

Spontaneous Emission Decay and Excitation in Photonic Time Crystals

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Over the last few decades, the predominant strategies for controlling spontaneous emission have involved tailoring the spatial surroundings of quantum emitters or atoms to create resonant or spatially periodic photonic structures. However, the rise of time-varying photonics has prompted a reevaluation of spontaneous emission in dynamically changing environments, especially within photonic time crystals, where optical properties undergo time-periodic modulation. Here, we apply classical light-matter interaction theory together with Floquet analysis to reveal a substantial enhancement of the spontaneous emission decay rate at the momentum gap frequency in photonic time crystals. Moreover, our findings suggest that photonic time crystals enable a nonequilibrium light-matter interaction process: the spontaneous excitation of an atom from its ground state to an excited state, accompanied by the concurrent emission of a photon, referred to as spontaneous emission excitation.

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Investigation of electromagnetic wave dynamics in space-time periodic media began in the 1950s [1–5], initially focusing on temporally growing instabilities in distributed parametric media. However, time-varying photonics gained widespread attention much later. A pioneering experiment with a time-periodic transmission line revealed a shallow yet definitive momentum gap, marking a key milestone despite limited initial recognition [6]. This finding spurred the extension of photonic crystals and metamaterials into the space-time domain, leveraging the additional temporal degree of freedom for enhanced dispersion and band structure engineering [7–12]. Advances in time-varying photonics have since enabled exploration of broadband nonreciprocity [13], one-way amplification [9], parametric

oscillation [14,15], pulse compression [16], harmonic generation [8], and even Hawking radiation mimicry [17].

Only recently has the Floquet-system framework been refined, deepening our understanding of photonic time crystals (PTCs), which are physically distinct from Wilczek's many-body *time crystal* phases [18,19]. By long-standing convention, however, the same class of time-periodic photonic media has been referred to as a *temporal photonic crystal* [6,20–23]. The effective Hamiltonian matrix for PTCs, derived from Maxwell's equations, is identified as pseudo-Hermitian, which leads to the photonic Floquet eigenmodes becoming nonorthogonal. Furthermore, the momentum gap has been proven to be the phase in which pseudo-Hermiticity is broken along the wave number axis, with its edges characterized as exceptional points (EPs) [15,24,25]. These findings highlight the need for a non-Hermitian theoretical framework to analyze classical PTCs, whose non-Hermitian dynamics strongly shape their light-matter interactions. Considering these non-Hermitian dynamics, we show that the spontaneous emission decay rate at the momentum gap frequency in PTCs is significantly enhanced, contradicting a recent study [26] that predicted its complete vanishing.

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To accurately quantify light-matter interactions in PTCs, we must account for time-periodicity-induced loss and gain regions in wave number frequency space, reflected in positive or negative momentum-resolved photonic density of states (kDOS). The negative kDOS observed in the gain region necessitates a gain-induced correction, prompting a reevaluation of the spontaneous emission decay rate [27–31]. Intriguingly, gain in PTCs can trigger spontaneous excitation of an atom, accompanied by photon emission, highlighting the rich spectrum of light-matter interactions available in nonequilibrium photonics. Additionally, we show that the non-orthogonality between photonic Floquet eigenmodes, quantified by the Petermann factor (PF) [32–35], increases both the spontaneous emission decay and excitation rates.

In this Letter, we employ classical light-matter interaction theory, treating a quantum emitter as an oscillating dipole while the electromagnetic field obeys Maxwell’s equations. By modeling the emitter as an oscillating point dipole, $\mathbf{p}(\mathbf{r}, t) = \text{Re}[\mathbf{p}\delta(\mathbf{r} - \mathbf{r}_0)\exp(-i\omega t)]$, we recast the inherently quantum interaction in a tractable classical framework [36,37]. Notably, the spontaneous emission decay rate in the weak coupling regime can be estimated from classical power flow or radiation reaction arguments, even in linearly amplifying media [27–29]. Hence, the problem can be analyzed by solving Maxwell’s equations for a medium with time-periodic permittivity, $\epsilon(t) = \epsilon(t + 2\pi/\Omega)$, where Ω is the modulation frequency [Fig. 1(a)]:

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu \frac{\partial}{\partial t} \mathbf{H}(\mathbf{r}, t), \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} [\epsilon(t)\mathbf{E}(\mathbf{r}, t)] + \mathbf{J}(\mathbf{r}, t) + \sigma\mathbf{E}(\mathbf{r}, t). \quad (2)$$

Here, vacuum permittivity and permeability are set to unity. The conductivity σ accounts for intrinsic loss ($\sigma > 0$) in light-matter interactions, although other loss models are possible [24] (see Supplemental Material D [38]). We include the dipole-induced current source $\mathbf{J} = \partial\mathbf{p}/\partial t$ in Maxwell’s equations for the PTC. Combining Eqs. (1) and (2) yields a single differential equation that can be cast as an eigenvalue problem with a Floquet Hamiltonian $\mathcal{H}_F$ (see Supplemental Material A [38]).

To obtain the photonic Floquet band structure, we solve the momentum-resolved eigenvalue problem. Figure 1(b) shows the resulting bands and highlights the momentum gap, where the imaginary part of the eigenfrequency bifurcates. For a sinusoidally modulated permittivity, $\epsilon(t) = \epsilon_0 + \epsilon_m \sin \Omega t$, the conductivity σ alone determines the maximum positive imaginary part of the eigenfrequency within the gap, denoted $\gamma_{\max}$. We define the critical conductivity σ_c as the value where $\gamma_{\max} = 0$. When $\sigma < \sigma_c$ ($\gamma_{\max} > 0$) the system lies in the small intrinsic-loss regime, whereas $\sigma > \sigma_c$ keeps all imaginary parts negative, corresponding to the large intrinsic-loss regime.

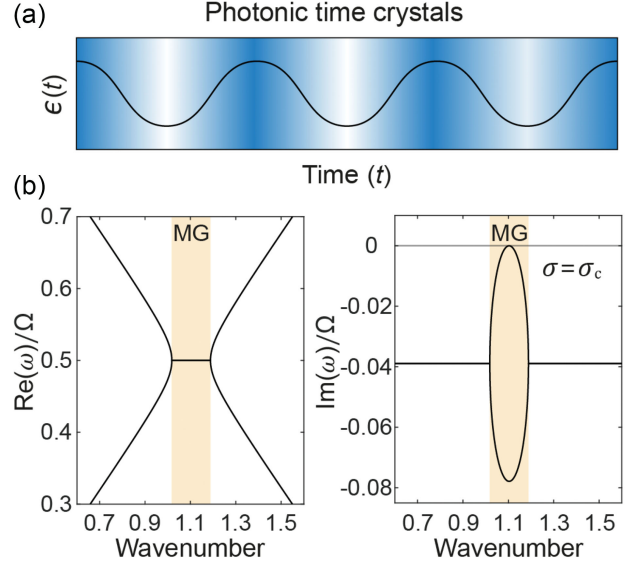


FIG. 1. (a) Time periodic permittivity modulation. (b) Floquet band structure of a photonic time crystal with $\epsilon(t) = \epsilon_0 + \epsilon_m \sin \Omega t$. The plot uses $\epsilon_0 = 5$, $\epsilon_m = 1.5$, $\mu = 1$, and $\Omega = 1$, with conductivity set to the critical value $\sigma = \sigma_c = 0.3715$. The momentum gap (MG) is highlighted in yellow.

Quantifying the spontaneous emission decay rate in a PTC begins with the time-averaged power radiated by a point dipole. In spatial Fourier space, this power equals the momentum integral of the contributions from extended dipole components (see Supplemental Material B [38]):

$$\begin{aligned} \bar{P}(\omega) &= \int_{\mathbb{K}} \bar{P}(\mathbf{k}, \omega) d^3\mathbf{k} \\ &= \frac{1}{(2\pi)^3} \frac{\pi\omega^2 |\mathbf{p}|^2}{4\epsilon_0} \int_{\mathbb{K}} \rho_{\mathbf{p}}(\mathbf{k}, \omega) d^3\mathbf{k}. \end{aligned} \quad (3)$$

Here, $\rho_{\mathbf{p}}(\mathbf{k}, \omega)$ is the momentum-resolved photonic density of states (kDOS) projected onto the dipole orientation,

$$\rho_{\mathbf{p}}(\mathbf{k}, \omega) \equiv \frac{2\epsilon_0\mu\omega}{\pi} \text{Im}[\mathbf{n}_{\mathbf{p}} \cdot \mathbb{G}_0(\mathbf{k}, \omega) \cdot \mathbf{n}_{\mathbf{p}}], \quad (4)$$

with $\mathbf{n}_{\mathbf{p}} = |\mathbf{p}|/\mathbf{p}$ and $\mathbb{G}_0$ the dyadic Green’s function. In this formulation, $\mathbb{G}_0$ is a projection of the full Green’s function of the Floquet Hamiltonian matrix, weighted by the Fourier components of $1/\epsilon(t)$,

$$\begin{aligned} \mathcal{G}_F(\mathbf{k}, \omega) &= [\omega \mathbf{I}_F - \mathcal{H}_F(\mathbf{k})]^{-1} \\ &= \sum_m \frac{1}{\omega - \omega_m(\mathbf{k})} \frac{|R_m(\mathbf{k})\rangle \langle L_m(\mathbf{k})|}{\langle L_m(\mathbf{k}) | R_m(\mathbf{k}) \rangle}, \end{aligned} \quad (5)$$

evaluated at the excitation frequency ω . Here, m indexes the modes, $\omega_m = \Omega_m - i\gamma_m$ are the complex quasideigenfrequencies, and $\langle L_m |$ and $|R_m\rangle$ are the left and right eigenvectors of $\mathcal{H}_F$ (see Supplemental Material A [38]).

Focusing on the two modes $\omega_{\pm} = \Omega_{\pm} - i\gamma_{\pm}$, which form the momentum gap at $\omega = \Omega/2$, and assuming $\mathbf{k} \perp \mathbf{n}_p$, the kDOS can be approximated as follows (for $\mathbf{k} \parallel \mathbf{n}_p$, see Supplemental Material B [38]):

$$\begin{aligned} \rho_p(\mathbf{k}, \omega) &\approx \sum_{\alpha=\pm} \rho_p^{\alpha}(\mathbf{k}, \omega) \\ &\approx \sum_{\alpha=\pm} \frac{1}{\pi} \frac{\gamma_{\alpha}}{(\omega - \Omega_{\alpha})^2 + \gamma_{\alpha}^2} \text{Re}[I_{\alpha}(\mathbf{k})] \\ &\quad + \frac{1}{\pi} \frac{\Omega_{\alpha} - \omega}{(\omega - \Omega_{\alpha})^2 + \gamma_{\alpha}^2} \text{Im}[I_{\alpha}(\mathbf{k})]. \end{aligned} \quad (6)$$

Here, $I_{\alpha}(\mathbf{k})$ is the complex, normalized field intensity projected onto the dipole; its real and imaginary parts weight the absorptive (symmetric) and dispersive (anti-symmetric) Lorentzian terms, respectively. Far from the momentum gap, the imaginary part of $I_{\alpha}(\mathbf{k})$ is negligible compared with its real part. In addition, $\text{Re}[I_{+}(\mathbf{k})]$ is positive, whereas $\text{Re}[I_{-}(\mathbf{k})]$ is negative. Hence, at any fixed wave number far from the gap, the kDOS reduces to the sum of two weighted, symmetric Lorentzian profiles of opposite sign. Each symmetric Lorentzian is weighted by $\text{Re}[I_{\alpha}(\mathbf{k})]$, which grows sharply as the wave number approaches the edge from within the band. It is also worth noting that, in the time-invariant limit ($\epsilon_m \rightarrow 0$), $\rho_p^-(\mathbf{k}, \omega)$ vanishes, and the kDOS reduces to that of the unmodulated medium.

We first consider the large intrinsic loss case, where $\sigma > \sigma_c$ (equivalently, $\gamma_{\max} < 0$). In Fig. 2(a), the kDOS map remains positive throughout the entire $(\mathbf{k}, \omega)$ plane. Figures 2(b) and 2(c) show the kDOS and its modal decompositions at $k = 0.9$ (outside the gap) and $k = 1.05$ (within the gap), respectively. Outside the gap, the kDOS attains a shallow minimum near the negative-frequency Floquet sideband owing to the negative contribution from ρ_p^- , yet it stays positive overall. Within the gap, $\text{Im}[I_{\alpha}(\mathbf{k})]$ becomes appreciable, so $\rho_p^{\alpha}(\mathbf{k}, \omega)$ departs from a purely symmetric Lorentzian (Fig. 2(c)). At $k = k_{EP-}$ (the left edge of the gap), the kDOS is well approximated by the sum of a symmetric Lorentzian and a term proportional to its frequency derivative [Fig. 2(d); see Supplemental Material G [38]]. In the large intrinsic loss regime, the kDOS spontaneous emission model remains valid, predicting an enhanced yet finite spontaneous emission decay rate near the momentum gap frequency [left panel of Fig. 2(a)]. For reference, the spontaneous emission decay rate in a homogeneous, time-invariant medium with permittivity ϵ_0 is shown as a black dashed line.

We now turn to the low loss regime, defined by $\sigma < \sigma_c$ (equivalently, $\gamma_{\max} > 0$). In this case, the kDOS outside the gap becomes negative near the negative-frequency Floquet sideband [Figs. 2(e) and 2(f)] and the negative region extends into the central gap [Figs. 2(f)–2(h)]. Negative local densities of states have been considered

in spontaneous emission from linear gain media [29,30] and in the spectral functions of driven dissipative quantum systems [31]. Here, the sign reversal arises from the net gain introduced by the time periodic permittivity modulation; Supplemental Material D [38] shows the same effect in a driven Lorentz oscillator with a time periodic spring constant. To evaluate the spontaneous emission decay rate, we calculate the time-averaged power radiated into the momentum domain $\mathbb{K}_D$ where the kDOS is positive,

$$\bar{P}_D(\omega) = \frac{1}{(2\pi)^3} \frac{\pi\omega^2 |\mathbf{p}|^2}{4\epsilon_0} \int_{\mathbb{K}_D} \rho_p(\mathbf{k}, \omega) d^3\mathbf{k}, \quad (7)$$

and compare it with the vacuum reference $P_0(\omega)$. The corrected decay rate then follows as $\Gamma_D(\omega) = \Gamma_0(\omega) \bar{P}_D(\omega)/P_0(\omega)$ [28,29].

Within a quantum framework, negative kDOS implies a spontaneous transition of a two-level atom from its ground to excited state [27], accompanied by photon emission, which we refer to as *spontaneous emission excitation*. In regions with a negative kDOS, the radiation reaction acting on an extended dipole source exhibits negative values, indicative of *negative damping*. We interpret this as a spontaneous emission process accompanied by the excitation of an atom. In a time-invariant medium, spontaneous emission excitation is forbidden, because energy conservation prevents an atom from gaining internal energy while emitting a photon when no external modulation is present. A PTC, however, makes this process possible: the temporal modulation supply discrete quanta that exactly match the energy deficit, thereby enabling the atom to draw energy from the modulation, emit a photon, and end in an excited state. The corresponding spontaneous emission excitation rate is given by $\Gamma_E(\omega) = \Gamma_0(\omega) \bar{P}_E(\omega)/P_0(\omega)$, where the time-averaged power absorbed by the dipole is

$$\bar{P}_E(\omega) = \frac{1}{(2\pi)^3} \frac{\pi\omega^2 |\mathbf{p}|^2}{4\epsilon_0} \int_{\mathbb{K}_E} |\rho_p(\mathbf{k}, \omega_0)| d^3\mathbf{k}, \quad (8)$$

with the integral restricted to the momentum domain $\mathbb{K}_E$ where the kDOS is negative. Figure 2(e) presents the kDOS map for the small intrinsic loss case, with the spontaneous emission decay rate shown in the left panel and the excitation rate in the right panel. The decomposition of the kDOS within the gap is provided in Supplemental Material E [38].

While the spontaneous emission decay and excitation rates follow from Eqs. (7) and (8), evaluating them at the gap frequency for the low loss regime is complicated by singularities in the full Green's function. Specifically, a pair of simple poles, k_{R-} and k_{R+} , appear on the real wave number axis within the gap [Fig. 3(a)], and at these points the full Green's function as well as the decay and excitation rates become unbounded. As the loss decreases, the simple pole moves toward the exceptional point, and in the lossless

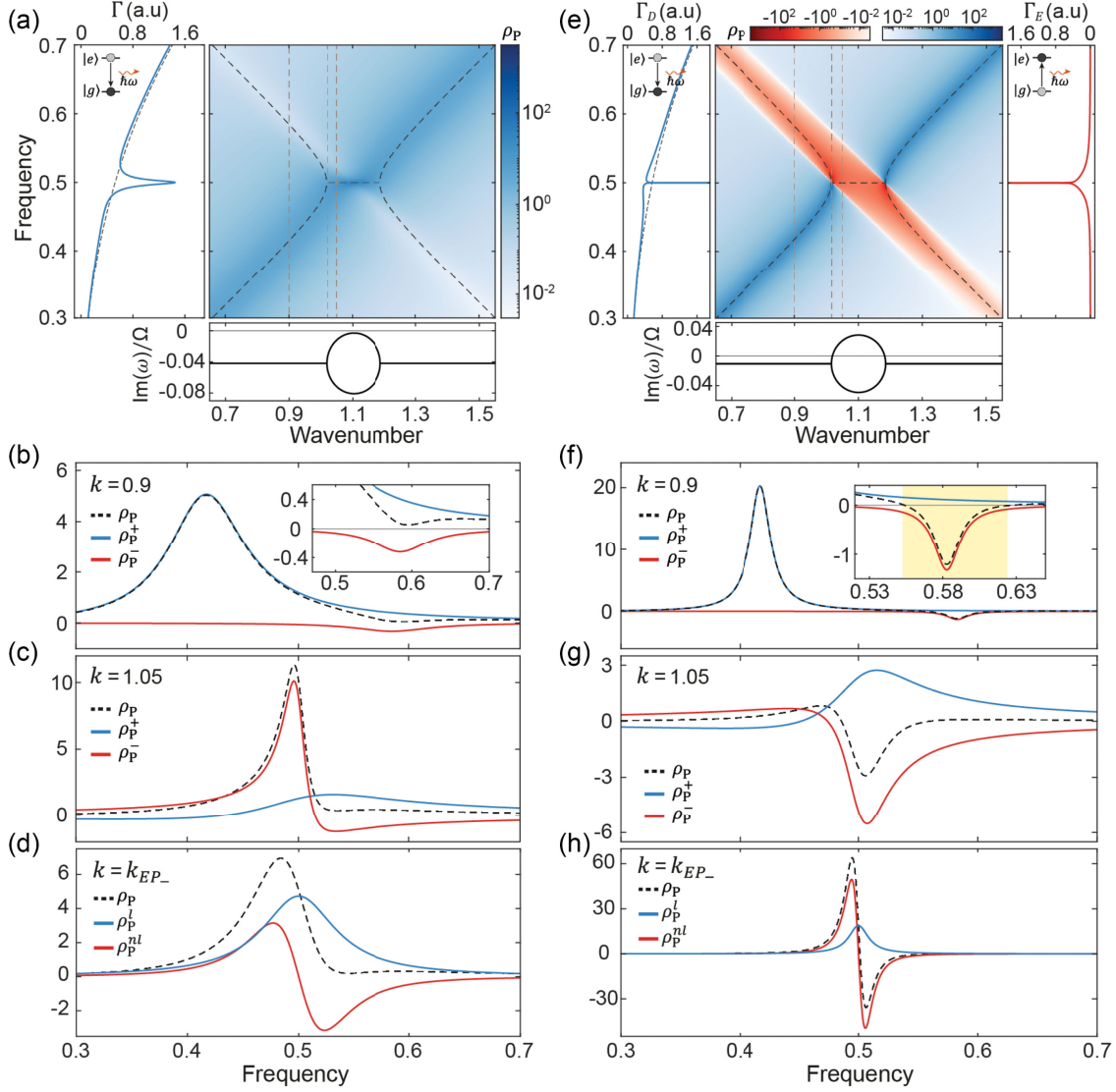


FIG. 2. (a) Map of the kDOS, the imaginary part of the quasieigenfrequency, and the spontaneous emission decay rate Γ for $\sigma = 0.4 > \sigma_c$ (large-loss case). (b)–(d) Plots of the kDOS and its modal decompositions versus frequency for $k = 0.9$ (outside the momentum gap), $k = 1.05$ (within the gap), and $k = k_{EP-}$. Outside the gap, the kDOS is well represented by the sum of two weighted, symmetric Lorentzian profiles of opposite sign, whereas inside the gap it becomes asymmetric. At $k = k_{EP-}$, the kDOS decomposes into a symmetric Lorentzian and a term proportional to its frequency derivative. (e) Map of the kDOS, the imaginary part of the quasieigenfrequency, and the spontaneous emission decay and excitation rates for $\sigma = 0.1 < \sigma_c$ (small loss case). (f)–(h) Corresponding plots of the kDOS and its decompositions at $k = 0.9$, 1.05 , and k_{EP-} . Outside the gap, the kDOS becomes negative near the negative-frequency Floquet sideband, and this negative region extends into the central gap. All calculations assume $\mathbf{k} \perp \mathbf{n}_p$ except for the decay and excitation curves. The spontaneous emission decay rate in a homogeneous, time-invariant medium with permittivity ϵ_0 is plotted as a black dashed line in (a) and (e).

limit a simple pole and a double pole coexist at that point. When the emitter frequency is detuned slightly from the gap frequency, the simple poles leave the real k axis and become branch points in the complex k plane [Fig. 3(b)]. Once these singularities lie off the real axis, the full Green's function is analytic along the integration path and the spontaneous emission rate integral converges.

Although the singularities make the rate formally divergent, the spontaneous emission decay rate in a lossless PTC

can still be estimated by omitting their contribution and taking the asymptotic limit $\sigma \rightarrow 0$. Under this approximation, the kDOS outside the gap ($|\mathbf{k}| < k_{EP-}$) becomes

$$\lim_{\sigma \rightarrow 0} \rho_p(\mathbf{k}, \omega) \approx \sum_{\alpha=\pm} \delta(\omega - \Omega_\alpha) \text{Re}[I_\alpha(\mathbf{k})], \quad (9)$$

from which the *remnant* spontaneous emission decay rate is given by

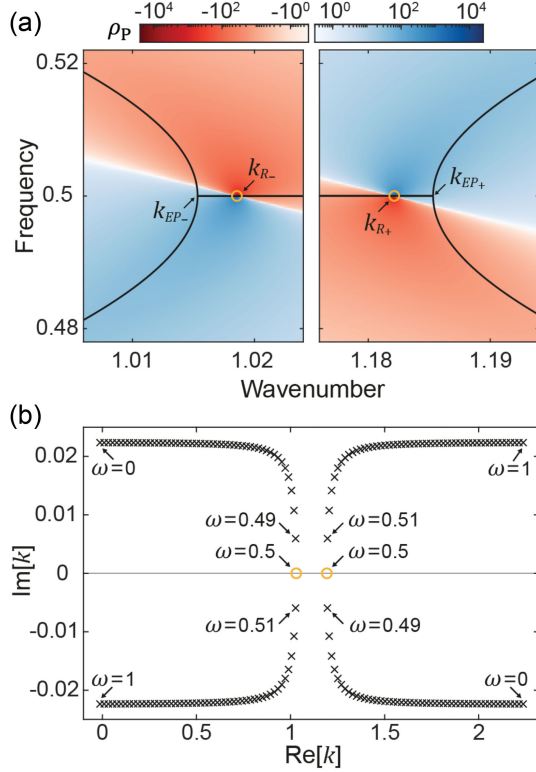


FIG. 3. (a) At the momentum gap frequency, the exceptional points are located at $k_{EP\pm}$, whereas the simple poles reside at $k_{R\pm}$. Blue (red) regions indicate positive (negative) kDOS, corresponding to decay (excitation). (b) Trajectories of the singularities in the complex- k plane as the emitter frequency is detuned. Each pole ($\circ$) leaves the real axis and becomes a branch point ($\times$), after which the rate integrals converge.

$$\Gamma_D(\omega) \approx \frac{\omega |\tilde{\mathbf{p}}|^2}{8\hbar\epsilon_0} |\mathbf{k}|^2 \text{Re}[\mathcal{I}_+] \frac{\sqrt{\text{PF}}}{d\Omega_+/d|\mathbf{k}|}. \quad (10)$$

Here, the Petermann factor is (see Supplemental Material F [38])

$$\text{PF} = \frac{\langle R_+(\mathbf{k}) | R_+(\mathbf{k}) \rangle \langle L_+(\mathbf{k}) | L_+(\mathbf{k}) \rangle}{|\langle L_+(\mathbf{k}) | R_+(\mathbf{k}) \rangle|^2}, \quad (11)$$

and $\mathcal{I}_+ \equiv I_+(\mathbf{k})/\sqrt{\text{PF}}$ is evaluated for $\mathbf{k} \perp \mathbf{n}_p$. As ω approaches $\Omega/2$ from below, both $\sqrt{\text{PF}}$ and the dispersion slope ($d\Omega_+/d|\mathbf{k}|$) diverge, yet their ratio remains finite as shown in Figs. 4(a) and 4(b). Equivalently, $\text{Re}[\mathcal{I}_+(\mathbf{k})]/(d\Omega_+/d|\mathbf{k}|)$ stays bounded (see Supplemental Material G for the kDOS at the exceptional points [38]). Hence, the spontaneous emission decay rate converges to a finite positive value [Fig. 4(b)]. This behavior contradicts the claim that the decay vanishes at the band edge because the dispersion slope is vertical [26]. The Petermann factor counterbalances the slope divergence and underscores the influence of nonorthogonal photonic Floquet modes. Consistent with our analysis, a nonzero

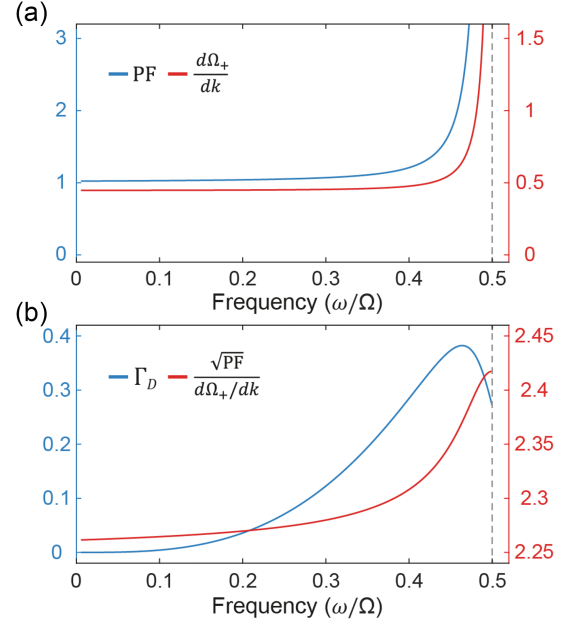


FIG. 4. (a) As ω approaches the momentum gap frequency, the Petermann factor PF and the dispersion slope $d\Omega_+/dk$ both diverge; their asymptotic trends are shown. (b) With the singular contributions removed, the spontaneous emission decay rate converges to a finite value at the gap frequency. The ratio $\text{PF}/(d\Omega_+/dk)$ is plotted alongside the rate and likewise converges near the gap frequency.

decay rate has already been confirmed, both theoretically and experimentally, in time-invariant photonic systems that exhibit similarly steep band edges [39,40].

The study of spontaneous emission in time-periodic photonic structures has deepened our understanding of nonequilibrium light-matter interactions. We show that tailoring the temporal environment enhances the spontaneous emission decay rate, particularly at the momentum gap frequency of PTCs, adding a temporal dimension to emission control beyond Hermitian regime [26,41,42]. This idea provides a new control knob for light-matter interaction, going beyond conventional approaches of past decades, which have relied solely on resonant or spatially periodic structures surrounding atoms or quantum emitters [43–60]. Time periodicity introduces loss, gain, and Floquet eigenmode nonorthogonality, revealing a richer physics than previously envisioned. Notably, the newly identified spontaneous excitation with photon emission challenges conventional theories, distinguishing equilibrium and nonequilibrium emission processes. These findings highlights the need for a more detailed quantum electrodynamic framework to fully describe PTCs.

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Data availability—The data that support the findings of this Letter are not publicly available. The data are available from the authors upon reasonable request.

- [1] F. R. Morgenthaler, Velocity modulation of electromagnetic waves, *IRE Trans. Microwave Theory Tech.* **6**, 167 (1958).
- [2] D. Holberg and K. Kunz, Parametric properties of fields in a slab of time-varying permittivity, *IEEE Trans. Antennas Propag.* **14**, 183 (1966).
- [3] E. S. Cassedy, Dispersion relations in time-space periodic media: Part II—Unstable interactions, *Proc. IEEE* **55**, 1154 (1967).
- [4] R. Fante, Transmission of electromagnetic waves into time-varying media, *IEEE Trans. Antennas Propag.* **19**, 417 (1971).
- [5] L. Felsen and G. Whitman, Wave propagation in time-varying media, *IEEE Trans. Antennas Propag.* **18**, 242 (1970).
- [6] J. R. Reyes-Ayona and P. Halevi, Observation of genuine wave vector (k or β) gap in a dynamic transmission line and temporal photonic crystals, *Appl. Phys. Lett.* **107**, 074101 (2015).
- [7] N. Chamanara, Z. L. Deck-Léger, C. Caloz, and D. Kalluri, Unusual electromagnetic modes in space-time-modulated dispersion-engineered media, *Phys. Rev. A* **97**, 063829 (2018).
- [8] M. R. Shcherbakov, K. Werner, Z. Fan, N. Talisa, E. Chowdhury, and G. Shvets, Photon acceleration and tunable broadband harmonics generation in nonlinear time-dependent metasurfaces, *Nat. Commun.* **10**, 1345 (2019).
- [9] E. Galiffi, P. A. Huidobro, and J. B. Pendry, Broadband nonreciprocal amplification in luminal metamaterials, *Phys. Rev. Lett.* **123**, 206101 (2019).
- [10] V. Pacheco-Peña and N. Engheta, Effective medium concept in temporal metamaterials, *Nanophotonics* **9**, 379 (2020).
- [11] J. Park and B. Min, Spatiotemporal plane wave expansion method for arbitrary space-time periodic photonic media, *Opt. Lett.* **46**, 484 (2021).
- [12] S. Lee, J. Park, H. Cho, Y. Wang, B. Kim, C. Daraio, and B. Min, Parametric oscillation of electromagnetic waves in momentum band gaps of a spatiotemporal crystal, *Photonics Res.* **9**, 142 (2021).
- [13] D. L. Sounas and A. Alù, Non-reciprocal photonics based on time modulation, *Nat. Photonics* **11**, 774 (2017).
- [14] X. Wang, M. S. Mirmoosa, V. S. Asadchy, C. Rockstuhl, S. Fan, and S. A. Tretyakov, Metasurface-based realization of photonic time crystals, *Sci. Adv.* **9**, eadg7541 (2023).
- [15] J. Park, H. Cho, S. Lee, K. Lee, K. Lee, H. C. Park, J.-W. Ryu, N. Park, S. Jeon, and B. Min, Revealing non-Hermitian band structure of photonic floquet media, *Sci. Adv.* **8**, eabo6220 (2022).
- [16] N. Chamanara, D. G. Cooke, and C. Caloz, Linear pulse compansion based on space-time modulation, in *Proceedings of the 2019 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting* (2019), pp. 239–240.
- [17] S. A. R. Horsley and J. B. Pendry, Quantum electrodynamics of time-varying gratings, *Proc. Natl. Acad. Sci. U.S.A.* **120**, e2302652120 (2023).
- [18] F. Wilczek, Quantum time crystals, *Phys. Rev. Lett.* **109**, 160401 (2012).
- [19] A. Shapere and F. Wilczek, Classical time crystals, *Phys. Rev. Lett.* **109**, 160402 (2012).
- [20] J. R. Zurita-Sánchez, P. Halevi, and J. C. Cervantes-Gonzalez, Reflection and transmission of a wave incident on a slab with a time-periodic dielectric function, *Phys. Rev. A* **79**, 053821 (2009).
- [21] M. Salem and C. Caloz, Temporal photonic crystals: Causality versus periodicity, in *Proceedings of the 2015 International Conference on Electromagnetics in Advanced Applications (ICEAA)* (IEEE, New York, 2015), pp. 490–493.
- [22] J. S. Martínez-Romero, O. M. Becerra-Fuentes, and P. Halevi, Temporal photonic crystals with modulations of both permittivity and permeability, *Phys. Rev. A* **93**, 063813 (2016).
- [23] J. S. Martínez-Romero and P. Halevi, Parametric resonances in a temporal photonic crystal slab, *Phys. Rev. A* **98**, 053852 (2018).
- [24] N. Wang, Z.-Q. Zhang, and C. T. Chan, Photonic floquet media with a complex time-periodic permittivity, *Phys. Rev. B* **98**, 085142 (2018).
- [25] Y.-X. Wang and A. A. Clerk, Non-Hermitian dynamics without dissipation in quantum systems, *Phys. Rev. A* **99**, 063834 (2019).
- [26] M. Lyubarov, Y. Lumer, A. Dikopoltsev, E. Lustig, Y. Sharabi, and M. Segev, Amplified emission and lasing in photonic time crystals, *Science* **377**, 425 (2022).
- [27] S. Franke, J. Ren, M. Richter, A. Knorr, and S. Hughes, Fermi's golden rule for spontaneous emission in absorptive and amplifying media, *Phys. Rev. Lett.* **127**, 013602 (2021).
- [28] J. Ren, S. Franke, B. VanDrunen, and S. Hughes, Classical purcell factors and spontaneous emission decay rates in a linear gain medium, *Phys. Rev. A* **109**, 013513 (2024).
- [29] J. Ren, S. Franke, and S. Hughes, Quasinormal modes, local density of states, and classical purcell factors for coupled loss-gain resonators, *Phys. Rev. X* **11**, 041020 (2021).
- [30] A. A. Vyshnevyy, Gain-dependent purcell enhancement, breakdown of Einstein's relations, and superradiance in nanolasers, *Phys. Rev. B* **105**, 085116 (2022).
- [31] O. Scarlatella, A. A. Clerk, and M. Schiro, Spectral functions and negative density of states of a driven-dissipative nonlinear quantum resonator, *New J. Phys.* **21**, 043040 (2019).

- [32] K. Petermann, Calculated spontaneous emission factor for double-heterostructure injection lasers with gain-induced waveguiding, *IEEE J. Quantum Electron.* **15**, 566 (1979).
- [33] M. V. Berry, Mode degeneracies and the Petermann excess-noise factor for unstable lasers, *J. Mod. Opt.* **50**, 63 (2003).
- [34] J. Zhang, B. Peng, Ş. K. Özdemir, K. Pichler, D. O. Krimer, G. Zhao, F. Nori, Y.-x. Liu, S. Rotter, and L. Yang, A phonon laser operating at an exceptional point, *Nat. Photonics* **12**, 479 (2018).
- [35] H. Wang, Y.-H. Lai, Z. Yuan, M.-G. Suh, and K. Vahala, Petermann-factor sensitivity limit near an exceptional point in a Brillouin ring laser gyroscope, *Nat. Commun.* **11**, 1610 (2020).
- [36] P. W. Milonni and W. A. Smith, Radiation reaction and vacuum fluctuations in spontaneous emission, *Phys. Rev. A* **11**, 814 (1975).
- [37] J. Vuckovic, O. Painter, Y. Xu, A. Yariv, and A. Scherer, Finite-difference time-domain calculation of the spontaneous emission coupling factor in optical microcavities, *IEEE J. Quantum Electron.* **35**, 1168 (1999).
- [38] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/5v2w-yg7v> for derivations of the dyadic Green's function, kDOS formulation and decomposition, proof of pseudo-Hermiticity, driven Lorentz oscillator analysis, nonorthogonality of Floquet eigenmodes, and kDOS at exceptional points.
- [39] A. Pick, B. Zhen, O. D. Miller, C. W. Hsu, F. Hernandez, A. W. Rodriguez, M. Soljačić, and S. G. Johnson, General theory of spontaneous emission near exceptional points, *Opt. Express* **25**, 12325 (2017).
- [40] L. Ferrier, P. Bouteyre, A. Pick, S. Cuff, N. H. M. Dang, C. Diederichs, A. Belarouci, T. Benyattou, J. X. Zhao, R. Su, J. Xing, Q. Xiong, and H. S. Nguyen, Unveiling the enhancement of spontaneous emission at exceptional points, *Phys. Rev. Lett.* **129**, 083602 (2022).
- [41] P. Garg, J. D. Fischbach, A. G. Lamprianidis, X. Wang, M. S. Mirmoosa, V. S. Asadchy, C. Rockstuhl, and T. J. Sturges, Inverse-designed dispersive time-varying nanostructures, *Adv. Opt. Mater.* **13**, 2402444 (2025).
- [42] M. M. Sadafi, A. F. da Mota, and H. Mosallaei, Time-varying Mie resonators for real-time manipulation of quantum emitter radiation, *Phys. Rev. B* **111**, 125419 (2025).
- [43] V. P. Bykov, Spontaneous emission in a periodic structure, *Sov. J. Exp. Theor. Phys.* **35**, 269 (1972), https://scholar.google.com/scholar?q=Spontaneous+Emission+in+a+Periodic+Structure&hl=ko&as_sdt=0%2C5&as_ylo=&as_yhi=1973.
- [44] V. P. Bykov, Spontaneous emission from a medium with a band spectrum, *Sov. J. Quantum Electron.* **4**, 861 (1975).
- [45] E. Yablonovitch, Inhibited spontaneous emission in solid-state physics and electronics, *Phys. Rev. Lett.* **58**, 2059 (1987).
- [46] J. Joannopoulos, P. R. Villeneuve, and S. Fan, Photonic crystals, *Solid State Commun.* **102**, 165 (1997).
- [47] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and E. F. Schubert, High extraction efficiency of spontaneous emission from slabs of photonic crystals, *Phys. Rev. Lett.* **78**, 3294 (1997).
- [48] E. P. Petrov, V. N. Bogomolov, I. I. Kalosha, and S. V. Gaponenko, Spontaneous emission of organic molecules embedded in a photonic crystal, *Phys. Rev. Lett.* **81**, 77 (1998).
- [49] M. Boroditsky, R. Vrijen, R. Coccioli, R. Bhat, and E. Yablonovitch, Spontaneous emission extraction and Purcell enhancement from thin-film 2-d photonic crystals, *J. Lightwave Technol.* **17**, 2096 (1999).
- [50] Z.-Y. Li, L.-L. Lin, and Z.-Q. Zhang, Spontaneous emission from photonic crystals: Full vectorial calculations, *Phys. Rev. Lett.* **84**, 4341 (2000).
- [51] P. Lodahl, A. Floris van Driel, I. S. Nikolaev, A. Irman, K. Overgaag, D. Vanmaekelbergh, and W. L. Vos, Controlling the dynamics of spontaneous emission from quantum dots by photonic crystals, *Nature (London)* **430**, 654 (2004).
- [52] D. Englund, D. Fattal, E. Waks, G. Solomon, B. Zhang, T. Nakaoka, Y. Arakawa, Y. Yamamoto, and J. Vučković, Controlling the spontaneous emission rate of single quantum dots in a two-dimensional photonic crystal, *Phys. Rev. Lett.* **95**, 013904 (2005).
- [53] H. Altug, D. Englund, and J. Vučković, Ultrafast photonic crystal nanocavity laser, *Nat. Phys.* **2**, 484 (2006).
- [54] G. Lecamp, P. Lalanne, and J. P. Hugonin, Very large spontaneous-emission β factors in photonic-crystal waveguides, *Phys. Rev. Lett.* **99**, 023902 (2007).
- [55] S. Noda, M. Fujita, and T. Asano, Spontaneous-emission control by photonic crystals and nanocavities, *Nat. Photonics* **1**, 449 (2007).
- [56] V. S. C. Manga Rao and S. Hughes, Single quantum dot spontaneous emission in a finite-size photonic crystal waveguide: Proposal for an efficient “on chip” single photon gun, *Phys. Rev. Lett.* **99**, 193901 (2007).
- [57] L. Novotny and B. Hecht, *Principles of Nano-Optics* (Cambridge University Press, Cambridge, England, 2012).
- [58] C. Sauvan, J. P. Hugonin, I. S. Maksymov, and P. Lalanne, Theory of the spontaneous optical emission of nanosize photonic and plasmon resonators, *Phys. Rev. Lett.* **110**, 237401 (2013).
- [59] Z. Lin, A. Pick, M. Lončar, and A. W. Rodriguez, Enhanced spontaneous emission at third-order Dirac exceptional points in inverse-designed photonic crystals, *Phys. Rev. Lett.* **117**, 107402 (2016).
- [60] W. L. Barnes, S. A. Horsley, and W. L. Vos, Classical antennas, quantum emitters, and densities of optical states, *J. Opt.* **22**, 073501 (2020).