

Lattice-QCD Validation of Hadron Mass and Trace-Anomaly Decomposition Sum Rules

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We present the first lattice-QCD validation of multiple sum rules associated with quark-gluon decomposition of hadron mass by computing all relevant tensor components of the quark and gluon energy-momentum tensor matrix elements from first principles. We achieve this through nonperturbative renormalization of the QCD energy-momentum tensor, including its trace, in a gradient-flow scheme, followed by continuum extrapolations, two-loop matching to the $\overline{\text{MS}}$ scheme, and zero-flow-time extrapolations. These ingredients enable a direct and simultaneous verification, in a common renormalization scheme and scale, of multiple energy-density-based and trace-based mass decomposition sum rules proposed in the literature. We demonstrate the framework for the η_c and J/ψ charmonia using three fine lattice spacings with a physical strange-quark and near-physical up- and down-quark masses. We present the first lattice-QCD results for the gravitational form factor $\tilde{C}$. We find sizable gluonic contributions to charmonia masses at the hadronic scale, $\sim 15\%$ in the Lorcé and Metz-Pasquini-Rodini decompositions. The trace-anomaly contribution in the Ji sum rule is $\sim 6\%$, while the gluonic component of the trace anomaly in the Hatta-Rajan-Tanaka sum rule is $\sim 35\%$. The method is general and can be straightforwardly adopted for lattice-QCD calculations of mass and spin decompositions as well as gravitational form factors of other hadrons and nuclei.

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Introduction—The mass of nearly all visible matter in the Universe arises from asymptotic bound states of quantum chromodynamics (QCD) [1–3]. Quarks and gluons—almost massless at the Lagrangian level—form massive hadrons through strong, nonperturbative dynamics. Because of color confinement, partons are never observed in isolation, and how their confined dynamics generate hadron masses remains a fundamental question.

The QCD energy-momentum tensor (EMT) provides the field-theoretic bridge between quark-gluon degrees of freedom and hadronic observables. Its hadronic matrix elements relate the hadron mass to quark and gluon energies, explicit quark-mass terms, and the trace anomaly [4–8]. Gauge-invariant mass decompositions and the associated trace- and energy-density sum rules quantify this connection [9–15]. These sum rules state that the hadron mass equals the sum of all quark and gluon contributions, providing a concrete

benchmark for how confined partons collectively generate mass.

To test these theoretical relations, two complementary approaches are essential: experimental access to EMT form factors and nonperturbative theoretical determination of EMT matrix elements. Experimentally, the 12 GeV program at Thomas Jefferson National Accelerator Facility and the forthcoming Electron-Ion Collider aim to probe gravitational form factors (GFFs) through deeply virtual exclusive processes, providing leading-twist information on the traceless EMT and its spatial structure [16–25]. However, only limited combinations of EMT components can be reconstructed, and the trace part governing the anomaly contribution remains inaccessible at leading twist.

A complete picture, therefore, requires first-principles information on all EMT components, including those beyond experimental reach. Lattice-QCD offers a gauge-invariant, systematically improvable, nonperturbative framework to address this question. However, the reduction of the continuum rotational to the lattice hypercubic symmetry induces mixing of the EMT trace with power-divergent lower-dimensional operators [26,27]. Additionally, conventional nonperturbative renormalization of lattice EMT using regularization-independent momentum-subtraction (RI/MOM)-type schemes are gauge dependent, infrared sensitive, and statistically noisy. As a result, past studies have largely been

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restricted to selected traceless components [28–40], with the trace-related component inferred using the relevant sum rules [7,11,41–45].

We overcome these challenges and provide the first direct lattice-QCD validation of hadron mass decompositions and the trace-anomaly sum rules by computing each of their constituent components. We employ nonperturbative renormalization of both traceless and trace parts of the EMT in the gradient-flow scheme, followed by continuum limit [46–49], two-loop matching to $\overline{\text{MS}}$, and zero-flow-time extrapolations [50–53]. This enables a consistent determination of all EMT components in a common scheme and scale, allowing a direct and simultaneous consistency check of multiple energy-density- and trace-based decompositions proposed in the literature [7–15]. Closely related gradient-flow constructions have also been explored for flowed twist-2 operators entering higher parton distribution function (PDF) moments [54–56].

$\overline{\text{MS}}$ -renormalized EMT via gradient flow—The symmetric QCD EMT in Euclidean formulation, related to its Minkowski counterpart by the Wick rotation $[T_{\rho\sigma}]_M = i^{\delta_{ip}+\delta_{i\sigma}}[T_{\rho\sigma}]_E$, is

$$T_{\rho\sigma}(x) = \mathcal{O}_{1,\rho\sigma}(x) - \frac{1}{4}\mathcal{O}_{2,\rho\sigma}(x) + \frac{1}{4}\mathcal{O}_{3,\rho\sigma}(x) - \frac{1}{2}\mathcal{O}_{4,\rho\sigma}(x) - \mathcal{O}_{5,\rho\sigma}(x), \quad (1)$$

where

$$\begin{aligned} \mathcal{O}_{1,\rho\sigma} &= \frac{2}{g_0^2} \text{Tr}^c [F_{\rho\omega} F_{\sigma\omega}], \\ \mathcal{O}_{2,\rho\sigma} &= \frac{2}{g_0^2} \delta_{\rho\sigma} \text{Tr}^c [F_{\omega\xi} F_{\omega\xi}], \\ \mathcal{O}_{3,\rho\sigma} &= \sum_f \mathcal{O}_{3,\rho\sigma,f} = \sum_f \bar{\psi}_f (\gamma_\rho \overleftrightarrow{D}_\sigma + \gamma_\sigma \overleftrightarrow{D}_\rho) \psi_f, \\ \mathcal{O}_{4,\rho\sigma} &= \delta_{\rho\sigma} \sum_f \bar{\psi}_f \overleftrightarrow{D} \psi_f, \quad \mathcal{O}_{5,\rho\sigma} = \delta_{\rho\sigma} \sum_f m_f \bar{\psi}_f \psi_f. \end{aligned} \quad (2)$$

Here, g_0 is the bare coupling, m_f is the bare quark mass, Tr^c denotes the color trace, f labels quark flavors, and the covariant derivative is $\overleftrightarrow{D}_\rho = \overrightarrow{D}_\rho - \overleftarrow{D}_\rho$. For analyzing hadron mass decompositions, the EMT is separated into quark and gluon parts [57–59]:

$$\begin{aligned} T_{\rho\sigma} &= \sum_{X=q,g} T_{X,\rho\sigma}, \quad \text{with } T_{q,\rho\sigma} = \frac{1}{4} \sum_x \mathcal{O}_{3,\rho\sigma}(x), \\ T_{g,\rho\sigma} &= \sum_x \left[\mathcal{O}_{1,\rho\sigma}(x) - \frac{1}{4} \mathcal{O}_{2,\rho\sigma}(x) \right]. \end{aligned} \quad (3)$$

The total EMT is conserved and, hence, does not require renormalization, but the individual components $T_{X,\rho\sigma}$ and $\mathcal{O}_{i,\rho\sigma}$ must be renormalized and mix under renormalization.

Since renormalization and taking traces do not commute [7,12,60], they must be renormalized first before taking its trace, which is essential for a consistent comparison of different mass decomposition formulations. The QCD equations of motion (EOM) imply $2\langle \mathcal{O}_{5,\rho\sigma,f} \rangle = -\langle \mathcal{O}_{4,\rho\sigma,f} \rangle$, where angular brackets denote matrix elements in a physical state.

The gradient flow [46–49] alleviates the problem of power-divergent mixing of lattice EMT [26,27] by evolving lattice fields along a fictitious flow time t_f , which smooths ultraviolet fluctuations and renders composite operators finite at nonzero t_f . The flowed operators $\tilde{\mathcal{O}}_{i,\rho\sigma}(t_f, x)$ are nonperturbatively renormalized in the gradient-flow scheme at flow scale t_f , providing a common operator basis for evaluating distinct mass decomposition sum rules on equal footing. Their continuum limits are obtained by extrapolating results at several lattice spacings, $a \rightarrow 0$. The continuum-extrapolated $\tilde{\mathcal{O}}_{i,\rho\sigma}(t_f, x)$ are then matched to the $\overline{\text{MS}}$ scheme at scale μ using perturbatively known coefficients $\mathcal{O}_i^{\overline{\text{MS}}}(\mu) = M_{ij}(t_f, \mu) \tilde{\mathcal{O}}_j(t_f) + O(t_f)$, with $i, j = 1, 2, 3, 4$, valid for $2t_f\mu^2 \sim 1$ [50–52,61]. The residual $O(t_f)$ corrections are finally removed by extrapolating $t_f \rightarrow 0$.

Lattice-QCD computations—We use three 2 + 1-flavor highly improved staggered quark (HISQ) gauge ensembles generated by the HotQCD Collaboration [62–64]. The light sea-quark masses correspond to a pion mass of about 160 MeV, and the strange sea-quark mass is tuned to its physical value. The ensembles span three lattice spacings, $a = 0.06, 0.05$, and 0.04 fm, with respective lattice sizes 64×48^3 , 64^4 , and 64^4 . For the valence charm quark, we employ the setup in Refs. [65–80]—a tree-level tadpole-improved Wilson-clover action, where the gauge links entering the Dirac and clover operators are one-step hypercubic (HYP) smeared [81].

We compute the η_c ($\Gamma = \gamma_5$) and J/ψ ($\Gamma = \gamma_{1,2,3}$) two-point correlation functions, $C_\Gamma^{2\text{pt}}(t) = \sum_x \langle \mathcal{O}_\Gamma(\mathbf{x}, t) \mathcal{O}_\Gamma^\dagger(\mathbf{0}, 0) \rangle$, using Gaussian-smeared sources and sinks, and meson interpolating operators $\mathcal{O}_\Gamma(\mathbf{x}, t) = \bar{c}(\mathbf{x}, t) \Gamma c(\mathbf{x}, t)$. The valence charm-quark mass is tuned by requiring the resulting η_c and J/ψ masses to reproduce their physical values [82] at the percent level.

Lattice gauge fields are evolved using the Zeuthen-flow discretization [83], and the fermion flow is implemented with a covariant Laplacian consistent with the valence action. Both flows are integrated with a third-order Runge-Kutta scheme [84], yielding $O(a^2)$ flow-time discretization effects. Measurements are performed at $t_f = n\epsilon_f$ with $\epsilon_f = 5.1984 \times 10^{-4}$ fm², for integers n spanning the window used in our continuum and $t_f \rightarrow 0$ extrapolations. We use clover discretization to construct the flowed operator $\tilde{\mathcal{O}}_{1,\rho\sigma}(t_f)$, and its trace $\tilde{\mathcal{O}}_{2,\rho\sigma}(t_f)$, for the flowed gauge fields. For the flowed operator $\tilde{\mathcal{O}}_{3,\rho\sigma}(t_f)$, and its trace $\tilde{\mathcal{O}}_{4,\rho\sigma}(t_f)$, we employ “ringed” fermion fields [51] to cancel wave-function renormalization.

Vacuum-subtracted hadronic matrix elements of flowed operators, $\langle H|\tilde{\mathcal{O}}_{i,\rho\sigma}|H\rangle(\mu)$ with $H = \eta_c, J/\psi$, are extracted from the three-point function $C_\Gamma^{\rho\sigma}(t, \tau) = \sum_{\mathbf{x}, \mathbf{y}} \langle \mathcal{O}_\Gamma(\mathbf{x}, t) \tilde{\mathcal{O}}_{i,\rho\sigma}(\mathbf{y}, \tau) \mathcal{O}_\Gamma^\dagger(\mathbf{0}, 0) \rangle - \langle C_\Gamma^{\rho\sigma}(t) \rangle \sum_{\mathbf{y}} \langle \tilde{\mathcal{O}}_{i,\rho\sigma}(\mathbf{y}, \tau) \rangle$, using source-sink separations $t/a = 22, 24, 26, 28, 30$, and 32 (see Supplemental Material [85]). Keeping our focus on hadron mass decompositions, we consider only hadrons at rest. The quark-line-connected three-point functions for $\tilde{\mathcal{O}}_{3,\rho\sigma}$ and $\tilde{\mathcal{O}}_{4,\rho\sigma}$ are computed using standard sequential-source methods. While computing the disconnected three-point functions for $\tilde{\mathcal{O}}_{1,\rho\sigma}$ and $\tilde{\mathcal{O}}_{2,\rho\sigma}$, we use the all-mode averaging [86] technique for the corresponding two-point correlation functions to improve statistical precision. The quark-line-disconnected contributions of the charm quark to $\tilde{\mathcal{O}}_{3,\rho\sigma}$ and $\tilde{\mathcal{O}}_{4,\rho\sigma}$ are found to be negligible. We omit all light-flavor (up, down, and strange) contributions entering solely through quark-line-disconnected diagrams for the charmonia states. As we shall see later, all sum rules are satisfied without these contributions, indicating that any omitted light-flavor disconnected terms are not resolved at our present level of precision.

We fit the spectral decomposition of the ratio $R_\Gamma^{\rho\sigma}(t, \tau) = C_\Gamma^{\rho\sigma}(t, \tau)/C_\Gamma^{2\text{pt}}(t)$ at each t_f using two-exponential ansatz to obtain the EMT matrix elements of η_c and J/ψ in the limit $t \rightarrow \infty$. We also employ summation fits, where $R_\Gamma^{\rho\sigma}(t, \tau)$ are summed over the operator insertion time τ and extrapolated to $t \rightarrow \infty$ through a linear fit in $1/t$. These complementary methods yield consistent results across ensembles and flow times, providing confidence that excited-state effects due to finite source-sink separations of the three-point functions are under control.

Hadronic matrix elements of $\overline{\text{MS}}$ -renormalized EMT— These procedures provide hadronic matrix elements of component $\tilde{\mathcal{O}}_{i,\rho\sigma}$, nonperturbatively renormalized in the gradient-flow scheme at multiple flow times t_f and for three lattice spacings. To obtain matrix elements of $\tilde{\mathcal{O}}_{i,\rho\sigma}$ renormalized in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV, we perform three sequential steps: (i) continuum, $a \rightarrow 0$, extrapolations of the gradient-flow-renormalized $\tilde{\mathcal{O}}_{i,\rho\sigma}$ matrix elements at each t_f ; (ii) one- and two-loop perturbative scheme conversion of the continuum-extrapolated matrix elements to the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV for each t_f ; and (iii) zero-flow-time, $t_f \rightarrow 0$, extrapolations of the resulting $\overline{\text{MS}}$ -renormalized matrix elements.

At each t_f , the $a \rightarrow 0$ extrapolation is carried out by fitting $\langle H|\tilde{\mathcal{O}}_{i,\rho\sigma}|H\rangle(t_f, a) = \langle H|\tilde{\mathcal{O}}_{i,\rho\sigma}|H\rangle(t_f, 0) + a^2 X_{i,\rho\sigma}(t_f)$ using uncorrelated bootstrap samples from the three ensembles to determine $\langle H|\tilde{\mathcal{O}}_{i,\rho\sigma}|H\rangle(t_f, 0)$. For all operators and t_f values investigated, the linear a^2 behavior is well supported by the data. Representative examples are shown in Fig. 1 for $\sum_{k=1}^3 \langle \eta_c|\tilde{\mathcal{O}}_{1,kk}/3|\eta_c\rangle/(2M_{\eta_c}^2)$ (upper) and $\sum_{k=1}^3 \langle \eta_c|\tilde{\mathcal{O}}_{3,kk}/3|\eta_c\rangle/(2M_{\eta_c}^2)$ (lower) at $t_f = 5\epsilon_f$.

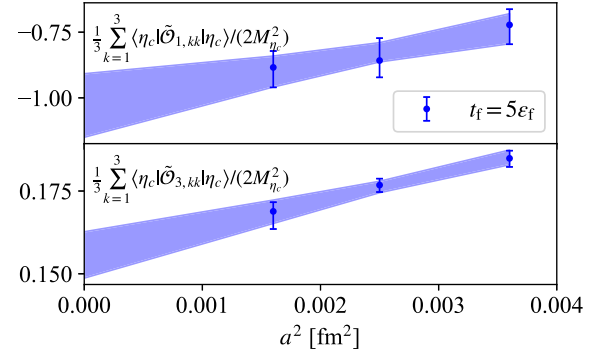


FIG. 1. Hadronic matrix elements of gradient-flow-renormalized EMT components are plotted as data points. The shaded bands indicate the continuum extrapolations.

The continuum-extrapolated matrix elements are then converted from the gradient-flow scheme to the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV using $\langle H|\mathcal{O}_i^{\overline{\text{MS}}}|H\rangle(\mu, t_f) = M_{ij}(t_f, \mu) \times \langle H|\tilde{\mathcal{O}}_j|H\rangle(t_f, 0)$, where $M_{ij}(t_f, \mu)$, with $i, j = 1, \dots, 4$, are the one- and two-loop perturbative matching coefficients [50–52,61] that incorporate both renormalization and operator mixing. Together with the QCD EOM, this yields $\overline{\text{MS}}$ -renormalized matrix elements of all five operators in Eq. (2). To ensure that the scheme conversion remains perturbatively reliable—the $\ln(2t_f\mu^2)$ terms in $M_{ij,\rho\sigma}$ remain small at $\mu = 2$ GeV—we restrict ourselves to a small interval $t_f = 0.00156 - 0.00416$ fm². As shown in Fig. 2, within this window the difference between one-loop and two-loop matched results stays small, indicating uncertainty due to higher-order perturbative corrections is under control.

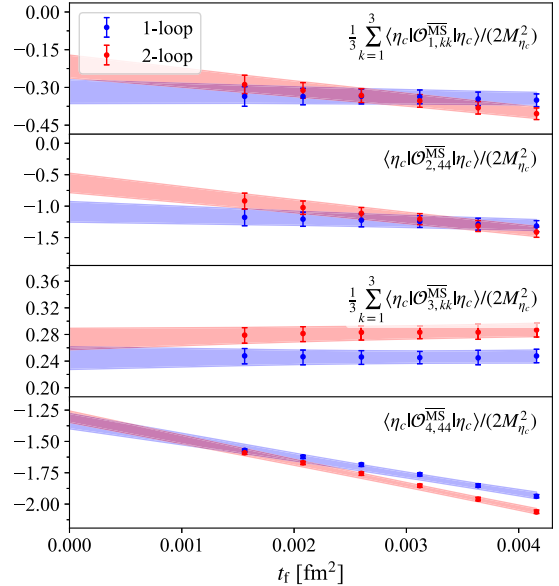


FIG. 2. Hadronic matrix elements of $\overline{\text{MS}}$ -renormalized EMT components at $\mu = 2$ GeV, obtained using one-loop (blue) and two-loop (red) perturbative matching, are plotted as data points. The shaded bands indicate the $t_f \rightarrow 0$ extrapolations.

Since gradient flow smooths the gluon and quark fields over an approximate radius $\sqrt{8t_f}$ [47–49], the small- t_f expansion of the flowed operators receives additional contributions from higher-dimensional operators, leading to residual t_f dependence of the matched matrix elements. To remove these contaminations, we perform a $t_f \rightarrow 0$ extrapolation of the matched matrix elements using $\langle H | \mathcal{O}_{i,\rho\sigma}^{\overline{\text{MS}}} | H \rangle(\mu, t_f) = \langle H | \mathcal{O}_{i,\rho\sigma}^{\overline{\text{MS}}} | H \rangle(\mu) + Y_{i,\rho\sigma} t_f$, with correlated fits to bootstrap samples over the $t_f = 0.00156 - 0.00416 \text{ fm}^2$ interval. As illustrated in Fig. 2, the data display a clear linear dependence in this range. Smaller flow times are excluded from the fit, since ultraviolet effects there are not yet regulated primarily by the flow, and the simple linear behavior in t_f is not clearly observed.

Gravitational form factors—From $\langle H | \mathcal{O}_{i,\rho\sigma}^{\overline{\text{MS}}} | H \rangle(\mu)$, we construct $\langle H | T_{X,\rho\sigma}^{\overline{\text{MS}}} | H \rangle(\mu)$ using Eq. (3), propagating correlated uncertainties using bootstrap samples.

From here on, we present all physical results in Minkowski space, with Euclidean lattice matrix elements converted accordingly.

For the pseudoscalar η_c meson, the rest-frame matrix elements of $\overline{\text{MS}}$ -renormalized EMT components [28,32],

$$\begin{aligned} \langle \eta_c | T_{X,00}^{\overline{\text{MS}}} | \eta_c \rangle &= 2M_{\eta_c}^2 [A_X(0) + \bar{C}_X(0)], \\ \sum_{k=1}^3 \langle \eta_c | T_{X,kk}^{\overline{\text{MS}}} | \eta_c \rangle &= -6M_{\eta_c}^2 \bar{C}_X(0), \end{aligned} \quad (4)$$

provide two independent relations to extract $\overline{\text{MS}}$ -renormalized GFFs $A_X(0)$ and $\bar{C}_X(0)$ at vanishing momentum transfer. The $A_X(0)$ gives the partonic momentum fraction and $\bar{C}_X(0)$ encodes the trace contribution.

For the vector J/ψ meson, the rest-frame matrix elements of $\overline{\text{MS}}$ -renormalized EMT components in the state J polarized along the γ_1 direction [32,87]:

$$\begin{aligned} \langle J | T_{X,00}^{\overline{\text{MS}}} | J \rangle &= 2M_{J/\psi}^2 \left[A_X(0) - \frac{1}{2} \bar{C}_X(0) + \frac{1}{4} \bar{f}_X(0) \right], \\ \langle J | T_{X,11}^{\overline{\text{MS}}} | J \rangle &= 2M_{J/\psi}^2 \left[\frac{1}{2} \bar{C}_X(0) + \frac{3}{4} \bar{f}_X(0) \right], \\ \sum_{k=2,3} \langle J | T_{X,kk}^{\overline{\text{MS}}} | J \rangle &= 4M_{J/\psi}^2 \left[\frac{1}{2} \bar{C}_X(0) - \frac{1}{4} \bar{f}_X(0) \right], \end{aligned} \quad (5)$$

determine three $\overline{\text{MS}}$ -renormalized GFFs, A_X , $\bar{C}_X$, and $\bar{f}_X$, at vanishing momentum transfer.

Table I presents the results of first direct lattice-QCD calculations of GFFs $\bar{C}$ and $\bar{f}$. We obtain $\bar{C}_c(0) + \bar{C}_g(0) = 0$ and, for the vector meson, $\bar{f}_c(0) + \bar{f}_g(0) = 0$ within uncertainties, validating the corresponding trace-anomaly sum rules [88]. We also find, within uncertainties, the longitudinal momentum sum rule for partons $A_c(0) + A_g(0) = 1$ for both η_c and J/ψ .

The σ term using QCD EOM—The contribution to the hadron mass due to explicit chiral-symmetry breaking for $m_f > 0$ is quantified by

$$\sigma_f = \frac{1}{2M_H} \langle H | \mathcal{O}_{5,00,f}^{\overline{\text{MS}}} | H \rangle = -\frac{1}{4M_H} \langle H | \mathcal{O}_{4,00,f}^{\overline{\text{MS}}} | H \rangle. \quad (6)$$

We utilize the QCD EOM for quark flavor f to eliminate $\mathcal{O}_{5,00,f}^{\overline{\text{MS}}}$ in favor of $\mathcal{O}_{4,00,f}^{\overline{\text{MS}}}$. Since $m_f > 0$ explicitly breaks the scale invariance of classical QCD, the σ term also contributes to the trace of QCD EMT in addition to anomaly generated through quantum corrections.

An example of $\overline{\text{MS}}$ -renormalized σ_c , obtained from $\langle H | \mathcal{O}_{4,00,c}^{\overline{\text{MS}}} | H \rangle$, is shown in Fig. 2, where it can be seen that the one- and two-loop matching give consistent results in the $t_f \rightarrow 0$ limit.

Hadron mass decompositions—Having determined all necessary $\overline{\text{MS}}$ -renormalized matrix elements, we directly validate the sum rules underlying several complementary decompositions of the hadron mass. The Hatta-Rajan-Tanaka (HRT) [7,89–91] decomposition based on the rest-frame matrix elements of the trace of $\overline{\text{MS}}$ -renormalized EMT is

$$M_H = \sum_{X=q,g} M_X^{\text{HRT}}, \quad \text{with } M_X^{\text{HRT}} = \frac{\langle H | (T_X^{\overline{\text{MS}}})^\rho_\rho | H \rangle}{2M_H}. \quad (7)$$

The Lorcé [11], Metz-Pasquini-Rodini (MPR) [45,92], and Ji [9,10,60,93] decompositions are all based on the rest-frame matrix element of $T_{00}^{\overline{\text{MS}}}$ but organize the quark and gluon contributions in different ways. The Lorcé’s decomposition in the $\overline{\text{MS}}$ scheme reads

$$M_H = \sum_{X=q,g} M_X^{\text{Lor}}, \quad \text{with } M_X^{\text{Lor}} = \frac{\langle H | T_{X,00}^{\overline{\text{MS}}} | H \rangle}{2M_H}. \quad (8)$$

The MPR decomposition explicitly separates out σ_f , giving

TABLE I. Gravitational form factors of η_c [Eq. (4)] and J/ψ [Eq. (5)] in the $\overline{\text{MS}}$ scheme and at the scale $\mu = 2 \text{ GeV}$.

	η_c		J/ψ		
	$A(0)$	$\bar{C}(0)$	$A(0)$	$\bar{C}(0)$	$\bar{f}(0)$
Gluon (g)	0.084(45)	0.058(18)	0.077(35)	−0.146(29)	−0.012(32)
Charm (c)	0.918(17)	−0.068(4)	0.926(34)	0.176(87)	0.023(5)
Total	1.000(46)	−0.010(18)	0.997(43)	0.030(30)	0.010(32)

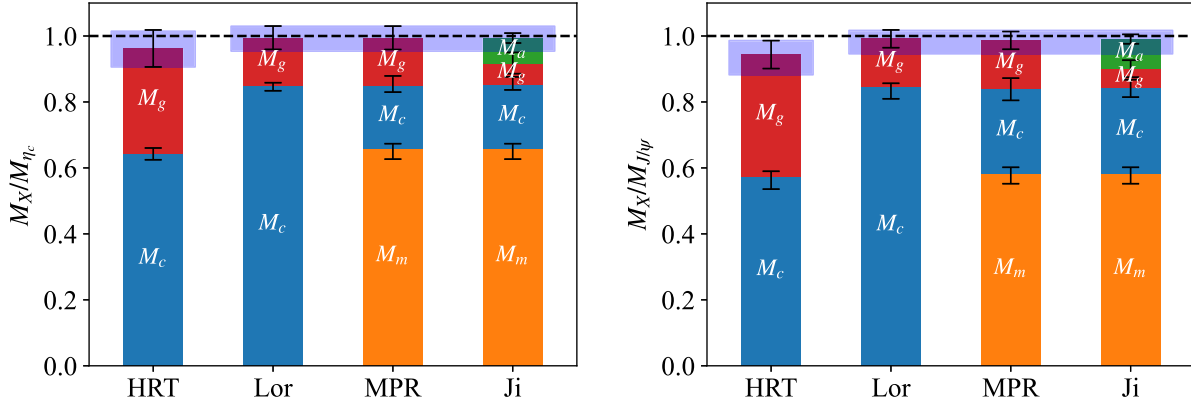


FIG. 3. Hatta-Rajan-Tanaka [HRT, Eq. (7)], Lorcé [Lor, Eq. (8)], Metz-Pasquini-Rodini [MPR, Eq. (9)], and Ji [Ji, Eq. (10)] decompositions of η_c (left panel) and J/ψ (right panel) masses. All contributions are presented at 2 GeV $\overline{\text{MS}}$ scale using two-loop gradient flow to $\overline{\text{MS}}$ matching. The black error bars indicate combined systematic and statistical uncertainty of each individual components. The blue error bands on top indicate combined systematic and statistical uncertainty of the sum over all components in each cases.

$$M_H = \sum_{X=q,g,m} M_X^{\text{MPR}}, \quad \text{with } M_m^{\text{MPR}} = \sum_f \sigma_f, \\ M_q^{\text{MPR}} = M_q^{\text{Lor}} - \sum_f \sigma_f, \quad M_g^{\text{MPR}} = M_g^{\text{Lor}}. \quad (9)$$

The Ji decomposition distinguishes the contributions from the traceless part of the EMT from the trace anomaly part, yielding

$$M_H = \sum_{X=q,g,m,a} M_X^{\text{Ji}}, \quad \text{with } M_m^{\text{Ji}} = M_m^{\text{MPR}}, \\ M_q^{\text{Ji}} = M_q^{\text{Lor}} - \frac{1}{4} M_q^{\text{HRT}} - \frac{3}{4} M_m^{\text{MPR}}, \\ M_g^{\text{Ji}} = M_g^{\text{Lor}} - \frac{1}{4} M_g^{\text{HRT}}, \\ M_a^{\text{Ji}} = \frac{1}{4} [M_q^{\text{HRT}} + M_g^{\text{HRT}} - M_m^{\text{MPR}}]. \quad (10)$$

Figure 3 presents HRT, Lorcé, MPR, and Ji mass decompositions, all at 2 GeV $\overline{\text{MS}}$ scale, of η_c (left panel) and J/ψ (right panel). For the quark sector, we include only the charm flavor and find all mass rules are satisfied within uncertainty, demonstrating the mutual consistency of these distinct decomposition schemes when evaluated in the same renormalization framework. This suggests that omitted light-flavor disconnected contributions are smaller than our current total precision, although a dedicated calculation of these terms is left to future work. The dominant contributions to $M_{\eta_c} = 2984.1$ MeV and $M_{J/\psi} = 3096.9$ MeV come from the charm quark: $M_c^{\text{HRT}} = 1918^{+53}_{-54}$ MeV, $M_c^{\text{Lor}} = 2536^{+25}_{-48}$ MeV, and $M_c^{\text{Ji}} + M_m^{\text{Ji}} = 2542^{+26}_{-40}$ MeV for η_c and $M_c^{\text{HRT}} = 1776^{+51}_{-117}$ MeV, $M_c^{\text{Lor}} = 2617^{+35}_{-110}$ MeV, and $M_c^{\text{Ji}} + M_m^{\text{Ji}} = 2614^{+40}_{-78}$ MeV for J/ψ . Gluons also provide sizable contributions: $M_g^{\text{HRT}} = 953^{+168}_{-167}$ MeV, $M_g^{\text{Lor}} = 426^{+111}_{-100}$ MeV, and $M_g^{\text{Ji}} = 188^{+102}_{-91}$ MeV for η_c and $M_g^{\text{HRT}} = 1144^{+134}_{-129}$ MeV,

$M_g^{\text{Lor}} = 455^{+82}_{-85}$ MeV, and $M_g^{\text{Ji}} = 179^{+78}_{-82}$ MeV for J/ψ . These first-principle QCD calculations demonstrate QCD confinement not only provides substantial gluon-energy contributions, but also generates non-negligible trace-anomaly contributions— $M_a^{\text{Ji}} = 231^{+45}_{-44}$ MeV for η_c and $M_a^{\text{Ji}} = 277^{+43}_{-45}$ MeV for J/ψ —to the hadron masses, even for deeply bound $c\bar{c}$ states.

Conclusions—In this Letter, we have presented the first direct lattice-QCD validation of sum rules associated with quark-gluon decomposition of hadron mass. By constructing the QCD energy-momentum tensor nonperturbatively in a gradient-flow scheme, we achieve a formulation that controls operator mixing and avoids power-divergent subtractions, enabling a well-defined continuum limit extrapolation, followed by two-loop matching to the $\overline{\text{MS}}$ scheme and zero-flow-time extrapolations. This establishes a common renormalization framework in which multiple mass decomposition sum rules can be verified simultaneously.

Applying this framework to the η_c and J/ψ charmonia, we demonstrate the consistency of energy-density-based and trace-based decompositions and quantify substantial contributions from gluonic energy and the QCD trace anomaly to heavy quark bound-state masses. The first lattice determination of the gravitational form factor $\bar{C}$ further illustrates the scope of the approach. Residual effects from finite volume, the partial quenching from the omission of a dynamical charm quark in the sea, and neglected light-flavor disconnected contributions are expected to be subleading at the present level of precision but should be quantified more systematically in future higher-precision studies.

Beyond charmonium, the methodology developed here provides a general foundation for first-principles studies of hadron mass, spin structure, and gravitational form factors. In particular, it enables access to components of the energy-momentum tensor, such as trace and higher-twist contributions, that are challenging to constrain experimentally,

thereby providing essential nonperturbative input complementary to ongoing and future programs at the Thomas Jefferson National Accelerator Facility and the future Electron-Ion Collider.

Work used GRID [94], GPT (Grid Python Toolkit) [95], and RunDec [96] software packages.

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