

Localization of the M2-Brane

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We study quantum M2-branes in holographic backgrounds and show that their moduli spaces of zero modes are localized according to an R-symmetry Killing vector. We discuss the relation with recent results in the context of equivariant localization in gauged supergravity and argue its origin within M-theory path integrals expanded in saddle points over M2-branes. We argue that the M2-brane partition function, including its nonperturbative corrections, should be compared to the field theory grand canonical partition function. Our results extend recent observations in the context of the giant graviton expansion of superconformal indices to generic supersymmetric anti-de Sitter boundary conditions. As a byproduct we predict nonperturbative corrections to a variety of supersymmetric observables of the Aharony-Bergman-Jafferis-Maldacena theory.

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Introduction—The strong coupling limit of string and M-theory has largely remained an enigma due to the lack of tools that allow for the explicit computation of physical observables. This parallels the early history of quantum field theories, although in recent decades there has been major progress through the development of modern tools such as integrability, bootstrap, and localization to name three. The latter has proved invaluable in the study of nonperturbative dynamics of supersymmetric quantum field theories, and allowed for the exact determination of partition functions, correlation functions, and expectation values of line operators [1].

The successes in supersymmetric quantum field theories can to some extent be translated to guiding principles in string and M-theory through top-down holographic dualities. A vital consistency check for the quantization of strings and branes is the comparison of their path integrals in a saddle point approximation to the field theory counterparts computed using, e.g., supersymmetric localization. One example where this interplay has proven particularly fruitful is the duality between M-theory on $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ spacetime and the $U(N)_k \times U(N)_{-k}$ Aharony-Bergman-Jafferis-Maldacena (ABJM) theory, with Chern-Simons level k [2]. After a map of parameters, the holographic duality states that

$$\log Z_{\text{QFT}} = \log \mathcal{Z}_{\text{M-theory}} \approx \sum_{\text{saddles}} Z_{1\text{-loop}} e^{-S_{\text{cl}}}, \quad (1)$$

where we have expressed the gravitational path integral as an infinite sum over saddle points. Following the intuition from the off-shell background field approach to string theory [3–5], the saddles are written in terms of M-branes with a classical on-shell action S_{cl} and the quantum effective action $Z_{1\text{-loop}}$ is truncated at one loop [6]. The leading saddle is given by the degenerate (pointlike) brane with a vanishing classical action and bosonic zero modes represented by the brane position. The one and higher loop partition functions integrated over the zero modes are expected to give rise to the supergravity action and its higher derivative corrections. Subleading saddles contribute nonperturbatively and represent brane instanton corrections. Although it is not clear that such an interpretation is valid in M-theory, there is computational evidence supporting it. For example, for 1/2-Bogomol’nyi-Prasad-Sommerfield (BPS) $S^2 \times S^1$ boundary conditions on AdS_4 , the exponentiation of (1) reproduces the giant graviton expansion of the 1/2-BPS index [7–9]. Another example is provided by the ABJM partition function on S^3 , which was recently shown to receive nonperturbative corrections that can be computed by quantizing an M2-brane instanton [10–13].

In this Letter we argue that brane expansions such as the one in (1) are applicable to much more general supersymmetric setups [14]. To showcase this we consider eleven-dimensional backgrounds that arise as uplifts of solutions to minimal gauged supergravity in four dimensions. We study the M2-brane partition function on these backgrounds in the saddle point approximation and show that supersymmetry implies that the branes are localized to

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the fixed points of a Killing vector associated with the background Killing spinor.

Let us start with the leading degenerate saddle, i.e., the on-shell supergravity action itself. In [16,17] it was argued that the four-dimensional supergravity action allows for an equivariant localization, reducing its evaluation to a sum over fixed points of the Killing vector. As we discuss in this Letter, this localization happens in the zero-mode sector of the degenerate M2-brane and constitutes the zeroth-order check of supersymmetric localization of M2-branes. We show that the same localization procedure also applies to the moduli space of zero modes in the subleading saddles given by nondegenerate M2-branes.

Having localized the nonperturbative M2-branes, we quantize them and compute their one-loop partition functions. Comparing to the dual S^3 partition function we find that the one-loop result agrees with the nonperturbative correction in the grand canonical ensemble. We argue that this one-loop exactness is a consequence of supersymmetric localization on the M2-brane.

This Letter aims to elucidate the existence of equivariant and supersymmetric localization in the context of M-theory path integrals, and its implications for nonperturbative M2-branes. Computational details are omitted here and will be presented in [18].

Background—We study M-theory backgrounds that can be consistently truncated down to four-dimensional $\mathcal{N} = 2$ gauged supergravity. The dynamics of the latter is controlled by the standard Einstein-Maxwell action with a cosmological constant [19],

$$S = -\frac{1}{16\pi G_N^{(4)}} \int \star_4 \left(R_4 + 6 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right), \quad (2)$$

where $F = dA$, and R_4 is the Ricci scalar and $\star_4$ denotes the Hodge star operator. The theory also contains a gravitino ψ_μ whose supersymmetry variation determines the preserved supersymmetry in the background,

$$\delta_\eta \psi_\mu = \left(\nabla_\mu - i \frac{A_\mu}{2} + \frac{1}{2} \gamma_\mu + \frac{i}{4} F \gamma_\mu \right) \eta = 0. \quad (3)$$

Here, ∇_μ denotes the spin-covariant derivative, γ_μ denotes the curved four-dimensional gamma matrices, $F = \frac{1}{2} \gamma^{\mu\nu} F_{\mu\nu}$, and η is the supersymmetry parameter.

Any solution to the equation of motion of the four-dimensional theory can be uplifted to a solution of eleven-dimensional supergravity as follows [20]:

$$\begin{aligned} ds_{11}^2 &= L^2 \left(ds_4^2 + 4 ds_{\text{KE}_6}^2 + (dy + 2\sigma + \frac{1}{2}A)^2 \right), \\ G_4 &= iL^3 (3\text{vol}_4 + 2J \wedge \star_4 F), \end{aligned} \quad (4)$$

where $ds_{\text{KE}_6}^2$ is a six-dimensional Kähler-Einstein metric with Kähler form $d\sigma = 2J$. In this Letter we focus on the

holographic dual to the ABJM theory in which case the seven transverse directions combine into a metric on $S^7/\mathbf{Z}_k$ deformed by the four-dimensional gauge field A . More precisely, we choose the following frames on the internal space:

$$\begin{aligned} e^5 &= 2L d\theta, \\ e^6 &= L \sin \theta d\theta_1, & e^7 &= L \sin \theta \sin \theta_1 d\phi_1, \\ e^8 &= L \cos \theta d\theta_2, & e^9 &= L \cos \theta \sin \theta_2 d\phi_2, \\ e^{10} &= L \sin \theta \cos \theta (2d\varphi + \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2), \\ e^{11} &= L \left(dy + 2\sigma + \frac{1}{2}A \right), \end{aligned} \quad (5)$$

where

$$2\sigma = \sin^2 \theta \cos \theta_1 d\phi_1 + \cos^2 \theta \cos \theta_2 d\phi_2 - \cos 2\theta d\varphi. \quad (6)$$

The coordinate ranges are $0 \leq \theta \leq \pi/2$, $0 \leq \theta_{1,2} \leq \pi$, $0 \leq y, \phi_{1,2} \leq 2\pi$, and $0 \leq \varphi \leq 4\pi/k$, where the latter implements the $\mathbf{Z}_k$ quotient. It is important to note that we do not quotient the y direction since for nontrivial A , we preserve four-dimensional $\mathcal{N} = 2$ supersymmetry and translations along y acts as the R symmetry. This will become apparent when we write down the Killing spinor associated with the solutions below.

If the four-dimensional solution preserves supersymmetry, then the eleven-dimensional solution preserves (at least) the same amount of supersymmetry. This can be explicitly demonstrated by constructing an eleven-dimensional Killing spinor ϵ for which the supersymmetry variation of the eleven-dimensional gravitino vanishes,

$$\delta_\epsilon \Psi_M = \left(\nabla_M + \frac{1}{24} \Gamma_M G_4 - \frac{1}{8} G_4 \Gamma_M \right) \epsilon = 0. \quad (7)$$

The Killing spinor ϵ can be identified using similar techniques as in [21]. Assuming that it is a product of the conformal Killing spinor on (round) S^7 and a four-dimensional spinor, we can reduce the vanishing of the supersymmetry variation along the seven internal coordinates $\delta_\epsilon \Psi_\alpha = 0$ to an algebraic condition on the seven-spinor [22]. This constraint reduces the number of independent components in the seven-spinor from 8 to 2, which can be expressed as

$$\chi = e^{i\gamma_{11}} \frac{1}{2} (1 - i\gamma_{8911}) \frac{1}{2} (1 - i\gamma_{51011}) \chi_0, \quad (8)$$

where χ_0 is a constant seven-dimensional spinor. This can be further split into positive and negative frequency modes along y using γ_{11} ,

$$\chi^\pm \equiv \frac{1}{2}(1 \pm \gamma_{11})\chi. \quad (9)$$

The eleven-dimensional spinor can now be written as

$$\epsilon = \eta \otimes \chi^+ + \eta^c \otimes \chi^-, \quad (10)$$

where η is the four-dimensional Killing spinor satisfying $\delta_\eta \psi_\mu = 0$ and η^c is its charge conjugate.

We should note that for some particularly symmetric four-dimensional backgrounds, there are more solutions to the eleven-dimensional BPS equation $\delta_\epsilon \Psi_M = 0$. Indeed, the round AdS_4 with $F = 0$ preserves all 32 supercharges available for $k = 1, 2$, and 24 supercharges if $k > 2$ in eleven-dimensional supergravity. The particular supercharges we have constructed play a special role however, they are the holographic duals to the supercharges used for supersymmetric localization on the field theory side. Indeed, we will see that it plays a similar role on the gravity side, where it localizes the zero-mode sector of instantonic supersymmetric M2-branes.

Localization in supergravity—In [16] the on-shell action for solutions of minimal gauged supergravity was shown to exhibit a structure reminiscent of supersymmetric localization in quantum field theory (QFT). This was further elucidated using the Berline-Vergne-Atiyah-Bott fixed point theorem in [17]. The argument goes as follows: any supersymmetric solution of the four-dimensional theory (2) is equipped with a Killing spinor η , which can be squared to construct a Killing vector ξ . From this Killing vector one defines an equivariant exterior derivative $d_\xi \equiv d - \xi \lrcorner$. A particular set of BPS observables, which includes the on-shell action, can now be reformulated as an integral of an equivariantly closed polyform. As a consequence of the Berline-Vergne-Atiyah-Bott theorem this integral reduces to a sum over the fixed points of ξ [23], which can either be isolated points (called nuts) or two-dimensional surfaces (called bolts) [17]. An essential feature of the fixed point sets is that they are in one-to-one correspondence with the locations where the Killing spinor is chiral (without vanishing) $\eta = \pm \gamma_{(4)} \eta$, with $\gamma_{(4)}$ the four-dimensional chirality operator.

Referring back to the saddle point expansion in (1), we would like to reinterpret the fixed point result of [16,17] in terms of M2-branes. Presumably, the two derivative supergravity action arises from the (suitably regularized) one-loop quantization of pointlike M2-branes. The position of the branes constitutes eleven bosonic zero modes that are integrated over. The fixed point formula tells us that instead of evaluating the full integral over the moduli space of zero modes, one merely has to evaluate their contribution at the fixed points of ξ . We would like to emphasize that this localization of zero modes to the fixed point set of ξ is established at the one-loop level (i.e., two-derivative supergravity) and may be corrected at higher loops. In [24] it was

observed that a modified fixed point formula still holds for the higher derivative action obtained in [25]. In order to prove that localization happens to all order we should follow the standard route of localization and add a chosen Q -exact functional to the action and take the $\hbar \rightarrow 0$ limit. This task seems a tall order that is currently out of reach. In the remainder of this Letter we will provide further evidence for localization in M-theory by studying M2-branes with finite classical action and their one-loop quantization.

Localization in M-theory—Let us now consider probe M2-branes in the eleven-dimensional geometry (4). Let us furthermore demand that the M2-brane is BPS with respect to (some of) the supercharges defined by (10). For a BPS brane configuration, the supersymmetry variation of the fermions should be cancelled by the local κ -symmetry transformation, which can be achieved if and only if [26]

$$(1 - iq\Gamma_{M2})\epsilon = 0, \quad (11)$$

where $q = \pm 1$ is the charge of the brane and $\Gamma_{M2} = (1/3!) \epsilon^{abc} \Gamma_{abc}$ such that $\Gamma_{M2}^2 = -1$.

Many different solutions exist for this equation, but since we are interested in M2-branes wrapping closed three-manifolds with finite classical action, we will restrict to submanifold of S^7 . The M2-brane projector can be rewritten in terms of the lower-dimensional gamma matrices as

$$\Gamma_{M2} = \gamma_{(4)} \gamma_{M2}, \quad (12)$$

where γ_{M2} is the product of the pull-back of seven-dimensional gamma matrices to the world volume. The BPS condition can be solved separately for the four-dimensional spinor and the seven-dimensional spinor,

$$i\gamma_{M2}\chi^\pm = s\chi^\pm, \quad \gamma_{(4)}\eta = sq\eta, \quad (13)$$

where $s = \pm 1$. Importantly, we find for the brane to be supersymmetric it must localise to the points where the four-dimensional spinor is chiral, i.e. at the fixed points of the Killing vector ξ . We thus find that the instantonic M2-branes localize on the nuts and bolts in four dimensions, just as the supergravity observables did!

The remaining task is to classify the solutions to the first equation in (13). We focus on dominant solutions that have the smallest on-shell action. There are two possible configurations that are not related by symmetries of the background that must be considered. Roughly they are in one-to-one correspondence with the two projectors that appear in (8). For one solution the M2-brane wraps $(\theta_2, \phi_2, \varphi)$ and $\theta = 0$, while for the other the brane wraps (θ, y, φ) . In both cases the M2 world volume is a round Lens space $S^3/\mathbb{Z}_k$ with radius $2L$. Naively a third solution exists where the M2-brane wraps $(\theta_1, \phi_1, \varphi)$ and $\theta = \pi/2$, but this is related to the first configuration by a $\mathbb{Z}_2$

symmetry of the background and should not be counted. Both inequivalent configurations have $s = 1$ and have the same on-shell action,

$$S_{\text{cl}} = \frac{1}{(2\pi)^2 \ell_p^3} \int \text{vol}_3 = 2\pi \sqrt{2N/k}, \quad (14)$$

where vol_3 is the volume form on the brane and we have used the relation between the gravity and field theory parameters $(L/\ell_p)^6 = Nk\pi^2/2$.

Quantized M2-branes—We now turn to the one-loop partition function of the M2-branes. This involves quantizing the fluctuations of the brane around its classical configuration. The degrees of freedom consist of four complex scalar fields and eight fermions living on the brane world volume. Since to this order the action is quadratic, the partition function is given by the determinant of kinetic operators,

$$Z_{\text{1-loop}} = Z_{\text{zero-modes}} \frac{\prod_f \sqrt{\det' \mathcal{D}}}{\prod_b \sqrt{\det' \mathcal{K}}}, \quad (15)$$

where we have separated out the zero modes from the determinants and collected into $Z_{\text{zero-modes}}$.

The kinetic operators $\mathcal{D}$ and $\mathcal{K}$ are computed by expanding the M2-brane action [27] to quadratic order. The general formula for the bosonic operators in terms of background quantities is presented in [28,29] and fermions can be treated using expressions in [29–31]. Using these, the derivation of the operators is straightforward and hence we will not go through the explicit computation here. A detailed derivation will be presented elsewhere [18].

The two supersymmetric M2-brane configurations discussed above lead to the same spectrum of fluctuation with identical kinetic operators. Their two partition functions therefore completely agree. For this reason we will focus on the first of the two configurations for which the world volume metric on $S^3/\mathbb{Z}_k$ takes the form

$$ds_{\text{M2}}^2 = L^2 \left(d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2 + (d\varphi - \cos \theta_2 d\phi_2)^2 \right). \quad (16)$$

For the present case, the M2-brane world volume is particularly symmetric and therefore the kinetic operators are restricted to the form

$$\mathcal{K} = -D^2 + M^2, \quad \mathcal{D} = iD + M, \quad (17)$$

where M denotes the mass of the respective field and D_a is a gauge covariant derivative that depends on a background gauge field that arises due to the curvature and four-form field strength present in the background geometry. In this case two gauge fields arise, and the charge of each field with respect to these two gauge fields is determined, like the mass, by expanding the M2-brane action (see [18]).

The two gauge fields are given by

$$\mathcal{A}_1 = \frac{1}{2}(d\varphi - \cos \theta_2 d\phi_2), \quad \mathcal{A}_2 = -\frac{1}{2}\cos \theta_2 d\phi_2, \quad (18)$$

and the covariant derivative is $D = \nabla - iQ_1 \mathcal{A}_1 - iQ_2 \mathcal{A}_2$.

When quantizing the fields, it is efficient to perform a mode expansion along φ for which the charge Q_1 shifts the mode numbers. The gauge field $\mathcal{A}_2$ on the other hand acts as a unit flux monopole field on the S^2 parametrized by (θ_2, ϕ_2) .

We are now in position to list the spectrum of fluctuations for the four scalar and eight fermionic fields. It turns out that the spectrum is completely determined by the self-dual or anti-self-dual component of the four-dimensional field strength evaluated on the brane. More precisely let us define

$$f = \frac{1}{2}|F + q\star_4 F|, \quad (19)$$

where the norm of a four-dimensional tensor is defined as $|T| = \frac{1}{2}\sqrt{T^{\mu\nu}T_{\mu\nu}}$ and this expression should be understood as being evaluated on the M2-brane world volume, which we recall is located where the four-dimensional spinor satisfies $\gamma_{(4)}\eta = q\eta$. As can be observed from Table I, the entire spectrum is controlled by f . The limit $f \rightarrow 0$, which is relevant for the AdS_4 background, recovers the spectrum computed in [13].

Zero modes: For $k > 2$, the zero-mode sector is independent of f , and as such identical to the zero modes studied in [12]. Namely, there are 12 bosonic and 12 fermionic zero modes. In [12] a supersymmetric deformation was employed to lift the zero modes and by taking the deformation to zero, their contribution was determined to equal $Z_{\text{zero-modes}} = 2$. Our analysis provides an alternative perspective on this result: imposing supersymmetry with respect to the spinor (10) leads to two equally contributing M2-brane configurations, explaining the factor 2.

The one-loop determinants: In order to evaluate the determinants in (15), we follow the same strategy as in [13] which was to expand in Fourier modes along φ and quantize the tower of 2D modes using the results of Sec. 2 in [12]. Although world volume supersymmetry is not manifest in the Green-Schwarz formalism, it does

TABLE I. Spectrum of scalar and fermion fluctuations.

Field	d.o.f.	Q_1	Q_2	ML
Scalars	4	f	0	$if/2$
	4	1	1	$i\sqrt{3}/2$
Fermions	4	0	0	$3q/4$
	2	$1 + f$	1	$-q(3/4 - f/2)$
	2	$1 - f$	1	$-q(3/4 + f/2)$

seem to reveal itself in the spectrum by the fact that there is an enormous cancellation between fermionic and bosonic contributions. The contribution of the remaining unpaired modes can be expressed in terms of the following infinite products [32]:

$$s(z) = \prod_{n=1}^{\infty} \left(\frac{n+z}{n-z} \right)^n, \quad t(z) = \prod_{n=1}^{\infty} \frac{k^2}{4} (n^2 - z^2), \quad (20)$$

which can be evaluated using ζ -function regularization,

$$s(z) = e^{\frac{i\text{Li}_2(e^{2\pi iz})}{2\pi} - \frac{i\pi}{12} - z \log[1 - e^{2\pi iz}] + \frac{i\pi z^2}{2}},$$

$$t(z) = \frac{4 \sin \pi z}{k z}. \quad (21)$$

We are now in position to write the full one-loop partition function of instantonic M2-branes by combining the zero modes and nontrivial determinants and summing over the fixed point set in four dimensions,

$$Z_{\text{M2}} = 2 \sum_{\text{fixed points}} \frac{s(\frac{2}{k})^{2k} s(x_+)^{-k} s(x_-)^{-k}}{t(x_+)t(x_-)} e^{-2\pi\sqrt{2N/k}}, \quad (22)$$

where we have introduced $x_{\pm} = (2/k)(1 \pm f)$ and the sum runs over the fixed points of the Killing vector ξ . We have not distinguished between isolated fixed points and fixed surfaces but the latter should be integrated over with a suitable measure that we will discuss momentarily. We would like to emphasize that this result holds for any supersymmetric background of minimal four-dimensional $\mathcal{N} = 2$ gauged supergravity. For a given four-dimensional solution and its fixed point set, the chirality constraint determines the M2-brane charge q , which in turn feeds into the definition of f and $x_{\pm}$ in (19).

Examples—Let us now apply our formula (22) to a few well known solutions of four-dimensional supergravity. We will be brief but a more complete treatment will appear in [18].

Anti-de Sitter (AdS) with a round S^3 boundary: The R-symmetry Killing vector has an isolated fixed point at the center of AdS, where both charges for the M2-brane are supersymmetric. For both charges $f = 0$ such that

$$Z_{\text{M2}} = \frac{1}{\sin^2(2\pi/k)} e^{-2\pi\sqrt{2N/k}}, \quad (23)$$

recovering the result of [13]. This answer matches the leading nonperturbative corrections to the ABJM partition function on the round sphere [11].

AdS with a squashed S_b^3 boundary, preserving a $U(1)^2$ isometry [33]: The fixed point of the Killing vector is at the center of AdS and the spinor has a definite chirality, allowing only a single charge for the M2-brane to be supersymmetric. The Yang-Mills field is nontrivial, such

that $f = [(b^2 - 1)/(b^2 + 1)]$ and

$$Z_{\text{M2}} = 2 \frac{s(\frac{2}{k})^{2k} s(\frac{4b^2/k}{b^2+1})^{-k} s(\frac{4/k}{b^2+1})^{-k}}{t(\frac{4b^2/k}{b^2+1}) t(\frac{4/k}{b^2+1})} e^{-2\pi\sqrt{2N/k}}. \quad (24)$$

Supersymmetric extremal AdS-Kerr-Newman black hole [34–36]: There are two fixed points, at the north and south pole of the horizon, where the Killing spinor has a definite chirality and $f = [(\omega - 1)/(\omega + 1)]$, with ω the angular chemical potential of the black holes, in the conventions of [37]. The sum in (22) therefore reduces to a factor 2 and the M2-brane partition function can be read off. The dual partition function is related to the superconformal index of the ABJM theory [38,39].

Supersymmetric thermal AdS, i.e., Euclidean AdS in global coordinates with a noncontractible time circle: In contrast to the black hole background, the Killing vector does not have fixed points, and thus there are no supersymmetric (instantonic) M2-brane configurations. This is consistent with recent results that showed that the dual partition function, which is related to the 1/2-BPS index of the ABJM theory, can be written as a giant graviton expansion of M5-branes [7–9].

Euclidean dyonic black hole with Riemann surface horizons of genus $g > 1$ [40]: The dual partition function is related to the topologically twisted index [41–43], which was argued to equal the superconformal index and the S_b^3 partition functions in the “Cardy limits” $\omega \rightarrow 0$ and $b \rightarrow \infty$ [44,45], up to a prefactor $(1 - g)$ [42,46]. This is consistent with the fact that $f = 1$ for all three associated backgrounds in these limits. The fixed point set in this case is the black hole horizon, where the spinor has a definite chirality, and the fixed point sum in (22) should be treated as an integral. We determine the integration measure from the aforementioned relations between the holographically dual partition functions, such that the integral evaluates to $(1 - g)$ [47].

Supersymmetric localization—A fascinating feature of the result (23) is that even though it comes from a one-loop quantization, it matches the leading nonperturbative correction to the field theory grand canonical S^3 partition function [48]

$$e^{J(\mu,k)} = \sum_{N=0}^{\infty} Z_{S^3}(N, k) e^{\mu N}. \quad (25)$$

A possible explanation is that in string and M-theory we compute the partition function for a fixed background, and thus for fixed length scale L/ℓ_p . The canonical ABJM partition function is instead computed for a fixed rank N and consequently for a holographic comparison one should perform a Legendre transform interchanging between keeping $\mu \sim (L/\ell_p)^3$ fixed and keeping the flux quantum N fixed.

The feature of one-loop exactness seems to persist more generally for nonperturbative corrections to the grand canonical potential $J(\mu, k)$ that can be mapped to (world sheet) instantons in the gravitational theory. Namely, the coefficients of these exponentially suppressed terms are determined by a topological string on local $\mathbf{P}^1 \times \mathbf{P}^1$, and factors of $\sin 2\pi/k$ [11]. As such they are independent of N and consequently are determined in terms of a one-loop quantization of instantonic M2-branes.

The fact that the one-loop result in (23) matches the nonperturbative correction in the field theory grand canonical partition function could be an artefact of the large amount of supersymmetry, which is not present for the more general setup studied here. Currently there are only a handful of ABJM observables that have been determined nonperturbatively. Notably, for the squashed sphere partition function with $\mathfrak{v} = \sqrt{3}$ of the $k = 1$ ABJM theory, results of [49] support the one-loop exactness of M2-branes we have discussed here. The reasons are similar as for the round S^3 partition function. Namely, there is a topological string description for the nonperturbative sector that dictates a simple N dependence of the nonperturbative effects in the grand canonical ensemble. Recently, similar results were obtained for a mass-deformed ABJM on the squashed sphere [50,51].

In this Letter we demonstrated that by picking a suitable supercharge the M2-brane is localized in target space eliminating many of the subtleties encountered in [12,13]. By analyzing a broad class of observables and performing explicit one-loop analysis in M-theory, we have uncovered a formula (22) that gives tantalizing hints at the presence of localization within quantum gravity. Recall that the supersymmetric configurations we studied involve M2-branes wrapping a lens space $S^3/\mathbb{Z}_k$ embedded in a flux background. The curvature and flux of the environment leads to a deformation of the world volume theory in the form of nontrivial background fields. We may speculate that at low energies the dynamics of these M2-branes are governed by a similarly deformed version of the ABJM theory and hence the M2-brane partition function may be obtained as the ABJM partition function that can be computed using localization. Our one-loop result in (22) features basic building blocks of 3D supersymmetric partition functions familiar from the localization literature [52–54]. A point we aim to address in the future is to directly reproduce (22) as an ABJM partition function.

We would like to emphasize that the unreasonably simple answer in (22) is a consequence of quantizing instantons directly in M-theory, instead of weakly coupled string theory. Indeed, in the latter this answer can be thought of as an infinite series over loop diagrams of world sheet instantons, corresponding to the large k expansion of (22). It will be very interesting to study whether localization is also applicable to the leading saddle in the quantum gravitational path integral (1). Our results suggest that this study might be under better control in the context of M-theory than type II

string theory and thus motivates us to revisit the approach of Fradkin and Tseytlin [3–5] in the context of degenerate M2-branes, for example along the lines of [55].

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