

Imprints of Asymptotic Freedom on Confining Strings

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We consider the Polyakov loop correlator in the confining phase of large N Yang-Mills theory in three and four dimensions. It can be computed by summing over the exchange of closed flux tubes winding around the thermal cycle. At short separations, the leading divergence is controlled by perturbation theory. Combining these two facts allows us to determine the asymptotic spectral density of string states contributing to the correlator. This sharply relates the weakly coupled UV of the gauge theory to the dynamics of highly energetic flux tubes. Then, in a toy integrable setting, we explore how this can bound the scattering data of the Goldstone modes on top of a long string. We derive a bound on the asymptotic behavior of the reflection amplitude of Goldstones against the flux tube boundary sourced by the Polyakov line, and rule out an asymptotically linear phase shift for the S matrix.

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Introduction—Understanding confinement requires connecting the dynamics of almost-free gluons at high energies to the behavior of emergent confining flux tubes sourced by far separated probe quarks. In this Letter, we describe some of the simplest consequences of asymptotic freedom [1,2] for the high-energy dynamics of confining strings in large N pure Yang-Mills theory in three and four dimensions.

In $D > 2$, it has proven useful to consider first a long confining string [3]. The low-energy structure of the worldsheet is quite rigid. This limit is described by $D - 2$ massless transverse string modes X^i whose interactions are constrained by the nonlinearly realized space-time Poincaré symmetry [9–13]. The Goldstone bosons are free to leading order, and their S matrix is entirely fixed up to order $(\ell_s E)^6$ in terms of the string tension ℓ_s^{-2} . The next several orders of the S matrix are determined in terms of only a handful of nonuniversal Wilson coefficients. These and other low-energy observables can be rigorously bounded by bootstrap techniques [14–18].

Both the effective theory analysis and most S -matrix bootstrap bounds are agnostic to the UV completion and thus apply to most theories with long, stable, stringlike excitations. These are very diverse and include, beyond the chromoelectric flux tubes, domain walls in the Ising model [19–21], and vortex strings in the Abelian Higgs model [22,23]. Without extra input, it is therefore natural that

these methods have almost nothing to say about the dynamics at energy scales larger than ℓ_s^{-1} .

A glimpse at the Yang-Mills flux tube structure at higher energies is provided by lattice simulations [24–27]. The primary observables are the energy levels of closed winding strings, sourced by Polyakov loops in $\mathbb{R}^{D-1} \times S_R^1$, as a function of the circle radius R . The resulting finite volume spectrum clearly shows the existence of a light-massive pseudoscalar resonance in 4D, while in 3D no signature of resonances is manifest [28]. This light-resonance spectrum has been speculated to reflect the high-energy structure of the theory, with the quantum numbers of the Goldstones and the axion (or absence thereof in 3D) relating to the low-dimension operator insertions in the Polyakov line [29,30].

Turning these qualitative expectations into sharp analytic predictions is nontrivial. The key challenge is to find flux tube observables whose high-energy limit is genuinely controlled by asymptotic freedom and thus accessible in weakly coupled Yang-Mills theory. Neither the worldsheet S matrix nor the finite-volume spectrum do the job: scattering of high-energy Goldstones occurs on top of a long string, thus making it hard to factorize the UV data from the IR background [29], while trying to probe short distances by shrinking R eventually triggers a transition out of the confining phase in which the string description applies [31].

In this Letter we point out that the two-point Polyakov loop correlator, $\langle W_{\square}(\tau, \vec{0}) W_{\square}(0, \vec{0}) \rangle$, fits the bill. This is, of course, the same object used in the lattice to extract finite-volume spectra. At large τ , R it is described by the long string effective field theory (EFT), while its finite R , small τ asymptotics is controlled by perturbative Yang-Mills. Importantly, the correlator is a smooth function of τ , with

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no phase transition, and thus interpolates between weakly coupled gluons and long confining strings. We discuss this in more detail.

At finite N , this interpolation is not captured by winding worldsheet states alone, as around $\tau \sim \ell_s^{-1}$, intermediate glueballs start to give important contributions. In the strict large N limit, though, the Polyakov line does not source glueballs and excited strings cannot decay, so the flux tube decouples from other bulk degrees of freedom. Hence, at large N the small τ asymptotics is to be reproduced entirely in terms of the exchange of highly excited closed strings. This is the topic of section.

We then perform a Cardy trick of sorts, using the perturbative quark-antiquark potential to infer the large mass spectral density of closed string states contributing to the Polyakov correlator, see Refs. [24,32–34] for a similar Cardy trick in the EFT. This allows us to establish an exponential decay of the coupling between high-mass string states and Polyakov lines, offsetting the Hagedorn growth of states, see Eqs. (11) and (12).

Next, we consider large R , and study the implications of the asymptotic scaling (11) for the Goldstone interactions, under the toy approximation of integrability. We show that causality implies a bound on the high-energy asymptotics of the form factor $K(p) \equiv \langle W|p, -p \rangle$ for the Wilson line to source pairs of Goldstones, see Eq. (17).

We complement the main text with Supplemental Material [35]. Besides details on computations, examples, and reviews on the topics covered in the main text, we establish the positivity of time delays for $2d$ massless elastic S matrices, and rule out an asymptotically linear phase shift for the S matrix on the worldsheet.

The Polyakov correlator—

Cylinder partition function from large N : We consider the Euclidean correlation function of two Wilson loops (one fundamental and one antifundamental) in $\mathbb{R}^{D-1} \times S_R^1$, $\langle W_\square(\tau, \vec{0}) W_\square(0, \vec{0}) \rangle$. The loops are parallel, separated by a Euclidean time τ , and wrapping the compact direction of length R (making them so-called Polyakov loops). We place them at the same position $\vec{0} \in \mathbb{R}^{D-2}$ in the transverse space. Throughout, we keep the length of the compact direction above the inverse of the deconfinement temperature, $R > 1/T_D$, so that the theory remains in the confining phase.

In this phase, fundamental Wilson loops source confining strings. At large N , the dominant contribution to the correlator comes from the exchange of single closed string states, whose internal dynamics we must sum over. In other words, the only worldsheet topology that survives the large N limit is that of a cylinder stretching between the two loops. This follows from a standard large N counting of diagrams *à la* 't Hooft [73]. Higher-genus topologies, corresponding to the exchange of multistrings (glueballs) states, are suppressed by powers of $1/N$.

The cylinder partition function $Z_{\text{cyl}}(\tau, R) \equiv \langle W_\square W_\square \rangle$ can be evaluated in two quantizations,

$$Z_{\text{cyl}}(\tau, R) = \langle W | e^{-H_{\text{cl}} \tau} | W \rangle = \text{Tr}_{\mathcal{H}_{\text{op}}} (e^{-H_{\text{op}} R}). \quad (1)$$

In the closed string channel, the Wilson line defines a state $|W\rangle$ in the Hilbert space $\mathcal{H}_{\text{cl}}$ on $\mathbb{R}^{D-2} \times S_R^1$, which is evolved by the Hamiltonian H_{cl} . At large N , $|W\rangle$ only couples to single closed string states that wind once around the S_R^1 . In the open string channel, the correlator is interpreted as the thermal trace over the Hilbert space $\mathcal{H}_{\text{op}}$ on $\mathbb{R}^{D-1}$ twisted by the Wilson line defects. At large N , the only states contributing to this trace are open strings stretching between the quarks.

Both representations should converge for any finite τ, R in the confining phase. While R must be kept sufficiently large to remain in the confined phase, we do not encounter any phase transitions as we dial τ , since Wilson lines are mere probes of the theory. This is backed by both lattice simulations and confining holographic models. Thus, τ provides a knob that allows us to smoothly interpolate between the UV and IR limits of the theory.

We now discuss the dependence of the partition function on the two dimensionless ratios τ/ℓ_s and R/ℓ_s . The decompositions (1) imply that

$$\log Z_{\text{cyl}}(\tau, R) \xrightarrow{\tau/\ell_s \rightarrow \infty} -\tau E_0^{\text{cl}}(R), \quad (2)$$

$$\log Z_{\text{cyl}}(\tau, R) \xrightarrow{R/\ell_s \rightarrow \infty} -RV_{q\bar{q}}(\tau), \quad (3)$$

where $E_0^{\text{cl}}(R)$ is the ground state energy of the winding flux tube, and $V_{q\bar{q}}(\tau)$ the energy of the open string ground state, identified here as the static quark-antiquark potential.

When both τ/ℓ_s and R/ℓ_s are large, the partition function can be evaluated in the long string EFT. The leading behavior is given by the area law

$$\log Z_{\text{cyl}}(\tau, R) \xrightarrow{\tau/\ell_s, R/\ell_s \rightarrow \infty} -\tau R/\ell_s^2 - 2m_q R, \quad (4)$$

which defines the scale ℓ_s^2 , and determines the leading behavior of $E_0^{\text{cl}}(R)$, $V_{q\bar{q}}(\tau)$ at large R , τ , respectively. We also included the leading boundary effect, corresponding to an effective quark mass, see Supplemental Material [35] for further discussion on boundary terms.

More important to us is that for $\tau \ll \ell_s$ asymptotic freedom kicks in, and the leading asymptotics of the correlator $\langle W_\square(\tau) W_\square(0) \rangle$ can be computed perturbatively; a computation that we now turn to.

UV limit from asymptotic freedom: Indeed, at small τ/ℓ_s with R/ℓ_s fixed we have (again)

$$\log Z_{\text{cyl}}(\tau, R) \xrightarrow{\tau/\ell_s \rightarrow 0} -RV_{q\bar{q}}(\tau), \quad (5)$$

where the quark antiquark potential $V_{q\bar{q}}$ is asymptotically Coulomb-like,

$$V_{q\bar{q}}(\tau) = \begin{cases} \frac{\lambda}{4\pi} \log(\tau) + O(\tau^0) & D = 3, \\ \frac{3\pi}{11} \frac{1}{\tau \log(\tau)} + O\left(\frac{\log[\log(\tau)]}{\tau \log(\tau)^2}\right) & D = 4. \end{cases} \quad (6)$$

The asymptotics in (5) is to be understood to the same order as (6) at small τ/ℓ_s fixed R/ℓ_s . Subleading terms in (5) need not be extensive in R and in general produce R -dependent corrections to $\log Z_{\text{cyl}}$.

Before discussing the origin of Eqs. (5) and (6), a few comments are in order. First, the potential is only defined up to an additive constant. The overall shift of the potential corresponds to a cosmological constant counterterm on the worldline of the defects. Second, note the absence of a physical scale in the logarithms. This choice is deliberate and serves to emphasize that the scale is set at subleading orders in the short-distance expansion.

In $D = 4$, the leading asymptotics in (6) are completely fixed with no undetermined parameters. This result is beautifully observed in the lattice, see for example the seminal work of Necco and Sommer [74] for the case of $SU(3)$. In (6) we expressed the $D = 4$ result directly at large N . In $D = 3$, the functional form (6) is under control but the λ coefficient, which reduces to the 't Hooft coupling $g^2 N$ at large N , must be determined nonperturbatively in terms of the physical scale ℓ_s . For $SU(3)$ it can be extracted from the data of Lüscher and Weisz [24] to be $\lambda = 4.8\ell_s^{-1}$, see also Ref. [75] for modern $SU(2)$ data. We are not aware of large N determinations.

From the diagrammatic perspective, (6) follows from the exponentiation of diagrams for the square Wilson loop [76–80], combined with renormalization group (RG) running of the coupling [1,2], which ensures that contributions to the potential beyond single-gluon exchange are subleading at small τ . For example, in $D = 4$ we write

$$\begin{aligned} V_{q\bar{q}}(\tau) &= \frac{\lambda(\tau)}{8\pi\tau}, \\ \tau \partial_\tau \lambda(\tau) &= \frac{11}{24\pi^2} \lambda(\tau)^2 + O(\lambda(\tau)^3), \end{aligned} \quad (7)$$

where in the RG equation we kept only the one-loop term, which is asymptotically correct at small τ . Solving (7) for small separations leads to (6). The $D = 3$ result in (6) is manifest in perturbation theory.

So far we have discussed the potential that naturally dominates the limit $R/\ell_s \gg 1$. It turns out that (6) also gives the correct small τ asymptotics at finite inverse temperature R , Eq. (5). This can be derived perturbatively, although the computation is more subtle [81–85]. We comment on it in Supplemental Material [35]. When the dust settles, the small τ expansion is again determined from the exponentiation of diagrams, with leading asymptotics

being controlled by hard gluons of momenta $|k| \simeq 1/\tau$, which are not affected by the finite thermal scale R . The lack of temperature dependence in the short distance limit of the quark antiquark free energy (5) can also be observed (for finite N) directly from measurements of the Polyakov loop correlator in the lattice. See for example [86] for some data in the confined regime. The asymptotics (5) continue to be valid above the deconfinement temperature, and there data is more plentiful [87,88].

To summarize, although (5) is not established non-perturbatively, its all-order perturbative argument combined with the lattice demonstration leaves little room for doubt regarding its correctness. The remainder of this letter is devoted to exploring the implications of observation (5) for the worldsheet theory of confining strings.

Imprints on the string spectrum—In the closed string channel, the short-distance limit, $\tau/\ell_s \rightarrow 0$, of the cylinder partition function is controlled by the exchange of highly excited string states. We can thus extract the asymptotic spectral density by inverting the partition function. This logic mimics the seminal work by Cardy on the asymptotic density of states of two-dimensional conformal field theory (CFTs) [89]. Rather than modular invariance, however, the asymptotic spectral density is controlled here by asymptotic freedom.

At large N , by inserting in (1) a complete set of closed flux tube states $|n, \vec{p}_\perp\rangle \in \mathcal{H}_{\text{cl}}$, labeled by their transverse center of mass momenta $\vec{p}_\perp$ and internal labels n , and summing over $\vec{p}_\perp$, we may write the partition function as [32,33]

$$Z_{\text{cyl}}(\tau, R) = \int_0^\infty dm \rho_v(m, R) 2\tau \left(\frac{m}{2\pi\tau}\right)^{\frac{D-1}{2}} K_{\frac{D-3}{2}}(m\tau). \quad (8)$$

Here,

$$\rho_v(m, R) \equiv \sum_n |v_n(R)|^2 \delta(m - M_n^{\text{cl}}(R)), \quad (9)$$

and $K_\nu(x)$ is the modified Bessel function of the second kind. The spectral density $\rho_v(m, R)$ counts the density of winding-one closed string states with mass m , weighted by their coupling $v_n(R) \equiv \langle W_\square | n, \vec{p}_\perp \rangle / \sqrt{2M_n^{\text{cl}}(R)}$ to the Polyakov loops. Each such state contributes with a $(D-1)$ -dimensional position-space propagator to the partition function (8). See Supplemental Material for details [35].

By Poincaré invariance, adding a transverse separation $\vec{x}_\perp$ to the Polyakov loops only changes the partition function to $Z_{\text{cyl}}(r \equiv \sqrt{\tau^2 + \vec{x}_\perp^2}, R)$. The spectral density can be extracted from (8) by passing to momentum space in the transverse direction, and performing an inverse Laplace transform in the rest frame, i.e.,

$$\rho_v(m, R) = \frac{1}{2\pi i} \int_{\tau_0 - i\infty}^{\tau_0 + i\infty} d\tau e^{m\tau} \int d^{D-2} \vec{x}_\perp Z_{\text{cyl}}(r, R), \quad (10)$$

where $\tau_0 > 0$. This inversion can be checked explicitly by replacing (10) into (8), which is done in Supplemental Material [35]. Note that a standard inverse Laplace transform would extract $\varrho(E, R)$ counting states by their (continuous) energy $E_{n, \vec{p}_\perp}^{\text{cl}}(R) = \sqrt{M_n^{\text{cl}}(R)^2 + \vec{p}_\perp^2}$, rather than their mass $M_n^{\text{cl}}(R)$.

In the large m limit, the integrals (10) are dominated by the region where r is small and Z_{cyl} is controlled by (5) and (6). Both the integral in $\vec{x}_\perp$ and the integral in τ can be evaluated by saddle point approximations. We obtain for the asymptotic density $\rho_v(m, R)$ as $m \rightarrow \infty$, (a) In $D = 3$, where the leading inversion integrals can actually be done exactly, the result is

$$\log \rho_v = \left(\frac{\lambda R}{4\pi} - 2 \right) \log m + O(m^0), \quad (11)$$

with the error stemming only from the corrections to the free energy (5). (b) In $D = 4$,

$$\log \rho_v = 2\sqrt{\frac{3\pi}{11} Rm} + O\left(\frac{\sqrt{m} \log(\log m)}{(\log m)^{\frac{3}{2}}}\right). \quad (12)$$

Here the leading error has two sources; corrections to the free energy (5) and corrections to the location of the saddle.

The detailed evaluation of the transforms and comments on subleading contributions can be found in Supplemental Material [35]. As usual, the saddle approximation requires understanding the discrete density ρ_v in an average sense, so that the methods are justified. This is especially important to subleading order. The results of the type (11) and (12) should be made sharp (perhaps with additional technical assumptions) by means of Tauberian results, following the CFT discussion [90–97].

In [98] and [99], a similar strategy was followed to derive the asymptotic spectral density of the partition function of parallel defects in a CFT. Instead of perturbation theory, the small- τ behavior was fixed there by the effective theory for the fusion of conformal defects [98–101]. For one-dimensional defects, the partition function grows as $Z_{\text{CFT}} \sim e^{(a_0 R/\tau)}$, and the corresponding asymptotic density is $\rho_{\text{CFT}}(E) \sim E^{-\frac{3}{4}} \exp(2\sqrt{a_0 R E})$, with a_0 the dimensionless Casimir energy. The $D = 3$ density in (11) differs significantly from this result because the Yang-Mills coupling is relevant and we have an explicit scale λ . The $D = 4$ result (12), on the other hand, only differs by log corrections because the Yang-Mills coupling is marginally relevant.

The spectral density $\rho_v(m, R)$ weights closed string states by their coupling $v_n(R)$ to the Polyakov loop. We can write it in terms of the density of states $\rho(m, R)$ as

$$\rho_v(m, R) = \bar{v}(m, R) \rho(m, R), \quad (13)$$

where $\bar{v}(m, R) \equiv (1/N_m) \sum_{n=1}^{N_m} |v_n(R)|^2$ is the average of couplings over all the states with mass m . Assuming a

Hagedorn growth [102] for the closed string density of states, $\rho(m, R) \sim \exp(m/T_H)$, we conclude that the averaged coupling must decay exponentially $\bar{v}(m, R) \sim \exp(-m/T_H)$ as $m \rightarrow \infty$ to recover the milder growths of (11) and (12). This suppression reflects the absence of a Hagedorn-like phase transition as we vary τ .

It is interesting to contrast this behavior with the critical string. For the critical bosonic string, the coupling of closed strings to D -branes is independent of the mass level and thus does not damp the Hagedorn growth. This leads to a divergence at finite τ associated to an open string tachyon. A similar divergence is found in type II superstrings stretching between a brane and an antibrane [103]. A separate mechanism for the absence of singularity in τ is available if the two boundaries of the cylinder are not conjugate to each other. In this case, states need not contribute positively to the spectral density, and the Hagedorn growth may be damped by virtue of cancellations between different states. This is the case for parallel D -branes in type II superstring [104].

Imprints on worldsheet dynamics—We now explore what consequences asymptotic freedom might have for the dynamics on the string worldsheet. We will focus on the case of $D = 3$ for the remainder of this Letter. In lieu of a Lagrangian, we describe dynamics through on-shell scattering data of excitations on top of a very long winding string. We consider the cylinder partition at large R , where it is dominated by $V_{q\bar{q}}(\tau)$. The dynamics of the worldsheet and its boundary is encapsulated in the S - and R matrices of the transverse Goldstone bosons X^i .

In order to sharply connect the finite volume open string ground state energy $V_{q\bar{q}}(\tau)$ to the scattering data, we will assume integrability of the Goldstone interactions. Of course, the worldsheet dynamics of pure Yang-Mills is definitely not integrable, and thus the toy models here are to be understood as proof of concept to motivate more general approaches, some of which we comment on in section.

In the $D = 3$ case, where the spectrum consists of a single Goldstone, the thermodynamic Bethe ansatz [105,106] expresses the open string ground state energy as

$$V_{q\bar{q}}(\tau) = \tau \ell_s^{-2} + \frac{1}{2\pi} \int_0^\infty dp \log \left(1 - |K(p)|^2 e^{-\epsilon(p)} \right), \quad (14)$$

where the pseudoenergy $\epsilon(p)$ is the solution to

$$\begin{aligned} \epsilon(p) = 2p\tau + \int_0^\infty \frac{dp'}{2\pi i} \frac{\partial \log S(pp')}{\partial p'} \\ \times \log \left(1 - |K(p')|^2 e^{-\epsilon(p')} \right). \end{aligned} \quad (15)$$

The crucial observation is that the kernel of (15) involves the Eisenbud-Wigner time delay $\Delta t = -iS^* \partial_p S$ [107,108]. In relativistic massless scattering, causality requires that

scattered particles come out inside the light cones of the incoming ones, implying that $\Delta t \geq 0$ [109,110]. The absence of phase transitions as we dial τ implies that the log in (14) and (15) remains negative throughout its domain. This, combined with the positivity of the kernel in (15), readily implies a bound on the pseudoenergy; $\epsilon(p) \leq 2p\tau$. The intuition behind this bound is clear; positivity of the time delay makes interactions attractive, which in turn lower the energy of the states compared to the free theory.

By plugging this bound into (14) and using $\log(1-x) \leq -x$, we conclude that

$$-V_{q\bar{q}}(\tau) \geq -\tau\ell_s^{-2} + \frac{1}{2\pi} \int_0^\infty dp |K(p)|^2 e^{-2p\tau}. \quad (16)$$

We interpret this result as a causality bound on thermodynamic data. It states that the free energy can only be larger than its two-particle contribution.

From (16) we have

$$-\frac{V_{q\bar{q}}(\tau) - \tau\ell_s^{-2}}{\log x} \geq \frac{e^{-2x\tau}}{2\pi \log x} \int_0^x dp |K(p)|^2,$$

where x is large. Recalling that $V_{q\bar{q}}(\tau)$ asymptotes to the Coulomb potential (6) as $\tau \rightarrow 0$, we now take $x \rightarrow \infty$ with τx fixed and then send $\tau x \rightarrow 0$. We obtain

$$\lambda \geq \lim_{x \rightarrow \infty} \frac{2}{\log x} \int_0^x dp |K(p)|^2. \quad (17)$$

This bounds $|K(p)|^2$ to decay no slower than $\lambda/2p$, on average, at large p .

Let us emphasize that $|K(p)|^2$ is not bounded by any fundamental principles of the scattering theory; this decay is a direct consequence of asymptotic freedom. Indeed, it rules out for example the asymptotic behavior of the R matrix describing interactions on the boundary of a long critical string, see Supplemental Material [35]. This asymptotic softness of the coupling between Goldstone bosons and the Wilson line is the microscopic reason for the decay of $\bar{v}(m, R)$ observed in the previous Section. Of course, we have shown this in the restricted setting of integrability, but it seems plausible that a similar mechanism may persist beyond that.

In Supplemental Material [35], we argue that one can also use (14) and (15) to constrain the high-energy behavior of the S matrix. We show that a “zig-zag” asymptotic behavior $\log S \sim i c s$ [12,29] is incompatible with the Coulomb asymptotics of $V_{q\bar{q}}(\tau)$ in (6), regardless of the choice of K . The interpretation is that the linear phase shift leads to such strong interactions that cannot be compensated by any integrable boundary to avoid a phase transition in τ . Of course, the linear phase shift may be saved in actual Yang-Mills if high-energy scattering is far from

integrable, e.g., if it is accompanied by a shower of soft Goldstones [29].

Discussion—In this Letter, we have initiated the systematic study of imprints that asymptotic freedom leaves on confining strings. By identifying a worldsheet observable that is controlled by perturbation theory, we have unlocked a door to connecting the flux tube theory with the underlying gauge theory. The asymptotic spectral densities from section, and the bound on the integrable R matrix from section are but its first consequences.

Two-point functions of local operators $\langle \mathcal{O}(x) \mathcal{O}^\dagger(0) \rangle$ have played an undeniable role in the study of confining gauge theories [111–113]. They probe the RG flow as we dial x and, through various sum rules, they let us connect asymptotic freedom to the spectrum of mesons or glueballs that they couple to. As we have seen, the Polyakov correlator $\langle W_\square(\tau) W_\square(0) \rangle$ probes the same flow, but it lets us explore instead the spectrum of flux tubes. It is natural to ask whether the asymptotics (5) can be used to derive sum rules on the low-lying flux tube spectrum.

An important tool in the study of the UV limit of local two-point functions is the OPE $\mathcal{O}_1(x) \mathcal{O}_2(0) \sim \sum_k c_k(x) \mathcal{O}_k(0)$. It predicts the structure of perturbative and nonperturbative corrections in the correlator. It would be very interesting to develop an analogous formalism for the fusion of Wilson lines, following the recent developments in the fusion of conformal defects [98–101]. This should classify the structure of corrections in (5). It is worth noting that in the phenomenology literature, this fusion is described in terms of potential Non Relativistic Quantum Chromodynamics (pNRQCD) [114,115], but its connection to conformal defect fusion is still lacking.

One of the results of section is Eq. (16). It is a bound on the finite-volume potential $V_{q\bar{q}}(\tau)$, thermodynamic data, stemming from causality. The logic for such a bound is the following. For a gas of particles (not necessarily dilute), thermodynamic potentials are built from the microscopic interactions of its constituents. If such interactions are relativistic, they are constrained by causality, which translates back into bounds for thermodynamic data. It would be fantastic to prove a generalization of this result beyond integrability. We speculate on this direction in Supplemental Material [35].

Finally, let us point out that an S -matrix bootstrap program for the boundary Wilson coefficients of the low-energy theory is hindered by the lack of a bound of $R(p)$ on the imaginary axis, see Supplemental Material for further discussion [35]. Assuming integrability, Eq. (17) provides such a bound asymptotically. It would be interesting if a refinement of this strategy could enable to connect the asymptotically free result (17) to the low energy EFT parameters of the QCD string.

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