

Floquet Engineering of Interactions and Entanglement in Periodically Driven Rydberg Chains

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Neutral atom arrays driven into Rydberg states constitute a promising approach for realizing programmable quantum systems. Enabled by strong interactions associated with Rydberg blockade, they allow for simulation of complex spin models and quantum dynamics. We introduce a new Floquet engineering technique for systems in the blockade regime that provides control over novel forms of interactions and entanglement dynamics in such systems. Our approach is based on time-dependent control of Rydberg laser detuning and leverages perturbations around periodic many-body trajectories as resources for operator spreading. These time-evolved operators are utilized as a basis for engineering interactions in the effective Hamiltonian describing the stroboscopic evolution. As an example, we show how our method can be used to engineer strong spin exchange, consistent with the blockade, in a one-dimensional chain, enabling the exploration of gapless Luttinger liquid phases. In addition, we demonstrate that combining gapless excitations with Rydberg blockade can lead to dynamic generation of large-scale multipartite entanglement. Experimental feasibility and possible generalizations are discussed.

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Introduction—Programmable quantum simulators provide unique insights into complex many-body systems. They can be used for explorations of strongly correlated quantum phases of matter [1–5], nonequilibrium quantum dynamics [6–15], many-body entanglement [8,16–20], and quantum metrology [13,14,21–24]. Neutral atom arrays are a promising approach to realizing programmable quantum simulators [10,25–27], where tunable atom trapping geometry along with strong interactions resulting in Rydberg blockade allow one to generate a variety of strongly correlated spin models. Coherent laser excitation into Rydberg states generates dynamics within the accessible Hilbert space, similar to that provided by a global transverse field in the Ising model. These constrained dynamics result in new physical phenomena such as quantum many-body scars [10,12,28], which evade thermalization starting from certain product initial states. At the same time, extending the toolbox of Rydberg quantum simulation in the blockade regime to dynamical generators beyond simple transversal fields is an open challenge. This is important, for instance, for realizing spin liquids [3,29–31] and lattice gauge theories [32–36], for steering the

dynamics of many-body states via counterdiabatic terms [37–39], for realizing new types of quantum optimization algorithms [40–42], and for recent efforts to generate metrologically useful entanglement in systems with quantum many-body scars [43,44].

Motivated by these considerations, in this Letter we introduce a technique for Floquet engineering [45–54] that employs time-dependent control to realize effective Rydberg-blockaded models with versatile interactions. While Floquet engineering is widely utilized for interacting spin systems [55–60], conventional analog techniques rely on local Pauli frame transformations that generally violate the blockade constraint, or are limited to digital few-body gates designed via optimal control techniques [61]. Our approach, illustrated in Fig. 1(b), leverages driven, periodic many-body trajectories—originally discovered in the context of stabilizing quantum many-body scars [12,62,63]—to steer effective dynamics of arbitrary initial states. The complex micromotion of these trajectories serves as a resource for programmable Hamiltonian engineering, since perturbations applied during the periodic drive act at stroboscopic times via an effective Hamiltonian generated by time-evolved operators [62]. The resulting class of time-evolved operators forms a basis for the realization of novel interactions with tunable coefficients, which are not accessible in the static native Hamiltonian. Using this approach, we show how blockade-consistent spin exchange interactions can be engineered, enabling the investigation of gapless phases with emergent particle number conservation [64]. Moreover, in this new regime, we demonstrate the dynamic generation of structured multipartite entanglement

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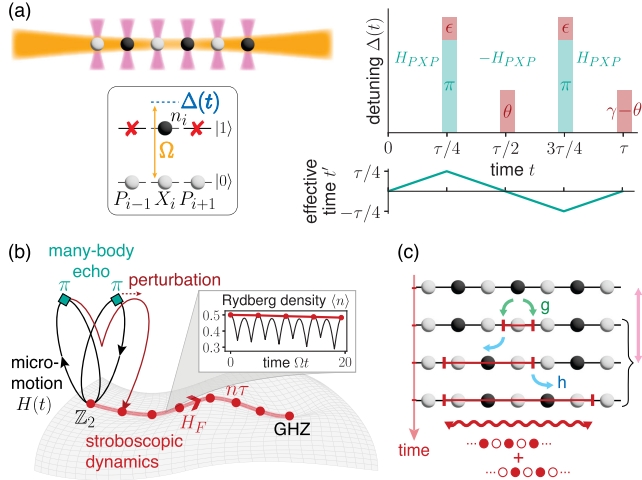


FIG. 1. Hamiltonian engineering. (a) We consider the driven PXP model as an approximate description of Rydberg atoms in optical tweezers, with Rabi frequency Ω and τ -periodic time-dependent global detuning $\Delta(t)$. Our protocol consists of π -pulses that realize a many-body echo (green) and deliberately placed perturbations (red). The echo realizes evolution under H_{PXP} for effective time $-(\tau/4) \leq t' \leq (\tau/4)$, returning to $t' = 0$ at stroboscopic times. (b) The resulting dynamics perturbs around the periodic trajectories of the many-body echo, illustrated as micromotion in the full blockade-constrained Hilbert space. At stroboscopic times $n\tau$, an effective Floquet Hamiltonian H_F generates evolution within a constant energy submanifold. In our main application, the stroboscopic dynamics evolves a Néel ($\mathbb{Z}_2$) initial state towards a highly entangled GHZ-like state. Inset: Micromotion (black) and stroboscopic dynamics (red) of the Rydberg density $\langle n \rangle$ on an $L = 16$ periodic chain. Despite large oscillations within Floquet cycles, the stroboscopic evolution approximately conserves $\langle n \rangle$. (c) Our approach realizes effective models H_F with tunable control over the dynamics of domain wall excitations on top of the Néel order. This includes a chemical potential J , blockade-consistent spin exchange interactions h that act as two-site hopping for domain walls, as well as creation or annihilation terms g of domain wall pairs. The regime of small g provides a mechanism for generating long-range multi-partite entanglement via growing superpositions of alternate $\mathbb{Z}_2$ orders.

from Néel ($\mathbb{Z}_2$) product initial states and explain this effect in terms of the dynamics of domain walls.

Hamiltonian engineering—The key idea of this work can be understood by considering the driven PXP model illustrated in Fig. 1(a), which describes a one-dimensional atom chain with periodic boundary conditions, driven into the Rydberg state with fixed Rabi frequency Ω under idealized nearest-neighbor Rydberg blockade, with time-dependent global detuning $\Delta(t)$:

$$H(t) = \frac{\Omega}{2} H_{\text{PXP}} - \Delta(t) N, \quad (1)$$

$$H_{\text{PXP}} = \sum_i P_{i-1} \sigma_i^x P_{i+1}, \quad N = \sum_i n_i. \quad (2)$$

Here the operators $P_i = [(1 + \sigma_i^z)/2] = |\circ\rangle_i \langle \circ|_i$ and $n_i = [(1 - \sigma_i^z)/2] = |\bullet\rangle_i \langle \bullet|_i$ project site i onto ground ($\circ$) and Rydberg ($\bullet$) states, respectively, while $\sigma_i^x = |\circ\rangle_i \langle \bullet|_i + |\bullet\rangle_i \langle \circ|_i$ generates Rabi oscillations. Our objective is to realize off-diagonal number conserving processes beyond single spin flips, utilizing the intrinsic controls Ω and $\Delta(t)$. In a conventional (static) approach ($\Delta(t) = \Delta$) to this problem, one typically relies on large detuning $\Delta \gg \Omega$, where multibody spin flips emerge perturbatively in powers of $(\Omega/\Delta)^n$. The relative strengths of such processes are generally weak and cannot be tuned independently.

Our dynamic protocol circumvents this restriction by modulating laser detuning $\Delta(t)$ and leveraging a many-body spin echo. Specifically, since H_{PXP} anticommutes with the operator $\prod_i \sigma_i^z = e^{i\pi N}$, π pulses of the global detuning effectively reverse its sign [28,62]: $e^{i\pi N} H_{\text{PXP}} e^{i\pi N} = -H_{\text{PXP}}$. This property enables a dynamical decoupling in the strongly interacting PXP model by means of a simple pulse sequence ($n \in \mathbb{N}$),

$$\Delta_0(t) = \pi \sum_n \delta\left(t - \frac{\tau}{4} - n\frac{\tau}{2}\right). \quad (3)$$

Within each Floquet period τ , the system evolves forward under H_{PXP} for $\tau/4$, backward for $\tau/2$, and forward again for $\tau/4$. At a given time $t \leq \tau$, the system has thus undergone an effective evolution by H_{PXP} for a time $t' = t'(t) = ||t - \tau/4| - \tau/2| - \tau/4$, where $t' \in [-(\tau/4), (\tau/4)]$, as shown in Fig. 1(a) (see Supplemental Material [65], Sec. IA). As $t'(\tau) = 0$, the system exhibits periodic revivals at stroboscopic times $n\tau$.

Our scheme utilizes complex micromotion along the periodic trajectory to generate nontrivial effective evolution, as illustrated in Fig. 1(b). Specifically, we introduce global detuning perturbations consisting of τ -periodic discrete pulses around the echo protocol of Eq. (3), $\Delta(t) = \Delta_0(t) + \tilde{\Delta}(t)$ with $\tilde{\Delta}(t) = \sum_{j,n} \tilde{\Delta}_j \delta(t - t_j - n\tau)$. At stroboscopic times, these perturbations translate into evolution under a static, local effective Floquet Hamiltonian H_F , which holds up to an exponentially long prethermal timescale $T_p \gtrsim (\tau/|\tilde{\Delta}|) e^{c_p/|\tilde{\Delta}|}$, where $|\tilde{\Delta}| = \sum_j |\tilde{\Delta}_j|$ and $c_p > 0$ [74,75]. In the interaction picture with respect to the perfect echo evolution for $\tilde{\Delta} = 0$, we obtain the leading contributions to H_F through a Floquet-Magnus expansion [62,76] (see [65], Secs. IB–IC), resulting in $H_F = \sum_{n=0}^{\infty} H_F^{(n)}$, with

$$H_F^{(0)} = -\sum_j \frac{\tilde{\Delta}_j}{\tau} \tilde{N}(t'_j), \quad H_F^{(1)} = \sum_{j>\ell} \frac{\tilde{\Delta}_j \tilde{\Delta}_\ell}{2i\tau} [\tilde{N}(t'_j), \tilde{N}(t'_\ell)]. \quad (4)$$

We note that $H_F^{(n)}$ are constructed from Rydberg number operators conjugated by evolution under H_{PXP} for the

effective times $t'_j = t'(t_j)$ of the echo protocol, $\tilde{N}(t') \equiv e^{it'\frac{\Omega}{2}H_{\text{PXP}}} N e^{-it'\frac{\Omega}{2}H_{\text{PXP}}}$. Consequently, operator spreading under the micromotion generated by H_{PXP} induces interactions in the form of n -nested commutators $(t')^n [H_{\text{PXP}}, \dots, [H_{\text{PXP}}, N] \dots]$ in $\tilde{N}(t')$ and thus H_F .

Coefficients of these terms in H_F are controlled by the locations t_j and weights $\tilde{\Delta}_j$ of the pulses. Pulses with $t'_j = 0$ couple to the bare Rydberg number operator N . Further, symmetric weights of pulses at $\pm t'_j$ ensures that all terms containing an *odd* number of commutators vanish in $H_F^{(0)}$, which thus conserves Rydberg number parity. Hence, for small t'_j the leading nontrivial contribution to $H_F^{(0)}$ appears at order $(t'_j)^2$, which contains nearest-neighbor pair flips.

Single spin flips appear only in $H_F^{(1)}$. Based on this intuition, we introduce the following detuning perturbations, parameterized in terms of the (dimensionless) variables γ, θ, ϵ [see Fig. 1(a)],

$$\tilde{\Delta}(t) = (\gamma - \theta) \sum_n \delta(t - n\tau) + \theta \sum_n \delta\left(t - \frac{\tau}{2} - n\tau\right) + \epsilon \sum_n \delta\left(t - \frac{\tau}{4} - n\tau\right) + \epsilon \sum_n \delta\left(t - \frac{3\tau}{4} - n\tau\right). \quad (5)$$

Here, the first two pulses both occur at effective time $t' = 0$, while the latter occur at $t' = \pm(\tau/4)$. Inserting into Eq. (4) and expanding $\tilde{N}(\tau/4)$ perturbatively for short periods $(\Omega\tau/4) \ll 1$, we obtain an approximate closed-form expression for the effective Hamiltonian,

$$H_F \approx -JN - hH_{\text{PXP}} + gH_{\text{PXP}} + \frac{h}{4}H_{\text{ZIZ}},$$

$$J = \frac{(\gamma + 2\epsilon)}{\tau} - \frac{3\epsilon\Omega^2\tau}{32}, \quad h = -\frac{\epsilon\Omega^2\tau}{32}, \quad g = -\frac{\epsilon(\theta + \epsilon)\Omega}{8}. \quad (6)$$

Here, we have kept all terms to quadratic order in $\Omega\tau, \epsilon, \gamma, \theta$ (see details in [65], Sec. I C); $H_{\text{PXP}} \equiv \frac{1}{2} \sum_i P_{i-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y) P_{i+2}$ is a blockaded nearest-neighbor spin-exchange interaction and $H_{\text{ZIZ}} \equiv \sum_i \sigma_i^z \sigma_{i+2}^z$. Our Floquet protocol provides flexible relative tunability of the coefficients J, h, g by controlling the period $\Omega\tau$ and parameters ϵ, γ, θ . Thus, our construction extends the capabilities of the Rydberg quantum simulator to blockade models with independent control over quasiparticle number conservation, motion, and creation or annihilation processes (for comparison with static approach see [65], Sec. I D). In particular, this enables access to exchange-dominated regimes $h \gtrsim g$, which we explore in the following.

Controlled multipartite entanglement—We consider dynamics starting from a Néel state, $|\Psi(t=0)\rangle = |\mathbb{Z}_2\rangle$. Excitations on top of this state can be viewed as domain

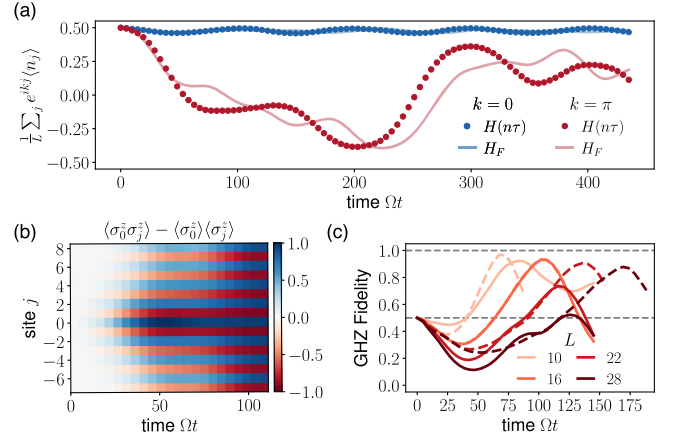


FIG. 2. Entanglement dynamics. (a) Stroboscopic evolution of an $L = 16$ periodic chain under a drive with period $(\Omega\tau/2\pi) = 0.77$ and perturbations set to $\epsilon = -0.45$, $\gamma = 1.0$, $\theta = 0.15$. Starting from a Néel initial state, the Rydberg number density (blue) remains high, while the $\mathbb{Z}_2$ order (red) washes out, in qualitative agreement with evolution under the effective Hamiltonian Eq. (6). (b) Dynamics under the drive $H(n\tau)$ generates spreading, connected σ^z -correlations with antiferromagnetic (AFM) spatial profile. (c) The driven dynamics realizes an AFM GHZ state for $L = 16$ once the connected σ^z correlations peak near unity at all distances. For fixed drive parameters, as system size increases, the peak in GHZ fidelity shifts approximately linearly with L , with decreasing height (solid). However, drive parameters can be tuned for each system size to maintain large GHZ fidelity (dashed, see text and [65], Sec. III B).

walls, created in pairs by H_{PXP} ; see Fig. 1(c). J sets a chemical potential and H_{PXP} generates (two-site) hopping of domain walls. We fix the parameters of the Floquet drive to $(\Omega\tau/2\pi) = 0.77$ (close to the scar period of the bare PXP model [10]), and $\epsilon = -0.45$, $\gamma = 1.0$, $\theta = 0.15$. This translates to an effective model of Eq. (6) with $J \approx 0.225\Omega$, $h \approx 0.068\Omega$, $g \approx -0.017\Omega$, where the rate of domain wall hopping is stronger than of creation or annihilation. As shown in Fig. 1(b), the average Rydberg density $\langle n(t) \rangle$ varies rapidly during micromotion, but evolves only slowly at stroboscopic times. Even though the parameters perturbing the echo are sizeable, the effective Hamiltonian Eq. (6) nonetheless provides a good description of the stroboscopic evolution as demonstrated in Fig. 2(a). We confirm in [65], Sec. IV B that quantitative discrepancies are dominated by higher order terms in the effective Hamiltonian rather than heating. Interestingly, while the density of Rydberg excitations remains high, the staggered magnetization $\sum_j (-1)^j \langle \sigma_j^z(t) \rangle$ washes out. As this happens, the system develops growing connected correlations $\langle \sigma_i^z(t) \sigma_j^z(t) \rangle - \langle \sigma_i^z(t) \rangle \langle \sigma_j^z(t) \rangle$; Fig. 2(b). Once these fluctuations reach the system size (here: $L = 16$), the state develops large overlap with an antiferromagnetic GHZ state, defined as $|\text{GHZ}\rangle = (1/\sqrt{2})(|\mathbb{Z}_2\rangle + e^{i\phi}|\mathbb{Z}_2'\rangle)$, which we quantify via the fidelity $\max_\phi |\langle \text{GHZ} | \Psi(t) \rangle|^2$ [18,77],

see Fig. 2(c). At maximum fidelity, we find $\phi = \pm(\pi/2)$ (see [65], Sec. III B).

This generation of multipartite entanglement can be understood based on the effective model Eq. (6), as sketched in Fig. 1(c): gH_{PXP} slowly creates a superposition of the initial $\mathbb{Z}_2$ state with states containing a pair of domain walls, which move rapidly via the strong hH_{PXP} interaction. The coherently propagating pair carries a growing string of the alternate $\mathbb{Z}'_2$ order, thus generating local patches of antiferromagnetic GHZ-like correlations. On a periodic chain, the pair may reannihilate at the antipodal point to form the state $|\text{GHZ}\rangle$. However, in large systems, we expect that after a time $t^* \sim 1/g$, interactions among the growing number of domain walls effectively limit the achievable multipartite entanglement to some finite length scale l^* , leading to a decreasing overlap with $|\text{GHZ}\rangle$ for increasing L in Fig. 2(c). To determine this length scale l^* , we note that under hH_{PXP} , individual domain walls form a band of quasiparticles with dispersion $\epsilon_k = -2h \cos(k)$; see Fig. 3(a). The initial state $|\mathbb{Z}_2\rangle$ couples to pairs $|k; -k\rangle$ of domain walls with opposite momenta $\pm k$. As the velocity of these domain walls is $v^* \sim h$, within the time $t^* \sim 1/g$ they will spread for a length $l^* = v^* t^* \sim h/g$, setting the achievable scale of multipartite entanglement.

We verify this relation in thermodynamically large systems via the quantum Fisher information (QFI) density, which quantifies the metrological potential of the pure state $|\Psi(t)\rangle$, evolving under H_F , with respect to a staggered z field: $F_Q/L \equiv (1/L) \text{Var}(\sum_j (-1)^j \sigma_j^z)_{|\Psi(t)\rangle}^2$. In particular, a QFI density $F_Q/L > m$ rigorously guarantees at least $(m+1)$ -body entanglement [79,80]. In Fig. 3(b), iTEBD simulations of an infinite chain for varying values of g and h demonstrate that the QFI obeys the predicted scalings for the formation time t^* and maximum size l^* of the multipartite entangled regions. As a consequence, we note that scaling $g \sim 1/L$ in finite systems implies $l^* \sim L$, and a GHZ state may form within *linear* time $t^* \sim L$. We verify this numerically in Fig. 2(c); for a detailed analysis of the parameters that yield high fidelity GHZ states; see [65], Sec. III B.

Luttinger liquid dynamics—Due to the small value of g , the entanglement features observed in the previous section may be understood as a dynamical probe of the low energy properties of the constrained model Eq. (6) with $U(1)$ symmetry at $g=0$. In particular, $H_F|_{g=0}$ is known to be integrable [81]. Its ground state phase diagram, which we calculate from Bethe ansatz integral equations (see [65], Sec. II A), is shown in Fig. 3(c). For $J/h \in (-3, 6)$, the system is in a Luttinger liquid phase [64] with gapless domain wall excitations. We explore the low-energy dispersion of collective excitations characteristic to Luttinger liquids in [65], Sec. II C. Outside this regime, the ground state transitions into a gapped paramagnetic ($J/h < -3$) or $\mathbb{Z}_2$ ($J/h > 6$) phase.

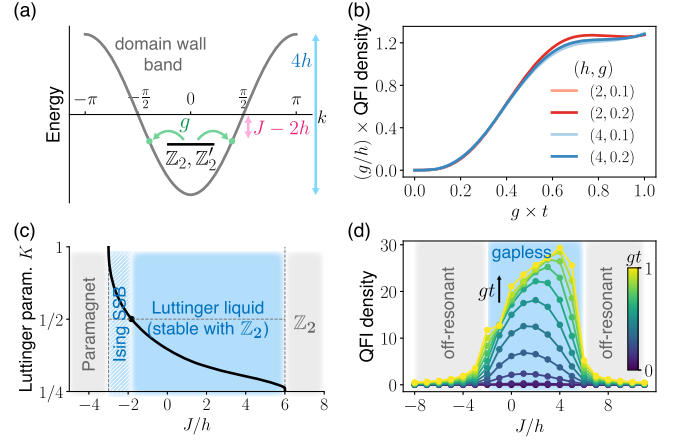


FIG. 3. Blockaded spin-exchange and approximate $U(1)$ symmetry. (a) The effective Hamiltonian of Eq. (6) for $g/h \lesssim 1$ weakly couples the Néel states to pairs of domain walls with momenta $\pm k$ and quasiparticle dispersion $\epsilon_k = -2h \cos k$. Depending on the energy offset between Néel states and the dispersive band, this coupling can be on or off resonant. (b) Dynamics under H_F for small g generates a maximum QFI density (see text) of order $\sim h/g$ within a time $t^* \sim 1/g$. This is seen through a scaling collapse for the dynamics of an infinite size chain simulated via iTEBD for different values of h , g at fixed $J = 2h$. (c) Phase diagram of H_F at the integrable point $g = 0$, with gapped paramagnetic and $\mathbb{Z}_2$ phases, as well as a Luttinger liquid that is stable for $K < 1/2$ (in the presence of $\mathbb{Z}_2$ number parity), and unstable for $K > 1/2$ (towards a gapped Ising SSB state [64,78]). The Luttinger parameter K (black curve) is obtained from the Bethe ansatz solution. (d) iTEBD simulation of the QFI density as a function of time t from a $\mathbb{Z}_2$ initial state for weakly broken integrability at fixed small g . The QFI functions as a dynamical probe of the transition between $\mathbb{Z}_2$ phase and Luttinger liquid in c), and grows at a rate tied to the domain wall dispersion.

We thus see that the protocol of the previous section corresponds to a quantum quench of the $\mathbb{Z}_2$ state into the Luttinger liquid phase. In particular, for small g , $|\mathbb{Z}_2\rangle$ couples *resonantly* to domain wall pairs $|k, -k\rangle$ mediating the entanglement dynamics when $J - 2h + 2\epsilon_k = \mathcal{O}(g)$, with matrix element $\langle k, -k | gH_{\text{PXP}} | \mathbb{Z}_2 \rangle \sim 2g \sin(k)$ proportional to the group velocity [see Fig. 3(a) and derivation in [65], Sec. II B]. For J/h outside the Luttinger liquid phase, domain walls can only be created virtually, suppressing the growth of the QFI. As a consequence, dynamics of multipartite entanglement quantified by the QFI density [82] could serve as a dynamical order parameter for the transition from the gapped $\mathbb{Z}_2$ phase to the gapless Luttinger liquid, with its early time growth reflecting the domain wall group velocity. We confirm this prediction numerically by computing the dynamics of the QFI under H_F of Eq. (6) using iTEBD for different values of J/h , see Fig. 3(d). The growth of the QFI accurately captures the phase boundary at $J/h = 6$ and shows an enhanced rate towards the center of the Luttinger liquid phase. We also

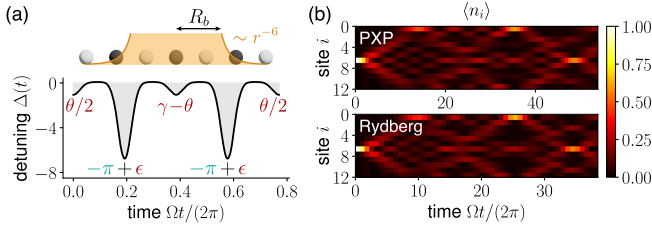


FIG. 4. Experimental applicability. (a) In experiment, Rydberg atoms at distance $r = |i - j|$ interact via V_{vdW} with strength $\Omega(R_b/r)^6$; here, blockade radius $R_b = 1.5$. In implementing the Floquet drive in (b), we use realistic Gaussian pulses with width $w = 0.046\tau$, ensuring each pulse integrates to the desired detuning (we apply $-\pi$ pulses to avoid blockade violations). A small constant detuning $\delta_{\text{MF}}/\Omega = 0.09$ is added to mitigate the long range tail of V_{vdW} [12]. (b) We benchmark Floquet evolution of the full Rydberg Hamiltonian under finite-width pulses, as described in (a), against the idealized PXP model driven by infinitely sharp pulses. We apply a drive $\epsilon = -\gamma/2 = -\theta = 0.45$ that generates an effective Hamiltonian with $h = (J/3) = -(\epsilon\Omega^2\tau/32)$, $g = 0$ to engineer a single-particle quantum walk on an $L = 12$ periodic chain. The dynamics of local Rydberg occupations $\langle n_i \rangle$ are consistent with this target; the effective hopping is slightly stronger in the Rydberg model.

verify these features in a direct simulation of the Floquet protocol for finite systems in [65], Sec. III A.

Robustness and implementation in Rydberg arrays—Successfully realizing and controlling H_F in Eq. (6) relies on the expansions in small perturbations ϵ , γ , θ and drive period $\Omega\tau$, which also tune the rate of dynamics. Thus, an optimal choice of drive parameters must consider the trade-off between a high-accuracy target Hamiltonian and the realistic constraint of observing significant dynamics within finite coherence time of an experimental device, an interplay we explore in [65], Sec. IV A. In addition, in experiment, Rydberg atoms interact via van der Waals interactions $V_{\text{vdW}} = \Omega \sum_{ij} (R_b/|i - j|)^6 n_i n_j$, with blockade radius R_b . To demonstrate that our approach applies qualitatively, we construct a pure spin exchange model ($-2\epsilon = \gamma = 2\theta \Rightarrow J = 3h \neq 0, g = 0$) and numerically evaluate the quantum walk of a single Rydberg excitation resulting from Floquet evolution including V_{vdW} at $R_b = 1.5$. We further add a small constant detuning to mitigate the long range tail of V_{vdW} [12] and use Gaussian pulse profiles suitable for realistic control hardware with limited rate of detuning modulation; Fig. 4(a). Comparison with the corresponding PXP result in Fig. 4(b) shows very good qualitative agreement over an extended duration. Quantitatively, the effective hopping is even stronger in the experimentally relevant scenario, likely due to residual contributions from finite pulse width and long range tails. A detailed analysis of these effects is left for future work.

Discussion and outlook—We have introduced a Floquet protocol for Rydberg atoms that exploits periodic

trajectories of quantum states and enables versatile Hamiltonian engineering with independent control over various interactions, achieved uniquely through our dynamic approach. As an application, we realized models with emergent particle number conservation and dominant blockade-consistent exchange interactions in one dimension, exploring previously inaccessible gapless Luttinger liquid phases. In particular, we found that combining Rydberg blockade and gapless domain wall excitations leads to the generation of long-range, multipartite entanglement upon evolving $\mathbb{Z}_2$ product states. Our scheme opens the door to investigating interacting Luttinger liquids on near-term quantum simulators, such as low-energy response functions [65,83]. In fact, it could be promising to explore the utility of our Floquet protocol as a novel tool for preparing such low energy states.

Although our discussion focuses on one-dimensional systems, generalization to other geometries is natural. For instance, models akin to Eq. (6) are relevant to Rydberg spin liquids in two dimensions [30,31], as well as to achieving quantum speedup in combinatorial optimization by enabling delocalization in the adiabatic algorithm [41]. Moreover, going beyond short evolution times of the operator $\hat{N}(t')$ provides access to even higher-body spin interactions, an approach we employ in Ref. [84] to study the dynamics of two-dimensional lattice gauge theories. We further emphasize that our Floquet scheme uses only simple global controls, but may be extended by incorporating site-resolved detuning fields [15], which could allow exploration of spatially inhomogeneous or chiral interactions [85]. It would also be interesting to study the connection of our protocol with time-dependent methods for state preparation, such as counterdiabatic driving [86–88] and trajectory optimization [39]. Finally, we note that our Floquet protocol can be extended to other quantum simulation platforms, contingent on time-dependent control to engineer non-trivial periodic trajectories, as is available in dipolar interacting systems [55–58,60], trapped ions [89,90], neutral atoms [91,92], or superconducting devices [93–95].

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