

Uncharted Logarithmic Structures in QCD Transverse-Energy Flow

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We investigate the QCD transverse-energy (E_T) flow distribution within an azimuthal region of phase space, defined by an angular interval $\Delta\phi$ on the plane transverse to a chosen reference axis. Vetoes on the resulting E_T are widely employed at the LHC to isolate missing transverse momentum in final states with invisible particles. We show that this observable has logarithmic structures never seen before in QCD calculations. Notably, its description involves the resummation of nonglobal and coherence-violating logarithmic contributions that are more singular than any reported to date. We analyze its all-orders behavior, providing analytical resummation for pp collisions at next-to-double-logarithmic accuracy and numerical resummation at leading-logarithmic accuracy in the Veneziano limit. We further present a computation of the leading coherence-violating correction, highlighting its novel structure. The intricate structure of vetoes in azimuthal gaps reveals new aspects of QCD dynamics and offers a novel probe of coherence-violating logarithmic terms in LHC observables.

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Introduction—In the past decades, the outstanding physics program of the Large Hadron Collider (LHC) has triggered a new wave of theoretical progress aimed at describing the complex structure of scattering observables with an unprecedented level of precision. These advancements concern different areas of quantum field theory, notably quantum chromodynamics (QCD), and are instrumental in deepening our understanding of non-Abelian gauge theories as well as optimizing the physics potential of the LHC and future collider experiments. In spite of the remarkable progress of the past years, the description of the rich structure of QCD dynamics in real-life scattering observables still constitutes, in general, a formidable problem. The use of exclusive kinematic cuts in experimental measurements often challenges the ability to make precise predictions for multiscale observables. An important example is that of nonglobal observables [1,2]. These measure the QCD activity in selected regions of the solid angle, designed in order to enhance the sensitivity to a specific physics effect of interest. Arguably, the best-known

variant of this class of observable [3] is the measurement of the fraction of events that survive a veto E_T on the energy flow transverse to a reference axis $\hat{n}$ (e.g., the beam axis in Drell-Yan production at hadron colliders) measured inside a region Ω of the solid angle,

$$\Sigma = \frac{1}{\sigma_0} \int_0^{E_T} dE_{T,\Omega} \frac{d\sigma}{dE_{T,\Omega}}, \quad E_{T,\Omega} = \sum_{k_i \in \Omega} k_{t,i}, \quad (1)$$

where σ_0 is the Born cross section for the process under consideration, and the $k_{t,i}$'s are the transverse momenta with respect to $\hat{n}$. In the limit where the veto scale E_T is much smaller than the hard scale Q of the underlying hard scattering process, the perturbative expansion of Σ features large radiative corrections enhanced by logarithms $L \equiv \ln Q/E_T \gg 1$. The presence of these large corrections impairs the convergence of the perturbative prediction and demands the all-order resummation of the corresponding series. The resummed perturbative expansion can be organized in two ways, either at the level of Σ ,

$$\Sigma = 1 + g_{\text{DL}}(\alpha_s L^2) + \frac{1}{L} g_{\text{NDL}}(\alpha_s L^2) + \dots, \quad (2)$$

or, for observables that satisfy the property of *exponentiation*, at the level of $\ln \Sigma$,

$$\ln \Sigma = L g_{\text{LL}}(\alpha_s L) + g_{\text{NLL}}(\alpha_s L) + \frac{1}{L} g_{\text{NNLL}}(\alpha_s L) + \dots \quad (3)$$

The former counting is equivalent to expanding Σ in powers of $\sqrt{\alpha_s}$ at fixed $\alpha_s L^2 \sim 1$, and the latter to expanding $\ln \Sigma$ in

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α_s at fixed $\alpha_s L \sim 1$. In the example of Eq. (1), the region Ω is typically defined by an angular cut away from the axis $\hat{n}$ along which the large energy flow of the event propagates. This is achieved by requiring that the rapidity of particles $|\eta| < \eta_{\text{cut}} \sim \mathcal{O}(1)$ and, optionally, by an additional cut on the azimuthal angle ϕ . In this case, the large logarithmic corrections originate from soft partons propagating with a wide angle with respect to $\hat{n}$, and for this reason the series of Σ is said to be *single logarithmic*; i.e., double-logarithmic (DL) terms are absent in Eqs. (2) and (3) ($g_{\text{DL}}(\alpha_s L^2) = g_{\text{LL}}(\alpha_s L) = 0$). At hadron colliders, observables of this type are known to feature color-suppressed, coherence-violating logarithms (CVLs) that start at $N^3\text{DL}$ in the counting of Eq. (2). In the counting of Eq. (3), these terms are superleading [4–6], and therefore, in this case, Eq. (3) is only valid in the leading-color (large- N_c) approximation.

For such observables, state-of-the-art resummations, commonly obtained using numerical methods based on the large- N_c limit of QCD, reach next-to-next-to-leading-logarithmic (NNLL) accuracy [7–12]; i.e., they control terms up to and including the function $g_{\text{NNLL}}(\alpha_s L)$ in the $\ln \Sigma$ counting of Eq. (3). Furthermore, in the NLL approximation, a variety of calculations were performed in the literature, both at high fixed perturbative orders [13–15] as well as at all orders in full color, neglecting CVLs [16–20]. This progress in understanding nonglobal observables resulted in several key applications to collider phenomenology (cf., e.g., Refs. [21–37]).

In this Letter, we initiate a study of the above observable in a different regime, that is when $L \ll \eta_{\text{cut}}$, effectively removing any rapidity constraint on emissions, while retaining a cut on the azimuthal angle with respect to the reference axis $\hat{n}$, i.e., $|\phi| < \Delta\phi/2$. As we will show, this seemingly simple modification of the observable introduces considerable complexity in the perturbative resummation that challenges the state-of-the-art methods used to tackle these calculations. One example of these challenges is an enhanced sensitivity to nonglobal logarithms, showing up already at the $g_{\text{NDL}}(\alpha_s L^2)$ and $g_{\text{LL}}(\alpha_s L)$ level in Eqs. (2) and (3), respectively.

Our investigation has important phenomenological implications. Indeed, such vetoes in azimuthal gaps are often used as an isolation cut in measurements involving missing transverse momentum at the LHC (for instance in monojet dark matter searches [38–41]). The precise understanding of this type of cut is therefore paramount to robustly predict the structure of the standard model in these key applications. Furthermore, at hadron colliders, the increased sensitivity to nonglobal logarithms necessarily implies the presence of large coherence-violating effects, making the observable introduced here a suitable probe for their study.

Azimuthal vetoes in Drell-Yan—We consider the production of a color-singlet system of invariant mass Q in pp

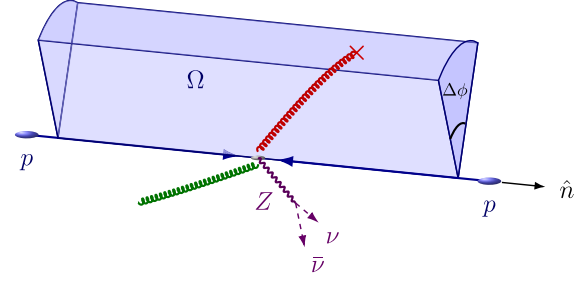


FIG. 1. The azimuthal-veto region Ω , of opening $\Delta\phi$ around the beam axis $\hat{n}$, in pp collisions. QCD radiation inside Ω contributes to $E_{T,\Omega}$ and is vetoed (red, $\times$), while emissions outside are unconstrained (green).

collisions. The observable is defined by vetoing QCD radiation in an azimuthal gap of width $\Delta\phi$ with respect to the beam axis; see Fig. 1. It is instructive to examine first the structure of the observable at $\mathcal{O}(\alpha_s)$; namely, we write $\Sigma = 1 + \Sigma^{(1)} + \mathcal{O}(\alpha_s^2)$. At this order, a single gluon is emitted from the initial-state quark line. The veto restricts only the azimuth of the emission relative to the beam, while its rapidity is integrated up to the kinematic bound $|\eta| \leq \ln(Q/k_t)$. With $f \equiv \Delta\phi/2\pi$ and $\bar{\alpha} \equiv \alpha_s/2\pi$, one finds, in the double-logarithmic approximation,

$$\Sigma_{\text{DL}}^{(1)} = -R_{\text{DL}}(L) = -4C_F f \bar{\alpha} L^2, \quad (4)$$

where the double-logarithmic contribution originates from vetoing soft collinear radiation in the gap Ω .

For nonglobal observables, one naively expects the LL tower $Lg_{\text{LL}}(\alpha_s L)$ in the $\ln \Sigma$ expansion of Eq. (3) to be reproduced by $-R(L)$, where the Sudakov radiator $R(L)$ is obtained from R_{DL} by promoting the coupling to its running counterpart,

$$\begin{aligned} R(L) &= \frac{4C_F f}{\pi} \int_0^L d\ell \alpha_s(Qe^{-\ell}) \ell \\ &= R_{\text{DL}}(L) + \mathcal{O}(\bar{\alpha}^2 L^3), \end{aligned} \quad (5)$$

with nonglobal corrections appearing only at NLL. As we now demonstrate, however, for the present observable the naive exponential $e^{-R(L)}$ does *not* suffice to reproduce Σ at LL: remarkably, nonglobal corrections enter the resummation already at this order, and must be accounted for alongside the radiator.

We now make this explicit through a fixed-order calculation. The leading nonglobal correction first appears at $\mathcal{O}(\alpha_s^2)$, in configurations where a soft parent emission lies outside the veto region Ω while one of its softer daughters enters Ω . The corresponding contribution reads,

$$\begin{aligned} \Sigma_{\text{NG}}^{(2)} &= - \int [dk_1][dk_2] \Theta(k_{t,1} - k_{t,2}) \tilde{\mathcal{M}}^2(k_1, k_2) \\ &\quad \times (\Theta(v_\Omega(k_1, k_2) - e^{-L}) - \Theta(v_\Omega(k_1 + k_2) - e^{-L})), \end{aligned} \quad (6)$$

where $\tilde{\mathcal{M}}^2(k_1, k_2) \equiv \mathcal{M}^2(k_1, k_2) - \mathcal{M}^2(k_1)\mathcal{M}^2(k_2)$ denotes the correlated part of the two-emission matrix element, and $\mathcal{M}^2(k_1, k_2)$ denotes the squared eikonal amplitude for two soft emissions normalized to the Born. The phase space measure reads $[dk_i] = d^4k/(2\pi)^3 \delta(k^2) \theta(k^0)$, and $v_\Omega(k_1, \dots, k_n) \equiv \sum_{k_i \in \Omega} k_{t,i}/Q$. Focusing on the terms relevant at LL, we approximate $\tilde{\mathcal{M}}^2$ with its strongly ordered soft limit, where it reduces to the standard [42] $C_F C_A$ angle-correlated kernel $\propto c(\phi_{12})/[\text{ch}(\eta_{12}) - c(\phi_{12})]$, where ϕ_i and η_i denote, respectively, the azimuth and rapidity of emission i with respect to $\hat{n}$, $\phi_{12} \equiv \phi_1 - \phi_2$ and $\eta_{12} \equiv \eta_1 - \eta_2$. Here, we have introduced the shorthand $c \equiv \cos$ and $\text{ch} \equiv \cosh$. The $\Delta\phi$ veto constrains only the azimuths, while the rapidities are integrated up to their kinematic bounds. The parent contribution in Eq. (6) can be approximated as $\Theta(v_\Omega(k_1 + k_2) - e^{-L}) \simeq \Theta(v_\Omega(k_1) - e^{-L})$, so that Eq. (6) has support only when $k_2 \in \Omega$ while $k_1 \notin \Omega$. One thus finds

$$\Sigma_{\text{NG}}^{(2)} = -16C_F C_A \bar{\alpha}^2 \int_{e^{-L}}^1 \frac{dv_1}{v_1} \int_{e^{-L}}^{v_1} \frac{dv_2}{v_2} \int_0^{\ln \frac{1}{v_1}} d\eta_1 \times \int_0^{\ln \frac{1}{v_2}} d\eta_2 \int_{k_1 \notin \Omega} \frac{d\phi_1}{2\pi} \int_{k_2 \in \Omega} \frac{d\phi_2}{2\pi} \frac{c(\phi_{12})}{\text{ch}(\eta_{12}) - c(\phi_{12})}. \quad (7)$$

The kernel is dominated by configurations with $\eta_{12} \simeq 0$. The two v_i integrals contribute the usual factors of L each; the additional logarithmic enhancement arises from the fact that the dominant configuration $\eta_{12} \simeq 0$ with $k_1 \notin \Omega$ and $k_2 \in \Omega$ can be realized all the way into the collinear region, so that the rapidity integration itself produces a further factor of L . The nonglobal correction therefore appears at $\mathcal{O}(\bar{\alpha}^2 L^3)$ rather than the $\mathcal{O}(\bar{\alpha}^2 L^2)$ of standard nonglobal logarithms, yielding the compact result

$$\Sigma_{\text{NG}}^{(2)} = -\frac{8}{3} C_F C_A I_{\text{NG}}(f) \bar{\alpha}^2 L^3, \quad (8)$$

where

$$I_{\text{NG}}(f) = \int_{k_1 \notin \Omega} \frac{d\phi_1}{2\pi} \int_{k_2 \in \Omega} \frac{d\phi_2}{2\pi} \int_{-\infty}^{\infty} d\eta \frac{c(\phi_{12})}{\text{ch}(\eta) - c(\phi_{12})}. \quad (9)$$

The result is shown in Fig. 2 as a function of f . We observe that the nonglobal correction vanishes for $f = 1/2$ ($\Delta\phi = \pi$) at this order.

The appearance of nonglobal logarithms already at LL, a remarkable novelty of this observable, implies a greater sensitivity to soft dynamics, which significantly complicates the resummation structure.

LL resummation at large N_c —We carry out the LL resummation in the Veneziano limit, $N_c \sim n_f \gg 1$, accounting for the full nonglobal tower of logarithms associated with multiple soft emissions at commensurate

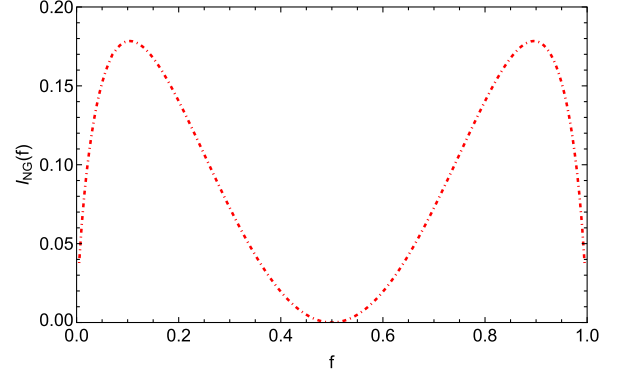


FIG. 2. The quantity $I_{\text{NG}}(f)$ as a function of the opening of the azimuthal gap.

angles and strong energy ordering. To this end, we derive a numerical resummation algorithm, implemented in the `Gnole` program [8,9]. The core modification with respect to the algorithm of Refs. [1,8,9] is the implementation of the correct kinematic bound on the emission rapidities, which captures the additional collinear enhancement; details are given in Supplemental Material [43], Sec. 1. The resulting numerical prediction for the LL function $g_{\text{LL}}(\alpha_s L)$ is shown in Fig. 3. The figure also shows the naive *global* result, simply the negative of the radiator in Eq. (5), which can be written in closed form as

$$g_{\text{LL}}^{\text{global}}(\lambda) = C_F f \frac{\ln(1 - 2\beta_0 \lambda) + 2\beta_0 \lambda}{\beta_0^2 \pi \lambda}, \quad (10)$$

where $\lambda = \alpha_s L$, $\beta_0 = (11C_A - 2n_f)/(12\pi)$. Since we work in the Veneziano limit, we set $C_F = C_A/2$. The difference between the two results amounts to nonglobal corrections, thus highlighting their sizable contribution to this observable already at LL order.

NDL resummation at full color—Given the complications present already at LL, it is interesting to inspect the

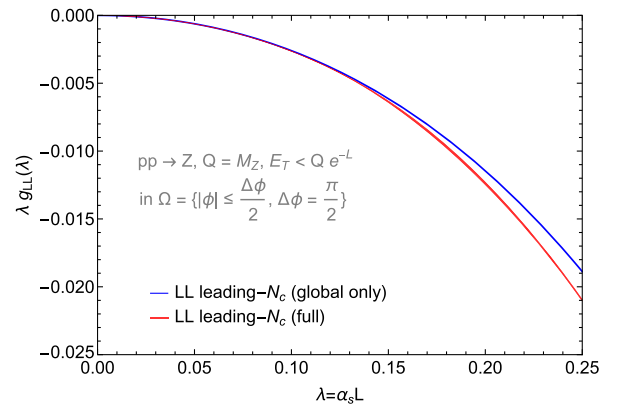


FIG. 3. The function $\lambda g_{\text{LL}}(\alpha_s L)$ vs the naive global result $\lambda g_{\text{LL}}^{\text{global}}(\alpha_s L)$. The value $\lambda \simeq 0.25$ corresponds to $E_T = 30$ GeV in a scenario where $Q = 500$ GeV.

structure of the cross section at higher orders. In order to retain the full color structure we adopt the counting of Eq. (2) and perform a calculation at NDL accuracy. At this level, nonglobal logarithms simply enter via the function $I_{\text{NG}}(f)$ given in Eq. (9), where we retain the full N_c dependence. Besides the nonglobal correction, at NDL we need to promote the radiator in Eq. (5) to its NLL counterpart

$$R(L) = 2C_F f \int_0^L d\ell \frac{\alpha_s(Qe^{-\ell})}{2\pi} (4\ell - 3), \quad (11)$$

where we included the hard-collinear contribution (the -3 piece in the integrand), needed at this order. Finally, at NDL, we need to account for the emission of a real hard-collinear gluon from the initial-state quarks. This leads to the following expression for the Drell-Yan cross section, accurate at NDL

$$\Sigma_{\text{NDL}} = e^{-R(L)} \left(1 - \frac{8}{3} C_F C_A I_{\text{NG}}(f) \bar{\alpha}^2 L^3 \right) - 2\bar{\alpha} C_F f e^{-R(L)} L \left(\frac{(\hat{P}^{(0)} \otimes h)_q(x_1, Q)}{h_q(x_1, Q)} + \{1 \leftrightarrow 2\} \right), \quad (12)$$

where $h_i(x, \mu)$ denotes the parton density for finding a parton of flavor i in the proton at a scale μ and carrying a longitudinal momentum fraction x . Here, $x_{1,2}$ are the longitudinal momentum fractions of the two incoming partons, $\otimes$ denotes the standard Mellin convolution in this fraction, and $\hat{P}^{(0)}$ denotes the regularized Altarelli-Parisi leading-order splitting function [44–46].

In Eq. (12), the first line takes account of soft emissions and hard-collinear virtual terms, encoded in the radiator. The second line encodes a real initial-state, hard-collinear emission, where the f prefactor indicates that the radiation is constrained by the azimuthal veto. While in a typical NDL resummation this type of effect can be entirely reabsorbed into the scale evolution of the parton densities, the presence of the $f \neq 1$ factor makes this impossible for the present observable. This highlights a further complication in the resummation of the present observable beyond the NDL order, in that the azimuthal cut interplays with hard-collinear dynamics (including, at higher orders, with spin correlations) and hence with the evolution of the parton densities.

Enhanced coherence-violating logarithms—The extension of Eq. (12) beyond NDL also requires the understanding of the impact of CVLs [4–6, 47–54], which enter the resummation of nonglobal observables at hadron colliders beyond the Veneziano limit. While CVLs are a generic feature of this class of observable, for the $\Delta\phi$ observable they are enhanced relative to other known cases: whereas in standard nonglobal observables they first

manifest at N^3DL , here the leading contribution arises already at NNDL . This enhancement can be understood from a fixed-order analysis analogous to that performed for the rapidity-gap-between-jets observable [5, 48, 49]. The relevant configuration involves two out-of-gap soft-collinear emissions along the beam direction, the exchange of two Glauber gluons, and the emission of a softer gluon that enters the azimuthal gap and is vetoed. We highlight that for the Drell-Yan case, it is necessary to have at least two out-of-gap soft-collinear emissions, as the color structure becomes trivial otherwise and the CVLs do not appear. In contrast to the rapidity-gap case, the vetoed gluon can be both soft *and* collinear, yielding an extra logarithmic enhancement.

Concretely, working with transverse momenta and rapidities defined with respect to the beam direction, we consider the incoming partons k_1 and k_2 and emissions k_3 and k_4 outside the gap together with a softer emission k inside the gap. Their transverse momenta are ordered as $k_{t,3} \gg k_{t,4} \gg k_t \gg E_T$, with kinematic rapidity bounds $|\eta_i| < \ln(Q/k_{t,i}) = \eta_i^{\text{max}}$. We use the color-space formalism [55] and denote the Born amplitude by $|m_0\rangle$ with $|m_0|^2 \equiv \langle m_0 | m_0 \rangle$. We further denote by $\hat{\sigma}_{\text{CVL}}$ the leading-order partonic cross section for the CVL correction, differential in the kinematics of the Drell-Yan pair. We begin by appealing to the parallel between this calculation and the analogous one for the gaps-between-jets (GBJ) observable [4, 5, 48, 49]. In the GBJ case, CVLs arise from configurations in which k_3 and k_4 straddle the rapidity gap, one emission on each side: the color matrix associated with the vetoed in-gap emission then fails to commute with the Glauber color matrix, leaving a nonvanishing residue in the color algebra. The analogous statement for the present observable is that the color matrix of the vetoed emission fails to commute with the Glauber color matrix *provided* its rapidity lies between η_3 and η_4 ; see Fig. 4. This rapidity slice of Ω plays the role of an *effective gap*, of size $|\eta_4 - \eta_3|$ and fixed azimuthal width $2\pi f$. Unlike the GBJ case, the size of the effective gap is itself a kinematic quantity; integrating over η_3, η_4 then yields the additional logarithm that enhances the leading CVL contribution to NNDL .

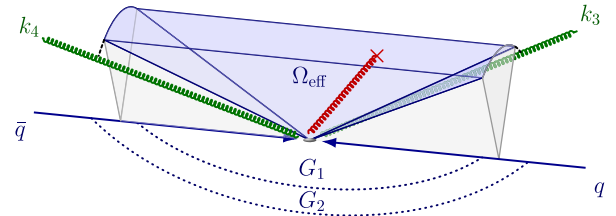


FIG. 4. CVL configuration: $q, \bar{q}$ exchange two Glauber gluons (dotted) and emit soft-collinear gluons k_3, k_4 (green). The shaded gap has polar boundaries set by the rapidities of k_3 and k_4 ; the faint-gray caps are the parts of Ω excluded by the rapidity ordering.

We now verify this picture by explicitly computing the leading $\mathcal{O}(\alpha_s^5)$ contribution, following [4,5]. It reads,

$$\begin{aligned} \hat{\sigma}_{\text{CVL}} = & \left(-\frac{2\alpha_s}{\pi}\right)^5 (1-f)^2 \int_{E_T}^Q \frac{dk_{t,3}}{k_{t,3}} \int_{-\eta_3^{\max}}^{\eta_3^{\max}} d\eta_3 \\ & \times \int_{E_T}^{k_{t,3}} \frac{dk_{t,4}}{k_{t,4}} \int_{-\eta_4^{\max}}^{\eta_4^{\max}} d\eta_4 \int_{E_T}^{k_{t,4}} \frac{dk_t}{k_t} \frac{1}{3} \ln^2\left(\frac{k_{t,4}}{E_T}\right) \\ & \times \left\langle m_0 \left| D_{a_3}^{\mu_3 \dagger} D_{a_4}^{\mu_4 \dagger} \left[\Gamma^{\text{I}}, \left[\Gamma^{\text{I}}, \Gamma^{\text{R}} \right] \right] D_{a_4}^{\mu_4} D_{a_3}^{\mu_3} \right| m_0 \right\rangle, \end{aligned} \quad (13)$$

where the prefactor $(1-f)^2$ enforces that k_3 and k_4 lie outside Ω . The factor $\ln^2(k_{t,4}/E_T)$ originates from integrating the transverse momentum of the two Glauber gluons between E_T and $k_{t,4}$. The real and imaginary parts of the anomalous dimension matrix are given by

$$\Gamma^{\text{R}} = -\frac{1}{2} \sum_{i < j}^n \mathbf{T}_i \cdot \mathbf{T}_j \int_{k \in \Omega} \frac{d\eta d\phi}{2\pi} \omega_{ij}, \quad (14)$$

$$\Gamma^{\text{I}} = i\pi \mathbf{T}_1 \cdot \mathbf{T}_2 + \frac{i\pi}{4} \left(\mathbf{T}_1^2 + \mathbf{T}_2^2 - \sum_{i=3}^n \mathbf{T}_i^2 \right), \quad (15)$$

where n denotes the number of partons in the final state and

$$\omega_{ij} \approx \Theta(\eta_i - \eta) \Theta(\eta - \eta_j) + \Theta(\eta_j - \eta) \Theta(\eta - \eta_i). \quad (16)$$

This approximation is valid for isolating the leading CVLs, since the large logarithms are generated from the limit of strongly ordered rapidities, as discussed in Supplemental Material [43], Sec. 2. In Eqs. (14) and (15), $\mathbf{T}_i$ denotes the color charge of parton i in the notation of Ref. [55] (with $\mathbf{T}_i \cdot \mathbf{T}_j \equiv \sum_a \mathbf{T}_i^a \mathbf{T}_j^a$).

The real-emission operators are

$$D_{a_\ell}^{\mu_\ell} = \sum_{i=1}^n \mathbf{T}_i^{a_\ell} h_{i,\ell}^{\mu_\ell}, \quad h_{i,\ell}^{\mu_\ell} = \frac{1}{2} k_{i,\ell} \frac{k_i^{\mu_\ell}}{k_i \cdot k_\ell}, \quad (17)$$

where a_ℓ, μ_ℓ are the color and Lorentz indices of the ℓ th emitted gluon.

The double commutator $[\Gamma^{\text{I}}, [\Gamma^{\text{I}}, \Gamma^{\text{R}}]]$ is exactly the same as the one obtained in the well-known rapidity-gap-between-jets calculation [5], and is responsible for the appearance of CVLs. The main differences for the $\Delta\phi$ observable are the limits of the rapidity integrals and the result for Γ^{R} , which can be easily derived from Eqs. (14) and (16). Simplifying the color algebra, e.g., using ColorMath [56], yields

$$\begin{aligned} & \left\langle m_0 \left| D_{a_3}^{\mu_3 \dagger} D_{a_4}^{\mu_4 \dagger} \left[\Gamma^{\text{I}}, \left[\Gamma^{\text{I}}, \Gamma^{\text{R}} \right] \right] D_{a_4}^{\mu_4} D_{a_3}^{\mu_3} \right| m_0 \right\rangle \\ & = \pi^2 C_F \frac{N_c^2}{8} f |m_0|^2 |\eta_4 - \eta_3|. \end{aligned} \quad (18)$$

Performing the remaining kinematic integrals and including the convolution with the parton densities, one obtains the leading-order CVL contribution

$$\Sigma_{\text{CVL}}^{(5)} = -\pi^2 C_F N_c^2 f (1-f)^2 \frac{8}{45} \bar{\alpha}^5 L^8, \quad (19)$$

that is, the whole CVL tower $\bar{\alpha}^n L^{2n-2}$ already contributes at NN DL, albeit coming with the usual suppression relative to leading-color terms. As anticipated, the extra power of L compared to the rapidity-gap case traces back to the factor $|\eta_4 - \eta_3|$ in Eq. (18), which yields the additional logarithm upon integration over rapidities. The enhancement of CVLs makes the observable introduced here valuable for investigations into their resummation and phenomenological impact. We leave this important avenue for future work, together with the investigation of other sources of NN DL corrections.

Phenomenological considerations—A key phenomenological application of such vetoes in azimuthal gaps is in searches of missing- E_T signals at the LHC, where any radiation within a $\Delta\phi$ azimuthal distance from the missing transverse momentum vector is vetoed as an isolation criterion (recent examples are given in Refs. [38–41]). In these applications, the azimuthal veto is also accompanied by a veto η_{cut} on the rapidity of QCD jets. In general, the description of experimental scenarios in which $\eta_{\text{cut}} \geq L$ will involve similar infrared structures to the ones studied here. For instance, in the analysis of Ref. [41], the selection involves events with at least 200 GeV of missing E_T in which jets with $p_T > 30$ GeV and $|\eta| < 5$ within an azimuthal distance $\Delta\phi = 0.5$ from the missing E_T are vetoed. This corresponds to a scenario in which $\eta_{\text{cut}} = 5 > L \simeq 1.9$, that falls into the domain considered in this Letter. Furthermore, we point out that in the setup of Ref. [41], where at least one hard jet is required, CVLs appear at $\mathcal{O}(\bar{\alpha}^4 L^6)$, i.e., still NN DL but one order lower in $\bar{\alpha}$.

An important theoretical tool to approach the complex cuts in realistic analyses, such as those referred to above, is given by parton-shower algorithms. An immediate question is then whether parton showers commonly used in experimental analyses, such as k_t -ordered dipole showers with dipole-local recoil, are capable of describing these observables, at least at the lowest logarithmic order. Following Ref. [57], in Supplemental Material [43], Sec. 2, we perform a dedicated study and find that this class of dipole showers fail to reproduce the LL behavior of the $\Delta\phi$ observable. This is a sharper failure than for standard observables, where such showers retain LL accuracy and break down only at NLL [57]. Our findings reinforce the broader case for higher-logarithmic-accuracy showers (cf. Refs. [57–75] for recent work) in LHC phenomenology, together with a better understanding of the logarithmic enhancements involved in general non-global observables.

Outlook—In this Letter, we have initiated the exploration of nonglobal observables at hadron colliders in a new kinematic regime, where the constraint η_{cut} on the rapidity of the radiation is much larger than the large logarithm L , such as in the case where one constrains the radiation only in an azimuthal region of phase space. This seemingly simple phase-space cut leads to profound complications for the accurate QCD resummation of this class of observable, highlighting several features in their infrared structures. We note that many of these features are present also at lepton colliders, where additional complications due to the definition of the reference axis arise. We provide a detailed analysis in Supplemental Material [43], Sec. 4, which includes also Refs. [76–78]. Aside from its theoretical novelty, our Letter is relevant for phenomenological analyses that impose azimuthal isolation cuts, for example, around the missing-transverse momentum in new-physics searches.

This Letter opens several possibilities for future investigations. First, from a theoretical viewpoint, it is interesting to understand the perturbative structure of this observable at higher logarithmic accuracy in light of the numerous features discovered here. It is also paramount to study such azimuthal vetoes in the context of the newly developed showers with higher logarithmic accuracy to establish whether they can be safely used in phenomenological applications. Finally, it will be important to study the potential impact of this class of resummations in phenomenological applications at the LHC, where accurate theory predictions are crucial to fully exploit the data.

Note added—While this Letter was being submitted, a new preprint [79] presented a calculation of CVLs for the one-jettiness variable. The NNDL order of CVLs observed in that article appears to be in line with our findings of Eq. (19). However, more studies are necessary to understand the connection between the two results.

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Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council

Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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