

Quantum Stroboscopy for Time Measurements

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Mielnik’s cannonball argument uses the Zeno effect to argue that *projective* measurements for time of arrival are impossible. If one repeatedly measures the position of a particle (or a cannonball!) that has yet to arrive at a detector, the Zeno effect will repeatedly collapse its wave function away from it: the particle never arrives. Here we introduce quantum stroboscopic measurements where we accumulate statistics of projective position measurements, *performed on different copies of the system* at different times, to obtain a time-of-arrival distribution. We show that, under appropriate limits, this gives the same statistics as time measurements of conventional “always on” particle detectors that bypass Mielnik’s argument using nonprojective, weak continuous measurements. In addition to time of arrival, quantum stroboscopy can describe distributions of general time measurements. It can also be adapted to obtain the conditional probability distribution of arrival times, given that the particle was not previously detected at the detector.

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If one performs a projective measurement of an observable repeatedly at a high rate, the first measurement will collapse the system to an eigenstate and the subsequent ones will freeze the system to that eigenstate, preventing further evolution: the Zeno effect [1,2]. This seems to prevent the possibility of describing time-of-arrival measurements, i.e., the time at which a particle arrives at the position of a detector, in the case in which it had a negligible probability of being at the detector position initially. Indeed, such a detector must measure the position repeatedly with a high repetition rate τ . But these measurements will collapse the position of the particle away from the detector, and the particle will never arrive there. Mielnik [3] points out that, by the same argument, one can even stop cannonballs! Barchielli *et al.* [4,5] showed that typical detectors *are* able to detect particles because they do not perform exact projective measurements: they measure a position “fuzzily” [6], with a variance σ that must scale at least as $1/\tau$ to avoid the Zeno effect. This scaling persists also in the continuous limit $\tau \rightarrow 0$, namely the product $\sigma\tau$ must be a constant κ , inversely proportional to the coupling strength $1/2\kappa$ between the apparatus and the system [4]. By contrast, strong projective precise measurements ($\sigma = 0$) have infinite coupling to the apparatus $\kappa \rightarrow \infty$. Many different models of continuous time measurements

(“always on” detectors) of this type have been studied, e.g., Refs. [6–11]. These are “weak” measurements due to the finite coupling κ or to the fuzziness σ . They have been used to obtain time distributions, e.g., the “waiting times” [12,13] or [14,15]. However, the coupling to the apparatus alters the dynamics of the system, yielding a master equation [16,17] with a dissipative-dispersive term of order $1/\kappa$ [4,5], so time measurements outcomes are typically distorted in a way that cannot be easily fixed.

In this Letter we introduce quantum stroboscopy: a procedure that overcomes both the Zeno effect and the distortions of continuous measurements, by leaving the system untouched up to a time t_m when a strong (projective) measurement is performed. By repeating on different identically prepared copies of the system for M different times t_m , and appropriately renormalizing the outcome statistics (Fig. 1), one can obtain *time distributions* that pertain to that measurement (time of arrival being one specific case). Under the above hypothesis that there is a negligible initial probability of finding the particle at the detector, the leading tails of the obtained distribution will give the earliest “time of arrival.” Our proposal differs from the stroboscopic procedures of [18–21], where repeated measurements are performed on the *same* system. Importantly, we show that the obtained statistics matches the one of the continuous measurements described above, in the limit where the effect of the coupling to the apparatus can be removed by accumulating sufficient statistics. Indeed, in continuous measurements, the apparatus disturbance typically grows with time, so the ratio between the measurement variances with and without apparatus is linear in the total duration of the experiment T , and it is

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proportional to the apparatus coupling $1/\kappa$. One can counteract this variance by repeating the procedure a number of times $M' \gtrsim T/\kappa$, so the variance $T/\kappa M'$ of the average becomes negligible. This is the same scaling as quantum stroboscopy where $M = T/\tau$. Indeed, if one takes also into account the fuzziness σ due to the avoidance of the Zeno effect, one must have $\kappa = \sigma\tau$, where the continuous measurements can be seen as a small- τ limit of a succession of τ -separated measurements with variance σ . To statistically remove the fuzziness σ , one needs to repeat the procedure an additional $M'' \propto \sigma$ times, so it is repeated $M'M'' \propto \sigma T/\kappa = T/\tau = M$ times, where we used $\kappa = \sigma\tau$, namely, quantum stroboscopy uses the same number of resources (experiment repetitions) $\propto T$ as the conventional “always on” detectors. But stroboscopy has the advantage that it does not introduce any difficult-to-remove distortions of the time distributions due to the apparatus coupling.

We also show that the quantum stroboscopy time distributions match the ones obtained from the recent “quantum clock” proposal for time measurements [22]. A modified version of quantum stroboscopy can also generate the “quantum flow” proposal for time measurements [23]. In the following we will focus mainly on time-of-arrival measurements, but our results can be immediately extended to other time measurements, e.g., “the time at which a spin is up” [22,24].

We first introduce quantum stroboscopy; then we show that its probabilities are the same as the quantum clock’s ones [22]; using the models in [7] and [8], we then show that they also match the ones from continuous measurements for time of arrival; and finally, we discuss more general measurements.

Quantum stroboscopy—Quantum stroboscopy consists of leaving the quantum system unperturbed up to a time t_m when a projective, ideally instantaneous, measurement is performed, and then the system is discarded, as also shown in [24,25]. Repeating the whole procedure for different times (whence the name “stroboscopy”) and then normalizing the measurement results over time, we obtain the conditional probability that the time is t given that the system has a specific value of the measured observable.

In more detail, quantum stroboscopy consists of the following: (1) Prepare the system at time $t = 0$ and let it evolve freely. (2) At time $t_m = m\tau + t_0$, with $m = 0, 1, \dots, M-1$ and t_0 the time of the first measurement, perform a projective (“strong”) measurement of the observable $A = \sum_{n=0}^{N-1} a_n P_n$, with a_n and P_n being its eigenvalues and projectors on eigenspaces, respectively. Then discard the system. (3) Repeat steps 1 and 2 on LM copies of the system with $L \gg N$, to obtain M accurate (strong) measurements of the observable A at M different times t_m [24–26]. (4) Distribute the outcomes on an $N \times M$ matrix (Fig. 1) whose element ℓ_{nm} is equal to the number of times the outcome a_n was obtained at time t_m . The time probability distribution, the “probability that time is t_m

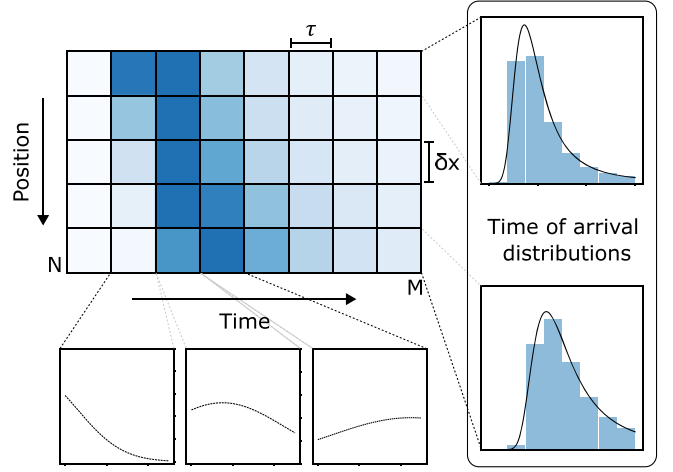


FIG. 1. Quantum stroboscopy for position. Divide the total measurement duration T into M intervals $\tau = T/M$. Then perform L position measurements at times $t_m = m\tau + t_0$ (with $m = 0, 1, \dots, M-1$), where $L \gg N$ is larger than the number N of histogram bins at each time. Populate the m th histogram with the outcomes (vertical lines): add an event to the n th row, m th column for a position result $n\delta x$ (with $n = 0, \dots, N-1$) at time t_m (δx is the spatial resolution). Normalize the rows, to obtain the time-of-arrival probabilities at positions $n\delta x$ as $p(t_m|n) = \ell_{nm} / \sum_m \ell_{nm}$, where ℓ_{nm} is the number of shots with outcome n at time t_m . By construction, this is normalized over time for all values of n . [It is undefined if there are rows n with no events: the particle never “arrives” at location $n\delta x$]. A similar procedure can produce time probabilities of arbitrary measurements. Here we simulated a Gaussian packet propagating downward. The color map and the lower graphs represent the probability $p(n|t_m)$ for each outcome at t_m . The right graphs show the time probability $p(t_m|n)$ (histograms) and their compatibility (proved in the main body) with the quantum clock (continuous line).

given that A has value a_n ,” is obtained by normalizing the rows of the ℓ_{nm} matrix:

$$p(t_m|a_n) = \ell_{nm} / \sum_{m=0}^{M-1} \ell_{nm}. \quad (1)$$

If row n has no entries, then the probability is undefined: the system is *never* measured at value a_n during the whole time interval $[t_0, t_0 + T]$ when it was probed. The number of points that fall into each histogram bin follows a multinomial distribution, and the statistical error of each bin can be estimated as $\sim 1/\sqrt{L}$, arising from the statistics of a binomial distribution, where success (failure) means falling inside (outside) the bin. Moreover, timing errors introduced by a noninstantaneous detector can be described replacing projective measurements with a positive operator-valued measure (POVM) [27]. See Appendix A.

In the above procedure, the choice of the time interval $[t_0, t_0 + T]$ can be done using some prior information on the system. For example, for periodic evolutions, T should be

larger than a period; for time of arrival it must contain the predicted time that the particle will be at the detector: the time t_0 must be smaller than the predicted arrival time of the first wave packet tail, T must be of the order of the wave packet duration Δt , and so on. If, instead, no prior information is present, one can quickly converge to the interesting T by starting with a large T and adjusting it rapidly by first running the above procedure with a small L (the binomial probability of finding an interesting outcome at some time ensures that it is sufficient), finding the interval where the outcomes are interesting (e.g., the particle has arrived). Then one can increase L only for such intervals.

We now apply it to time of arrival. There are two inequivalent reasonable definitions [3,23–26,28–45]: (i) the time at which a particle is detected at the detector (quantum stroboscopy). In the case in which the probability of being detected there is initially negligible, then the leading tail of the obtained distribution gives a notion of the time of “first arrival”; (ii) “first passage time,” i.e., the time at which the particle is detected at the detector conditioned upon the fact it had not been detected there before (conditional quantum stroboscopy). In the first case, the observable A is a projector P_x onto the detector position x , and $N = 2$ corresponding to “the particle is ($n = 0$) or is not ($n = 1$) at the detector.” In the second case, the above procedure must be slightly modified to consider repeated measurements on the same system, namely “conditional quantum stroboscopy” (see also Ref. [18]): (1) Prepare the system at time $t = 0$. (2) At every time step τ perform a projective measurement *on the same system* at the detector position until the particle is detected, recording the time $t = m\tau$, or until a cutoff time $t_{\max}$ is reached. (3) Repeat the procedure on different copies of the system collecting the statistics of detections t . The histogram of such t s will provide the conditional probability of the time at which the particle arrives at the detector, given that it was not detected there previously. As discussed above, to avoid the Zeno effect, the product $\sigma\tau$ must be constant (decreasing τ must correspond to increasing the measurement fuzziness σ).

Quantum clock—We now show that the procedure (i) gives the same distribution as the “quantum clock” proposal of Ref. [22], which uses the Bayes rule to define the probability distribution for time t given that the outcome of the measurement of A is a_n , as

$$p_{qc}(t|a_n) = \langle \psi(t) | P_n | \psi(t) \rangle / \int_{t_0}^{t_0+T} dt \langle \psi(t) | P_n | \psi(t) \rangle, \quad (2)$$

where $|\psi(t)\rangle$ is the system state at time t and T is the total time that the procedure is run for; see Eq. (9) of Ref. [22]. Consider the limit $L \rightarrow \infty$ of infinite measurements at each time step; the Born rule implies that, for each m , $\ell_{nm}/L \rightarrow \langle \psi(t) | P_n | \psi(t) \rangle$. Taking also $M \rightarrow \infty$,

$$\frac{\tau}{L} \sum_{m=0}^{M-1} \ell_{nm} \rightarrow \int_{t_0}^{t_0+T} dt \langle \psi(t) | P_n | \psi(t) \rangle, \quad (3)$$

where we used the fact that $\tau = T/M \rightarrow dt$, namely, $\ell_{nm}/(\tau \sum_m \ell_{nm}) \rightarrow p_{qc}(t_m|a_n)$, so

$$p_{qc}(t_m|a_n)dt = \lim_{L,M \rightarrow \infty} \ell_{nm} / \sum_m \ell_{nm}, \quad (4)$$

where the dt reminds us that p_{qc} is a probability *distribution*, namely $p_{qc}(t|a_n)dt$ is the (adimensional) probability that t is in the interval $[t, t + dt]$, whereas p_{qc} by itself has dimensions of t^{-1} , as is clear from Eq. (2). The quantum clock probability distributions were derived under the assumption that one can quantize time [46]. However, the above equivalence with quantum stroboscopy implies that this assumption is not really necessary.

A *different* notion of arrival at time t uses the *flow* of probability, i.e., the marginal change over time of the number of particles that reached the detector position [23–25,45]. In this case, the arrival time distribution is $\pi_x(t) \propto |\partial_t \int_{-\infty}^x \rho_t(u) du|$, with $\rho_t(x)$ being the distribution of the measured position at time t , given by the Born rule [23]. In quantum stroboscopy, this can be cast by replacing Eq. (1) with $\hat{\pi}_x(t_m) \propto |\sum_{n/x_n < x} \ell_{nm} - \ell_{n(m-1)}|$, normalized over all the possible times t_m . Similarly, π_x can be obtained by taking L and M to infinity.

Continuous measurements—We now show that, for the unconditioned time of arrival, the quantum stroboscopy distribution can be obtained from continuous position measurements. To this aim we use the Caves-Milburn model [7] of continuous position measurement. While this is a specific model, it is optimal; it satisfies the general bounds of [4,5]: $\kappa = \sigma\tau$ is constant in the limit of $\tau \rightarrow 0$.

The model uses a simple von Neumann measurement where the position x of the particle is coupled to M single particle memories with the coupling time-dependent Hamiltonian $H_{\text{int}} = \sum_{m=0}^{M-1} \delta(t - m\tau) x p_m$, where p_m is the momentum of the m th memory. Each memory is initially prepared in a Gaussian wave packet with variance σ (it induces an exact position measurement for $\sigma \rightarrow 0$). We consider nonzero σ a fuzzy position measurement. Taking the limit $\tau \rightarrow 0$ of continuous measurements, the effect of the measurement becomes a nonunitary coupling to an environment (the memories), which can be described by the master equation [7]

$$d\rho_t/dt = -\frac{i}{\hbar} [H_0, \rho_t] - \gamma \left(x \rho_t x - \frac{1}{2} \{x^2, \rho_t\} \right), \quad (5)$$

with ρ_t being the state of the particle, H_0 being its free Hamiltonian (in the absence of coupling to the measurement apparatus), and $\gamma = 1/(2\kappa)$ with $\kappa = \lim_{\tau \rightarrow 0} \sigma\tau$. Its effect is a dissipative dynamics that diffuses the particle's

momentum: $\Delta^2 p(t) = \Delta^2 p_0(t) + \hbar^2 t / 2\kappa$, where $\Delta^2 p$ is the overall variance in the momentum at time t and $\Delta^2 p_0$ is the variance due to the free evolution only [7]. In the free, mass m_p particle case, Eq. (5) gives equations of motion $d\langle x \rangle_t / dt = \langle p \rangle_t / m_p$, $d\langle p \rangle_t / dt = 0$, and [47]

$$\Delta^2 x(t) = \Delta^2 x_0 + c_0 t + \Delta^2 p_0 t^2 / m_p^2 + \hbar^2 t^3 / 6\kappa m_p^2, \quad (6)$$

with $\Delta^2 x_0$, $\Delta^2 p_0$ being the initial variances and c_0 being an integration constant; see Appendix B. Both position and momentum uncertainties grow with time (Heisenberg uncertainty notwithstanding) because the evolution [Eq. (5)] is nonunitary (dissipative) due to the disturbance induced by the apparatus. It can be easily counterbalanced by repeating the procedure M times and averaging, so the variance on the average is reduced by a factor M (central limit theorem). In the absence of apparatus disturbance ($\kappa \rightarrow \infty$), Eq. (6) gives a quadratic position spread. Recovering a quadric spread in Eq. (6), i.e., in the presence of disturbance, requires $M \propto t$, which recovers a $\propto t^2$ variance in the average position. This just recovers the scaling: there is a distortion of the recovered distribution because it is not governed by the (expected) constant $\Delta^2 p_0$ but by a different one. By contrast, quantum stroboscopy does not suffer from this distortion and will indeed capture the correct $\Delta^2 p_0 t^2 / m_p^2$ scaling.

So, in the limit in which one can neglect these distortions the two procedures both give a distribution whose variance grows quadratically. Indeed, the continuous measurement obtains M particle trajectories $\langle x_j \rangle_t$ (which have a nonzero probability when probed with suitable test functions [5]). One then must invert them to find the times t_{kj} when $\langle x_j \rangle_{t_{kj}}$ is equal to the detector position. Distributing them on a histogram, one can obtain the probability that time is t given that the particle is measured at the detector position, the same as for quantum stroboscopy. [From the average trajectory, one can also obtain other (trajectory-based) notions of time of arrival, e.g., Refs. [39,44].]

Trajectories are not necessary: in the framework of continuous measurements one can consider a measurement that continuously checks whether a particle has arrived at a certain location x [8,11,48,49]. For example, one can use the following interaction between a localized mode c_x of the particle's field and a bosonic memory mode b_t (a different memory for each time t): $H_{\text{int}}(t) = i\sqrt{\kappa}(c_x b_t^\dagger - c_x^\dagger b_t)$ [8], a time dependent Hamiltonian which couples c_x to b_t only in the interval $[t, t + dt]$. It is convenient to introduce modes $B_t \equiv b_t \sqrt{dt}$ which satisfy $[B_t, B_t^\dagger] = 1$. The effect of this coupling on the evolution of the system is again a dissipative master equation [8],

$$d\rho_t / dt = -\frac{i}{\hbar} [H_0, \rho_t] - \gamma \left(c_x \rho_t c_x^\dagger - \frac{1}{2} \{ c_x^\dagger c_x, \rho_t \} \right). \quad (7)$$

By monitoring the number of photons N_t in the memory modes B_t , one sees a stream of clicks N_t : a photon $N_t = 1$ in mode B_t implies that the particle has arrived at time t (to first order in dt one never sees more than 1 photon in B_t) and the probability of seeing a photon at t is $\langle N_t \rangle = \gamma dt \beta$ with $\beta = \text{Tr}[\rho_t^\dagger c_x^\dagger c_x]$, where $\rho_t^\dagger$ is the state of the system *conditioned* by its past history of clicks and N_t describes a Poisson increment [8]. This implies that, if the particle is with certainty at the detector position, namely $\beta = 1$, for a time Δt (a particle with a rectangular time-of-arrival profile), then the click statistic in that interval is Poissonian with expectation value and variance $\gamma \Delta t = \Delta t / 2\kappa$. Typical (nonrectangular) situations will entail that the click statistics will be a product of a Poissonian times the temporal wave packet profile. In both cases, to counteract the Poissonian variance, one needs to repeat the experiment at least a number $M \propto \Delta t / \kappa$ of times, where Δt is the interval when the distribution is substantially nonzero. (This M guarantees, for example, that the statistical fluctuations of the average time of arrival will not be affected much by the Poissonian spread.) The outcome of this procedure will be a string of times t_j at which $N_{t_j} = 1$. To obtain a probability, one must arrange them into a histogram. In the limit in which the apparatus disturbance in (7) can be neglected, it gives the probability that time is t given that the particle was detected at the detector position, the same as quantum stroboscopy (which does not require such a limit by construction: the particle is unperturbed until it is measured). If $T \simeq \Delta t$ (as discussed above), we find that the number of times M one must repeat the continuous measurements is again of order T , the same as quantum stroboscopy.

The above arguments can be used to treat all continuous measurements where the variance $\Delta^2 A_{\text{time step}}$ of the considered observables A at each time t are independent and constant at each time step $t \rightarrow t + dt$, namely, measurements where the error at different times is uncorrelated. In this case, it is clear that the error over a finite interval T grows linearly with T : $\Delta^2 A_{\text{tot}} = T \Delta^2 A_{\text{time step}}$, and one can remove the apparatus disturbance by increasing the statistics M by choosing $M \propto T$, which, again, is the same scaling as quantum stroboscopy.

General time measurements—We now consider measurements beyond time of arrival [22,24], e.g., the time at which a driven two level atom is excited or the time at which a precessing spin is up. In the first case, the continuous measurement scenario involves a highly non-trivial interplay between the atom and the driving field that plays the role of the apparatus [see Eq. (2.14) of Ref. [50]], which prevents a straightforward deconvolution to recover the free-evolution temporal characteristics, although a numerical optimization of the apparatus specs recovers some of them [6], e.g., their oscillatory behavior. Instead, a stroboscopic measurement can easily recover the full free-evolution distribution of the “probability that time

is t_m given that the atom is excited”: $p(t_m|\text{excited}) = \cos^2(\Omega t_m/2) / \sum_{m=0}^{M-1} \cos^2(\Omega t_m/2)$ (with Ω being the Rabi frequency), which, for $M \rightarrow \infty$ gives the quantum clock distribution $\cos^2(\Omega t/2) / \int_0^T dt \cos^2(\Omega t/2)$ [22]. It closely tracks the free evolution (Rabi flopping), namely the “probability that the atom is excited given that time is t ”: $p(\text{excited}|t) = \cos^2(\Omega t/2)$.

Conclusions—In conclusion, we have introduced quantum stroboscopy that produces time distributions for measurements of arbitrary observables A . It returns the same time probability of the quantum clock proposal [22], while a suitably modified version returns the time probability of the “quantum flow” proposal [23]. It also returns the same distributions of the “always on” detectors, if the intrinsic noise of such detectors can be bypassed using sufficient statistics. In this case, the resource count of quantum stroboscopy matches the one of the “always on” detectors: they both require a number of repetitions M , linear in the total measurement time T . Continuous measurements require M repetitions to reduce the apparatus error by averaging the outcomes, whereas quantum stroboscopy requires M repetitions because it is composed of M projective measurements at M different times on M copies of the system (one measurement per copy). If, instead, the intrinsic noise of the “always on” detectors cannot be bypassed (as for Rabi flopping), then these detectors become useless for time distributions, while quantum stroboscopy still manages to correctly track the free evolution. For time of arrival, quantum stroboscopy describes the unconditioned distribution of finding the particle at the detector. A slight modified procedure, the “conditioned quantum stroboscopy,” returns the conditional distribution of finding the particle at the detector given that it was not there previously (first passage).

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Data availability—The data that support the findings of this article are openly available [27].

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End Matter

Appendix A: Nonideal detectors—In this appendix, we analyze quantum stroboscopy for nonideal time measurements with finite resolution and timing errors. Timing errors typically consist of a jitter that introduces a random offset to each time measurement outcome, whereas a finite timing resolution implies that the measurement outcome has discrete values $\{t_m\}$ with integer m . We model such errors by replacing projective (strong instantaneous) measurements used in the main text with the POVM,

$$\{\Pi_m = \int dt \phi(m|t) |t\rangle\langle t| \text{ with } m = 0, \dots, M-1\}, \quad (\text{A1})$$

$$\Pi_{\text{fail}} = \mathbb{1} - \sum_m \Pi_m, \quad (\text{A2})$$

where $\{\phi(m|t)\}$ is a sequence of equal-width Gaussians (whose widths represent the jitter) and whose averages identify the time bins $\{t_m\}$. Notice that t and m label the values of a continuous (time) and discrete (bin) random variable, respectively.

We consider different regimes where the POVM elements have significantly overlapping or disjoint supports. We analyze this scenario by performing a Monte Carlo simulation that consists of the following steps. (1) Draw the ideal outcomes from the continuous stroboscopic distribution of Eq. (1), e.g., via inverse random sampling or rejection sampling (whenever inverting the cumulative distribution is computationally challenging). (2) For each sample, compute the cumulative distribution $F_\phi(m|t) = \sum_{\tilde{m} \leq m} \phi(\tilde{m}|t)$ under the normalization condition $\max_t \sum_m \phi(m|t) \leq 1$. (3) Draw the real outcomes by

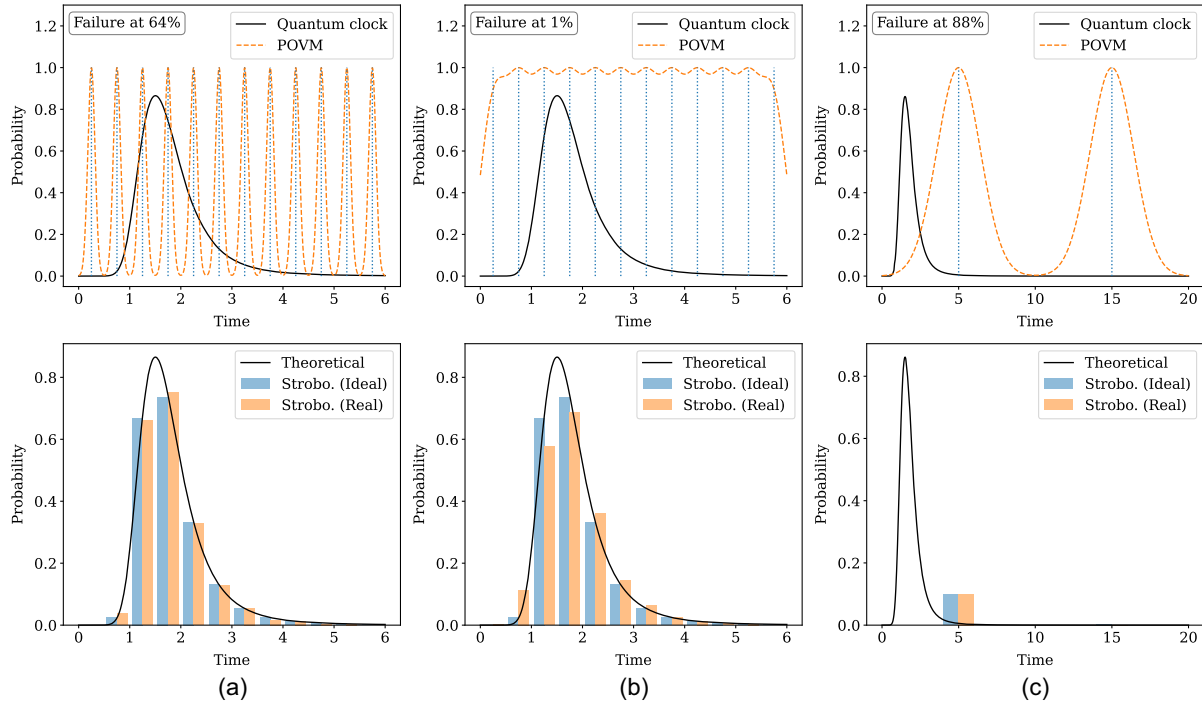


FIG. 2. Monte Carlo simulation of noninstantaneous stroboscopic measurements. The upper panel shows the theoretical distribution (solid line) compared against the sequence of Gaussians that forms the POVM $\{\phi(m|t)\}$ (dashed line). Each vertical axis tracks a different stroboscopic bin. The lower panel compares the histograms of the ideal (projective) stroboscopic measurement against the noninstantaneous (POVM) one. (a) Stroboscopy for nonoverlapping POVM. The real outcome closely resembles the projective one, with a 64% probability of detection failure. (b) Stroboscopy with overlap. The real outcome significantly differs from the ideal one, with failure probability of 1%. (c) Worst-case scenario in which the POVM resolution is lower than the system time scale. Both the ideal and the noninstantaneous measurement collapse to a single bin.

applying inverse transform sampling on $F_\phi(m|t)$. In case of detection failure, i.e., when the particle is not collected by one of the bins, renormalize the distribution to successful events.

Our simulations exhibit two different regimes. When the overlap decreases, the outcome distribution better approximates the ideal one with a non-negligible probability of failure. Conversely, a smaller overlap reduces the probability of failure but progressively dampens the distribution toward the uniform one. See Fig. 2.

Appendix B: Derivation of Eq. (6)—In this appendix, we analyze the diffusion of the particle's position in the Caves-Milburn model; see also Ref. [47]. Consider the master equation describing the apparatus disturbance, which can be equivalently written as

$$d\rho_t/dt = -\frac{i}{\hbar}[H_0, \rho_t] - \frac{1}{4\kappa}[x, [x, \rho_t]]. \quad (\text{B1})$$

Consider an observable A with variance $\Delta^2 A = \langle A^2 \rangle - \langle A \rangle^2$. In Schrödinger's picture, the observable expectation value evolves as $d\langle A \rangle/dt = \text{Tr}(A d\rho_t/dt)$, namely

$$d\langle A \rangle_t/dt = -\frac{i}{\hbar}\langle [A, H_0] \rangle - \frac{1}{4\kappa}\langle [x, [x, A]] \rangle, \quad (\text{B2})$$

with variance $d\Delta^2 A_t/dt = d\langle A^2 \rangle_t/dt - 2\langle A \rangle_t d\langle A \rangle_t/dt$. From now on, we neglect the subscript t , which explicitly denotes time dependence, and we denote I the identity operator. Since $[x, p^2] = 2i\hbar p$, the equations of motions (i.e., the time evolution of the first-order moments) read as $d\langle x \rangle/dt = \langle p \rangle/m_p$ and $d\langle p \rangle/dt = 0$.

The second-order moment and the variance of x and p can be obtained as follows. From $[x, [x, p^2]] = -2\hbar^2 I$ we get $d\langle p^2 \rangle/dt = \hbar^2/2\kappa$. By substitution and integration, the momentum variance reads as

$$\Delta^2 p = \Delta^2 p_0 + \hbar^2 t/2\kappa, \quad (\text{B3})$$

where $\Delta^2 p_0$ accounts for the free evolution only. Moreover $[x^2, p^2] = 2i\hbar\{x, p\}$, yielding $d\langle x^2 \rangle/dt = \langle \{x, p\} \rangle/m_p$. Taking the second-order derivative of $\Delta^2 x$ gives

$$d^2 \Delta^2 x/dt^2 = \frac{1}{m_p} d\langle \{x, p\} \rangle/dt - \frac{2}{m_p^2} \langle p \rangle^2, \quad (\text{B4})$$

where the second term is obtained by imposing the equations of motions. From $[\{x, p\}, p^2] = 4i\hbar p^2$, we get

$d\langle\{x, p\}\rangle/dt = 2\langle p^2\rangle/m_p$. From this, we obtain a direct relation between $\Delta^2 p$ [given by (B3)] and the second-order derivative of $\Delta^2 x$, which integrated twice yields

$$\Delta^2 x = \Delta^2 x_0 + c_0 t + \Delta^2 p_0 t^2/m_p^2 + \hbar^2 t^3/6m_p^2\kappa, \quad (\text{B5})$$

with $\Delta^2 x_0, \Delta^2 p_0$ being the initial variances, without disturbance, and c_0 being an integration constant representing the initial position-momentum covariance. This is Eq. (6) of the main text.