

Black Hole States in Quantum Spin Chains

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We define a black hole state in a spin chain by studying thermal correlators in holography. Focusing on the Heisenberg model we investigate the thermal and complexity properties of the black hole state by evaluating its entanglement entropy, emptiness formation probability, and Krylov complexity. The entanglement entropy grows logarithmically with effective central charge $c \simeq 5.2$. We also find evidence for thermalization at infinite temperature.

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The black hole state—At the heart of the gauge-gravity duality lies a remarkable identification of a thermal ensemble in field theory with a black hole in the dual gravitational description [1]. The infall of a particle into the black hole permits local operators to exist in isolation and gives rise to their thermal one-point functions that thereby serve as sensitive probes of the black-hole interior [2].

In a more refined holographic picture, particles are replaced by strings, which in turn admit an effective spin-chain description. The key advantage of the latter is manifest integrability [3]. The one-point functions (in other setups where they arise) map to overlaps of the spin-chain boundary states and can be efficiently studied by integrability methods [4,5] (see Refs. [6,7] for a review). Since black holes are chaotic systems, we expect the integrability to be broken in the thermal ensemble, and we will later confirm this expectation.

The resulting boundary state nonetheless exhibits remarkable features which we believe are of interest on their own. Our ultimate goal is to study boundary states that arise in AdS/CFT, but this requires extra formalism while succinct features can be illustrated in a simpler setting of a spin-1/2 Heisenberg model,

$$H = \sum_{\ell=1}^L (\boldsymbol{\sigma}_{\ell} \cdot \boldsymbol{\sigma}_{\ell+1} + \lambda \boldsymbol{\sigma}_{\ell} \cdot \boldsymbol{\sigma}_{\ell+2}). \quad (1)$$

We will construct what we call the “black-hole states” in this model using simple plausibility arguments and later show that their direct counterparts govern thermal one-point functions in AdS/CFT. Other approaches to defining

black-hole states based on discrete models and thermal correlation functions can be found in [8], where the black hole emerges from the 2D Ising model, and in [9–11] where it occurs through a discretization of hyperbolic space.

Since the spin chain of AdS/CFT is integrable, we mostly concentrate on the integrable case without the next-to-nearest interactions. Nonzero λ breaks integrability, making the model fully chaotic for $\lambda \gtrsim 0.5$. We will use this to see how much thermalization of the black-hole state depends on the chaotic nature of the Hamiltonian.

The black-hole state should be a spin singlet (black holes have no hair) and it should be sufficiently generic to reflect black holes’ chaotic nature. In order to systematically enumerate singlets we invoke well-known results from the quantum-mechanical theory of valence, originally due to Rumer [12,13].

The singlet formed by two spins is their antisymmetric combination: $\epsilon_{\alpha_1\alpha_2}$, where $\alpha_i \in \{\uparrow, \downarrow\}$. Four spins can be split in pairs in three possible ways, but only two of them are linearly independent because of the identity $\epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_3\alpha_4} + \epsilon_{\alpha_1\alpha_4}\epsilon_{\alpha_2\alpha_3} = \epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}$ [12] illustrated in Fig. 1(a). Rumer’s identity can be used to eliminate crossings for any number of spins. The noncrossing (“kreuzungslose”) pairings are linearly independent and form a complete albeit non-orthogonal basis [13]. The number of pairings, for L spins, is the Catalan number $C_{L/2}$, and this is precisely the dimension of the singlet subspace for the spin chain of length L [14].

We define the black-hole state as the sum of all non-crossing pairings taken with equal weights. The state can be built recursively from the relation

$$|\text{BH}_L\rangle = \sum_{n=1}^{L/2} \sum_{\alpha\beta} \epsilon_{\alpha\beta} |\alpha\rangle \otimes |\text{BH}_{2n-2}\rangle \otimes |\beta\rangle \otimes |\text{BH}_{L-2n}\rangle, \quad (2)$$

as illustrated in Fig. 1(b). Catalan numbers, we mention in passing, satisfy pictorially equivalent recursion.

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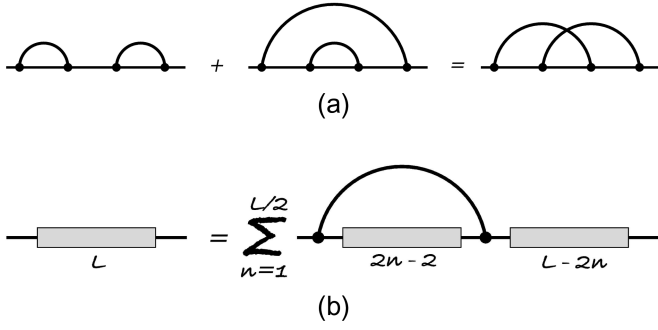


FIG. 1. (a) Rumer relation. (b) Recursion relation defining the black-hole states.

The noncrossing pairings are identical to 't Hooft's planar diagrams, and the wave function of the black-hole state can be alternatively computed from a matrix integral,

$$\text{BH}^{\alpha_1 \dots \alpha_L} = \lim_{N \rightarrow \infty} \int d\Psi^\dagger d\Psi^\downarrow e^{-\text{tr} \bar{\Psi} \Psi} \text{tr} \Psi^{\alpha_1} \dots \Psi^{\alpha_L}. \quad (3)$$

The integration here is over two $N \times N$ Hermitian matrices with Grassmann entries. The bar denotes Majorana conjugation,

$$\bar{\Psi}_\beta = \Psi^\alpha \varepsilon_{\alpha\beta}. \quad (4)$$

In the exponent therefore stands the standard action for a zero-dimensional Majorana fermion. Integration has to be Grassmann; bosonic matrices would be incompatible with the cyclicity of the trace and antisymmetry of $\varepsilon_{\alpha\beta}$. The Wick contractions among Ψ^{α_ℓ} produce all possible spin pairings and in the large- N limit only planar contractions survive. The recursion relation is simply the Schwinger-Dyson equation of the matrix integral.

The black-hole state is manifestly $\text{SU}(2)$ -invariant, changes sign under cyclic permutations (has momentum π), and covers evenly the singlet subspace of the spin chain's Hilbert space. In some sense it is just the familiar matrix product state but with bond matrices drawn at random from the Gaussian ensemble. Random matrix product states have been introduced recently in a quite similar context [15].

The Heisenberg model is integrable, its Hamiltonian commutes with an infinite number of extra conserved charges starting with $Q_3 = i \sum_\ell \sigma_\ell \cdot [\sigma_{\ell+1} \times \sigma_{\ell+2}]$. A boundary state is compatible with this structure if annihilated by Q_3 (and higher odd charges as well) [16,17]. Integrable boundary states have remarkable mathematical properties, in particular, their overlaps with the physical eigenstates of the Hamiltonian can be systematically computed by Bethe ansatz [18–20]. The black-hole state, however, breaks integrability. It is easy to check that $Q_3|\text{BH}\rangle \neq 0$ at length six and higher. We therefore expect that the black-hole state exhibits some chaotic features even if the underlying Hamiltonian is completely integrable.

We will probe the black-hole state with the standard diagnostics of quantum chaos: entanglement entropy, Krylov complexity, and eigenvalue thermalization. For comparison, we also consider a singlet valence-bond state (VBS), which is known to preserve integrability [21],

$$\text{VBS}^{\alpha_1 \dots \alpha_L} = \varepsilon^{\alpha_1 \alpha_2} \dots \varepsilon^{\alpha_{L-1} \alpha_L}. \quad (5)$$

It has the same quantum numbers as the black-hole state upon averaging over translations by one site: $\langle \text{VBS}_\pi | = \langle \text{VBS} | (1 - T)$. As we shall see it has very different entanglement properties and much lower Krylov complexity.

Entanglement entropy—The Von Neumann entanglement entropy can be defined for a state of a quantum system whose Hilbert space factorizes into a tensor product,

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B. \quad (6)$$

If the state of the total quantum system is described by the density matrix ρ on $\mathcal{H}$, one defines the reduced density matrix corresponding to the subspace $\mathcal{H}_A$ as

$$\rho_A = \text{Tr}_B \rho. \quad (7)$$

The entanglement entropy of subsystem A with respect to B is then

$$S(A) = -\text{Tr} \rho_A \log \rho_A. \quad (8)$$

Even if the total system is in a pure state $\rho = |\psi\rangle\langle\psi|$, entanglement between subsystems generates an entropy.

A simple recipe for computing the entanglement entropy for a (normalized) spin-chain state, $|\psi\rangle$, is to express the state in a product basis as follows (see, e.g., Ref. [22]):

$$|\psi\rangle = \sum_{i,j} M_{ij} |i\rangle_A |j\rangle_B, \quad (9)$$

where the $|i\rangle_A$ and the $|j\rangle_B$ constitute an orthonormal basis of, respectively, the subspace A and the subspace B . Then it holds that

$$\rho_A = M M^\dagger \quad (10)$$

and

$$S(A) = -\sum_k \lambda_k \log \lambda_k \quad (11)$$

for the nonvanishing eigenvalues λ_k of $M M^\dagger$.

We will consider the entanglement entropy of a subspace of the spin chain of length ℓ relative to the full chain of length L , denoted in the following as $S(\ell)$, for, respectively, the black-hole state and the integrable valence-bond state.

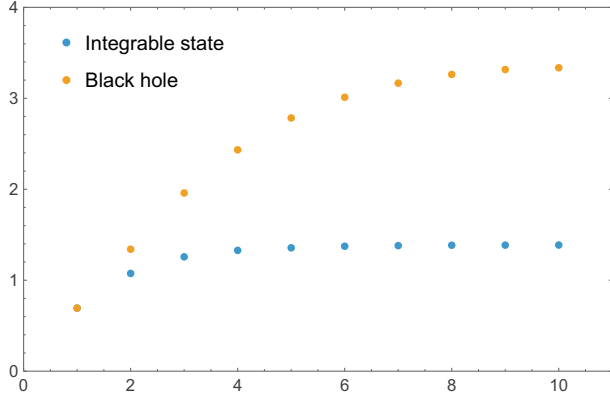


FIG. 2. The entanglement entropy as a function of ℓ for the integrable and black-hole states at $L = 20$.

In a critical (gapless) system, the entanglement entropy of the ground state can be computed by 2D conformal field theory (CFT) methods and for periodic boundary conditions is expected to behave as [23]

$$S(\ell) = \frac{c}{3} \log \left(\frac{L}{\pi} \sin \frac{\pi \ell}{L} \right) + c'_1, \quad (12)$$

where c is the central charge of the CFT and c'_1 is a nonuniversal numerical constant. This is in contrast to a gapped ground state, which will have constant entanglement entropy and thermal states for which the entanglement entropy grows linearly with ℓ [24]. The behavior in (12) was confirmed for the ground state of the critical Heisenberg $\text{XXX}_{1/2}$ chain [25–27]. The central charge in this case equals one: $c = 1$.

In Fig. 2 we show the entanglement entropy $S(\ell)$ as a function of ℓ for, respectively, the integrable and the black-hole state for $L = 20$. We remark that $S(\ell)$ for $\ell = 1$ is always equal to $\log 2$ for the states that we consider due to their singlet nature. Furthermore, we notice that the integrable two-site singlet product state has an entanglement entropy that saturates at $\log 4$ originating from the fact that only two singlet bonds are broken when we cut out an interval of length ℓ . In contrast the entropy of the black-hole state continues to grow in almost perfect agreement with the CFT prediction (12).

In Fig. 3 we fit the entanglement entropy for the black-hole state to the functional form of Eq. (12). The fit is remarkably accurate and produces the following values for c and c'_1 :

$$c = 5.2, \quad c'_1 = 0.164. \quad (13)$$

The physical interpretation of this result is unclear to us. We do not know if it is meaningful to assign a CFT to a state, and what this CFT can be.

Thermalization—One possible formulation of the eigenvalue thermalization hypothesis posits that time averages in

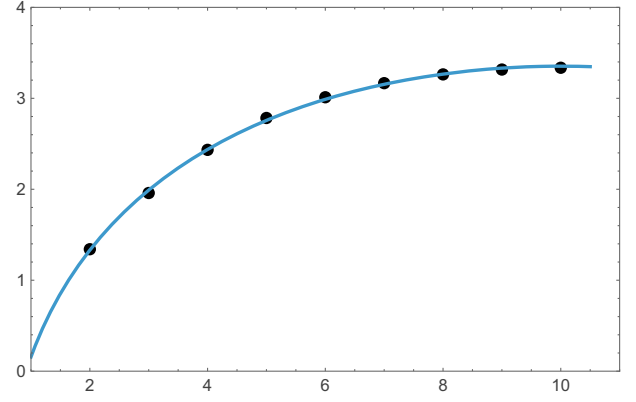


FIG. 3. The von Neumann entanglement entropy as a function of ℓ for the black-hole state for $L = 20$ and the corresponding fit to the functional form given in Eq. (12).

a sufficiently generic state thermalize [28–30],

$$\overline{\langle \Psi | A(t) | \Psi \rangle} = \text{tr} \rho A, \quad \rho = \frac{1}{Z} e^{-\beta H}. \quad (14)$$

The time average, in absence of accidental degeneracies, reduces to the statistical average in the diagonal ensemble,

$$\overline{\langle \Psi | A(t) | \Psi \rangle} = \sum_n |\langle \Psi | n \rangle|^2 \langle n | A | n \rangle. \quad (15)$$

Of course the diagonal ensemble does not literally coincide with the grand-canonical one, but course-graining in the infinite-volume limit converts, it is believed, one to the other provided the initial state $|\Psi\rangle$ is sufficiently generic [31]. The grand-canonical ensemble of an integrable model contains infinitely many chemical potentials, one for each conserved charge, and the eigenvalue thermalization hypothesis does not imply such a remarkable reduction in the degrees of freedom as for chaotic systems. Surprisingly, the black-hole state seems to thermalize without any dependence on the higher conserved charges, not even on the Hamiltonian itself, and moreover this happens for integrable as well as nonintegrable Hamiltonians.

We will compare the diagonal ensemble with the infinite-temperature one. The density matrix of the latter is the projector onto the singlet subspace,

$$\rho_\infty = \frac{1}{C_{L/2}} \Pi_s, \quad \Pi_s = \int_{\text{SU}(2)} dg g \otimes \dots \otimes g, \quad (16)$$

where dg is the Haar measure on the $\text{SU}(2)$ group manifold. The black-hole state obviously cannot cover the whole Hilbert space of the spin chain, having overlap only with singlet eigenstates.

To probe the diagonal ensemble we study the emptiness formation probability (EFP) [32],

$$P(n) = \prod_{\ell=1}^n \frac{1 + \sigma_{\ell}^3}{2}, \quad (17)$$

which projects n consecutive spins onto the up state. The expectation value of EFP decays exponentially with the length of the string: $\langle P(n) \rangle \sim e^{-fn}$ with the exponent given by the free energy density [33–35]. The latter is temperature-dependent, making EFP an ideal probe of thermalization.

It is easy to calculate the EFP in the infinite-temperature ensemble (16). In the parametrization $g = n_0 + i\sigma^i n_i$ with a unit four-vector n_{μ} the group integrals become averages over the round three-dimensional sphere—for instance,

$$\text{tr} \Pi_s = \frac{1}{2\pi^2} \int d^3 n (2n_0)^L = C_{L/2}, \quad (18)$$

reconfirming that the total number of singlets is the Catalan number. Here, we have taken into account that $\text{tr} g = 2n_0$. By the same token, $\text{tr} g(1 + \sigma^3)/2 = n_0 + in_3$, and the EFP in the infinite-temperature ensemble is

$$\langle P(n) \rangle_{\infty} = \frac{1}{2\pi^2 C_{L/2}} \int d^3 n (2n_0)^{L-n} (n_0 + in_3)^n \quad (19)$$

$$= \frac{(L-n)!(L/2)!}{L!(L/2-n)!} \stackrel{L \rightarrow \infty}{\simeq} e^{-f(n/L)L} \quad (20)$$

with

$$f(x) = x \ln 2 + \left(\frac{1}{2} - x\right) \ln(1-2x) - (1-x) \ln(1-x). \quad (21)$$

In Fig. 4 we compare this prediction with the numerical results for the diagonal ensemble of the black-hole state. The agreement is reasonably good, given relatively small length of the spin chain. Interestingly, the results do not differ much for integrable and nonintegrable Hamiltonians: in the lower plot we compare the models with $\lambda = 0$ and $\lambda = 1$. The former is integrable while the latter is fully chaotic by the standard level-repulsion test, nonetheless the difference is minuscule, within the accuracy set by finite-size effects.

Krylov complexity—Krylov complexity measures to what extent an operator or a state explores a system’s Hilbert space under time evolution, i.e., when repeatedly acted upon by the Hamiltonian [36,37]. An extensive recent review of the virtues and applicability of the concept can be found in [38,39]. The Krylov complexity of a state, also known as the spread complexity, depends both on the Hamiltonian and on the state itself [39–45]. We will be interested in studying the Krylov complexity of the black-hole state (2). For comparison we also calculated the Krylov complexity of the integrable valence-bond state (5). Krylov complexity of coherent states of relevance for the semiclassical limit of AdS/CFT integrable models was

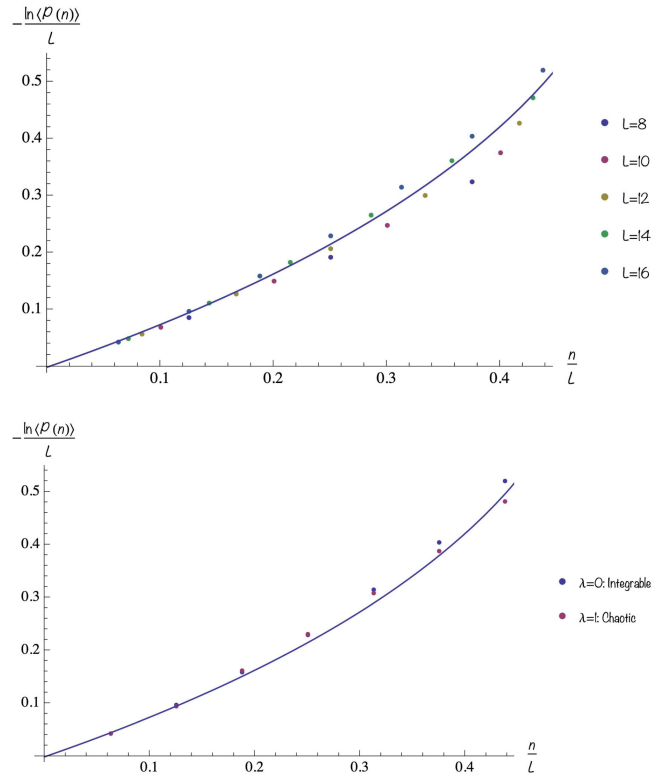


FIG. 4. The emptiness formation probability in the diagonal ensemble with Boltzmann weights $\langle \text{BH} | n \rangle^2$, compared to the statistical average at infinite temperature: the solid line is (21). The upper plot is for the Heisenberg Hamiltonian, $\lambda = 0$ in (1). In the lower plot the Heisenberg model is compared to a non-integrable case with $\lambda = 1$ for the spin chain of length 16.

discussed in [46]. Furthermore, holographic Krylov complexity was argued to be directly extractable from string motion in anti-de Sitter [47].

Starting from a given state $|\psi_0\rangle$ one defines the Krylov space of order K as the space spanned by the states $\{|\Psi_0\rangle, H|\Psi_0\rangle, H^2|\Psi_0\rangle, \dots, H^{K-1}|\Psi_0\rangle\}$. Using the Lanczos algorithm one builds from this set an orthonormal basis $\{|\psi_0\rangle, |\psi_1\rangle, \dots, |\psi_{K-1}\rangle\}$, and in the same process generates an expression for the Hamiltonian, in terms of Lanczos coefficients,

$$H = \begin{pmatrix} a_0 & b_1 & & & \\ b_1 & a_1 & b_2 & & \\ & b_2 & a_2 & b_3 & \\ & & b_3 & a_3 & \ddots \\ & & & \ddots & \ddots & b_{K-1} \\ & & & & b_{K-1} & a_{K-1} \end{pmatrix}. \quad (22)$$

Subsequently one studies the time evolution of $|\psi_0\rangle$ in this new basis,

$$|\psi(t)\rangle = \sum_{n=0}^{K-1} \psi_n(t) |\psi_n\rangle, \quad |\psi(0)\rangle = |\psi_0\rangle. \quad (23)$$

The Schrödinger equation for $|\psi(t)\rangle$ amounts to the following set of coupled differential equations for the functions $\psi_n(t)$:

$$i \frac{\partial}{\partial t} \psi_n(t) = b_n \psi_{n-1}(t) + a_n \psi_n(t) + b_{n+1} \psi_{n+1}(t), \quad (24)$$

with the understanding that $\psi_{-1}(t) = \psi_K(t) = 0$ and with the initial condition

$$\psi_n(t)|_{t=0} = \delta_{n,0}. \quad (25)$$

Introducing a Krylov space position operator $\hat{n}$ as

$$\hat{n} = \sum_{n=1}^{K-1} |\psi_n\rangle \langle \psi_n|, \quad (26)$$

the Krylov complexity (or the spread complexity) of the initial state is defined as

$$C_K(t) = \langle \psi(t) | \hat{n} | \psi(t) \rangle = \sum_{n=0}^{K-1} n |\psi_n(t)|^2. \quad (27)$$

A more elaborate version can be found in [37]. Time averaging is often invoked to get rid of short-scale oscillations,

$$\overline{C_K}(t) = \int_0^t C_K(s) ds / t. \quad (28)$$

In Fig. 5 we show the time averaged Krylov complexity of the black-hole state compared to that of the integrable two-site singlet, for $L = 16$. We remark that the part of the spin-chain Hilbert space available for the black-hole state amounts to 69 parity even states out of the 1430 states with noncrossing pairings for $L = 16$. While the limitation on the available Hilbert space does not allow us to make strong conclusions and calls for more investigations, we do observe that the complexity of the black-hole state grows more rapidly and saturates and has a higher value compared to the integrable state. The oscillations, not shown in the plot, are also more pronounced for the integrable state.

Thermal one-point functions—We finally return to our original motivation of studying thermal one-point functions in AdS/CFT. We consider the full set of scalar operators in the $\mathcal{N} = 4$ super-Yang-Mills theory,

$$\mathcal{O} = \Psi^{i_1 \dots i_L} \text{tr} \Phi_{i_1} \dots \Phi_{i_L}, \quad (29)$$

where $i_1, \dots, i_L \in \{1, \dots, 6\}$. The one-loop dilatation operator acting on these states can be identified with the

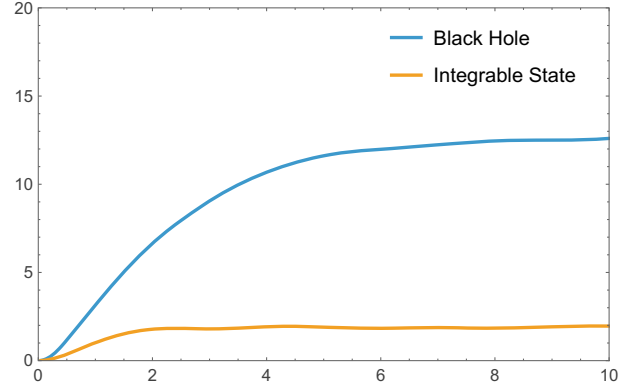


FIG. 5. The time averaged Krylov complexity for, respectively, the integrable and the black-hole state for $L = 16$.

Hamiltonian of an $\text{SO}(6)$ spin chain: $H = \sum_{\ell} h_{\ell, \ell+1}$ with $h = 2 - 2P + K$, where P and K are the permutation and trace operators [3], and this Hamiltonian is integrable.

To compute the one-point functions in the thermal ensemble, to the leading order in perturbation theory, we need to contract identical fields pairwise with the thermal propagator

$$D_{\beta}(x) = \frac{1}{4\pi^2} \sum_{n=-\infty}^{+\infty} \frac{1}{(t + n\beta)^2 + \mathbf{x}^2}, \quad (30)$$

evaluated at coincident points. Thermal effects are encoded in the difference $D_{\beta} - D_{\infty}$, which results in omitting the $n = 0$ term. Each bubble then contributes

$$\frac{1}{4\pi^2} \sum_{n \neq 0} \frac{1}{n^2 \beta^2} = \frac{1}{12\beta^2}. \quad (31)$$

The full planar contribution to the one-point function is accounted for by the following spin-chain overlap:

$$\langle \mathcal{O} \rangle = \frac{1}{\sqrt{L}} \left(\frac{\pi T}{\sqrt{3}} \right)^L \frac{\langle \text{BH} | \Psi \rangle}{\langle \Psi | \Psi \rangle^{\frac{1}{2}}}, \quad (32)$$

where the prefactor accounts for the unit normalization of the two-point function. The boundary state is the direct counterpart of the black-hole state that we have studied in the Heisenberg model,

$$\text{BH}^{i_1 \dots i_L} = \lim_{N \rightarrow \infty} \int d\phi^i e^{-\text{tr} \phi^i \phi^i} \text{tr} \phi^{i_1} \dots \phi^{i_L}. \quad (33)$$

The integration here is over six Hermitian $N \times N$ matrices with the usual numerical entries. The state can be also constructed from a recursion relation,

$$|\text{BH}_L\rangle = \sum_{n=1}^{L/2} \sum_{i=1}^6 |i\rangle \otimes |\text{BH}_{2n-2}\rangle \otimes |i\rangle \otimes |\text{BH}_{L-2n}\rangle. \quad (34)$$

This state shares many features with its $SU(2)$ counterpart. In particular, it breaks integrability: one can check that $Q_3|BH\rangle \neq 0$ for $Q_3 = i\sum_{\ell}[h_{\ell,\ell+1}, h_{\ell+1,\ell+2}]$. In contradistinction to the Heisenberg model, the planar contractions do not constitute a full basis of singlets for the $SO(6)$ spin chain, as the Rumer identity does not hold for $SO(6)$ spins.

Conclusion—We have introduced the concept of a black-hole state in a quantum spin chain motivated by thermal correlators in holography. As an initial step we concentrated on the black-hole states in the Heisenberg spin chain and provided evidence that they thermalize at infinite temperature. In addition, we demonstrated that their entanglement entropy grows logarithmically with length as is typical for critical, gapless systems. It would be important to develop a systematic explanation for this observation.

The black-hole state that arises directly from thermal correlators in holography is also not integrable. We leave a comprehensive investigation of its thermal and complexity properties to future work. Besides giving information about the infall of a particle into the black hole [2] and thermal phase transitions [1], thermal one-point functions have an important role to play as input to the thermal bootstrap program [48–50], not least in relation to the investigation of two-point correlators as probes sensitive to the internal structure of the space-time singularity [51–53].

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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