

Robust Detection of an Entanglement Transition in the Projective Transverse-Field Ising Model

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We propose a scalable and noise-resilient protocol for the detection of the entanglement transition in a projective version of the transverse-field Ising model. Entanglement transitions are experimentally difficult to observe due to the inherent randomness of projective measurements and noise in large-scale experimental settings. Our approach combines error correction algorithms with classical shadow tomography to overcome both problems. This allows for experimentally accessible upper and lower bounds on the entanglement transition without postselection or full state tomography. These bounds remain robust under noise and their sharpness is a measure of the noise rate.

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Introduction—The creation of entanglement is typically associated with multiqubit unitaries, whereas projective single-qubit measurements cause its destruction. Remarkably, the study of *random quantum circuits* revealed that this competition can drive nonequilibrium phase transitions between phases with volume-law and area-law entanglement [1–5]. Similar transitions can also occur between different area-law phases, and can be driven by the competition of projective measurements only [6–8]. Entanglement transitions are characterized by changes in the scaling of pure-state entanglement entropies averaged over many random quantum circuits [2,6,9]—while there is no signature in the density matrix of the ensemble of trajectories itself. This makes entanglement transitions experimentally elusive [10–12]: measuring entanglement requires multiple copies of a quantum state, whereas preparing such copies with random quantum circuits is exponentially unlikely due to the inherent randomness of measurements. This *postselection problem* bars straightforward observations of entanglement transitions [11,13]. In this Letter, we address this issue by combining shadow tomography [14] with tailored error correction algorithms [15] to construct protocols for probing a particular entanglement transition reliably in large, noisy systems.

Various strategies have been suggested to either alleviate or bypass the postselection problem, each with benefits and downsides. This includes unitary “stand-ins” that map

to projective time evolutions [12,16,17], quantum steering via unitary feedback [10,18,19], signatures in more accessible quantities like cross entropies [20–22], studying special nonlocal models [23–25], and exploiting emergent quantum error correction abilities [15,26–30]. Following a different route, shadow tomography [31–33] has been proposed as a tool to measure upper and lower bounds on entanglement entropies [14,34]. Despite this progress, experimentally feasible protocols for probing entanglement transitions that are both scalable and noise-resilient are still lacking.

Here, we combine efficient error correction algorithms [15] with classical shadow tomography [14] to devise explicit, platform-agnostic, robust protocols for measuring upper and lower bounds on entanglement transitions. We motivate and develop these protocols for the *projective transverse-field Ising model* [6,7], which features a transition between two area-law phases and is both experimentally accessible and efficiently simulatable. We show how decoding quantum information can be used to measure a *lower bound* on the entanglement transition, and then construct a protocol based on error correction and shadow tomography [14] to measure an *upper bound* as well. In combination, both procedures yield an experimentally accessible, robust *interval* that constrains the entanglement transition and probes the noise rate.

Setting—The projective transverse-field Ising model (PTIM) [6,7] is defined on an open chain of L qubits that are initialized in a state $|\Psi_0\rangle$. Let X_i and Z_i denote Pauli matrices acting on qubit $i \in \{1, \dots, L\}$. A *trajectory* $|\Psi_t\rangle$ of the PTIM is generated by a discrete time evolution: let the system be in state $|\Psi_t\rangle$ at time step t . The next state $|\Psi_{t+1}\rangle$ is obtained by first looping over all sites and measuring with probability $p \in [0, 1]$ the operators $E_i \equiv X_i$ projectively, and then looping over all pairs of adjacent sites

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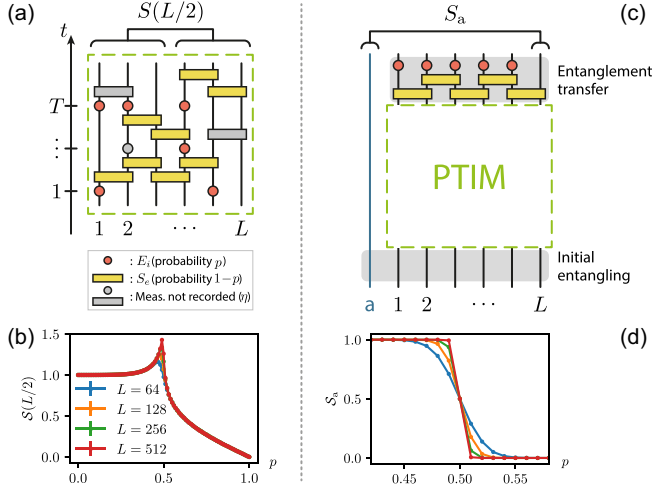


FIG. 1. Entanglement transition of the PTIM. (a) Exemplary trajectory of a PTIM. Circles (boxes) correspond to projective $E_i \sim X$ ($S_e \sim ZZ$) measurements applied with probability p ($1-p$) per site and time step. Noise manifests as unrecorded measurements (gray circles and boxes). (b) The PTIM features a transition in the trajectory-averaged half-system EE $S(L/2)$ in the steady state for $T \rightarrow \infty$ (here, $T \gtrsim 10L$) at $p_c = 0.5$ between two area-law phases. (c) Alternative protocol to detect the transition utilizing an initially fully entangled ancilla qubit. (d) The transition in the survival probability of an initial Bell cluster is measured by the EE S_a of the ancilla for $L = T$. All simulations in this Letter are based on 10^5 trajectories.

$e = (i, i+1)$ (edges) and measuring $S_e \equiv Z_i Z_{i+1}$ with probability $1-p$. This is repeated for T time steps to produce the final state $|\Psi_T\rangle$. The measurements of a trajectory are recorded, where $M^p \equiv (E^p, S^p)$ denotes the measurement *pattern* (locations and times of measurements) and $M^r \equiv (E^r, S^r)$ contains their *results*. In an experiment, we expect the sampling of a trajectory to be error-prone. Here, we assume that errors manifest as unrecorded measurements: with probability $\eta \in [0, 1]$ (noise rate), projective measurements E_i and S_e that were applied are *not recorded* in the history $M^{p,r}$. Hence the generated ensemble of trajectories depends on the initial state $|\Psi_0\rangle$, the spacetime dimensions L and T , and the measurement probability p , while the noise rate η only affects the measurement record $M^{p,r}$. A *sample* $s \equiv (|\Psi_T\rangle, M^{p,r})$ is then given by the final state and the measurement record. An exemplary trajectory of a noisy PTIM is shown in Fig. 1(a).

Consider a subsystem $A \subseteq \{1, \dots, L\}$ of $|A|$ qubits. For each sample s , we define the reduced density matrix $\rho_{s,A} := \text{Tr}_{\bar{A}}[|\Psi_T\rangle\langle\Psi_T|]$ and the von Neumann entanglement entropy (EE) $S_s(A) := -\text{Tr}[\rho_{s,A} \log_2 \rho_{s,A}]$. We are interested in the average of entanglement entropies $S(A) \equiv \langle S_s(A) \rangle$ over many ($\sim 10^5$) samples. In the following, quantities of single samples are labeled by an index s while sample averages are denoted by $\langle \cdot \rangle$ and calligraphic symbols. The entanglement transition of the PTIM manifests in the singular behavior of the half-system EE $S(L/2) \equiv S(\{1, \dots, L/2\})$

as a function of the measurement probability p in the steady state limit $T \rightarrow \infty$ of large systems; see Fig. 1(b) for simulations initialized in a global GHZ state $|\Psi_0\rangle \propto |0\dots 0\rangle + |1\dots 1\rangle$. The critical point $p_c = 0.5$ corresponds to a bond-percolation transition and is described by a conformal field theory [6,7]; it separates two area-law phases with ($p < p_c$) and without ($p > p_c$) long-range entanglement.

PTIM trajectories can be simulated efficiently via the stabilizer formalism [35–39]; however, since we are only interested in the entanglement of states, simulating a *colored cluster model* is even more efficient [6]. This method assumes initial states of the form $|\Psi_0\rangle \propto |0\dots 0\rangle \pm |1\dots 1\rangle$ and exploits that the PTIM only produces product states of *Bell clusters* $|m\rangle_A \pm |\bar{m}\rangle_A$, with m the configuration of some subset A of qubits and $\bar{m}$ its bitwise inverted. Indeed, E_i measurements “poke holes” into Bell clusters by projecting qubit i into a single-site Bell cluster $|\pm\rangle_i \propto |0\rangle_i \pm |1\rangle_i$, whereas S_e measurements merge adjacent Bell clusters. Since entanglement is independent of the configurations m and relative signs $\pm$, the entanglement structure can be tracked by “coloring” qubits according to the Bell cluster to which they belong at any given time, and evolving this pattern by updating colors according to the described effects of measurements. In the entangling phase of the PTIM ($p < p_c$) the survival probability of the initial Bell cluster $|\Psi_0\rangle \propto |0\dots 0\rangle \pm |1\dots 1\rangle$ approaches one for $L = T \rightarrow \infty$, whereas in the disentangling phase ($p > p_c$) it goes to zero.

This motivates an alternative probe of the entanglement transition, Fig. 1(c). A single *ancilla qubit* $i = a$ is added to the L system qubits and included in the initial Bell cluster $|\Psi_0\rangle \propto |0\rangle_a |0\dots 0\rangle_L + |1\rangle_a |1\dots 1\rangle_L$ by an initial entangling step. After this initialization (which we assume to be perfect), the ancilla remains unaffected by the PTIM evolution. After $T = L$ time steps, the trajectory is finalized by an (also perfect) *entanglement transfer*, where first all S_e measurements on system qubits are applied (merging all Bell clusters), followed by all but the last E_i measurements ($i \neq a, L$). We record the outcomes of these additional measurements in $M^r = (E^r, S^r)$ for later use. As a final step, the EE $S_s(\{a\}) = S_s(\{L\})$ of the ancilla is evaluated. The rationale is that the initial Bell cluster survives the PTIM evolution *if and only if* the ancilla remains entangled with the system, i.e., the sample average $S_a \equiv \langle S_s(\{a\}) \rangle$ directly probes the survival probability of the initial Bell cluster. Note that the entanglement transfer step is not needed for the measurement of S_a but becomes important later for determining bounds on the entanglement transition via shadow tomography. We show simulations of S_a in Fig. 1(d). In contrast to the half-system EE $S(L/2)$, the ancilla EE features a crossing for different system sizes at $p_c = 0.5$, clearly indicating the transition. Note that S_a only requires the measurement of the EE of a single qubit.

Postselection problem—Numerically, the PTIM is easily accessible. However, to verify the predictions in Figs. 1(b) or 1(d) *experimentally*, one must measure entanglement entropies of each final state $|\Psi_T\rangle$. This is notoriously difficult, as it generally requires full state tomography, which, in turn, is only accessible through many copies of states $|\Psi_T\rangle$ [40,41]. Unfortunately, these states are produced by a process that is inherently probabilistic for two reasons. The measurement *pattern* M^p is random, but this randomness is (for perfect measurements) assumed to be under the experimentalist's control. By contrast, the measurement *results* M^r are random according to the Born rule and cannot be controlled *in principle*. Since final states $|\Psi_T\rangle$ depend on these measurement outcomes, it is exponentially unlikely (in $T \sim L$) to prepare multiple copies, which renders entanglement entropies experimentally inaccessible. In the remainder of this Letter, we show how to alleviate this *postselection problem* and make the entanglement transition of the PTIM experimentally accessible, even for unreliable measurement records $M^{p,r}$.

Lower bound via decoding—It is known that entangling phases of random quantum circuits behave as emergent quantum codes [26,27], i.e., amplitudes suitably encoded in the initial state survive for exponentially long times—despite finite rates of projective measurements! Indeed, the initial state of the PTIM can encode a single logical qubit via $|\Psi_0\rangle = \alpha|0\dots 0\rangle + \beta|1\dots 1\rangle$ with amplitudes $|\alpha|^2 + |\beta|^2 = 1$. As shown in Ref. [6], these amplitudes survive the projective dynamics if an initial Bell cluster would, that is, in the entangling phase ($p < p_c$). In this case, and if the trajectory is finalized by one round of deterministic S_e measurements to merge all Bell clusters, the final state has the form $|\Psi_T\rangle \propto \alpha|\mathbf{m}\rangle + \beta|\bar{\mathbf{m}}\rangle$, where the basis configuration $\mathbf{m} \in \{0,1\}^L$ depends on the measurement pattern and results of the trajectory. By contrast, in the disentangling phase ($p > p_c$), the probability for the amplitudes to be destroyed approaches one for $L = T \rightarrow \infty$ and the final state has the form $|\Psi_T\rangle = |\mathbf{m}\rangle \pm |\bar{\mathbf{m}}\rangle$. This suggests that one can identify the entangling phase by successfully retrieving information from the final state $|\Psi_T\rangle$ that was encoded in the initial state $|\Psi_0\rangle$. Since this decoding can fail even when the amplitudes survive, such a protocol provides in general an experimentally accessible *lower bound* (in p) on the entanglement transition.

We now describe such a protocol for the PTIM [Fig. 2(a)]. For each sample s , we randomly encode one bit $m_s^* \in \{0,1\}$ into the initial state $|\Psi_0\rangle_s = |m_s^*, \dots, m_s^*\rangle$ [42], and then perform a PTIM evolution for $T = L$ steps. Each trajectory is finalized by a perfect measurement of all S_e operators, so that $|\Psi_T\rangle_s = |\mathbf{m}_s\rangle$ with a random bit pattern $\mathbf{m}_s$ if the amplitudes survive, and $|\Psi_T\rangle_s \propto |\mathbf{m}_s\rangle \pm |\bar{\mathbf{m}}_s\rangle$ otherwise. Finally, we measure Z_1 on the first qubit. Thus, for each sample, we possess the following data: (1) the encoded bit m_s^* , (2) the (potentially incomplete) measurement record $M_s^{p,r}$, and (3) the outcome $z_{s,1} \in \{\pm 1\}$ of the

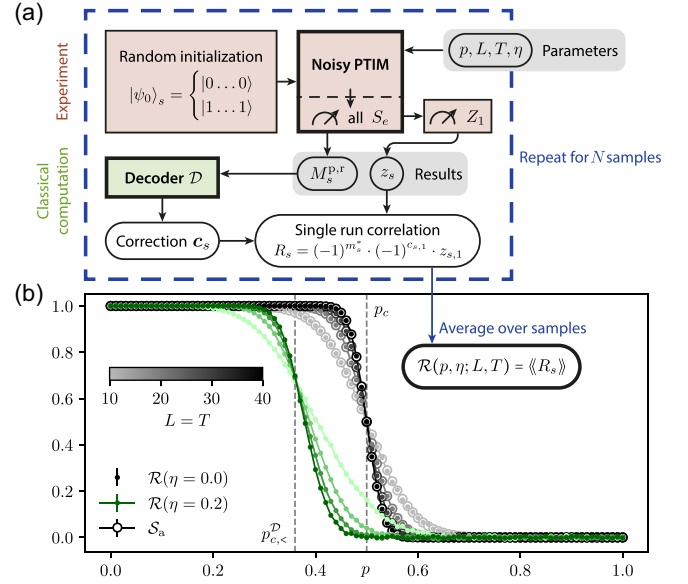


FIG. 2. Lower bound via decoding. (a) Protocol to measure the decoding transition of the PTIM as described in the text. (b) Sample-averaged correlation as a function of p for system sizes $L = 10, \dots, 40$ with (green) and without (black) noise. In a noisy circuit, the transition is shifted to the left and provides a lower bound on the entanglement transition.

final Z_1 measurement. Our task is to recover m_s^* from $M_s^{p,r}$ and $z_{s,1}$ (referred to as “decoding”). If decoding succeeds with high probability, the system must be in the entangling phase. A *decoder* is an algorithm $\mathcal{D}(M_s^{p,r}) = \mathbf{c}_s$ that uses the measurement record to compute a *correction* $\mathbf{c}_s$ such that $\mathbf{c}_s \oplus \mathbf{m}_s = (m_s^*, \dots, m_s^*)$ with high probability (if the amplitudes survive). Here, $\oplus$ denotes bitwise addition modulo 2. We can quantify the success of decoding by evaluating the correlation

$$R_s := (-1)^{m_s^*} \times (-1)^{c_{s,1}} \times z_{s,1}, \quad (1)$$

averaged over many samples: $\mathcal{R}(p, \eta; L, T) \equiv \langle R_s \rangle$. For perfect measurement records ($\eta = 0$), decoding is trivial because the trajectory can be simulated exactly and efficiently via the stabilizer formalism [or an even more efficient *extended colored cluster simulation*; see Sec. S1.A.2 of Supplemental Material (SM) [43]]. In Fig. 2(b) we plot $\mathcal{R}$ for this case. As expected, the (accessible!) decoding transition of $\mathcal{R}$ perfectly matches the (inaccessible) entanglement transition of S_a .

Decoding with noise—If the measurement record $M_s^{p,r}$ is *incomplete* ($\eta \neq 0$), a straightforward simulation is impossible since the results M_s^r can be inconsistent with the recorded pattern M_s^p . To fix this, we extend the decoder $\mathcal{D}$ (used for the noise-free case) by an additional preprocessing step $\mathcal{E}$ that augments the observed history $M_s^{p,r}$ to a hypothetical history $\tilde{M}_s^{p,r}$ that is both *likely* and *consistent*; the computation of the correction $\mathbf{c}_s$ (again via an extended

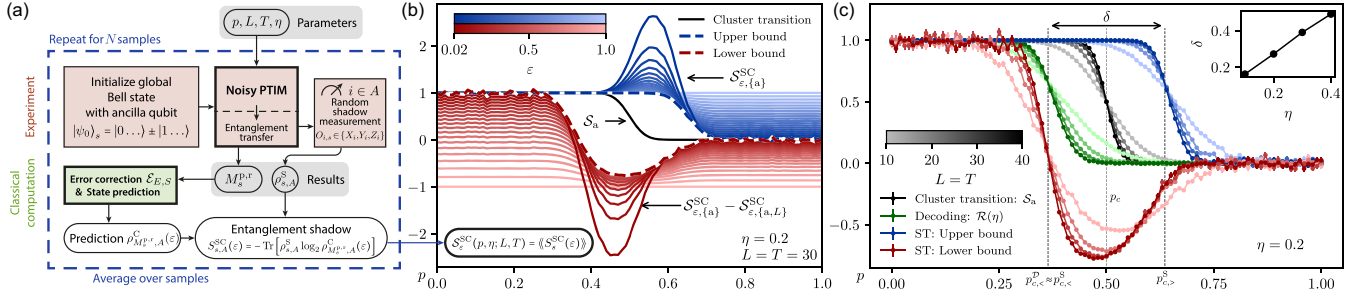


FIG. 3. Upper and lower bound via shadow tomography. (a) Protocol to measure the shadow of the entanglement entropy of a subsystem A on the noisy PTIM, based on a decoding algorithm for classical state prediction (see text). (b) Optimization over the regularization $0 < \varepsilon \leq 1$ yields an upper (dashed blue) and lower bound (dashed red) on the ancilla EE S_a (solid black), shown for a finite noise rate $\eta = 0.2$ and $L = T = 30$. (c) Comparison of the lower bounds from decoding (green) and ST (red), and the upper bound from ST (blue). For $L = T = 10, \dots, 40$ we find clear crossings that provide experimentally accessible lower and upper bounds on the entanglement transition at $p_c = 0.5$ (black). The simulations suggest that the two lower bounds share the same threshold. The interval δ between upper and lower bounds can be used to measure the noise rate η (inset).

colored cluster simulation) is then based on this consistent history. We denote this error-corrected decoder by $\mathcal{D}_E$. Since the *outcomes* of hypothetical measurements cannot be predicted (this follows from Pauli measurements either commuting or anticommuting) and c_s is computed from the results of S_e measurements, it is sufficient to augment $E_s^p \mapsto \tilde{E}_s^p$ by hypothetical E_i measurements (without results). To construct a consistent pattern $\tilde{M}_s^p \equiv (\tilde{E}_s^p, S_s^p)$ with as few unobserved E_i measurements as possible, the error correction algorithm $\mathcal{E} \equiv \mathcal{E}_E$ employs *minimum-weight perfect matching* (MWPM) [46,47]; see Sec. S1.A of SM [43].

Using this decoder for the protocol in Fig. 2(a), we can again measure $\mathcal{R}$; results for $\eta = 0.2$ are shown in Fig. 2(b). The decoding threshold $p_{c,<}^D$ of the MWPM-based decoder $\mathcal{D}_E$ is clearly indicated by a crossing of $\mathcal{R}(p)$ for $L = T \rightarrow \infty$. Since a necessary (but not sufficient) condition for successful decoding is the survival of an initial Bell cluster, it is $\mathcal{R}(p) \leq S_a(p)$ in the limit of many samples, with equality only for $\eta = 0$. Hence $p_{c,<}^D \leq p_c$ provides a robust, experimentally accessible lower bound on the entanglement transition. The deviation $|p_{c,<}^D - p_c|$ depends both on the noise η and the decoding algorithm. Since $\mathcal{D}_E$ is not necessarily a maximum likelihood decoder, smaller deviations might be achievable (though significant improvements are unlikely [15,48]).

Upper bound via shadow tomography—For an experimentally accessible *upper* bound, we employ the shadow tomography (ST) protocol proposed by Garratt and Altman [14] and combine it with efficient MWPM-based error correction for resilience against noise. We consider the setting depicted in Fig. 1(c) with an initially entangled ancilla qubit. The goal is to reliably bound its entanglement entropy S_a from above by an experimentally accessible quantity that can be obtained by the protocol in Fig. 3(a). For every trajectory, the system is initialized in a fully entangled Bell state and evolved under a PTIM circuit for

$L = T$ time steps. A final entanglement transfer [Fig. 1(c)] ensures that the entanglement entropy S_a of the ancilla with the system qubit $i = L$ encodes the survival of the initial Bell cluster; we denote the final state of these two qubits as $|\Psi_T^{\{a,L\}}\rangle_s$. Because of the postselection problem, S_a is not experimentally accessible. Instead, we evaluate two auxiliary quantities that combined yield an upper bound on $S_a = \langle S_a \rangle$.

First, we randomly pick $O_{a,s} \in \{X_a, Y_a, Z_a\}$ and measure this operator on the ancilla in the final state with outcome $o_s \in \{\pm 1\}$; from this, we compute the *shadow* $\rho_{s,\{a\}}^S := \frac{3}{2}(\mathbb{1} + o_s \cdot O_{a,s}) - \mathbb{1}$ [32]. Second, we use the history $M_s^{p,r}$ to compute a *classical prediction* $\rho_{M_s^{p,r},\{a\}}^C$ for the state $\rho_{s,\{a\}} \equiv \text{Tr}_{\{L\}}[|\Psi_T^{\{a,L\}}\rangle\langle\Psi_T^{\{a,L\}}|_s]$ of the ancilla. We describe the algorithm for this prediction below. Finally, the *shadow of the ancilla EE* is defined as

$$S_{s,\{a\}}^{\text{SC}} := -\text{Tr}[\rho_{s,\{a\}}^S \log_2 \rho_{M_s^{p,r},\{a\}}^C]. \quad (2)$$

Note that for pure $\rho_{M_s^{p,r},\{a\}}^C$ this quantity diverges, which is avoided by regularizing the prediction $\rho_{M_s^{p,r},\{a\}}^C(\varepsilon) := (1 - \varepsilon)\rho_{M_s^{p,r},\{a\}}^C + (\varepsilon/2)\mathbb{1}$ via a depolarizing channel of strength ε . The regularized shadow (2) is then averaged over trajectories to evaluate $S_{\varepsilon,\{a\}}^{\text{SC}}(p, \eta; L, T) \equiv \langle S_{s,\{a\}}^{\text{SC}}(\varepsilon) \rangle$. Crucially, in the limit of many samples, this quantity upper bounds the ancilla entropy [14],

$$S_a \leq S_{\varepsilon,\{a\}}^{\text{SC}}, \quad (3)$$

independent of ε and the algorithm used for $\rho_{M_s^{p,r},\{a\}}^C$. This follows from the properties of the shadow $\rho_{s,\{a\}}^S$ and Klein's inequality [39]. Equality in (3) is approached for ideal state prediction, $\rho_{M_s^{p,r},\{a\}}^C \equiv \rho_{s,\{a\}}$, in the limit $\varepsilon \rightarrow 0$ and

infinitely many samples (possible only for perfect measurement records, $\eta = 0$).

The challenge is to make Garratt and Altman's protocol robust against noise ($\eta \neq 0$) for large systems ($L \rightarrow \infty$). To this end, we introduce an efficient algorithm that combines classical simulations with error correction (see Sec. S2 of SM [43] for technical details): to compute $\rho_{M_s^{p,r},\{a\}}^C$, we trace out the system qubit $i = L$ from a prediction for $|\Psi_T^{\{a,L\}}\rangle_s$. For this state, the PTIM evolution allows *eight* possible stabilizer groups, namely $\langle \pm Z_a Z_L, \pm X_a X_L \rangle$ (entangled ancilla) or $\langle \pm X_a, \pm X_a X_L \rangle$ (disentangled ancilla). To decide on the group and the signs, we employ the error correction algorithm $\mathcal{E}_E$ from above, and combine it with the “dual” algorithm $\mathcal{E}_S$, which uses results of E_i measurements to augment $M_s^{p,r}$ by unobserved S_e measurements (again via MWPM; see Sec. S1.B of SM [43]). If the algorithm concludes that the sign of a stabilizer cannot be derived, the latter is dropped and a mixed state is predicted. This avoids false predictions that lead to infinities for $\varepsilon \rightarrow 0$ (and thereby bad upper bounds).

We plot $\mathcal{S}_{\varepsilon,\{a\}}^{\text{SC}}$ for $\eta = 0.2$ in Fig. 3(b) for different regularizations $0 < \varepsilon \leq 1$. Large ε suppress statistical fluctuations, but lead to less sharp bounds on the entanglement entropy if state predictions are reliable. Small ε can yield tighter bounds, but false state predictions are punished more severely [causing the peak in Fig. 3(b)]. Hence the optimal upper bound is the *lower envelope* of the upper bounds computed for different $0 < \varepsilon \leq 1$. Figures 3(b) and 3(c) demonstrate that this protocol provides a clear upper bound $p_{c,>}^S$ (indicated by a crossing for $L = T \rightarrow \infty$) on the entanglement transition at $p_c = 0.5$. Notably, the protocol requires only a single-site Pauli measurement $O_{a,s}$ on the final state of each sample. In Sec. S3 of SM [43] we demonstrate that sophisticated error correction is indeed crucial for this result.

Alternative lower bound via shadow tomography—Shadow tomography can also be used to *lower-bound* $\mathcal{S}_a$ [14]. Here, we compare this bound with our decoding bound $p_{c,<}^D$. To compute the ST-based lower bound, one measures on $|\Psi_T^{\{a,L\}}\rangle_s$ also the system qubit $i = L$ in a random basis to construct its shadow $\rho_{s,\{L\}}^S$. In combination with the ancilla shadow, this yields the two-qubit shadow $\rho_{s,\{a,L\}}^S := \rho_{s,\{a\}}^S \otimes \rho_{s,\{L\}}^S$. The (regularized) state prediction $\rho_{M_s^{p,r},\{a,L\}}^C(\varepsilon)$ for *both* qubits is already provided by the error-corrected algorithm sketched above. One can then generalize Eq. (2) to the shadow $\mathcal{S}_{s,\{a,L\}}^{\text{SC}}(\varepsilon)$ of the EE of the *combined system* $\{a, L\}$, and find for its sample average the lower bound $\mathcal{S}_{\varepsilon,\{a\}}^{\text{SC}} - \mathcal{S}_{\varepsilon,\{a,L\}}^{\text{SC}} \leq \mathcal{S}_a$ [14], where $\mathcal{S}_{\varepsilon,\{a\}}^{\text{SC}}$ is the shadow used for the upper bound. Simulations for $\eta = 0.2$ and $0 < \varepsilon \leq 1$ are shown in Fig. 3(b). Here, the *upper envelope* yields the best lower bound on $\mathcal{S}_a$. This envelope exhibits a clear crossing at $p_{c,<}^S$ for $L = T \rightarrow \infty$;

Fig. 3(c). The data suggest that the critical points measured by decoding and shadow tomography coincide: $p_{c,<}^S \approx p_{c,<}^D$. Note that both protocols rely on the same error correction algorithm $\mathcal{E}_E$.

Discussion—We introduced experimentally accessible protocols to lower and upper bound the entanglement transition of the projective transverse-field Ising model, which is not directly accessible due to the postselection problem. Figure 3(c) compares the experimentally accessible quantities from all protocols with the inaccessible ancilla entanglement entropy. All exhibit a clear crossing for increasing system size $L = T \rightarrow \infty$, indicating the critical values $p_{c,<}^D \approx p_{c,<}^S \leq p_c \leq p_{c,>}^S$. Notably, the interval $\delta := p_{c,>}^S - p_{c,<}^S$ depends approximately linearly on the noise rate η (inset) and therefore provides an independent experimental probe of the latter. Note that the error model can be straightforwardly generalized to local *unitary* errors that respect the global $X = \prod_i X_i$ symmetry of the PTIM without modifying the error correction algorithms. By contrast, robustness against faulty measurement *outcomes* may require protocols with more than $\mathcal{O}(L^2 \sim LT)$ measurements, similar to fault-tolerant decoding schemes for surface codes [49].

Our protocols combine information retrieval and shadow tomography with sophisticated error correction algorithms to cope with faulty measurement records. While the specific algorithms used in this Letter are tailored to the PTIM [43], we expect the protocols themselves to be applicable to any model allowing for efficient state prediction and error correction.

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Data availability—The data that support the findings of this article are openly available [50].

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