$ with an exponentially small correction ϵ . The longtime dynamics becomes $\rho(t) \approx (1 - e^{\lambda_1 t} + \epsilon e^{\lambda_1 t}) r_0 + e^{\lambda_1 t} (-1 + \epsilon) \delta_r$, intrinsically requiring a time-scale $\tau \sim |\text{Re}(\lambda_1)|^{-1}$ to distinguish the target state r_0 . One possible way to accelerate the relaxation dynamics is to carefully choose an initial state such that $c_1 = 0$, which would lead to a much shorter relaxation time $\tau' \sim |\text{Re}(\lambda_2)|^{-1}$ [77]. However, such shortcuts are not readily applicable to the AEES considered here: as numerically exemplified in the SM [42] for the dissipative 3SAT solver, we find that $\kappa^{-1} l_1$ is positive semidefinite, with a single zero mode $|\psi_{\text{sol}}\rangle$ separated from other positive eigenvalues by a gap that is constant in N . This situation excludes the possibility of finding easily accessible initial states ρ_{ini} such that $c_1 = 0$, as the only solution would be to (tautologically) start from the target state r_0 . This further corroborates that the dynamical obstruction to preparing nontrivial target states manifests in the scaling properties of AEES, thus reflecting the intrinsic complexity of dissipative state preparation.

Conclusion—We have revealed the existence of asymptotic exceptional steady states and discussed their intriguing connection to the computational complexity of state preparation tasks. The AEES is found to be near an exact EP in an extended parameter space and to play a vital role in relaxation dynamics to target states. Moreover, we note that AEES exhibit a subtle relation to the steady-state phase diagram of a dissipative system. In particular, it is important to distinguish between critical steady states in the sense of quantum phases of matter and the criticality of Liouvillian spectra. The latter, which has the dynamical meaning of critical slowdown and has been the focus of our present discussion of AEES, contains information both about the target state and the way to approach it. Another interesting future direction is to identify Hamiltonian or dissipative perturbations that are compatible with the targeted steady state but mitigate the scaling behavior of the AEES so as to speed up the state preparation task. Such perturbations may be seen as a dissipative counterpart to quantum catalysts in adiabatic quantum computing [13].

Note added—While preparing this Letter for submission, we became aware of a somewhat related preprint [78] which discusses Liouvillian EPs in a few-level system with $W \neq 1$, but without relating to the large N phenomenon of AEES introduced in our present Letter.

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Data availability—The data associated with this Letter are available in a Zenodo repository [79].

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