

Universality Class of the First Levels in Low-Dimensional Gravity

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(Received 3 June 2025; accepted 12 August 2025; published 15 September 2025)

We investigate the physics of a small group of quantum states defined above the sharply defined ground state of a chaotic ensemble. This “universality class of the first levels” is realized in the majority of “synthetic” random matrix models but, for all we know, in only one microscopically defined system: low-dimensional gravity. We discuss the physical properties of these states, notably their exceptional rigidity against external perturbations, as quantified by the so-called quantum state fidelity. Examining these structures through the lenses of random matrix and string theory, we highlight their relevance to the physics of low-dimensional holographic principles.

DOI: [10.1103/5tn7-knd7](https://doi.org/10.1103/5tn7-knd7)

Introduction—The advent of the Sachdev-Ye-Kitaev model [1] has made systems exhibiting “maximal chaos” all the way down to their ground states a focus of attention. For generic representatives of this class, changes in comparatively few system parameters (think of the interaction constants of a many-body Hamiltonian) affect exponentially many levels, turning the spectrum, and in particular the ground state itself into broadly distributed objects [2,3]. The situation is different for systems that are “dense” in the sense that the number of statistically independent parameters is comparable to their Hilbert space dimension, D [4]. For these systems, the spectral edge and the first few states above define a miniature “universality class of the first levels” (UFL for brevity, throughout), with physical properties not realized elsewhere in the spectrum: both the position of the UFL levels and, as we are going to show in this Letter, their wave functions display exceptional rigidity under configurational averaging. Intuitively, this behavior results from the squeezing of mutually repelling levels between a nonfluctuating edge and the bulk of the spectrum. Representing fine-grained system properties at scales of the many-body level spacing, these are

nonperturbative structures, which cannot be captured by semiclassical $1/D$ expansion schemes.

Dense systems harboring a UFL are mostly of synthetic nature—examples include random matrix ensembles [5], quantum graphs [6], and quantum circuits subject to “dense” Haar distributed randomness [7]. However, a prime realization of a naturally occurring dense system is low-dimensional gravity. Considered as a sum over geometries, lattice discretizations [8] define structures with about as many ensemble parameters as degrees of freedom, which is the dense scenario.

For the paradigmatic example of two-dimensional Jackiw-Teitelboim (JT) gravity this view has been made concrete by demonstrating quantitative equivalence between the expansion of the gravitational path integral over geometries of ascending topological complexity and the $1/D$ expansion of a random matrix ensemble [9]. Perturbative in $1/D$, these probes are not yet fine-grained enough to witness the presence of the UFL. However, it has turned out that an extension of the semiclassical gravitational path integral to topological string theory provides the required precision [10,11].

While previous analyses of chaos in the gravitational context have focused on spectral distributions, recent work [12,13] has pointed out that the “geometry of quantum states” includes more sensitive information. Probes such as the quantum geometric tensor [14,15], the adiabatic gauge potential [16,17], or fidelity susceptibilities [18,19] describe how typical wave functions deform under the influence of external parameter variations, providing highly resolved portraits of specific chaotic quantum systems.

In this Letter, we apply this framework to a characterization of UFL quantum states. Focusing on the distribution

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of fidelity susceptibilities, we will demonstrate resilience to perturbations parametrically exceeding that of generic bulk states. While in the bulk ensemble variations allow only for statistical characterization averaged over many level spacings, these findings apply to an $\mathcal{O}(1)$ group of almost completely pinned states. In this sense, our present analysis may be the first description of black hole quantum mechanics tuned to the precision of individual states (if in the toy context of two-dimensional gravity). We will also discuss the implications of our findings for the construction of holographic boundary shadows of two-dimensional gravity.

Signatures of the spectral edge—The physics of the UFL is intimately linked to the interpretation of the spectral edge as a symmetry breaking phase transition, with the energy from the edge, ϵ , as the control parameter, and the average spectral density $\langle \rho(\epsilon) \rangle$ an order parameter. The broken symmetry is causality, i.e., the difference between retarded and advanced resolvents, $G^\pm(\epsilon) \equiv (\epsilon \pm i0 - H)^{-1}$, which outside the edge at $\epsilon \equiv 0$ vanishes on average $\langle \text{tr}[G^-(\epsilon) - G^+(\epsilon)] \rangle = 0$ (up to corrections exponentially small in Hilbert space dimension), while inside it is proportional to the spectral density order parameter.

The universal “ ϕ^4 theory” of this phase transition is the Kontsevich model [20], an integral over low-dimensional “flavor” matrices, A , with action

$$Z(X) \equiv \int dA e^{S(A)}, \quad S(A) = c \text{str} \left(XA + \frac{1}{3} A^3 \right), \quad (1)$$

where $X = \text{diag}(\epsilon_1, \dots, \epsilon_m | \epsilon_1, \dots, \epsilon_n)$ is a diagonal graded matrix containing m (“bosonic”) and n (“fermionic”) energies at which the system is probed, A are supermatrices (matrices containing Grassmann and commuting variables [21]) of dimension $m + n$, and the integration is over a flat measure. The symbol “str” denotes the supermatrix trace operation, and c is a coupling constant.

The Kontsevich model offers complementary perspectives: in quantum chaos, it defines an extension of the nonlinear σ model [21] from the bulk spectrum to the edge [22], providing a description of both energy levels and wave functions. For example, for $m, n = 1$, with just one probe energy $\epsilon_1 = \epsilon_1 = \epsilon$, a straightforward stationary phase analysis yields the estimate $\langle \rho(\epsilon) \rangle \simeq (c/\pi) \epsilon^{1/2}$, defining a semiclassical approximation for the spectral density. By contrast, the exact integral over two-dimensional supermatrices yields $\langle \rho(\epsilon) \rangle = c^{2/3} (\tilde{\epsilon} \text{Ai}^2(-\tilde{\epsilon}) + [\text{Ai}'(-\tilde{\epsilon})]^2)$, $\tilde{\epsilon} = \epsilon c^{2/3}$, where the appearance of Ai is explained by the resemblance of $\int_A \exp[S(A)]$ to integral representations of the Airy function [23]. Physically, the refined result characterizes the spectral edge with oscillatory near-edge spectral density, qualitatively indicated in Fig. 1, where oscillation maxima mark the rigid positions of UFL energy levels, and minima describe their repulsion. Their statistical distribution, witnessed by the rounded profile of the average spectral density,

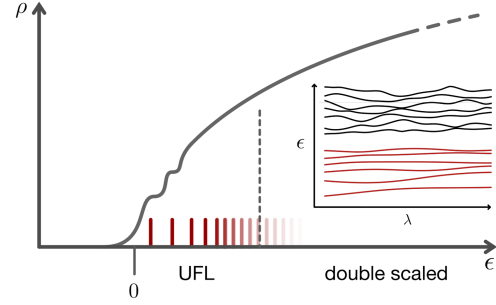


FIG. 1. Schematic structure of the near edge spectrum of a dense system. Close to the edge, the level spacing increases and statistical fluctuation of levels freeze out, as witnessed by the Airy-oscillatory average spectral density. Individual realizations of sparse systems show a similar edge structure; however, large fluctuations of the edge position, indicated as $\epsilon = 0$, remove these signatures in average quantities and correlation functions. Inset: energy levels as a function of parametric perturbations demonstrating the rigidity of near edge states (bottom) compared to generic bulk states (top).

is probed by correlation functions such as $\langle \rho(\epsilon_1) \rho(\epsilon_2) \rangle$, which can be computed with a little more effort from a $m, n = 2$ version of the model [4].

Within the gravitational context, the perturbative expansion of the Kontsevich quantitatively matches the topological expansion of the gravitational path integral [24,25]. Beyond perturbation theory it features as the effective theory describing correlations in topological string theory [11,26] at the level required to describe and physically interpret the UFL. In the following, we apply this framework to describe the rigidity of ground state wave functions to parameter variations in terms of the so-called quantum geometric tensor. We will first compute the distribution of diagonal elements of the quantum geometric tensor employing a variant of Eq. (1), then interpret our findings in terms of string fluctuation degrees of freedom, and finally comment on implications for the holographic correspondence.

State geometry and fidelity susceptibility—Quantum state geometry, i.e., the metrics according to which individual states $|n\rangle$ of a system deform under parameter variations, is encoded in an object known as the quantum geometric tensor, $g^{(n)} = g_{\alpha\beta} d\lambda^\alpha d\lambda^\beta$. For a Hamiltonian perturbed as $H \rightarrow H + \sum_\alpha \lambda_\alpha H_\alpha$, with eigenstates $|n\rangle \rightarrow |n_\lambda\rangle$, its elements are defined as

$$g_{\alpha\beta}^{(n)} = \langle \partial_\alpha n | \partial_\beta n \rangle - \langle \partial_\alpha n | n \rangle \langle n | \partial_\beta n \rangle, \quad (2)$$

where $|\partial_\alpha n\rangle = \partial_{\lambda_\alpha} |n_\lambda\rangle|_{\lambda=0}$. We here focus on the diagonal elements $g_{\alpha\alpha}$, the “fidelity susceptibilities” (FSs), as the most direct measure for the response of states to perturbations. Straightforward first order wave function perturbation theory yields the alternative representation,

$$g_{\alpha\alpha}^{(n)} = \sum_{m \neq n} \frac{|\langle n | H_\alpha | m \rangle|^2}{(E_n - E_m)^2}, \quad (3)$$

highlighting the combined influence of spectral (denominator) and wave-function (numerator) correlations on this observable, and energies E_n are measured with respect to the band center. Reflecting the stochastic nature of both, the FSs are distributed variables, and we aim to compute their distribution,

$$P_E(g) = \sum_n \langle \delta(E - E_n) \delta(g - g_{\alpha\alpha}^{(n)}) \rangle, \quad (4)$$

for the quantum states of a dense chaotic system with its lower edge at $E = -E_0$ in response to a generic deformation λ_α . Here, $\langle \dots \rangle$ denotes ensemble averaging over Gaussian distributed perturbations H_α , and over a distribution of H , where it will be interesting to compare three different settings: (i) diagonal matrices with randomly distributed matrix elements, (ii) generic states E inside the spectrum of invariantly distributed H 's, i.e., distributions satisfying $P(H)dH = P(UHU^\dagger)dH$, and (iii) the same, but for E level spacing close to the edge, the UFL. These cases represent opposite extremes in that (i) has neither spectral nor state correlations, representing an ensemble of “integrable systems,” while (ii,iii) are maximally correlated or chaotic.

For each of these settings, a sequence of nontrivial but standard operations [27–29] leads to the integral representation $P_E(g) \propto \int_{-\infty}^{\infty} dz z Z(z) e^{-Dgz^2}$, where “ $\propto$ ” means equality up to normalization, and the integration is over a z contour infinitesimally shifted into the upper complex plane. The kernel $Z(z) = \lim_{x_i \rightarrow 0} Z(X)$, with

$$Z(X) = \left\langle \frac{\prod_{i=1}^{2+\beta} \det(H - E + x_i)}{\prod_{s=\pm} \det(H - E + sz)} \right\rangle, \quad (5)$$

effectively defines a probability generating function with “source matrix” $X = \text{diag}(z, -z|x_1, \dots, x_n)$. Here, $\beta = 0$ for case (i), while the presence of two extra determinants, $\beta = 2$, for (ii,iii) reflects the higher level of correlations in these cases. For (i), the averaging of H 's diagonal elements over a box distribution and energy values E in the center of the band is straightforward, and so is the subsequent Gaussian z integral. As a result, one obtains $P(g) \propto g^{-3/2} e^{-D/g}$, a so-called Levy stable distribution characterized by fat tails at large g , and the nonexistence of statistical moments. This tendency toward large susceptibilities is a consequence of the statistical independence of close-by energy values in the model (i) of uncorrelated energy levels.

Supersymmetry—To compute the fidelity distribution of the correlated models (ii,iii), we employ an adapted variant of the Kontsevich model. Referring for details to Supplemental Material [29], we here highlight cornerstones

of the computation that afford a gravitational perspective to be discussed below. Our starting point is a representation of the determinants in Eq. (5) as Gaussian integrals over complex commuting (denominator) and Grassmann (numerator) variables, respectively. The subsequent average over the distribution of H generates correlations between these variables, which can be decoupled by one of several available variants of Hubbard-Stratonovich transformations [30,31] in terms of a six-dimensional supermatrix, A . In the immediate vicinity of the edge, the supermatrix integral reduces to Eq. (1), with $c = D$ the Hilbert space dimension and $X = (z, -z|x_1, x_2, x_3, x_4)$.

Turning to the computation of that integral, its structure suggests working in polar coordinates, $A = TWT^{-1}$, with radial degrees of freedom contained in $W \equiv \text{diag}(u_1, u_2 | -w_1, -w_2, -w_3, -w_4)$ and angular matrices T , which can be integrated out via a supervariant of the Itzykson-Zuber integration identity [32]. As a result we obtain the intermediate representation

$$Z(X) \propto \int DW \frac{s\Delta(W)}{s\Delta(X)} e^{S(W)}, \quad (6)$$

where at the edge the action reduces to the Kontsevich action (1) evaluated on the radial matrix $A = W$, DW is the flat measure of radial coordinates, and $s\Delta$ the super-Vandermonde determinant [33].

The final integral over radial variables now depends on whether we are probing states inside the spectrum, $E + E_0 \equiv \epsilon \gg \Delta_e \sim D^{-2/3}$, where Δ_e is the near-edge level spacing, model (ii), or values inside the domain of the UFL, $\epsilon = \mathcal{O}(\Delta_e)$, model (iii). In the former case, the integral can be evaluated by stationary phase methods, leading to the “correlated Levy distribution” [19,27]

$$P(g) \propto p(gD/\rho_\epsilon^2) e^{-\rho_\epsilon^2/Dg},$$

$$p(x) = \frac{3}{4} x^{-5/2} + x^{-7/2} + x^{-9/2}, \quad (7)$$

with $\rho_\epsilon \equiv \langle \rho(\epsilon) \rangle \propto D\epsilon^{1/2}$. Compared to model (i), the stronger attenuation at large g values, $P(g) \sim g^{-5/2}$, $g \gg D$, reflects the level repulsion of chaotic systems, which makes anomalously large contributions to the rhs of Eq. (3) less likely [12,13].

This principle finds a yet more radical manifestation as we turn to the edge, model (iii). While saddle point analysis no longer is an option, the integral Eq. (6) turns out exactly doable and yields $Z(z)$ in terms of Airy functions [29]. Gaussian integration then yields our main result,

$$P(g) \propto p\left(\frac{g}{D^{1/3}}\right) e^{-\frac{D}{12g^3}},$$

$$p(x) = c_5 x^{-5/2} + c_7 x^{-7/2} + \dots + c_{17} x^{-17/2}, \quad (8)$$

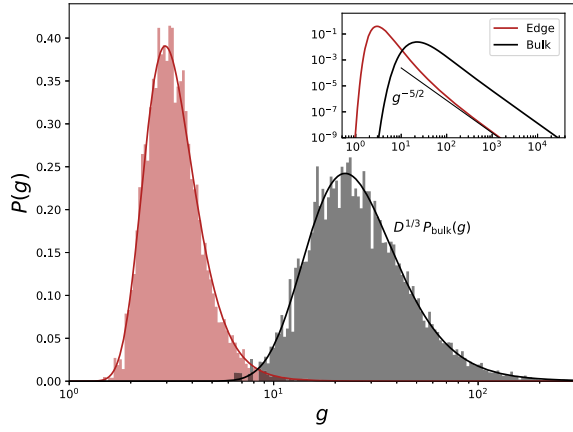


FIG. 2. FS distributions for near-edge (red) and bulk (black) states are compared to histograms generated from an ensemble of 5000 matrices of dimension $D = 1000$. (Bulk distribution scaled by $D^{1/3}$ for better visibility). Inset: FS distributions for near-edge and bulk states, with asymptotic scaling $g^{-5/2}$ added for reference.

and numerical coefficients of $\mathcal{O}(1)$ [29]. This is again a heavy tailed distribution, where the dominant power law $x^{-5/2}$ is identical to that of model (ii), indicating the same high level of correlation between neighboring states. Crucially, however, this power law is defined with reference to the scaling variable $x = gD^{-1/3}$, implying that even large values $x \gtrsim 1$ correspond to parametrically smaller values of the susceptibility than in the bulk. This higher level of state inertia reflects the rigid spacing of a sparse collection of states at the edge. Conversely, the dependence of the exponent on the same scaling combination prevents anomalously *small* values $g \lesssim D^{1/3}$, leading to a distribution parametrically narrower than in the bulk.

Figure 2 shows that these results are in excellent agreement with numerical simulations [34]. Figure 3 illustrates how wave functions described by this statistics maximize entropy in Hilbert space (the m axis) in terms of Gaussian distributions whose sensitivity to perturbations is reduced for the few terminal states forming the UFL.

String theory—We conclude by taking a look at the fidelity distribution from a gravitational perspective. As mentioned above, JT gravity is an integral over fluctuating surface geometries; see Fig. 4 for a cartoon representation of a surface with a single handle and two boundaries with circle topology, as required for the computation of spectral two-point correlation functions [9]. Reading these structures from left to right, they describe the generation and recombination of circular “miniuniverses” via cubic scattering amplitudes, as indicated by the cut structure. These processes correspond to the likewise cubic matrix scattering amplitudes described by the Kontsevich model [35], a connection made quantitative through so-called ribbon-diagram constructions [4]. However, they fall short of representing quantum states of the UFL. First, the ratios

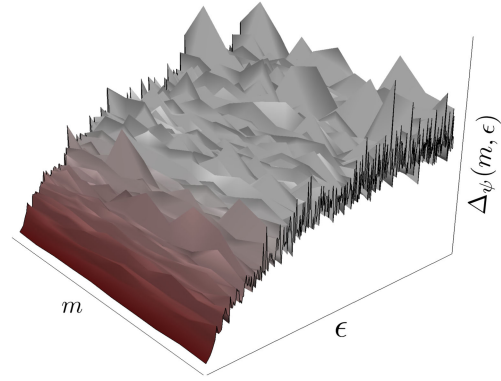


FIG. 3. Moduli of wave-function differences $\Delta_\psi(m, \epsilon) \equiv ||\psi_m(0, \epsilon) - \psi_m(\lambda, \epsilon)||$ for a fixed small perturbation $\lambda = 10^{-7}$ as a function of wave-component m and energy ϵ , here for a system of dimension $D = 1000$, averaged over 100 samples.

of determinants featuring in Eq. (5) do not afford a representation in terms of surfaces with finite number of boundary insertions. Second, an expansion in terms of surface topologies, equivalent to a perturbative summation of matrix diagrams, does not resolve individual boundary states.

Both problems are elegantly solved by an effective string theory, possessing JT gravity as its semiclassical limit [10]. This framework, portrayed in more detail in the End Matter, describes two-dimensional gravity as a conformal field theory of a scalar field Φ , obtained by dimensional reduction from a six-dimensional parent geometry. The integration over open string excitations involved in this reduction introduces the equivalent of an ensemble average, and splitting of closed string excitations describes the miniverse generation mentioned above. Crucially for us, the representation of the observable Eq. (5), and its nonperturbative analysis are concrete options within this framework. As detailed in the End Matter, the intermediate steps of this analysis quantitatively reproduce Eq. (6), providing its supermathematical structures with a physical interpretation in terms of (anti)brane correlations. The main conclusion of this analysis is that the string theoretical framework provides access to the quantum states of the UFL in terms of correlation functions that do not afford an interpretation in terms of semiclassical geometry fluctuations, but one in terms of string correlations.

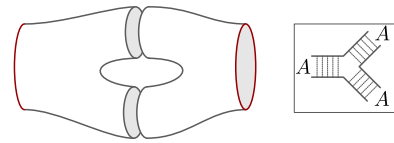


FIG. 4. Cartoon illustrating the generation and recombination of “miniuniverses” in the gravitational context, and its representation through the cubic vertices of the matrix theory Eq. (1).

Discussion—The above construction shows the quantitative equivalence of three cornerstones in the theory of dense chaotic systems: the quantum state distribution Eq. (5) evaluated for ensembles of random matrix Hamiltonians, the same observable in a theory of topological strings, and the equivalence of both to the Kontsevich matrix model Eq. (1). In this way, we have found that the universality class of the first levels above the edge is described by a maximum entropy distribution distinguished by exceptional resilience to perturbations.

The emergence of that universality class in two-dimensional holography is owed to the confluence of two individually unique principles. First, the presence of a nonfluctuating edge, and second the fact that JT incorporates its own ensemble. In two-dimensional gravity, the latter is realized through the integration over “hidden” open string degrees of freedom integrated out upon dimensional reduction of JT’s parent Kodaira-Spencer theory. These structures are not present in naturally defined quantum boundary theories such as ensembles of Sachdev-Ye-Kitaev models, where “natural” means theories defined in terms of a few-body chaotic Hamiltonians. Referring back to the beginning of the Letter, these are *sparse* theories where the averaging over polynomially few parameters in an exponentially high dimensional Hilbert space leads to large edge fluctuations eradicating all the structures discussed above. In this sense, the fine-grained string theoretical probe reveals a weak link in the low-dimensional holographic principle that requires further investigation.

Acknowledgments—T.M. acknowledges financial support by Brazilian agencies CNPq and FAPERJ, and J. T. M. financial support by Brazilian agency CAPES. J. v. d. H. acknowledges support from the National Science and Engineering Research Council of Canada (NSERC) and the Simons Foundation via a Simons Investigator Award. Work supported by CRC183 of the Deutsche Forschungsgemeinschaft (DFG) Project No. 277101999, CRC 183 (Project No. A03). The work is supported by a Discovery grant from NSERC.

Data availability—The data that support the findings of this Letter are openly available [36].

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to-one correspondence with ribbon graphs associated to the Kontsevich matrix integral.

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End Matter

Gravity perspective of the UFL—We here discuss how the UFL presents itself from the perspective of gravity and how the latter connects to the concepts discussed in the main text. The theory of gravitation relevant to us is called Jackiw-Teitelboim (JT) gravity, two-dimensional Einstein gravity coupled to a scalar (“dilaton”) field. This theory famously describes the perturbative contents of a matrix model with near-edge spectral density $\tilde{\rho}(\epsilon) = (1/4\pi^2) \sinh 2\pi\sqrt{\epsilon} \sim \sqrt{\epsilon}$ exhibiting the characteristic nonanalyticity of a dense model. [Here, $\tilde{\rho}(\epsilon)$ is normalized to 1.] The correspondence is established by comparing the expansion of the JT path integral in its coupling constant $\exp(S_0)$, where $S_0 \sim 1/G_N$ is the gravitational constant—essentially an expansion in geometries of ascending topological complexity, a.k.a. miniuniverse expansion—to the D^{-1} expansion of a matrix theory with dimension $D \sim \exp(S_0)$ [9]. Intriguingly, JT gravity describes ensemble averaged properties of the matrix theory, while individual members of the ensemble do not have a gravitational counterpart at this level.

As discussed in the main text, the UFL is perturbatively invisible, and hence does not have a gravitational representation within JT theory. However, there exists an extended framework, known as Kodaira-Spencer (KS) theory [39], which reproduces the topological expansion of the JT path integral in perturbation theory and beautifully includes the UFL beyond. KS theory is obtained from a theory of topological strings propagating in a six-dimensional noncompact Calabi-Yau (CY) manifold defined as the solution set of the complex equation

$$H(x, y) - uv = 0, \quad (\text{A1})$$

where our present discussion applies to the choice $H(x, y) \approx y^2 - x$. The two-dimensional reduction of KS theory [40] relevant to our comparison to JT gravity and matrix theory is obtained by integrating out string degrees of freedom. It results in an effective theory of a field $\Phi(x)$ living on the two-dimensional *spectral curve*, Σ , defined as the solution set of the equation $H(x, y) = 0 \Rightarrow y = y(x)$. Technically, Φ is a chiral boson, governed by an action containing a cubic interaction.

All elements of this framework, i.e., the boson field operators, its action, the spectral curve, and the degrees of

freedom integrated over in the reduction process carry definite physical meaning in connection to gravity and its matrix theory description. Referring to Ref. [10] for a detailed discussion, we here provide a quick glossary summarizing the most important structures:

The coordinate x locally parametrizing the spectral curve has the physical interpretation of (complex) “energy.” Considered as a Riemann surface, the geometry of the spectral curve encodes information on the spectral density. Specifically, the branch points defined by the equation $y^2 - x = 0$ know about the spectral edge, $x = 0$, and the spectral density $y(x) \sim \sqrt{x}$ in its vicinity [41].

The topological strings defined in the six-dimensional CY target space geometry come in two variants, closed and open. “Open strings” end on geometric structures known as branes, and we here need to distinguish between a large set of D compact “color branes” and an $\mathcal{O}(1)$ set of noncompact “flavor branes.” The data comprising the $\mathcal{O}(D \times D)$ open string endings on color branes $i, j = 1, \dots, D$ defines a “matrix,” H_{ij} , and the integration over it the equivalent of an ensemble average [42,43]. This structure naturally explains why gravity obtained by dimensional reduction resembles ensemble averaged theories. By contrast, flavor branes act as probes into the theory. As subsets of the CY, they are defined by fixing coordinates, $x, z, \dots$, on the spectral curve and setting either u or v to zero to define noncompact brane or antibrane submanifolds of the CY parametrized by (x, v) or (z, u) . Within the field theory framework, these objects are then represented by vertex operators $\psi(x) \equiv \exp[\Phi(x)]$, and $\psi^\dagger(z) \equiv \exp[-\Phi(z)]$, respectively [26,44]. These operators probe the information otherwise obtained by the insertion of matrix determinants $\det(x - H)$ or $\det(z - H)$ into the numerator or denominator of an ensemble averaged correlation function. Finally, “closed strings” are interpreted as circular miniuniverses, and in the field theory are represented as primaries $\partial\Phi(x)$. The cubic interaction of the boson with coupling constant $\lambda_{\text{KS}} \sim \exp(-S_0)$ describes the dynamical creation and annihilation of miniuniverses in the gravitational context.

Turning to a more concrete level, the discussion above indicates that the state geometry correlation function Eq. (5) affords a representation,

$$Z \equiv \left\langle \left\{ \psi^\dagger(z) \psi^\dagger(-z) \psi(x_1) \cdots \psi(x_4) \right\} \right\rangle_{\text{KS}}, \quad (\text{A2})$$

in terms of brane and antibrane insertions into KS field theory, where normal ordering $\{\dots\}$ removes divergent terms originating in the operator product expansion of the chiral boson $\Phi(x)\Phi(x') \sim \log(x-x') + \text{reg.}$ Note that the geometric setup for studying properties of states is different from the one utilized to calculate spectral quantities in Ref. [11], as Eq. (A2) involves an asymmetric configuration of branes and antibranes. Furthermore, the flavor branes have finite separation that signals that the quantum state statistics is not accessible through a perturbative closed string expansion, but requires knowledge of the open string sector. In the following we outline how this correlation function in KS field theory quantitatively agrees with that previously discussed in random matrix theory, including for brane coordinates z in the immediate vicinity of the edge, i.e., Σ 's branch point, $z = 0$.

We first note that the bookkeeping of all Wick contractions required by the normal ordering of exponentiated fields generates a super-Vandermonde determinant $s\Delta(X)$ of the brane coordinates $X = (z, -z|x_1, x_2, x_3, x_4)$, which was already featured in our previous analysis, Eq. (6),

$$Z = \frac{1}{s\Delta(X)} \left\langle \psi^\dagger(z) \psi^\dagger(-z) \psi(x_1) \cdots \psi(x_4) \right\rangle_{\text{KS}}, \quad (\text{A3})$$

now without normal ordering. To proceed, we apply a Fourier transform to pass from the energylike spectral curve coordinates (z, x_i) to timelike variables (u, w_i) . The transformed version of the correlation reads as

$$\begin{aligned} Z &= \frac{1}{s\Delta(X)} \prod_{i=1}^4 \int_{\mathcal{C}_0} dw_i \int_{\mathcal{C}_+} du_1 \\ &\times \int_{\mathcal{C}_-} du_2 e^{\frac{1}{\lambda_{\text{KS}}} (z(u_1 - u_2) + \sum_{i=1}^4 w_i x_i)} \\ &\times \left\langle \hat{\psi}^\dagger(u_1) \hat{\psi}^\dagger(u_2) \hat{\psi}(-w_1) \cdots \hat{\psi}(-w_4) \right\rangle_{\text{KS}}, \quad (\text{A4}) \end{aligned}$$

where the integration contours $\mathcal{C}_0, \mathcal{C}_\pm$ are shown in Fig. 1 of Supplemental Material [29]. The architecture of this

representation indicates that the diagonal matrix of timelike variables $W \equiv \text{diag}(u_1, u_2 | -w_1, -w_2, -w_3, -w_4)$ assumes a role analogous to that of the radial variables in the Kontsevich framework. To corroborate this analogy, we reintroduce normal ordering by multiplying by the super-Vandermonde $s\Delta(W)$. What then remains to evaluate is

$$\hat{Z} \equiv \left\langle \left\{ \hat{\psi}^\dagger(u_1) \hat{\psi}^\dagger(u_2) \hat{\psi}(-w_1) \cdots \hat{\psi}(-w_4) \right\} \right\rangle_{\text{KS}}.$$

Thanks to normal ordering, the vertex operators can be combined into a single exponential,

$$\begin{aligned} \hat{Z} &= \left\langle \left\{ \exp \left[-\sum_i \hat{\Phi}(u_i) + \sum_j \hat{\Phi}(-w_j) \right] \right\} \right\rangle_{\text{KS}} \\ &\simeq \exp \left(-\frac{1}{\lambda_{\text{KS}}} \left(\sum_i \langle \hat{\Phi}(u_i) \rangle_0 - \sum_j \langle \hat{\Phi}(-w_j) \rangle_0 \right) \right), \end{aligned}$$

where the replacement $\langle \dots \rangle_{\text{KS}} \rightarrow \langle \dots \rangle_0$ indicates that to leading order in $\lambda_{\text{KS}} \sim \exp(-S_0)$, the field expectation values are computed in the free theory. This final computation, detailed in Ref. [11], yields $\langle \hat{\Phi}(w) \rangle_0 = -\int^w x(w') dw'$, in terms of solutions $x(w)$ of the spectral curve equation $H(x, w) = 0$. Considering the near-edge $H(x, w) = w^2 - x$, and doing the integral, we obtain

$$\hat{Z} = \exp \left(\frac{1}{3\lambda_{\text{KS}}} \left(\sum_i u_i^3 + \sum_j w_j^3 \right) \right),$$

Substituting this result into Eq. (A4) we obtain an expression equal to Eq. (6), with the identification of coupling constants $c = \lambda_{\text{KS}}^{-1} = \exp(S_0)$ and the size of the flavor matrix determined by the number of (anti)branes. In this way, we establish the nonperturbative equivalence between the theory of topological strings and the matrix integral framework in the description of (gravitational) wave functions near the edge.