

Quantum Carpets of Higgs Quasiparticles in a Supersolid

K. Mukherjee^{1,*}, M. Schubert¹, R. Klemt², T. Bland¹, T. Pfau², and S. M. Reimann¹

¹*Mathematical Physics and NanoLund, LTH, Lund University, Box 118, 22100 Lund, Sweden*

²*Physikalisches Institut and Center for Integrated Quantum Science and Technology, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany*

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Supersolids formed from dipolar Bose Einstein condensates (BECs) exhibit spontaneous density modulation while maintaining global phase coherence. This state of matter supports gapped amplitude (Higgs) excitations featuring a quadratic dispersion relation. While Higgs modes are typically strongly damped due to coupling with other amplitude and phase modes, imposing an experimentally realistic toroidal geometry allows us to numerically study the time evolution and dispersion of a localized Higgs quasiparticle excitation, with minimal residual coupling to sound modes. Strikingly, the quadratic dispersion leads to the occurrence of (fractional) revivals, similar to those seen in the optical Talbot effect or the so-called quantum carpets. The revival times provide a novel method for determining the effective mass of the Higgs quasiparticle through a nonspectroscopic approach. These results pave the way for further studies of coherent Higgs dynamics and mutual interactions between Higgs quasiparticle.

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According to Goldstone's theorem, breaking a continuous symmetry necessarily gives rise to a massless scalar particle [1]. In contrast, the massive Higgs amplitude mode—another key excitation at a phase transition—requires additional symmetry protection [2,3]. In high-energy physics, this is provided by Lorentz invariance [3], while in non-relativistic quantum systems, approximate particle-hole symmetry is typically needed [4].

Despite this constraint, the Higgs mode has been observed across a wide range of platforms, including superconductors [5–9], quantum antiferromagnets [10], and ultracold atomic systems such as fermionic condensates [11–14], bosons near the superfluid Mott transition [15,16], spinor condensates [17], and light-coupled Bose gases [18]. It has also been predicted [19] and observed [20] in few-body fermionic systems. In BCS superfluids, the Higgs mode is tied to the pairing gap and typically exhibits flat dispersion [11], whereas in bosonic systems with lattices or emergent periodicity, the mode acquires a momentum-dependent dispersion, with the gap closing near a quantum critical point [16,21].

Experimental detection has so far relied mostly on spectroscopic techniques [11–13,15,16], such as modulating the pairing gap [11] or the lattice depth [16], and has

typically identified a single Higgs mode. However, spatial inhomogeneities can obscure these signals by coupling amplitude and phase modes [22–24]. Directly exciting a single Higgs quasiparticle given by coherent superposition of many Higgs-modes and tracking its real-space dynamics has remained out of reach, yet such capability would open the door to exploring coherent motion and even Higgs scattering. This challenge is especially compelling in the context of supersolids, which—once considered elusive [25]—have now been realized in several systems [26–28], including magnetic atoms [29–32] (see also Refs. [33–35]). These systems support a rich excitation spectrum [36–40], where Higgs excitations coexist with two sound excitations arising from the breaking of distinct continuous symmetries [41–46].

In this Letter, we numerically show that a localized Higgs quasiparticle in a dipolar supersolid can be realized in a toroidal trap [44,47,48] by exciting multiple Higgs modes, and that its evolution produces a quantum carpet interference pattern due to the quadratic dispersion and confinement. This enables interferometric detection of the Higgs quasiparticle, observation of its time evolution, extraction of its effective mass from revival times, and confirmation of the quadratic dispersion in real space.

Previous work [46] demonstrated that in toroidal confinement, amplitude and sound modes decouple across a wide interaction range, even away from criticality. This decoupling allows coherent superposition of multiple Higgs-branch excitations to form a localized wave packet that behaves as a free quantum particle. The resulting dynamics resemble the temporal Talbot effect [49,50], where a train of optical pulses re-forms periodically in a dispersive medium,

*Contact author: koushikphysics21@gmail.com

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and thus directly probe the Higgs dispersion. Originally observed in optics [51–53], Talbot self-imaging has since appeared in plasmonics [54,55], water waves [56,57], acoustics [58,59], and matter waves [60–63]. In BECs confined to optical lattices, this effect has been used to measure intersite phase coherence via density self-imaging [64–67]. A natural extension is to systems with spontaneously emerging periodicity, such as the dipolar supersolid studied here. Our approach thus offers a novel route to detect Higgs modes and their real-space dispersion, going well beyond standard spectroscopic methods.

Theoretical framework—We consider a dilute, weakly interacting gas of ultracold dipolar atoms, where, in the presence of a homogeneous magnetic field, the interatomic interactions are described by the two-body pseudopotential $U(\mathbf{r}) = (4\pi\hbar^2 a_s/M)\delta(\mathbf{r}) + (3\hbar^2 a_{dd}/M)[1 - 3\cos^2\Theta/r^3]$, where M is the atomic mass, a_s is the s -wave scattering length (tunable via Feshbach resonances [68,69]), and a_{dd} is the fixed dipolar length. The dipoles are polarized along the z axis, and Θ is the angle between $\mathbf{r}$ and the polarization direction.

The system is confined in a toroidal trap described by the potential $V_{\text{trap}} = M\omega_r^2[(r - r_0)^2 + \lambda^2 z^2]/2$, where $r_0 = 8\text{ }\mu\text{m}$ is the radius of the torus, and $\lambda = \omega_z/\omega_r$ is the aspect ratio. For conciseness, in the following we assume parameters corresponding to bosonic ^{162}Dy atoms ($a_{dd} = 130a_0$, where a_0 is the Bohr radius), as well as experimentally realistic trap frequencies of $(\omega_r, \omega_z)/2\pi = (100, 120)$ Hz. However, we emphasize that the results are not species specific and can be generalized to other atomic species with sufficiently large magnetic dipole moments.

We solve the extended Gross-Pitaevskii equation (eGPE) for the macroscopic wave function $\psi(\mathbf{r}, t)$ in imaginary time to obtain the ground state, incorporating the beyond-mean-field Lee-Huang-Yang correction within the local density approximation [70–73]. The interplay between short-range and long-range interactions leads to the emergence of a toroidal supersolid featuring multiple density modulations. To access excitation modes across a wide range of quasimomenta, we present our results in a regime with $n_d = 16$ density peaks and for the s -wave scattering length tuned to $a_s = 94.5a_0$, using a total atom number of $N = 1.5 \times 10^5$, as in Fig. 1. Note that further details of the model and generalizations of these results to other ring radii are presented in Supplemental Material [74].

To compute the excitation spectrum, we linearize the eGPE around the stationary state ψ_0 using the ansatz $\psi(\mathbf{r}, t) = [\psi_0(\mathbf{r}) + \epsilon(u_q(\mathbf{r})e^{-i\Omega_q t} + v_q^*(\mathbf{r})e^{i\Omega_q t})]e^{-i\mu t/\hbar}$, which yields the Bogoliubov–de Gennes (BdG) equations. Here, the modes are labeled by quantum number q , such that q/r_0 is the quasimomentum, and μ is the chemical potential of the ground state of the system. Solving these numerically provides the excitation energies $\hbar\Omega_q$ and mode functions $f_q = u_q + v_q$, corresponding to density fluctuations $\delta n_{\text{gs}} = \psi_0^* f_q$.

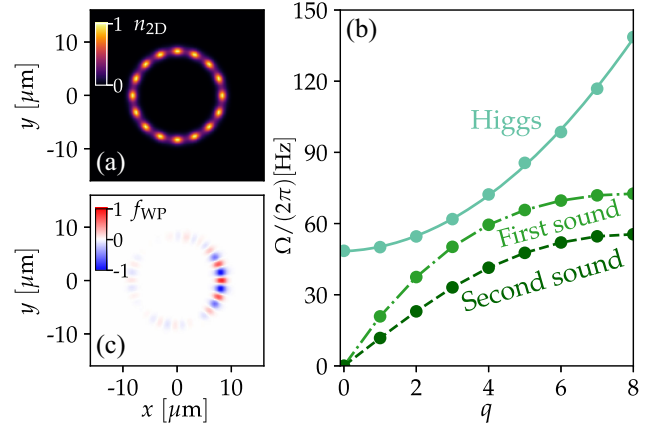


FIG. 1. Constructing a Higgs wave packet in a toroidal supersolid. (a) Normalized column density of an exemplar supersolid state, confined to a toroidal trap with $n_d = 16$ density peaks. (b) First three branches of the elementary excitation spectrum corresponding to the state in (a), plotted as a function of mode number q , with the Brillouin-zone edge located at $q = n_d/2 = 8$. The modes (circles) are connected by lines to guide the eyes. (c) Real-space structure of a Higgs wave packet, f_{WP} , formed by a linear superposition of the first five Higgs-mode eigenfunctions (see text).

The excitation spectrum of the supersolid ground state is shown in Fig. 1(b). The superfluid-to-supersolid phase transition is driven by a $q = n_d$ angular roton mode, leading to a periodic density modulation and the Brillouin zone (BZ) structure with $q \in [-n_d/2, n_d/2]$. The first two branches correspond to gapless sound modes that flatten near the BZ edge; the third is a gapped Higgs branch associated with amplitude oscillations.

We extract the effective mass of the Higgs quasiparticle from the curvature of its dispersion near $q = 0$,

$$M_H = \frac{\hbar}{4\pi r_0^2} \left(\frac{\partial^2 \Omega}{\partial q^2} \right)^{-1}. \quad (1)$$

This mass governs the inertial and dispersive properties of a Higgs wave packet: a larger M_H implies slower spreading and weaker momentum curvature.

The gap between the sound and Higgs branches enables construction of a localized quasiparticle by superposing BdG modes $f_{WP} = \sum_{q=-\mathcal{M}}^{\mathcal{M}} c_q f_q^H$, where f_q^H are Higgs-branch modes and c_q can be found by mapping a Gaussian function onto our basis states (see Ref. [74]). An example using five low- q modes ($\mathcal{M} = 5$) is shown in Fig. 1(c). The resulting fluctuation is peaked near $\theta = 0$, with minor residuals due to the finite system size.

Higgs dispersion and revivals of the wave packet—To generate a Higgs wave packet in our time-dependent simulations in an experimentally realistic approach, we imprint its spatial profile via a pulsed potential $V_{\text{pulse}}(\mathbf{r}) = \mu f_{WP}(\mathbf{r})/4$ applied for a brief duration τ . When τ (500 ns in

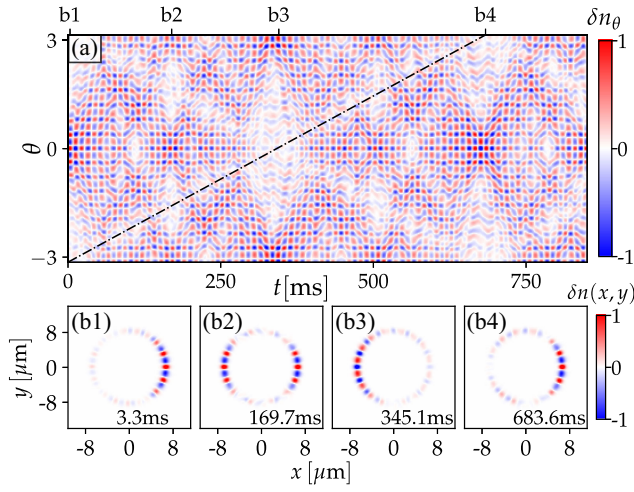


FIG. 2. Full and partial revivals of Higgs-mode-induced density fluctuations, forming a quantum carpet structure. (a) Time evolution of the density fluctuation, δn_θ , as a function of the azimuthal angle θ . The dashed line indicates the analytically determined position of a canal, $\theta_{\min}^1$ (see text). (b1)–(b4) show the normalized density fluctuations at different time instants [highlighted above (a)], corresponding to the full and fractional revivals of the wave packet. Initial state as in Fig. 1.

this Letter) is much shorter than the timescales of collective modes, the density remains unchanged while the wave function acquires a spatially varying phase $\phi(\mathbf{r}) = V_{\text{pulse}}(\mathbf{r})\tau/\hbar$. This phase gradient induces local currents that form a propagating Higgs wave packet. Similar techniques were recently employed in experiments on supersolids [80] and fermionic superfluids [81].

The ensuing dynamics are shown in Fig. 2. In (a), we compute the time-evolved density fluctuation $\delta n(\mathbf{r}, t) = |\psi(\mathbf{r}, t)|^2 - |\psi(\mathbf{r}, 0)|^2$, and display its azimuthal component, $\delta n_\theta(\theta)$, as a function of angle θ . This reveals interference features and temporally repeating structures in the density. After the application of V_{pulse} , the wave packet becomes fully formed near $t \approx 3.3$ ms, as shown in Fig. 2(b1), with a near-perfect overlap to the one shown in Fig. 1(c). In the subsequent dynamics, the wave packet rapidly disperses around the ring, and undergoes self-interference, as well as interference with the underlying supersolid structure. The initial pattern reappears at $t = 683.6$ ms $\approx T_B$ [Fig. 2(b4)]—thus defining full revival time (here referred to as the Talbot time) T_B .

Consistent with this interpretation, we observe localized structures at fractional Talbot times. For example, at $T_B/2$, the wave packet relocalizes around $\theta = \pi$ [Fig. 2(b3)], and at $T_B/4$, it exhibits multiple localized density peaks [Fig. 2(b2)]. This pattern of full and fractional revivals arises from the underlying quadratic dispersion and persists at long times (not shown here). The resulting $\delta n_\theta(\theta, t)$ displays a strikingly ordered structure, with canals [57,82–85] of minimum density fluctuation and recurring localization quantum carpet reminiscent of the iconic Talbot effect.

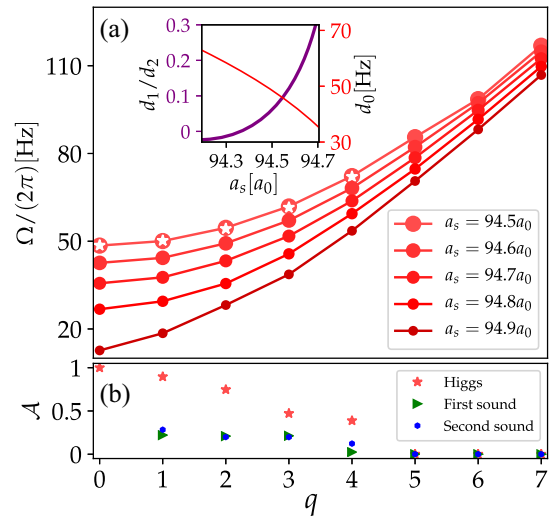


FIG. 3. Dispersion and spectral weight of Higgs modes. (a) Higgs-mode dispersion relations from BdG analysis for various scattering lengths a_s . Star markers indicate frequencies dynamically extracted at $a_s = 94.5a_0$, showing excellent agreement. The dispersion is well described by a polynomial of second order: $\Omega(q) = d_0 + d_1|q| + d_2q^2$. Inset: Dependence of fit parameters d_1/d_2 (left axis) and d_0 (right axis) on a_s , extracted using the first five Higgs modes. (b) Normalized spectral weight $\mathcal{A}$ of individual Higgs and sound modes excited by the phase-imprinting protocol at $a_s = 94.5a_0$.

Since the wave packet’s center does not drift in our system, the group velocity $v_g = d\Omega/dq \rightarrow 0$, as $q \rightarrow 0$. This also ensures symmetric spreading of positive and negative q components. In Fig. 3(a), we examine the Higgs-mode dispersion across a range of scattering lengths, noting that the density modulation appears for $a_s/a_0 \lesssim 94.92$ under the chosen parameters.

The excitation spectrum becomes increasingly linear near the Brillouin zone edge, with this linearity extending to smaller q as a_s increases. Near the superfluid–supersolid transition, this reflects the softening and eventual closure of the Higgs gap [41]. To construct a dispersive wave packet, we focus on momenta away from the BZ edge and interactions deep in the supersolid regime. We quantify the Higgs-mode curvature by fitting the BdG spectrum to $\Omega(q) = d_0 + d_1|q| + d_2q^2$, where d_0 is the gap at $q = 0$. The inset of Fig. 3(a) shows how d_0 and d_1/d_2 evolve with a_s , based on the five lowest- q Higgs modes. For $a_s < 94.7a_0$, the condition $d_1/d_2 < 0.3$ confirms a dominantly quadratic character.

Although our phase-imprinting protocol is designed to selectively excite the Higgs branch, it is important to assess whether sound modes are also inadvertently populated, particularly since Higgs and sound modes at fixed q ($q \geq 1$) belong to the same symmetry class [46]. To evaluate this, we compute the overlap between the time-evolved density fluctuation $\delta n(\mathbf{r}, t)$ and the BdG mode

functions f_q , defined as $O_q(t) = \int \delta n(\mathbf{r}, t) f_q(\mathbf{r}) d^3\mathbf{r} / \sqrt{\int \delta n^2(\mathbf{r}', t) d^3\mathbf{r}' \int f_q^2(\mathbf{r}'') d^3\mathbf{r}''}$. We extract the spectral content of each mode via a Fourier transform, $A_q(\Omega) = \int O_q(t) e^{i\Omega t} dt$, where the maximum amplitude $\mathcal{A} = \max |A_q(\Omega)|$ quantifies the modes strength in the dynamics, and the corresponding frequency Ω gives its characteristic excitation energy.

Figure 3(b) shows $\mathcal{A}$ for both Higgs and sound modes across various quasimomenta q . The Higgs modes clearly dominate the dynamics, while sound-mode contributions remain below 30% of their Higgs counterparts at all q . This confirms that, under our conditions, the dynamics observed in Fig. 2 are indeed dominated by Higgs-mode contributions, and remain robust after long times (verified up to 5 s). In fact, the sound-modes remain negligible except very close to the phase transition ($a_s \geq 94.8a_0$). Since V_{pulse} includes only the first five Higgs mode functions, the $\mathcal{A}$ vanishes for $q > 4$, also highlighting that the Higgs modes do not couple with each other.

Moreover, the dominant frequencies extracted from $A_q(\Omega)$ at $a_s = 94.5a_0$ closely match the Higgs-mode dispersion obtained from BdG analysis [see star markers in Fig. 3(a)], providing further evidence of the Higgs origin of the quantum carpet patterns in Fig. 2(a). This highlights the precision of our protocol, which excites Higgs modes at their natural frequencies—even though the pulse encodes only spatial information. In contrast to spectroscopic methods based on parametric modulation, which require frequency matching to induce a resonance, our approach relies purely on spatial phase imprinting to selectively initiate amplitude-mode dynamics.

Higgs mass from the revival times—To gain insight into the quantum carpet structure, we approximate the profiles of the Higgs-mode functions along the azimuthal directions as $f_q^H(\theta) \propto \cos(n_d\theta) \cos(q\theta)$. The total wave packet takes the form $f_{\text{WP}}(\theta) \approx \mathcal{N} \cos(n_d\theta) \Lambda(\theta, 0)$, with $\Lambda(\theta, t)$, describing the envelope of the azimuthal density fluctuation [74], and $\mathcal{N}$ is a normalization factor. The evolution of $\Lambda(\theta, t)$ is governed by an effective Schrödinger equation. From this, the time-dependent fluctuation follows [74],

$$\delta n(\theta, t) \propto \psi_0 \cos(n_d\theta) \sum_q c_q \cos \left[q\theta + \frac{\hbar q^2}{2M_H r_0^2} t \right], \quad (2)$$

revealing that the pattern arises from the quadratic dispersion relation and closed confinement. The positions of canals visible in Fig. 2(a), are determined by

$$\theta_{\min}^q = \pm \left(\frac{t\hbar q}{2M_H r_0^2} + \frac{\pi}{q} \right). \quad (3)$$

When the canals contributed by the two modes coincide with each other, partial revivals takes place, with the

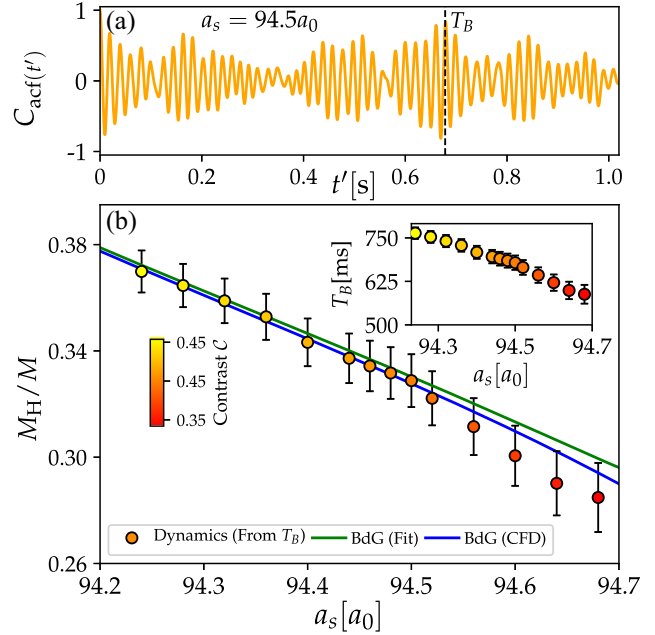


FIG. 4. Extracting the effective Higgs mass from the Talbot time. (a) Time evolution of the autocorrelation function of the density fluctuations, $C_{\text{acf}}(t')$, at $a_s = 94.5a_0$. The black dashed line marks the Talbot time T_B . (b) Effective Higgs mass M_H as a function of a_s , extracted via three methods: quadratic fit to the BdG dispersion (green line), central finite difference (CFD, blue line), and from real-time dynamics (colored markers) by measuring T_B (see inset), derived from C_{acf} . Error bars reflect the uncertainty associated with the fast oscillation scale $1/d_0$. Marker shading encodes the density contrast $\mathcal{C}$ of the corresponding ground state.

corresponding revival time being determined by

$$t_r^{q_1, q_2} = -\frac{2\pi M_H r_0^2}{q_1 q_2 \hbar}. \quad (4)$$

Here, q_1 and q_2 have opposite signs, featuring two counterpropagating modes. Similar to the optical Talbot effect, at half Talbot time $T_B/2$, where $T_B = 4\pi M_H r_0^2/\hbar$, the initial maximally localized pattern reappears, albeit shifted along the torus by πr_0 . In addition, at fractional Talbot times partial revivals occur, featuring an increased periodicity of the pattern, i.e., multiple evenly spaced copies of the initial pattern. The above discussion is also consistent with the observation in Fig. 2, see also Ref. [74]. The torus perimeter therefore serves as the analog of the grating spacing, with the quadratic dependence on periodicity governing the self-imaging revival in both cases. Furthermore, this analytic result links the revival dynamics to the effective mass of the Higgs quasiparticle. We thus see that, importantly, either measuring full or partial revival times—i.e., dynamically tracking the positions of canals in the density fluctuations—provides a direct probe of M_H , representing the central result of this Letter.

With these insights, we propose extracting the Higgs mass experimentally by measuring the Talbot time via the autocorrelation function of the density fluctuations:

$$C_{\text{acf}}(t') = \int dt \frac{\int d\mathbf{r} \delta n(\mathbf{r}, t) \delta n(\mathbf{r}, t + t') d^3\mathbf{r}}{\int d\mathbf{r} \delta n^2(\mathbf{r}, t) d^3\mathbf{r}}. \quad (5)$$

This function quantifies the temporal self-similarity of the density fluctuations, with peaks in $|C_{\text{acf}}(t')|$ signaling Talbot revivals. The full Talbot time T_B can be extracted from the position of the dominant revival peak, with an uncertainty of order $1/d_0$, where d_0 is the Higgs-mode gap. In Fig. 4(a), we show $C_{\text{acf}}(t')$ for $a_s = 94.5a_0$, with the Talbot revival marked by a dashed line. Scanning across a_s , we then compute the effective mass of the Higgs quasiparticle from the measured Talbot time, as shown in Fig. 4(b). We observe an increase in mass with decreasing scattering length, consistent with predictions from $O(N)$ field theories, where the Higgs mass similarly evolves with the critical parameter [86,87]. Furthermore, a correspondence between this behavior and the ground state structure can be established by calculating the density contrast $\mathcal{C} = (n_{\text{max}} - n_{\text{min}})/(n_{\text{max}} + n_{\text{min}})$. Here n_{max} and n_{min} are the local maximum and minimum densities, respectively. This contrast quantifies the strength of spontaneous density modulation, interpolating from $\mathcal{C} \rightarrow 0$ in the unmodulated (superfluid) limit to $\mathcal{C} \rightarrow 1$ in a fully modulated (crystalline) limit, analogous to the behavior of the superfluid fraction [29,31,88]. As $\mathcal{C}$ increases, the Higgs quasiparticle becomes heavier, resulting in longer revival times.

Finally, we compare the Higgs mass extracted dynamically with that obtained from the excitation spectrum via Eq. (1), using both finite differences between discrete BdG modes and a least-squares fit to a quadratic dispersion. As shown in Fig. 4(b), the two approaches agree remarkably well, deviating only when the dispersion ceases to be quadratic [Fig. 3(a)]. We also validate the robustness of our approach by examining Talbot dynamics and Higgs mass extraction across different numbers of density sites, as detailed in the Supplemental Material [74].

Summary—We have introduced an interferometric scheme based on the temporal Talbot effect to investigate the Higgs modes in a dipolar supersolid. Unlike the spectroscopic techniques, this method reveals signatures of amplitude modes directly in real space. Using toroidal confinement to decouple amplitude and phase modes, we construct a localized Higgs wave packet from low-lying excitations exhibiting a well-defined quadratic dispersion. This dispersion directly manifests in the partial revivals of the wave packet after a phase-imprinting pulse initiates the dynamics. The effective mass of the Higgs particle is directly encoded in the Talbot time, which can be extracted from the autocorrelation of the evolving density

fluctuations. In experiments, averaging multiple density snapshots could improve signal-to-noise ratio for the Talbot time [65].

This approach offers a powerful alternative to spectroscopic methods for detecting amplitude modes in quantum many-body systems. By linking the dynamics of a collective excitation to its effective mass, our Letter opens a new route to precision measurements of Higgs-like modes. Looking forward to future experiments, this protocol may inspire interferometric detection schemes in dipolar supersolids, and potentially in other platforms such as light-mediated supersolids [26,28,89], dipolar molecules [90–92], and exciton-polariton condensates [93,94]. Beyond the supersolid paradigm, any system exhibiting multiple spontaneously broken symmetries—one of which must be translational to enable real-space interferometry—confined in a ring-shaped geometry could, in principle, exhibit the same effect.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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