

Higher-Spin Effects in Black Hole and Neutron Star Binary Dynamics: Worldline Supersymmetry beyond Minimal Coupling

Domenico Bonocore^{1,*}, Anna Kulesza^{2,†} and Johannes Pirsch^{2,‡}

¹*TUM School of Natural Sciences, Technische Universität München,*

Physik Department, James-Franck-Straße 1, D-85748 Garching, Germany

²*Institut für Theoretische Physik, Universität Münster, Wilhelm-Klemm-Straße 9, D-48149 Münster, Germany*



(Received 30 May 2025; revised 9 September 2025; accepted 14 October 2025; published 18 November 2025)

The inclusion of spin effects in the binary dynamics for black holes and neutron stars is crucial for the computation of gravitational wave observables. Worldline supersymmetric models have been shown to be particularly efficient at this task up to quadratic order in spin, but progress at higher orders has been hampered by no-go theorems. In this Letter, we propose a novel approach to overcome this problem by extending the supersymmetry beyond minimal coupling. We demonstrate the potential of this approach by computing an “all-order-in-spin,” “linear in curvature,” manifestly supersymmetric Hamiltonian, as well as a “cubic-order-in-spin” Hamiltonian in arbitrary spacetime dimensions. In doing so, we identify a criterion that uniquely determines the Kerr geometry in terms of worldline supersymmetry. Equipped with these Hamiltonians, we demonstrate the exponentiation of three-point and Compton amplitudes using the recently proposed generalized Wilson line approach.

DOI: [10.1103/58z2-rypz](https://doi.org/10.1103/58z2-rypz)

Introduction—A fascinating aspect of future gravitational wave astronomy lies in the ability to infer fundamental properties of black holes and neutron stars from the detected waveforms. The upcoming third generation of gravitational wave detectors will enable reaching unprecedented precision for observable measurements [1–7], among them spin and tidal deformability. Such progress requires accurate waveform templates that, in particular, capture spin effects in the binary dynamics.

A wealth of analytical results have been derived in the post-Minkowskian (PM) expansion, i.e., the expansion around weak gravitational fields in Newton’s constant G . The driving force behind this breakthrough is largely due to a modern formulation of the two-body problem in general relativity (GR) with quantum field theory (QFT) techniques [8–67] together with advances in integration methods for Feynman integrals [68–76]. Importantly for this program, an effective two-body potential governs both the dynamics of the unbound scattering scenario and dynamics of an inspiraling binary, a connection that takes a particularly elegant form in the so-called bound-to-boundary map [77–79].

In this QFT-inspired approach, the rotation of black holes and neutron stars is represented as the spin s of a point particle, while finite size effects are captured using effective operators modeling the internal structure of the objects [11,80–87]. Correspondingly, apart from the expansion in G , one should also consider an expansion in the macroscopic spin variable S , defined as the conjugate momentum of the angular velocity. The current state of the art in the $G^n S^m$ double expansion is the combined $n + m = 5$ accuracy [29,40,42,53,55,57–61,88,89]. Additionally, considerable progress has been made at higher orders in spin [58,66,90–103], where, for example, results are available for the sectors $\mathcal{O}(GS^\infty)$, $\mathcal{O}(G^2S^\infty)$, and $\mathcal{O}(G^3S^4)$. In fact, GR provides an upper bound for the spin of black holes, via $S \leq Gm^2$, while neutron stars seem to be restricted to slower rotations [104]. Considering this scaling with G , higher orders in spin are elevated to the same level of importance as higher orders in the PM expansion, and are thus of primary relevance for improving the theoretical precision.

While the need to model spin effects is evident, a QFT description of these classical effects is not without challenges. Taking the classical limit in the scattering amplitude approach is quite involved [14,105,106] and the identification of the particle spin s with the macroscopic spin variable S requires interacting higher-spin fields [11]. Worldline models improve on these aspects considerably by efficiently reorganizing classical parts of QFT amplitudes into terms that have a well-defined $\hbar \rightarrow 0$ limit [10,36,107–110]. Crucially, the worldline approach enables to represent particles of spin $s = \frac{1}{2}$ or $s = 1$ minimally

*Contact author: domenico.bonocore@tum.de

†Contact author: anna.kulesza@uni-muenster.de

‡Contact author: johannes.pirsch@uni-muenster.de

coupled to gravity with a $\mathcal{N} = 2s$ supersymmetric action [111–114], capturing spin effects through $O(S^{2s})$, most prominently explored in the worldline QFT approach [29,59,60,107,115] and more recently using generalized Wilson lines (GWLs) [109,116]. The identification of the classical spin variables is particularly clear in the GWL approach, thanks to the synergy of the supersymmetric formulation and the soft expansion [108,109,117]. Extending a worldline description beyond quadratic order in spin, however, has proven challenging, although promising progress has recently been reported [100,101,118]. Supersymmetric formulations, in particular, are hindered by well-known no-go theorems [119–122]. One possibility that has recently been explored in [101] is to abandon supersymmetry in favor of bosonic fields for the spin tensor [123]. In this Letter, we approach the problem from a different angle by exploring nonminimal coupling on the supersymmetric worldline.

Nonminimal coupling has a long and extensive history. It emerges straightforwardly in the context of effective field theories, where higher-dimensional operators are added to the action to model physics below a certain energy scale, while the UV physics is absorbed into Wilson coefficients. For classical gravity, in particular, worldline effective field theories are formulated in terms of operators containing arbitrary powers of the curvature tensor, which therefore couple the point particle to the metric nonminimally. As such, they represent deviation from geodesic motion, and are thus associated with the finite extent of the source [80,84,86,124–126]. The operators that contribute at lowest order in the PM expansion are self-induced tidal effects. They correspond to the multipole moments of the object's stress-energy tensor that couple the spin S of the particle to derivatives of the curvature. For spinning black holes, the numerical value of the corresponding Wilson coefficients is unity, while values different from one capture the complicated internal structure of neutron stars instead.

A fresh look at nonminimal coupling comes from recent developments in on-shell methods for scattering amplitudes. Specifically, in [127], on-shell amplitudes for massive particles of arbitrary spin were introduced. While massless amplitudes exhibit a unique structure that corresponds to the usual notion of minimal coupling in QFT, massive amplitudes admit several distinct tensor structures. This suggests that massive higher-spin interactions naturally lead to higher-dimensional operators. One of these structures is special and matches the properties of the massless minimally coupled amplitude in the high-energy limit. In fact, it has been shown in [97,128] that these minimally coupled higher-spin amplitudes reproduce precisely the multipole moments that uniquely define the Kerr black hole geometry [84,129]. These multipole moments have a representation on the worldline as corrections to the dynamical mass of the particle. We will show in the following how they naturally emerge from the supersymmetry algebra.

In this Letter, we propose a nonminimally coupled supersymmetric worldline model that describes the gravitational interaction beyond quadratic order in spin. After introducing nonminimal coupling, in the next section we enforce the supersymmetry algebra order by order in spin, leading to a formal expression for the higher-spin Hamiltonian. Following that, we identify the subclass of nonminimal couplings that define the Kerr geometry on the worldline, enabling to provide for the first time two higher-spin Hamiltonians: first, the “all-order-in-spin” and “linear-in-curvature” Hamiltonian describing the dynamics of Kerr black holes in four spacetime dimensions, and second, the “cubic-order-in-spin” Hamiltonian for general extended bodies in generic spacetime dimensions d . The d -dimensional expressions are interesting from a theoretical perspective [130–133] and computationally useful for constructing amplitude integrands, making them free of Levi-Civita tensors and therefore unambiguous in dimensional regularization. Lastly, we demonstrate, using the GWL approach, that these Hamiltonians lead to the exponentiation of the higher-spin three-point and Compton amplitudes, which are the essential building blocks for the computation of observables.

Spin through worldline supersymmetry—Free massless particles of spin s are characterized in flat space by $2s + 1$ conserved charges on phase space. These are the $2s$ Grassmann valued charges $\mathcal{Q}_i$ for $i \in \{1, \dots, 2s\}$ and the Hamiltonian $\mathcal{H}$ related to the charges via a $\mathcal{N} = 2s$ -fold supersymmetry algebra under Poisson brackets (PB),

$$\{\mathcal{Q}_i, \mathcal{Q}_j\}_{\text{PB}} = -2i\delta_{ij}\mathcal{H}. \quad (1)$$

The supersymmetry algebra implements a worldline analog of the $2s$ -fold tensor product of the Clifford algebra written in terms of momenta $\{p_\mu\gamma^\mu, p_\nu\gamma^\nu\} = 2p^2$. Minimal coupling to gravity is achieved by using the charge

$$\mathcal{Q}_i = i\psi_i^a e_a^\mu \pi_\mu \equiv i\psi_i^a e_a^\mu \left(p_\mu + \frac{i}{2} \omega_\mu^{bc} \psi_b^i \psi_c^i \right), \quad (2)$$

where p_μ is the canonical momentum, ω_μ^{ab} is the spin connection, e_a^μ is the tetrad, and the Grassmann multiplet ψ_i^a is related to the GR spin tensor $S^{\mu\nu}$ through

$$S^{\mu\nu} = ie_a^\mu e_b^\nu \psi_i^a \psi_i^b. \quad (3)$$

This charge fails to obey the supersymmetry algebra for $s > 1$ [119]. In order to describe particles of higher-spin we consider the nonminimally coupled charge

$$\mathcal{Q}_i = i\psi_i^a e_a^\mu P_\mu \equiv i\psi_i^a e_a^\mu (\pi_\mu + q_\mu), \quad (4)$$

with yet to be determined q_μ . Correspondingly,

$$\{\mathcal{Q}_i, \mathcal{Q}_j\}_{\text{PB}} = i\delta_{ij}P^2 - \psi_i^a \psi_j^b T_{ab}, \quad (5a)$$

$$T_{ab} = \{e_a^\mu P_\mu, e_b^\nu P_\nu\}_{\text{PB}} - 4e_{[a}^\mu \frac{\delta P_\mu}{\delta(i\psi_i^c \psi_i^{[b]})} P_\nu e^{\nu]}, \quad (5b)$$

where the q_μ dependence is implicit in the new term T_{ab} and P^2 . Clearly, for vanishing T_{ab} the brackets are diagonal and thus $\mathcal{H}$ can be defined via Eq. (1), i.e., $\mathcal{H} = -\frac{1}{2}P^2$. The inclusion of a mass for the particle is simple in this particular case [134] and changes the Hamiltonian to

$$\mathcal{H} = \frac{1}{2}(m^2 - P^2) \equiv \frac{1}{2}(\mathcal{M}^2 - \pi^2), \quad (6)$$

where in the last step we introduced the dynamical mass $\mathcal{M}$, which collects all dependence over q_μ . The strategy we follow is therefore to solve $T_{ab} = 0$ for q_μ order by order in curvature and spin, hence enforcing supersymmetry up to the corresponding order in the expansions. To do so, we first expand q_μ into a power series in the spin tensor

$$q_\mu = \sum_{n=0}^{\infty} q_{\mu,(n)} S^{(n)}, \quad (7)$$

where (n) is short for the $2n$ indices $a_1 b_1 \dots a_n b_n$. Quite generally, the coefficients $q_{\mu,(n)}$ depend on the momentum π_μ and a collection of spacetime functions $\phi_I = \{R_{\mu\nu\rho\sigma}, \nabla_\alpha R_{\mu\nu\rho\sigma}, \dots\}$, where the ellipses denote higher orders in the derivatives and curvature tensor. As such, they encode the coupling between the spin and the background spacetime.

Since the $\mathcal{N} = 2s$ model encodes the spin dynamics only to order S^{2s} in the spin tensor, we can truncate the series in Eq. (7) at that order by requiring that the $q_{\mu,(n)}$ fulfil the Bianchi identity for any three indices a_i, b_i, a_j . This leads to the explicit constraint

$$\begin{aligned} T_{ab} = & \frac{1}{2} R_{abcd} S^{cd} - \{q_a, q_b\}_{\text{PB}} + R_{[a|c|ef} S^{ef} \frac{\delta q_{|b]}}{\delta \pi_c} \\ & - 2 \sum_{n=0}^{2s} S^{(n)} \left(2(n+1) \pi^r q_{[a|r|b](n)} - \nabla_{[a|} \phi_I \frac{\delta}{\delta \phi_I} q_{|b](n)} \right) \\ & + 2q_c e_f^\mu \omega_\mu^{dc} \frac{\delta}{\delta \pi_f} \eta_{d[b} q_{a]} + 2q_c \frac{\delta q_b}{\delta S_c^a} - 2q_c \frac{\delta q_a}{\delta S_c^b} = 0. \end{aligned} \quad (8)$$

In the following section, we present two solutions to Eq. (8), which yield the corresponding higher-spin Hamiltonians of the supersymmetric model.

Higher-spin Hamiltonians—Linear curvature all order in spin in $d = 4$: Restricting our attention to the first order in curvature and $d = 4$, we first define the spin vector s_μ and mass dipole D_μ in terms of the spin tensor of Eq. (3) according to

$$D_\beta + i s_\beta \equiv r^\alpha S_{\alpha\beta} + i \frac{1}{2} r^\alpha \epsilon_{\alpha\beta\gamma\delta} S^{\gamma\delta}, \quad (9)$$

where the unit vector $r^\mu = (\pi^\mu / \sqrt{\pi^2})$. Both D_μ and s_μ carry three degrees of freedom, due to orthogonality to r_μ . Since the spin tensor captures spatial rotations, only three of the six degrees of freedom among s_μ and D_μ are physical. The unphysical ones are removed with an additional constraint called “spin supplementary condition” (SSC), which in the supersymmetric model is enforced as $\mathcal{Q}_i = 0$. However, in order to maintain the supersymmetry of the action, all components of s_μ and D_μ are necessary at this stage.

Demanding that $q_{\mu,(n)}$ is traceless, i.e.,

$$q_{\mu,(n)} \eta^{a_i a_i} = 0, \quad (10)$$

the equation $T_{ab} = 0$ dramatically simplifies and a solution for q_μ can be written down to all orders in s_μ and D_μ . The resulting Hamiltonian then reads

$$\begin{aligned} \mathcal{H} = & \frac{m^2 - \pi^2}{2} - \sum_{n=2}^{\infty} \sum_{k=0}^n \binom{n}{k} \frac{1}{n! |\pi|^{n-2}} (D^{b_i})^k (s^{b_j})^{n-k} \\ & \times \begin{cases} (-1)^{\frac{3n-k}{2}} E_{(b_1 \dots b_{n-2}, b_{n-1} b_n)} & n-k \text{ even} \\ (-1)^{\frac{3n-k+1}{2}} B_{(b_1 \dots b_{n-2}, b_{n-1} b_n)} & n-k \text{ odd} \end{cases}, \end{aligned} \quad (11)$$

where

$$E_{a_1 \dots a_n, bc} = \hat{\pi}^x \hat{\pi}^y \nabla_{a_1} \dots \nabla_{a_n} R_{xbyc}, \quad (12a)$$

$$B_{a_1 \dots a_n, bc} = \frac{1}{2} \epsilon_{yczd} \hat{\pi}^x \hat{\pi}^y \nabla_{a_1} \dots \nabla_{a_n} R_{xbzd}. \quad (12b)$$

Equation (11) is a new result, as it captures the all-order dependence on the mass dipole, i.e., it does not rely on a specific choice of the SSC. Truncating Eq. (11) at order $\mathcal{O}(D)$ and making use of the differential Bianchi identity, we recover the result derived in [101] [see Eq. (3.20) there, where the mass dipole is referred to as Z_μ].

Importantly, this expression captures the multipole moments of Kerr black holes. In order to show this, we isolate the $\mathcal{O}(D^0)$ term

$$\begin{aligned} \mathcal{H} = & \frac{m^2 - \pi^2}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{s^a \nabla_a}{|\pi|} \right)^{2n-1} B_{ab} s^a s^b \\ & - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \left(\frac{s^a \nabla_a}{|\pi|} \right)^{2n-2} E_{cd} s^c s^d, \end{aligned} \quad (13)$$

which agrees with the literature [84,118,129]. We stress that the Kerr geometry emerges from the conditions that $q_{\mu,(n)}$ is traceless and fulfils the Bianchi identity.

Cubic in spin for generic d : For generic spacetime dimensions d and aiming at non-Kerr solutions, we relax

the traceless condition and make an ansatz for $q_{\mu,(n)}$ up to $n = 3$ containing the curvature and its derivative. The resulting dynamical mass is conveniently expressed in terms of the mass dipole D_μ and the physical spin tensor $\tilde{S}_{\mu\nu} = S_{\mu\nu} - 2r_{[\mu}D_{\nu]}$. For the $D_\mu = 0$ SSC, terms that are quadratic in the mass dipole are irrelevant, and we consider the linearized Hamiltonian

$$\begin{aligned} \mathcal{H} = & \frac{m^2 - \pi^2}{2} + \pi^2 C_1 \tilde{S}_{ab} \tilde{S}^{ab} + \frac{1}{2} C_{D\hat{\pi}ab} \tilde{S}^{ab} \\ & - \frac{C_{ES^2} + \tilde{C}_{ES^2}}{8} \tilde{S}^{ab} \tilde{S}^{cd} (1 - |\pi|^{-1} \nabla_D) C_{abcd} \\ & - \frac{\tilde{C}_{ES^2}}{2} \tilde{S}_c^a \tilde{S}^{cb} (1 - |\pi|^{-1} \nabla_D) E_{ab} \\ & - \frac{C_{BS^3}}{6|\pi|} \nabla_a C_{\hat{\pi}bcd} \tilde{S}^{ad} \tilde{S}^{be} \tilde{S}_e^c - \frac{\tilde{C}_{BS^3}}{6|\pi|} \nabla_{\hat{\pi}} C_{aD\hat{\pi}b} \tilde{S}^{ac} \tilde{S}_c^b \\ & + \frac{1}{|\pi|} (\tilde{C}_{ES^2} + \tilde{C}_{BS^3}) E_{a,bc} \tilde{S}^{ac} (D \cdot \tilde{S})^b + \mathcal{O}(D^2). \end{aligned} \quad (14)$$

The coefficients C_{ES^2} , $\tilde{C}_{ES^2}$, C_{BS^3} , and $\tilde{C}_{BS^3}$ are introduced as shorthands for combinations of coefficients coming from the ansatz [see Eqs. (A1)–(A3)] and are identified such that in $d = 4$ we recover the usual couplings in terms of C_{ES^2} , C_{BS^3} (see, e.g., Ref. [84] for an excellent overview). This Hamiltonian is a new result and extends effective higher-spin Hamiltonians in the literature to generic space-time dimensions. In particular, the Wilson coefficients $\tilde{C}_{ES^2}$ and $\tilde{C}_{BS^3}$ are new for generic d . They cancel or vanish in the special case $d = 4$, where

$$\begin{aligned} \mathcal{H} \stackrel{d=4}{=} & \frac{m^2 - \pi^2}{2} - 2\pi^2 C_1 s^a s_a + B_{Ds} \\ & + \frac{C_{ES^2}}{2} (1 - |\pi|^{-1} \nabla_D) E_{ss} + \frac{C_{BS^3}}{6|\pi|} B_{s,ss} + \mathcal{O}(D^2), \end{aligned} \quad (15)$$

which is in agreement with [101,136]. Interestingly, the supersymmetric model also includes mass corrections [137] parametrized by the C_1 coefficient multiplying the constant s^2 . In generic d , the Wilson coefficients are not fixed completely in the Kerr limit, but rather depend on a single unconstrained parameter D ,

$$C_{ES^2} = \frac{d^2 - 4}{12} + Dd(d-4)(d+1), \quad (16a)$$

$$\tilde{C}_{ES^2} = \frac{d+4}{12} + d(d+1)D, \quad (16b)$$

$$C_{BS^3} = \frac{6}{2(d-1)}, \quad (16c)$$

$$\tilde{C}_{BS^3} = -\frac{1}{4} - 3(d+1)D. \quad (16d)$$

We interpret this free parameter as an indication of a larger number of Kerr-like solutions to Einstein's equation in $d > 4$. As the uniqueness theorems do not apply there, we should not expect all multipole moments to be determined, which agrees with findings in [130,138]. In $d = 4$, however, the Kerr limit fixes all coefficients to unity as expected for Kerr black holes.

Wilson lines and exponentiation in gravitational scattering—The classical scattering of compact astrophysical objects is modeled by the exchange of soft gravitons $h_{\mu\nu}$, defined as usual as a perturbation around the flat metric $\eta_{\mu\nu}$, i.e., $\kappa h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, with $\kappa = \sqrt{32\pi G}$. Observables can then be derived, for example, from the eikonal phase χ , which consists of resummed subsets of graviton exchanges. This is achieved very elegantly in the GWL approach [108,109], where it can be shown that subsets of graviton interactions relevant in the classical limit exponentiate, thus resumming the classical part of the scattering amplitude to all orders in the coupling κ .

In particular, the higher-spin GWLs are defined through the path integral expression

$$\tilde{W}(p, \eta, b) = \int \mathcal{D}[x, \psi] \exp \left\{ i \int_{-\infty}^{\infty} dt L[x, \psi] \right\}, \quad (17)$$

with the Lagrangian

$$-\frac{\dot{x}^2}{2} + \frac{i}{2} \psi_i^a \frac{D}{dt} \psi_i^a - \frac{1}{2} h_{\mu\nu} (p + \dot{x})^\mu (p + \dot{x})^\nu + \frac{1}{2} (m^2 - \mathfrak{M}^2). \quad (18)$$

Here, $\mathfrak{M}$ is the dynamical mass in configuration space evaluated at position $b + pt + x$, where b and p refer to the impact parameter and momentum of the particle, respectively. The boundary conditions for the Grassmann fields $\psi_i^\mu(-\infty) + \psi_i^\mu(\infty) = 2\eta_i^\mu$ generate the classical spin tensor $S^{\mu\nu} = i\eta_i^\mu \eta_i^\nu$. The Green functions for worldline fields and the procedure for solving the path integral have been discussed thoroughly in [108,109].

The eikonal phase χ can be computed from the vacuum expectation value of two GWLs with regard to the Einstein-Hilbert action S_{EH} , i.e.,

$$e^{i\chi} = \int \mathcal{D}h_{\mu\nu} \tilde{W}(p_1, \eta_1, b_1) \tilde{W}(p_2, \eta_2, b_2) e^{iS_{\text{EH}}}, \quad (19)$$

which results in a diagrammatic expansion in terms of the classical amplitudes that make up the GWL (i.e., three-point, Compton, and higher-point amplitudes). The perturbative PM expansion of observables follows from the expansion of the Lagrangian in Eq. (18) to the corresponding order. Using the Hamiltonians from the previous section, we can compute two distinct GWLs: the first one in $d = 4$ to all order in spin and limited to 1PM

observables, and the second one to cubic order in spin, generic d , and suited for 2PM observables.

GWL at 1PM, all orders in spin, and $d = 4$: Computing the 1PM Wilson line requires no propagators of worldline fields. Linearizing the all-order-in-spin Hamiltonian in κ and selecting the covariant SSC for the background parameters $p_\mu n_i^\mu = 0$ yields

$$\begin{aligned} \tilde{W}(p, \eta, b) = \exp \left\{ \left(-\frac{ikm}{2} \right) \int_k \tilde{h}_{\mu\nu}(k) e^{-ikb} \hat{\delta}(vk) \right. \\ \left. \times \left(v^\mu v^\nu \cosh(\mathbf{a} \cdot k) + i \frac{v^{(\mu} \mathcal{S}^{\nu)} \sigma k_\sigma}{m(\mathbf{a} \cdot k)} \sinh(\mathbf{a} \cdot k) \right) \right\}, \end{aligned} \quad (20)$$

where $v^\mu = \frac{1}{m} p^\mu$ and $\mathbf{a}_\mu = \frac{1}{2} v^\nu \epsilon_{\nu\mu\varrho\sigma} \mathcal{S}^{\varrho\sigma}$. The exponent encapsulates the on-shell Kerr black hole three-point amplitude [97,99,128], while off-shell terms were omitted. This amplitude can also be obtained by considering a generalization of minimal coupling for higher-spin massive particles [127], offering a new perspective on the notion of minimal coupling on the worldline. Specifically, the Kerr limit singles out a class of couplings on the worldline via Eq. (10), thus suggesting an analog of minimal coupling for amplitudes on the supersymmetric worldline.

Observables are obtained by differentiating the eikonal phase of Eq. (19) with regard to b^μ or $\mathbf{a}^\mu$, and they agree with the results in the literature [49,139]. The all-order-in-spin GWL is a new result. It explicitly demonstrates for the first time the exponentiation of the Kerr three-point amplitude. The nontriviality of this result can be appreciated from the perspective of the amplitude approach, where already the exponentiation of the linear-in-spin three-point amplitude is quite involved [140].

GWL at 2PM, cubic order in spin, and generic d : At 2PM and cubic order in spin, the GWL for generic d takes the form

$$\begin{aligned} \tilde{W}(p, \eta, b) = \exp \left\{ -\frac{im}{2} \left(\kappa \int_k \tilde{h}_{\mu\nu} e^{-ikb} \hat{\delta}(kv) V_3^{\mu\nu} \right. \right. \\ \left. \left. + \kappa^2 \int_{k,l} \frac{\tilde{h}_{\mu\nu}(k) \tilde{h}_{\varrho\sigma}(l)}{2} e^{-ib(k+l)} \right. \right. \\ \left. \left. \times \hat{\delta}(v(k+l)) V_4^{\mu\nu\varrho\sigma} \right) \right\}. \end{aligned} \quad (21)$$

Here, $V_3^{\mu\nu}$ is the three-point amplitude, which reads

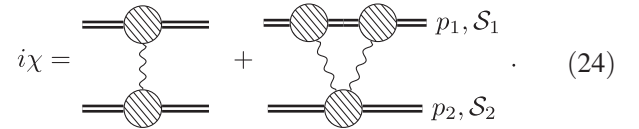
$$\begin{aligned} V_3^{\mu\nu} = v^\mu v^\nu + \frac{i}{m} v^{(\mu} (\mathcal{S} \cdot k)^{\nu)} \left(1 + \frac{C_{BS^3}}{6m^2} (k \cdot \mathcal{S} \cdot \mathcal{S} \cdot k) \right) \\ + \frac{\tilde{C}_{ES^2}}{2m^2} v^\mu v^\nu (k \cdot \mathcal{S} \cdot \mathcal{S} \cdot k) \\ - \frac{C_{ES^2} + \tilde{C}_{ES^2}}{2m^2} (\mathcal{S} \cdot k)^\mu (\mathcal{S} \cdot k)^\nu, \end{aligned} \quad (22)$$

where we have again omitted off-shell contributions. The Compton amplitude $V_4^{\mu\nu\varrho\sigma}$ can be written compactly by including the three-graviton vertex contribution and contracting with the graviton polarization vectors. Introducing the gauge invariant combinations $F_i^{\mu\nu} = k_i^\mu \epsilon_i^\nu - k_i^\nu \epsilon_i^\mu$, we find

$$\begin{aligned} V_4^{\mu\nu\varrho\sigma} \epsilon_{1,\mu} \epsilon_{1,\nu} \epsilon_{2,\varrho} \epsilon_{2,\sigma} \\ = 2 \frac{V_{(0)} + V_{(1)} + V_{(2)} + V_{(3)}}{(k_1 + k_2)^2} \text{Re} \left(\frac{1}{[v(k_1 - k_2) + i\epsilon]^2} \right), \end{aligned} \quad (23)$$

where the terms $V_{(i)}$ are listed in Eq. (B1).

The eikonal phase can be constructed from the following diagrams in terms of the GWL amplitudes (omitting mirrored expressions):



$$i\chi = \text{[Diagram 1]} + \text{[Diagram 2]}. \quad (24)$$

We have checked the eikonal and scattering angle against available results [42,58,107].

In the GWL approach the exponentiation of the amplitudes is manifest, demonstrating that also the tidal dependence in the form of the coefficients C_{ES^2} , $\tilde{C}_{ES^2}$, and C_{BS^3} exponentiates (though $\tilde{C}_{BS^3}$ does not appear at this order in spin and G).

Conclusions—In this Letter, we have proposed non-minimal coupling to gravity for supersymmetric worldline models as a promising framework to access higher-spin effects in the binary dynamics of black holes and neutron stars. In search for a suitable higher-spin Hamiltonian, we have deformed the supersymmetry charge and derived the necessary equation [Eq. (8)] that enforces supersymmetry. We then solved this equation by truncating the expansion either at a certain order in the Riemann tensor or a certain order in the spin tensor. Both of these paths have proven to be a constructive way to build higher-spin Hamiltonians that preserve the spin supplementary condition.

At linear order in the curvature, we have shown that there exists a unique solution in $d = 4$ to all orders in spin [Eq. (11)]. This new result captures all orders in the mass dipole, leaving supersymmetry manifest. At leading order in the mass dipole, it reduces to the well-known Hamiltonian of Kerr black holes expressed in terms of the spin vector [84,129]. Moreover, the way that the Hamiltonian emerges from the supersymmetry algebra provides a new perspective on the notion of minimal coupling at higher orders in spin, in parallel to on-shell methods [97,127,128].

At cubic order in spin, we have provided for the first time the Hamiltonian for generic d with full tidal dependence

[Eq. (14)]. In particular, we find two new Wilson coefficients and tensor structures entering the Hamiltonian, while in the $d \rightarrow 4$ limit we recover the known tidal operators with coupling C_{ES^2} and C_{BS^3} .

For both Hamiltonians, Eq. (10) sets all free Wilson coefficients to unity in $d = 4$, hence singling out the Kerr geometry from a generic compact object. In this way, the supersymmetric model provides a mechanism to obtain the Kerr geometry without any prior knowledge about this specific solution to Einstein's equation. Equipped with the two Hamiltonians and using the GWL approach, we demonstrated for the first time the explicit exponentiation of the Kerr three-point amplitude and the three-point and Compton amplitudes for generic compact objects [Eqs. (20) and (21)].

We believe the synergy between supersymmetry and nonminimal coupling provides a promising framework for increasing the theoretical accuracy of higher-spin gravitational wave observables. For instance, it paves the way for obtaining the still unknown terms beyond quadratic order in spin at 3PM. We also expect the all-order-in-spin solution to generalize to higher orders in the Riemann tensor. This would provide new insights into the dynamical multipole moments of Kerr black holes, which are under active investigation [42,141–143] and contribute to the higher-spin Compton amplitude.

Acknowledgments—This work was supported by the Deutsche Forschungsgemeinschaft (DFG) through the Research Training Group “GRK 2149: Strong and Weak Interactions—from Hadrons to Dark Matter” and the Excellence Cluster ORIGINS under Grant No. EXC-2094-390783311. J.P.’s work is also supported by the “DAAD—Forschungsstipendium für Doktorand*innen.” J.P. would like to thank Professor Radu Roiban and the Department of Physics at Penn State for their hospitality during the final stages of this work.

Data availability—No data were created or analyzed in this Letter.

[1] M. Punturo *et al.*, *Classical Quantum Gravity* **27**, 194002 (2010).
 [2] A. Abac *et al.*, [arXiv:2503.12263](https://arxiv.org/abs/2503.12263).
 [3] P. Amaro-Seoane *et al.* (LISA Collaboration), [arXiv:1702.00786](https://arxiv.org/abs/1702.00786).
 [4] P. A. Seoane *et al.* (LISA Collaboration), *Living Rev. Relativity* **26**, 2 (2023).
 [5] B. P. Abbott *et al.* (LIGO Scientific Collaboration), *Classical Quantum Gravity* **34**, 044001 (2017).
 [6] K. Chamberlain and N. Yunes, *Phys. Rev. D* **96**, 084039 (2017).
 [7] S. W. Ballmer *et al.*, [arXiv:2203.08228](https://arxiv.org/abs/2203.08228).
 [8] T. Damour, *Phys. Rev. D* **97**, 044038 (2018).
 [9] A. Koemans Collado, P. Di Vecchia, and R. Russo, *Phys. Rev. D* **100**, 066028 (2019).

[10] G. Mogull, J. Plefka, and J. Steinhoff, *J. High Energy Phys.* **02** (2021) 048.
 [11] Z. Bern, A. Luna, R. Roiban, C.-H. Shen, and M. Zeng, *Phys. Rev. D* **104**, 065014 (2021).
 [12] D. Neill and I. Z. Rothstein, *Nucl. Phys.* **B877**, 177 (2013).
 [13] A. Luna, I. Nicholson, D. O’Connell, and C. D. White, *J. High Energy Phys.* **03** (2018) 044.
 [14] D. A. Kosower, B. Maybee, and D. O’Connell, *J. High Energy Phys.* **02** (2019) 137.
 [15] A. Cristofoli, R. Gonzo, D. A. Kosower, and D. O’Connell, *Phys. Rev. D* **106**, 056007 (2022).
 [16] N. E. J. Bjerrum-Bohr, J. F. Donoghue, and P. Vanhove, *J. High Energy Phys.* **02** (2014) 111.
 [17] N. E. J. Bjerrum-Bohr, P. H. Damgaard, G. Festuccia, L. Planté, and P. Vanhove, *Phys. Rev. Lett.* **121**, 171601 (2018).
 [18] Z. Bern, C. Cheung, R. Roiban, C.-H. Shen, M. P. Solon, and M. Zeng, *Phys. Rev. Lett.* **122**, 201603 (2019).
 [19] Z. Bern, C. Cheung, R. Roiban, C.-H. Shen, M. P. Solon, and M. Zeng, *J. High Energy Phys.* **10** (2019) 206.
 [20] N. E. J. Bjerrum-Bohr, L. Planté, and P. Vanhove, *J. High Energy Phys.* **03** (2022) 071.
 [21] C. Cheung and M. P. Solon, *J. High Energy Phys.* **06** (2020) 144.
 [22] N. E. J. Bjerrum-Bohr, P. H. Damgaard, L. Planté, and P. Vanhove, *J. High Energy Phys.* **08** (2021) 172.
 [23] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, *Phys. Lett. B* **811**, 135924 (2020).
 [24] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, *J. High Energy Phys.* **07** (2021) 169.
 [25] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, *Phys. Lett. B* **818**, 136379 (2021).
 [26] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, *Phys. Lett. B* **843**, 138049 (2023).
 [27] C. Heissenberg, *Phys. Rev. Lett.* **131**, 011603 (2023).
 [28] G. U. Jakobsen, G. Mogull, J. Plefka, and B. Sauer, *J. High Energy Phys.* **10** (2022) 128.
 [29] G. U. Jakobsen and G. Mogull, *Phys. Rev. Lett.* **128**, 141102 (2022).
 [30] B. Bellazzini, G. Isabella, and M. M. Riva, *J. High Energy Phys.* **04** (2023) 023.
 [31] F. Bastianelli, F. Comberiati, and L. de la Cruz, *J. High Energy Phys.* **02** (2022) 209.
 [32] T. Damour, *Phys. Rev. D* **102**, 124008 (2020).
 [33] E. Herrmann, J. Parra-Martinez, M. S. Ruf, and M. Zeng, *J. High Energy Phys.* **10** (2021) 148.
 [34] P. H. Damgaard, K. Haddad, and A. Helset, *J. High Energy Phys.* **11** (2019) 070.
 [35] P. H. Damgaard, L. Plante, and P. Vanhove, *J. High Energy Phys.* **11** (2021) 213.
 [36] P. H. Damgaard, E. R. Hansen, L. Planté, and P. Vanhove, *J. High Energy Phys.* **09** (2023) 059.
 [37] R. Aoude, K. Haddad, and A. Helset, *J. High Energy Phys.* **05** (2020) 051.
 [38] M. Accattulli Huber, A. Brandhuber, S. De Angelis, and G. Travaglini, *Phys. Rev. D* **103**, 045015 (2021).
 [39] A. Brandhuber, G. Chen, G. Travaglini, and C. Wen, *J. High Energy Phys.* **10** (2021) 118.
 [40] Z. Bern, J. Parra-Martinez, R. Roiban, M. S. Ruf, C.-H. Shen, M. P. Solon, and M. Zeng, *Phys. Rev. Lett.* **126**, 171601 (2021).

- [41] Z. Bern, J. Parra-Martinez, R. Roiban, M. S. Ruf, C.-H. Shen, M. P. Solon, and M. Zeng, *Phys. Rev. Lett.* **128**, 161103 (2022).
- [42] Z. Bern, D. Kosmopoulos, A. Luna, R. Roiban, and F. Teng, *Phys. Rev. Lett.* **130**, 201402 (2023).
- [43] Z. Bern, D. Kosmopoulos, A. Luna, R. Roiban, T. Scheopner, F. Teng, and J. Vines, *Phys. Rev. D* **109**, 045011 (2024).
- [44] M. Levi, A. J. Mcleod, and M. Von Hippel, *J. High Energy Phys.* **07** (2021) 115.
- [45] C. Dlapa, G. Kälin, Z. Liu, and R. A. Porto, *Phys. Lett. B* **831**, 137203 (2021).
- [46] C. Dlapa, G. Kälin, Z. Liu, and R. A. Porto, *Phys. Rev. Lett.* **128**, 161104 (2022).
- [47] P. H. Damgaard, E. R. Hansen, L. Planté, and P. Vanhove, *J. High Energy Phys.* **09** (2023) 183.
- [48] A. Brandhuber, G. R. Brown, G. Chen, S. De Angelis, J. Gowdy, and G. Travaglini, *J. High Energy Phys.* **06** (2023) 048.
- [49] A. Brandhuber, G. R. Brown, G. Chen, J. Gowdy, and G. Travaglini, *J. High Energy Phys.* **02** (2024) 026.
- [50] S. De Angelis, P. P. Novichkov, and R. Gonzo, *Phys. Rev. D* **110**, L041502 (2024).
- [51] A. Herderschee, R. Roiban, and F. Teng, *J. High Energy Phys.* **06** (2023) 004.
- [52] S. Caron-Huot, M. Giroux, H. S. Hannesdottir, and S. Mizera, *J. High Energy Phys.* **01** (2024) 139.
- [53] F. Febres Cordero, M. Kraus, G. Lin, M. S. Ruf, and M. Zeng, *Phys. Rev. Lett.* **130**, 021601 (2023).
- [54] L. Bohnenblust, H. Ita, M. Kraus, and J. Schlenk, *J. High Energy Phys.* **11** (2024) 109.
- [55] Z. Bern, E. Herrmann, R. Roiban, M. S. Ruf, A. V. Smirnov, V. A. Smirnov, and M. Zeng, *J. High Energy Phys.* **10** (2024) 023.
- [56] M. Alaverdian, Z. Bern, D. Kosmopoulos, A. Luna, R. Roiban, T. Scheopner, and F. Teng, *Phys. Rev. Lett.* **134**, 101602 (2025).
- [57] D. Akpinar, F. Febres Cordero, M. Kraus, M. S. Ruf, and M. Zeng, *J. High Energy Phys.* **03** (2025) 126.
- [58] L. Bohnenblust, L. Cangemi, H. Johansson, and P. Pichini, *arXiv:2410.23271*.
- [59] G. U. Jakobsen, G. Mogull, J. Plefka, B. Sauer, and Y. Xu, *Phys. Rev. Lett.* **131**, 151401 (2023).
- [60] G. U. Jakobsen, G. Mogull, J. Plefka, and B. Sauer, *Phys. Rev. Lett.* **131**, 241402 (2023).
- [61] M. Driesse, G. U. Jakobsen, G. Mogull, J. Plefka, B. Sauer, and J. Usovitsch, *Phys. Rev. Lett.* **132**, 241402 (2024).
- [62] S. Foffa, P. Mastrolia, R. Sturani, C. Sturm, and W. J. Torres Bobadilla, *Phys. Rev. Lett.* **122**, 241605 (2019).
- [63] S. Foffa and R. Sturani, *Phys. Rev. D* **100**, 024047 (2019).
- [64] S. Foffa, R. A. Porto, I. Rothstein, and R. Sturani, *Phys. Rev. D* **100**, 024048 (2019).
- [65] T. Adamo, R. Gonzo, and A. Ilderton, *J. High Energy Phys.* **05** (2024) 034.
- [66] D. Akpinar, F. Febres Cordero, M. Kraus, A. Smirnov, and M. Zeng, *Phys. Rev. Lett.* **135**, 041602 (2025).
- [67] D. Akpinar, V. del Duca, and R. Gonzo, *Phys. Rev. D* **112**, 084014 (2025).
- [68] M. Beneke and V. A. Smirnov, *Nucl. Phys.* **B522**, 321 (1998).
- [69] T. Gehrmann and E. Remiddi, *Nucl. Phys.* **B580**, 485 (2000).
- [70] S. Laporta, *Int. J. Mod. Phys. A* **15**, 5087 (2000).
- [71] A. V. Smirnov, *J. High Energy Phys.* **10** (2008) 107.
- [72] J. M. Henn, *Phys. Rev. Lett.* **110**, 251601 (2013).
- [73] J. Klappert, F. Lange, P. Maierhöfer, and J. Usovitsch, *Comput. Phys. Commun.* **266**, 108024 (2021).
- [74] C. Dlapa, J. Henn, and K. Yan, *J. High Energy Phys.* **05** (2020) 025.
- [75] E. Gardi, F. Herzog, S. Jones, and Y. Ma, *J. High Energy Phys.* **08** (2024) 127.
- [76] L. de la Cruz, D. A. Kosower, and P. P. Novichkov, *arXiv:2410.18014*.
- [77] G. Kälin and R. A. Porto, *J. High Energy Phys.* **01** (2020) 072.
- [78] G. Kälin and R. A. Porto, *J. High Energy Phys.* **02** (2020) 120.
- [79] G. Cho, G. Kälin, and R. A. Porto, *J. High Energy Phys.* **04** (2022) 154; **07** (2022) 2.
- [80] W. D. Goldberger and I. Z. Rothstein, *Phys. Rev. D* **73**, 104029 (2006).
- [81] R. A. Porto, *Phys. Rev. D* **73**, 104031 (2006).
- [82] R. A. Porto and I. Z. Rothstein, *Phys. Rev. D* **78**, 044013 (2008); **81**, 029905(E) (2010).
- [83] R. A. Porto and I. Z. Rothstein, *Phys. Rev. D* **78**, 044012 (2008); **81**, 029904(E) (2010).
- [84] M. Levi and J. Steinhoff, *J. High Energy Phys.* **09** (2015) 219.
- [85] M. Levi, *Rep. Prog. Phys.* **83**, 075901 (2020).
- [86] G. Kälin and R. A. Porto, *J. High Energy Phys.* **11** (2020) 106.
- [87] G. Kälin, J. Neef, and R. A. Porto, *J. High Energy Phys.* **01** (2023) 140.
- [88] C. Dlapa, G. Kälin, Z. Liu, J. Neef, and R. A. Porto, *Phys. Rev. Lett.* **130**, 101401 (2023).
- [89] M. Driesse, G. U. Jakobsen, A. Klemm, G. Mogull, C. Nega, J. Plefka, B. Sauer, and J. Usovitsch, *Nature (London)* **641**, 603 (2025).
- [90] M.-Z. Chung, Y.-t. Huang, J.-W. Kim, and S. Lee, *J. High Energy Phys.* **05** (2020) 105.
- [91] R. Aoude, K. Haddad, and A. Helset, *Phys. Rev. Lett.* **129**, 141102 (2022).
- [92] F. Alessio and P. Di Vecchia, *Phys. Lett. B* **832**, 137258 (2022).
- [93] P. H. Damgaard, J. Hoogeveen, A. Luna, and J. Vines, *Phys. Rev. D* **106**, 124030 (2022).
- [94] W.-M. Chen, M.-Z. Chung, Y.-t. Huang, and J.-W. Kim, *J. High Energy Phys.* **08** (2022) 148.
- [95] Y. F. Bautista, A. Guevara, C. Kavanagh, and J. Vines, *J. High Energy Phys.* **03** (2023) 136.
- [96] Y. F. Bautista, A. Guevara, C. Kavanagh, and J. Vines, *J. High Energy Phys.* **05** (2023) 211.
- [97] A. Guevara, A. Ochirov, and J. Vines, *J. High Energy Phys.* **09** (2019) 056.
- [98] C. Cheung, J. Parra-Martinez, I. Z. Rothstein, N. Shah, and J. Wilson-Gerow, *Phys. Rev. Lett.* **132**, 091402 (2024).
- [99] E. Skvortsov and M. Tsulaia, *J. High Energy Phys.* **02** (2024) 202.
- [100] M. Ben-Shahar, *J. High Energy Phys.* **03** (2024) 108.
- [101] K. Haddad, G. U. Jakobsen, G. Mogull, and J. Plefka, *J. High Energy Phys.* **02** (2025) 019.

- [102] N. E. J. Bjerrum-Bohr, G. Chen, C. Su, and T. Wang, [arXiv:2503.20538](#).
- [103] F. Alessio, R. Gonzo, and C. Shi, [arXiv:2506.03249](#).
- [104] F. Cipolletta, C. Cherubini, S. Filippi, J. A. Rueda, and R. Ruffini, *Phys. Rev. D* **92**, 023007 (2015).
- [105] B. Maybee, D. O'Connell, and J. Vines, *J. High Energy Phys.* **12** (2019) 156.
- [106] Z. Capatti and M. Zeng, [arXiv:2412.10864](#).
- [107] G. U. Jakobsen, G. Mogull, J. Plefka, and J. Steinhoff, *J. High Energy Phys.* **01** (2022) 027.
- [108] D. Bonocore, A. Kulesza, and J. Pirsch, *J. High Energy Phys.* **03** (2022) 147.
- [109] D. Bonocore, A. Kulesza, and J. Pirsch, *J. High Energy Phys.* **05** (2025) 034.
- [110] S. Ajith, Y. Du, R. Rajagopal, and D. Vaman, [arXiv:2409.17866](#).
- [111] V. D. Gershun and V. I. Tkach, *JETP Lett.* **29**, 288 (1979), [https://inspirehep.net/literature/145354](#).
- [112] L. Brink, P. Di Vecchia, and P. S. Howe, *Nucl. Phys.* **B118**, 76 (1977).
- [113] L. Brink, S. Deser, B. Zumino, P. Di Vecchia, and P. S. Howe, *Phys. Lett.* **64B**, 435 (1976); **68B**, 488(E) (1977), [https://inspirehep.net/literature/109550](#).
- [114] M. Henneaux and C. Teitelboim, *Ann. Phys. (N.Y.)* **143**, 127 (1982).
- [115] L. Bohnenblust, H. Ita, M. Kraus, and J. Schlenk, [arXiv:2505.15724](#).
- [116] K. Fernandes and F.-L. Lin, *J. High Energy Phys.* **06** (2024) 015.
- [117] D. Bonocore, *J. High Energy Phys.* **02** (2021) 007.
- [118] J. Vines, D. Kunst, J. Steinhoff, and T. Hinderer, *Phys. Rev. D* **93**, 103008 (2016); **104**, 029902(E) (2021).
- [119] P. S. Howe, S. Penati, M. Pernici, and P. K. Townsend, *Phys. Lett. B* **215**, 555 (1988).
- [120] F. Bastianelli, R. Bonezzi, O. Corradini, and E. Latini, *J. Phys. Conf. Ser.* **1208**, 012006 (2019).
- [121] F. Bastianelli, O. Corradini, and E. Latini, *J. High Energy Phys.* **02** (2007) 072.
- [122] X. Bekaert, N. Boulanger, and P. Sundell, *Rev. Mod. Phys.* **84**, 987 (2012).
- [123] K. Hallowell and A. Waldron, *Nucl. Phys.* **B724**, 453 (2005).
- [124] R. A. Porto, *Phys. Rep.* **633**, 1 (2016).
- [125] Z. Liu, R. A. Porto, and Z. Yang, *J. High Energy Phys.* **06** (2021) 012.
- [126] W. D. Goldberger, [arXiv:2206.14249](#).
- [127] N. Arkani-Hamed, T.-C. Huang, and Y.-t. Huang, *J. High Energy Phys.* **11** (2021) 070.
- [128] M.-Z. Chung, Y.-T. Huang, J.-W. Kim, and S. Lee, *J. High Energy Phys.* **04** (2019) 156.
- [129] T. Scheopner and J. Vines, *J. High Energy Phys.* **12** (2024) 060.
- [130] C. Gambino, P. Pani, and F. Riccioni, *Phys. Rev. D* **109**, 124018 (2024).
- [131] A. Cristofoli, P. H. Damgaard, P. Di Vecchia, and C. Heissenberg, *J. High Energy Phys.* **07** (2020) 122.
- [132] M. Bianchi, C. Gambino, P. Pani, and F. Riccioni, *Phys. Rev. D* **111**, 084013 (2025).
- [133] M. Bianchi, C. Gambino, F. Riccioni, and V. Zevola, [arXiv:2505.04800](#).
- [134] Strictly speaking, this is only true for classical dynamics, which is enough to consider here. Quantizing the gauged $SO(N)$ symmetry is nontrivial due to an anomaly [135]. However, this anomaly does not hinder the classical model and a mass can be introduced by adding the term $im\theta_i$ to Q_i , such that $\{\theta_i, \theta_j\}_{\text{PB}} = i\delta_{ij}$ produces the m^2 contribution to $\mathcal{H}$.
- [135] P. S. Howe, S. Penati, M. Pernici, and P. K. Townsend, *Classical Quantum Gravity* **6**, 1125 (1989).
- [136] S. Marsat, *Classical Quantum Gravity* **32**, 085008 (2015).
- [137] J. Steinhoff, *Fund. Theor. Phys.* **179**, 615 (2015).
- [138] In fact, it appears that by comparing with Eq. (6.38) in [133] D can be related to the stress quadrupole of the Myers-Perry black hole in $d = 5$. A detailed investigation of this relation is left for future work.
- [139] A. Guevara, A. Ochirov, and J. Vines, *Phys. Rev. D* **100**, 104024 (2019).
- [140] K. Haddad, *Phys. Rev. D* **105**, 026004 (2022).
- [141] R. Aoude, K. Haddad, and A. Helset, *J. High Energy Phys.* **07** (2022) 072.
- [142] R. Aoude, K. Haddad, and A. Helset, *Phys. Rev. D* **108**, 024050 (2023).
- [143] L. Cangemi, M. Chiodaroli, H. Johansson, A. Ochirov, P. Pichini, and E. Skvortsov, *Phys. Rev. Lett.* **131**, 221401 (2023).

End Matter

Ansatz for q_μ at $\mathcal{O}(S^2)$ —We present here the ansatz and the method for identifying the Wilson coefficients $C_{ES^2}, \tilde{C}_{ES^2}$ used to construct the Hamiltonian in Eq. (14) to order S^2 . All possible tensor structures at order S^2 can be captured by the ansatz

$$\begin{aligned}
 q_{c,(2)}S^{(2)} = & |\pi| \left(-C_1 \hat{\pi}^c S_{ab} S^{ab} + C_3 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c S_a^d S_{bd} - C_2 \hat{\pi}^a S_{ab} S^{cb} \right) + \frac{1}{|\pi|} \left(B_9 \hat{\pi}^c S^{ab} S^{de} C_{abde} \right. \\
 & + B_{10} \hat{\pi}^a S^{cb} S^{de} C_{abde} + B_7 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c S_a^f S^{de} C_{aebf} + B_8 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^d S_a^f S^{ce} C_{bedf} \\
 & - B_6 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c S_a^d S^{ef} C_{bedf} + B_1 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^d S_a^f S_e^f C_{bdf}^c - B_4 \hat{\pi}^a S_b^{bd} C_{dae}^c \\
 & \left. + B_2 \hat{\pi}^a S_a^{bd} C_{dbe}^c - B_3 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^d S_a^e S_b^f C_{edf}^c + B_5 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c \hat{\pi}^d S_a^f S_b^h C_{dfeh} \right). \quad (\text{A1})
 \end{aligned}$$

Plugging Eq. (A1) into Eq. (8), one can solve for eight of the thirteen coefficients. The resulting $q_{a,(2)}S^{(2)}$ that preserves the supersymmetry is then given by

$$\begin{aligned}
 q_{a,(2)}S^{(2)} = & |\pi| \left(-C_1 \hat{\pi}^c S_{de} S^{de} + 2C_1 \hat{\pi}^c \hat{\pi}^d \hat{\pi}^e S_d^f S_{ef} \right) + \frac{1}{|\pi|} \left(\frac{1}{24} (2 + 3B_1 - 3B_3 + 3B_6 + 3B_8) \hat{\pi}^c S^{ab} S^{de} C_{abde} \right. \\
 & + \frac{1}{12} (-1 - 6B_1 + 6B_3 - 6B_8) \hat{\pi}^a S^{cb} S^{de} C_{abde} + \frac{1}{2} (B_1 + B_8) \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c S_d^f S^{de} C_{aebf} \\
 & + B_8 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^d S_a^f S^{ce} C_{bedf} - B_6 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c S_a^d S^{ef} C_{bedf} \\
 & + B_1 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^d S_a^e S_b^f C_{bdf}^c - \frac{1}{6} (1 - 6B_3) \hat{\pi}^a S_b^e S^{bd} C_{dae}^c + \frac{1}{2} \hat{\pi}^a S_a^b S^{de} C_{dbe}^c \\
 & \left. - B_3 \hat{\pi}^a \hat{\pi}^b \hat{\pi}^d S_a^f S_b^c C_{edf}^c + \frac{1}{2} (-B_3 + B_6 - 2B_8) \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c \hat{\pi}^d S_a^f S_b^h C_{dfeh}^c \right), \tag{A2}
 \end{aligned}$$

while $\mathcal{H}$ is given by

$$\begin{aligned}
 \mathcal{H} = & \frac{m^2 - \pi^2}{2} + C_1 S_{ab} S^{ab} - 2C_1 \hat{\pi}^a \hat{\pi}^b S_a^c S_{bc} + \frac{1}{24} (-2 - 3B_1 + 3B_3 - 3B_6 - 3B_8) S^{ab} S^{cd} C_{abcd} \\
 & + \frac{1}{6} (1 - 3B_1 - 6B_3 - 3B_8) \hat{\pi}^a \hat{\pi}^b S_c^e S^{cd} C_{adbe} + \frac{1}{2} (3B_3 - B_6) \hat{\pi}^a \hat{\pi}^b \hat{\pi}^c \hat{\pi}^d S_a^e S_b^f C_{cedf} \\
 & + \left(-\frac{1}{3} + B_1 - B_3 + B_6 + B_8 \right) \hat{\pi}^a \hat{\pi}^b S_a^c S^{de} C_{bdce} + \mathcal{O}(S^3). \tag{A3}
 \end{aligned}$$

A comparison with Eq. (14) reveals the relation between C_{ES^2} , $\tilde{C}_{ES^2}$ and the coefficients B_i . The C_{BS^3} coefficients follow from the analogous construction at $\mathcal{O}(S^3)$.

Compton amplitude—We list here the terms appearing in Eq. (23).

$$V_{(0)} = 2(v \cdot F_1 \cdot F_2 \cdot v)^2 \tag{B1a}$$

$$V_{(1)} = i(v \cdot F_1 \cdot F_2 \cdot v)(k_1 v(F_1 \cdot F_2)^{\mu\nu} + (k_2 \cdot F_1 \cdot v)F_2^{\mu\nu} + (1 \leftrightarrow 2))S_{\mu\nu} \tag{B1b}$$

$$\begin{aligned}
 V_{(2)} = & -\frac{1}{2} \left((k_1 \cdot F_2 \cdot v)(k_2 \cdot F_1 \cdot v)F_1^{\mu\nu} S_{\mu\nu} F_2^{\sigma\rho} S_{\rho\sigma} + F_1^{\mu\nu} S_{\mu\nu} (v \cdot F_1 \cdot F_2 \cdot k_1)(v \cdot F_1 \cdot S \cdot k_1) \right. \\
 & + F_2^{\mu\nu} S_{\mu\nu} (v \cdot F_2 \cdot F_1 \cdot k_2)(v \cdot F_2 \cdot S \cdot k_2) \left. \right) - 2\tilde{C}_{ES^2} (v \cdot F_1 \cdot F_2 \cdot v)(k_1 \cdot k_2)(v \cdot F_1 \cdot S \cdot S \cdot F_2 \cdot v) \\
 & + (C_{ES^2} + \tilde{C}_{ES^2}) \left(-(k_2 \cdot F_1 \cdot v)(v \cdot F_1 \cdot S \cdot F_2 \cdot S \cdot F_2 \cdot k_1) - (k_1 \cdot F_2 \cdot v)(v \cdot F_2 \cdot S \cdot F_1 \cdot S \cdot F_1 \cdot k_2) \right) \\
 & + \frac{C_{ES^2} + \tilde{C}_{ES^2}}{4} \left(-(k_1 \cdot F_2 \cdot v)^2 (F_1^{\mu\nu} S_{\mu\nu})^2 - (k_2 \cdot F_1 \cdot v)^2 (F_2^{\mu\nu} S_{\mu\nu})^2 + 4(v \cdot F_1 \cdot F_2 \cdot v)(k_1 + k_2) \cdot S \cdot S \cdot (k_1 + k_2) \right. \\
 & - 4(k_1 \cdot S \cdot k_2)(F_1 \cdot F_2)^{\mu\nu} S_{\nu\mu} \left. \right) + \frac{C_{ES^2} + \tilde{C}_{ES^2}}{2} \left(-(k_1 \cdot F_2 \cdot v)F_1^{\mu\nu} S_{\mu\nu} [(v \cdot F_1 \cdot F_2 \cdot S \cdot k_1) + (v \cdot F_2 \cdot F_1 \cdot S \cdot k_2) \right. \\
 & - (v \cdot F_1 \cdot S \cdot F_2 \cdot k_1)] + (k_1 \cdot F_2 \cdot v)(v \cdot F_1 \cdot S \cdot k_2)(F_1 \cdot F_2)^{\mu\nu} S_{\nu\mu} - (v \cdot F_1 \cdot S \cdot F_2 \cdot k_1)^2 \\
 & \left. + (v \cdot F_1 \cdot F_2 \cdot k_1)(v \cdot F_1 \cdot S \cdot F_2 \cdot S \cdot k_1) + F_1^{\mu\nu} S_{\nu\mu} (v \cdot F_2 \cdot F_1 \cdot k_2)(v \cdot F_2 \cdot S \cdot k_2) + (1 \leftrightarrow 2) \right) \tag{B1c}
 \end{aligned}$$

$$\begin{aligned}
 V_{(3)} = & -i\tilde{C}_{ES^2}((k_2 \cdot F_1 \cdot v)(k_1 \cdot \mathcal{S} \cdot F_1 \cdot k_2)(v \cdot F_2 \cdot \mathcal{S} \cdot \mathcal{S} \cdot F_2 \cdot v) \\
 & + 2(k_2 \cdot F_1 \cdot v)(v \cdot F_1 \cdot \mathcal{S} \cdot k_1)(k_1 \cdot F_2 \cdot \mathcal{S} \cdot \mathcal{S} \cdot F_2 \cdot v) + 2(v \cdot F_1 \cdot F_2 \cdot k_1)(v \cdot F_1 \cdot \mathcal{S} \cdot k_1)(v \cdot F_2 \cdot \mathcal{S} \cdot \mathcal{S} \cdot k_2)) \\
 & + i\tilde{C}_{ES^2}^2((v \cdot F_1 \cdot F_2 \cdot v)(k_1 \cdot k_2)(v \cdot F_2 \cdot \mathcal{S}^3 \cdot F_1 \cdot v) + (k_1 \cdot k_2)(v \cdot k_1)(v \cdot F_2 \cdot \mathcal{S}^2 \cdot F_1 \cdot v)(v \cdot F_1 \cdot \mathcal{S} \cdot F_2 \cdot v)) \\
 & + i2\tilde{C}_{ES^2}(C_{ES^2} + \tilde{C}_{ES^2})(-(v \cdot F_1 \cdot F_2 \cdot k_1)(v \cdot F_1 \cdot \mathcal{S} \cdot k_1)(v \cdot F_2 \cdot \mathcal{S}^2 \cdot k_1) \\
 & + (k_2 \cdot F_1 \cdot v)(k_1 \cdot F_2 \cdot v)(k_1 \cdot \mathcal{S} \cdot F_1 \cdot \mathcal{S}^2 \cdot F_2 \cdot v) + (k_2 \cdot F_1 \cdot v)(k_2 \cdot \mathcal{S} \cdot F_2 \cdot k_1)(v \cdot F_1 \cdot \mathcal{S}^2 \cdot F_2 \cdot v)) \\
 & - \frac{i}{4}(C_{ES^2} + \tilde{C}_{ES^2})(k_2 \cdot F_1 \cdot v)(k_2 \cdot \mathcal{S} \cdot F_2 \cdot k_1)(F_1^{\mu\nu} \mathcal{S}_{\mu\nu})(F_2^{\mu\nu} \mathcal{S}_{\mu\nu}) \\
 & - i(C_{ES^2} + \tilde{C}_{ES^2} - 1)^2(v \cdot F_2 \cdot \mathcal{S} \cdot k_2)(v \cdot F_1 \cdot \mathcal{S}^2 \cdot F_2 \cdot v)(k_1 \cdot \mathcal{S} \cdot F_1 \cdot k_2) \\
 & - \frac{i}{4}(C_{ES^2} + \tilde{C}_{ES^2})^2(2(k_1 \cdot F_2 \cdot v)(k_2 \cdot \mathcal{S} \cdot F_2 \cdot \mathcal{S} \cdot F_1 \cdot k_2)(F_1^{\mu\nu} \mathcal{S}_{\mu\nu}) \\
 & + 2(k_1 \cdot \mathcal{S} \cdot F_1 \cdot k_2)(v \cdot F_1 \cdot \mathcal{S} \cdot F_2 \cdot k_1)(F_2^{\mu\nu} \mathcal{S}_{\mu\nu}) + (k_1 \cdot \mathcal{S} \cdot k_2)(v \cdot F_1 \cdot F_2 \cdot k_1)(F_1^{\mu\nu} \mathcal{S}_{\mu\nu})(F_2^{\mu\nu} \mathcal{S}_{\mu\nu})) \\
 & - \frac{i}{3}C_{BS^3}(k_1 \cdot F_2 \cdot v)((v \cdot F_2 \cdot \mathcal{S} \cdot q_2)(v \cdot F_1 \cdot \mathcal{S}^2 \cdot F_1 \cdot k_1) + (v \cdot F_1 \cdot \mathcal{S} \cdot k_1)[v \cdot F_1 \cdot F_2 \cdot \mathcal{S}^2 \cdot (k_1 + k_2)] \\
 & + (v \cdot F_1 \cdot \mathcal{S} \cdot k_2)(v \cdot F_1 \cdot F_2 \cdot \mathcal{S}^2 \cdot k_2) + (k_2 \cdot \mathcal{S} \cdot F_1 \cdot k_2)(v \cdot F_2 \cdot \mathcal{S}^2 \cdot F_1 \cdot v) \\
 & + (k_1 \cdot \mathcal{S} \cdot k_2)(v \cdot F_1 \cdot \mathcal{S}^2 \cdot F_2 \cdot F_1 \cdot v) + (k_2 \cdot F_1 \cdot v)[v \cdot \mathcal{S} \cdot (F_2 \cdot \mathcal{S} + \mathcal{S} \cdot F_2) \cdot \mathcal{S} \cdot (k_1 + k_2)] \\
 & + (v \cdot F_1 \cdot F_2 \cdot v)\{[k_1 \cdot \mathcal{S} \cdot F_1 \cdot \mathcal{S}^2 \cdot (k_1 + k_2)] + (k_1 \cdot \mathcal{S}^2 \cdot F_1 \cdot \mathcal{S} \cdot k_2)\} - 2(v \cdot F_1 \cdot \mathcal{S} \cdot k_2)(v \cdot F_1 \cdot F_2 \cdot \mathcal{S}^2 \cdot k_1)) \\
 & + \frac{i}{3}C_{BS^3}((k_1 \cdot k_2)(v \cdot F_2 \cdot \mathcal{S} \cdot F_1 \cdot k_2)(v \cdot F_2 \cdot \mathcal{S}^2 \cdot F_1 \cdot v) + (k_1 \cdot \mathcal{S} \cdot k_2)(v \cdot F_2 \cdot F_1 \cdot k_2)(v \cdot F_1 \cdot \mathcal{S}^2 \cdot F_2 \cdot v) \\
 & - (k_1 \cdot \mathcal{S}^2 \cdot k_2)(v \cdot F_2 \cdot F_1 \cdot k_2)(v \cdot F_1 \cdot \mathcal{S} \cdot F_2 \cdot v) - (k_2 \cdot \mathcal{S}^2 \cdot k_2)(v \cdot F_2 \cdot F_1 \cdot k_2)(v \cdot F_1 \cdot \mathcal{S} \cdot F_2 \cdot v) \\
 & + (k_1 \cdot \mathcal{S} \cdot k_2)(v \cdot F_1 \cdot F_2 \cdot v)((k_1 + k_2) \cdot \mathcal{S}^2 \cdot F_2 \cdot F_1 \cdot v) \\
 & - 2(v \cdot F_2 \cdot F_1 \cdot k_2)(v \cdot F_2 \cdot \mathcal{S} \cdot k_2)[v \cdot F_1 \cdot \mathcal{S}^2 \cdot (k_1 + k_2)]) + (1 \leftrightarrow 2). \tag{B1d}
 \end{aligned}$$

We note that the way to write the Compton amplitude in terms of contractions of the F_i tensors is not unique. The Compton amplitude is on-shell and therefore the number of spacetime dimensions d does only enter implicitly through the various d -dimensional tensors.