

Regge Spectral Generator and Form Factors from Hard Exclusive Amplitudes in Holographic QCD

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(Received 29 March 2026; revised 20 July 2026; accepted 18 August 2026; published 9 September 2026)

We show that the infinite tower of hard exclusive amplitudes in holographic light-front QCD leads to a spectral generator $G(\alpha, \lambda)$ that encodes the full Regge spectrum. The construction assumes a Poisson distribution of Fock-state components, where λ represents the average parton multiplicity above the valence configuration. The resulting generator yields a Regge spectrum invariant under continuous λ deformations. Its even- and odd-twist projections yield the meson and baryon form factors with a single value of λ , fixed by the pion charge radius, which in turn determines the full Q^2 dependence. The constituent counting rules for both sectors follow from the structure of the spectral generator, which also provides a compact analytic representation of parton distributions.

DOI: [10.1103/55pw-29c6](https://doi.org/10.1103/55pw-29c6)

Introduction—Unlike deep inelastic scattering, where the proton is typically dissociated through incoherent processes, exclusive reactions at large virtuality leave the target hadron intact [1]. At high Q^2 the virtual probe resolves short-distance structure inside the hadron, providing a high-resolution view of its internal dynamics, while the scattering amplitude remains coherent and preserves the bound state. Exclusive amplitudes at large Q^2 obey the QCD constituent counting rules [2,3]. The same power-law behavior also emerges in the gauge-gravity framework, where Polchinski and Strassler showed that hard scattering in a confining background reproduces the twist scaling expected from QCD [4].

This observation laid the foundation for holographic light-front QCD [5,6], which rests on two complementary routes: the mapping of the Drell-Yan-West form factor [7,8]—the light-front overlap of hadronic wave functions summed over Fock components—onto the equivalent bulk-to-boundary expression in anti-de Sitter (AdS) space [5], and the mapping of the semiclassical light-front Hamiltonian eigenvalue equation onto the AdS wave equation, from which the Regge spectrum emerges [6]. Exclusive processes are described in this framework in

terms of coherent contributions from the Fock-state components of the hadron, allowing holographic methods to be applied even at large virtuality, where each twist amplitude exhibits hard scaling at high Q^2 [9]; however, the coefficients of the Fock expansion have so far required phenomenological input [10].

Motivated by the coherent structure of the light-front Fock expansion for exclusive processes at arbitrary virtuality, we introduce in this Letter a Poisson sum over the infinite tower of twist amplitudes. It defines a closed-form Regge spectral generator $G(\alpha, \lambda)$, where α is the Regge trajectory at fixed spin S of the external current and λ represents the average parton multiplicity above the valence configuration. The function $G(\alpha, \lambda)$ is analytic in α , except for simple poles, which generate the full Regge spectrum for arbitrary spin. Its Mittag-Leffler expansion shows that the pole positions in the spectral sums are independent of λ , which enters only through the residues, implying that the Regge spectrum is invariant under continuous λ deformations.

The even- and odd-twist projections of the spectral generator yield the form factors and parton distributions of mesons and baryons, respectively, reflecting their $q\bar{q}$ and qqq valence content. Both sectors share the same Regge poles of the exchanged trajectory and reproduce the leading constituent counting rules at large Q^2 through explicit residue sum rules, with a single Poisson parameter λ characterizing the probe. The meson-baryon distinction is thus determined by the coupling to even- or odd-twist states of a common exchange tower, with the pion form factor providing a quantitative test.

Holographic light-front QCD and Regge trajectories—In holographic light-front QCD, the hadron spectrum is obtained from a semiclassical approximation to the

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light-front invariant Hamiltonian equation $P^2 = M^2 \geq 0$. This leads to frame-independent relativistic wave equations and form factors with the same structure as their counterparts in AdS space, thereby establishing a one-to-one correspondence between the light-front Hamiltonian formalism and a gravity dual description [5,6].

Superconformal symmetry, a graded extension of conformal symmetry [11,12], provides a natural mechanism for the emergence of a confinement scale in holographic light-front QCD. This is particularly significant since QCD, in the limit of massless quarks, is classically conformal and contains no intrinsic scale. The superconformal construction determines the form of the effective interaction potential, including nontrivial constant terms, and leads to hadronic spectra with $M^2 \sim n + L$, where n and L denote the radial and orbital quantum numbers. It also predicts supersymmetric relations linking mesons and baryons, with the pion emerging as a massless state in the chiral limit, thereby breaking hadronic supersymmetry [13,14].

In holographic light-front QCD, Regge trajectories emerge dynamically as solutions of the holographic light-front wave equations rather than being introduced phenomenologically. Mesons and baryons organize along linear Regge trajectories, $\alpha(s) = \alpha(0) + \alpha' s$, with their mass spectrum determined by the on-shell condition

$$\alpha(s = M^2) = J.$$

For mesons, the total angular momentum is $J = L + S$, where S is the quark spin, and the Regge trajectory is given by

$$\alpha(s) = \left(\frac{S}{2} - n - \frac{\Delta M^2}{4\kappa^2} \right) + \alpha' s, \quad \alpha' = \frac{1}{4\kappa^2}, \quad (1)$$

for fixed quantum number n .

The Regge slope α' is universal, a direct consequence of the underlying superconformal structure, whereas the intercept $\alpha(0)$ differs for mesons and baryons and is lowered by the quark-mass correction ΔM^2 [15]. For the light-quark vector trajectory this shift is small but not negligible at the present level of precision: the analysis of Ref. [16] gives $\alpha(0) = 0.483$ for the ρ trajectory, together with the spectral value $\kappa = 0.534$ GeV, the inputs used in the determination of the Poisson parameter λ below.

Form factors in holographic light-front QCD—In the gauge-gravity duality framework, hadronic form factors are obtained from the overlap of a conserved current propagating in AdS space and normalizable bound-state wave functions [17]. Holographic light-front QCD relates the twist dimension τ , corresponding to the number of constituents in a given Fock-state component of the hadron, to the power-law behavior of the AdS bound-state wave function, $\psi_\tau(z) \sim z^\tau e^{-\kappa^2 z^2/2}$, near the Minkowski conformal boundary. The resulting electromagnetic form factor for a

spin-one external current and an arbitrary twist- τ Fock component of the target hadron can be written as [10]

$$F_\tau(t) \sim \int_0^\infty \frac{dz}{z^3} \psi_\tau(z) V(t, z) \psi_\tau(z), \quad (2)$$

where z , the holographic variable, is the fifth-dimensional coordinate of AdS space, $V(t, z)$ is the AdS current and $t = -Q^2$ the momentum transfer variable. Performing the z integration in (2) using the integral representation of $V(t, z)$ [18] leads to [19,20]

$$F_\tau(t) \sim \int_0^1 (1-x)^{\tau-2} x^{-\alpha' t} dx = B(\tau-1, 1-\alpha' t), \quad (3)$$

where $\alpha' = 1/4\kappa^2$.

Our final expression for the twist- τ component of the form factor

$$F_\tau(t) \propto B[\tau-1, S-\alpha(t)], \quad (4)$$

incorporates the spin S of the external current and $\alpha(t)$, the relevant Regge trajectory in the t -channel exchange. For the electromagnetic form factor of the proton, for example, $S = 1$, $\alpha(t)$ corresponds to a vector meson trajectory, and τ is the number of quarks probed by the external current in a given Fock-state component of the struck proton [20]. For the gravitational form factor $A(t)$, $S = 2$ and the Regge trajectory $\alpha_p(t)$ corresponds to the Pomeron [21]. Equation (4) is written in terms of the full Regge trajectory, $\alpha(t) = \alpha(0) + \alpha' t$, thereby incorporating the Regge intercept $\alpha(0)$, which is absent in soft-wall models [22]; its inclusion shifts the poles in the form factor to their physical locations [10].

A similar expression was introduced soon after the development of the Veneziano model, itself constructed to realize the duality between resonances and Regge behavior [23,24], by replacing the s -channel dependence with a fixed pole [25–28],

$$F_\gamma(t) \propto B[\gamma, 1-\alpha(t)] \rightarrow |t|^{-\gamma}, \quad (5)$$

where γ is determined by the decrease rate of the form factor at large $|t|$. However, prior to the establishment of the constituent counting rules in QCD [2,3], no relation between integer values of γ and the number of quark constituents could be established.

In holographic light-front QCD γ is identified with the twist through $\gamma = \tau - 1$ [20] in the Fock expansion. For integer twist $\tau = N \geq 2$, Eq. (4) can be written as a multipole amplitude, the product of $\tau - 1$ simple poles in the Regge variable,

$$F_\tau(t) \sim \prod_{n=0}^{\tau-2} \frac{1}{n + S - \alpha(t)}. \quad (6)$$

This expression exhibits the characteristic twist scaling $F_\tau(t) \sim 1/t^{\tau-1}$ at large t , with pole positions at

$$\alpha = S, S+1, \dots, S+\tau-2.$$

Equivalently, Eq. (6) can be written in terms of the radial mass spectrum of the probe by using the on-shell condition $\alpha(t = M_n^2) = S+n$, for fixed S . We find

$$F_\tau(t) = \prod_{n=0}^{\tau-2} \frac{M_n^2}{M_n^2 - t}, \quad (7)$$

with $F_\tau(0) = 1$, and the spectral relation

$$M_n^2 = \frac{1}{\alpha'}(n + S - \alpha(0)), \quad (8)$$

where $\alpha(0) = (S/2) - (\Delta M^2/4\kappa^2)$ is the leading ($n=0$) intercept. The full hadron form factor is the coherent sum over the infinite number of Fock components,

$$F_H(t) = \sum_\tau C_\tau \prod_{n=0}^{\tau-2} \frac{M_n^2}{M_n^2 - t}, \quad (9)$$

where the normalization $F_H(0) = 1$ requires $\sum_\tau C_\tau = 1$ if all possible states are included [10].

Regge spectral generator—The constituent counting rules predict the power-law falloff of amplitudes for exclusive scattering processes according to the number of constituents that must share the large momentum transfer in order to keep the hadron intact [1–3]. The full exclusive amplitude therefore receives contributions from the entire tower of twist components in the Fock basis, each contributing a product of $\tau-1$ poles with hard scaling $f_\tau \sim 1/t^{\tau-1}$ for large t .

We define a spectral generator $G(\alpha, \lambda)$ as the Poisson-weighted sum of the infinite tower of twist amplitudes $f_\tau(\alpha)$,

$$G(\alpha, \lambda) = \sum_{\tau=2}^{\infty} \frac{\lambda^{\tau-2} e^{-\lambda}}{(\tau-2)!} B(\tau-1, S-\alpha), \quad (10)$$

with $f_\tau(\alpha) \equiv B(\tau-1, S-\alpha)$. The parameter λ is the mean of the Poisson distribution $P_n(\lambda) = \lambda^n e^{-\lambda}/n!$, the average number of partons above the minimal valence configuration, thus $\langle \tau \rangle = 2 + \lambda$. The Poisson generator $G(\alpha, \lambda)$ can be expressed in closed form in terms of the confluent hypergeometric function ${}_1F_1$,

$$\begin{aligned} G(\alpha, \lambda) &= e^{-\lambda} \int_0^1 (1-x)^{S-\alpha-1} e^{\lambda x} dx \\ &= \frac{e^{-\lambda}}{S-\alpha} {}_1F_1(1; S-\alpha+1; \lambda), \end{aligned} \quad (11)$$

as shown in the Appendix.

In the Drell-Yan-West frame the current does not couple to states with different particle numbers [7,8,29]. As a result the elastic form factor is a sum over Fock components of definite constituent number, each contributing independently. Likewise, in holographic light-front QCD [6,10] the semiclassical wave equations describe independent oscillator modes: their coherent superposition should carry Poisson-distributed occupation numbers, in direct analogy with the driven harmonic oscillator.

Mittag-Leffler expansion—The Regge spectrum generated by $G(\alpha, \lambda)$ becomes explicit through its Mittag-Leffler expansion in the complex α plane. In fact, Eq. (11) shows that the spectral generator is a meromorphic function of α with simple poles. It admits the Mittag-Leffler decomposition (see the Appendix)

$$G(\alpha, \lambda) = \sum_{n=0}^{\infty} \frac{R_n(\lambda)}{n + S - \alpha}, \quad R_n(\lambda) = \frac{(-\lambda)^n}{n!}, \quad (12)$$

where the residues $R_n(\lambda)$ are determined by the Poisson parameter λ . Equation (12) exhibits the Veneziano-like structure of an infinite sequence of equidistant zero-width simple poles, $\alpha = S+n$ with $n=0, 1, 2, \dots$, which are independent of λ : the confinement spectrum is invariant under continuous λ deformations of the Poisson distribution.

Structurally, the Poisson generator (10) reorganizes the tower of fixed-twist amplitudes, each a finite product of poles, into an infinite sum of simple Regge poles (12), thereby providing an analytic mechanism through which the confining structure of individual Fock components generates the full Regge mass spectrum of radial excitations (8) via the spectral condition $\alpha(M_n^2) = S+n$.

Even-odd projection of the spectral generator—As the virtual photon propagates through the confinement domain at distances $\sim 1/\kappa$, it undergoes strong interactions and fluctuates into intermediate vector states: the current hadronizes into $q\bar{q}$ pairs and their radial excitations. This dressing of the current is encoded in the universal spectral generator $G(\alpha, \lambda)$, which contains all the states of the Regge trajectory in the exchange channel, Eq. (12), independently of the target hadron. The target selects the components matching its own light-front Fock expansion, leading to a distinct effective coupling for each hadron class, even though the coupling of the electromagnetic current at the quark level is universal.

In the semiclassical approximation, gluonic degrees of freedom are sublimated into the effective confining potential and do not appear as constituents [30]. The Poisson

distribution weights each twist component of the dressed current; the target singles out even twists for a meson, whose Fock tower $|q\bar{q}\rangle, |q\bar{q}q\bar{q}\rangle, \dots$ carries $\tau = 2, 4, \dots$, and odd twists for a baryon, with $|qqq\rangle, |qqqq\bar{q}\rangle, \dots$ and $\tau = 3, 5, \dots$.

We consider first the case where the probe couples to the even-twist states of a target meson (see the Appendix),

$$\begin{aligned} G_E(\alpha, \lambda) &= \sum_{\substack{\tau=2 \\ \tau \text{ even}}}^{\infty} \frac{\lambda^{\tau-2} e^{-\lambda}}{(\tau-2)!} B(\tau-1, S-\alpha) \\ &= e^{-\lambda} \int_0^1 (1-x)^{S-\alpha-1} \cosh(\lambda x) dx \\ &= \frac{e^{-\lambda}}{2(S-\alpha)} [{}_1F_1(1; S-\alpha+1; \lambda) \\ &\quad + {}_1F_1(1; S-\alpha+1; -\lambda)]. \end{aligned} \quad (13)$$

Likewise, for the coupling of the probe to the odd-twist states of a target baryon we find

$$\begin{aligned} G_O(\alpha, \lambda) &= \sum_{\substack{\tau=3 \\ \tau \text{ odd}}}^{\infty} \frac{\lambda^{\tau-2} e^{-\lambda}}{(\tau-2)!} B(\tau-1, S-\alpha) \\ &= e^{-\lambda} \int_0^1 (1-x)^{S-\alpha-1} \sinh(\lambda x) dx \\ &= \frac{e^{-\lambda}}{2(S-\alpha)} [{}_1F_1(1; S-\alpha+1; \lambda) \\ &\quad - {}_1F_1(1; S-\alpha+1; -\lambda)], \end{aligned} \quad (14)$$

with $G(\alpha, \lambda) = G_E(\alpha, \lambda) + G_O(\alpha, \lambda)$.

Since ${}_1F_1$ is an entire function of its argument λ , and the sign of λ enters (13) and (14) only through this argument, without modifying the $S - \alpha$ dependence, both projections inherit the full pole ladder of the Mittag-Leffler expansion (12). Mesons and baryons couple to the complete tower of states of the exchanged current, $\alpha = S + n$, with distinct couplings given by the residues $R_n^{E,O}$ (see the Appendix),

$$\begin{aligned} G_{E,O}(\alpha, \lambda) &= \sum_{n=0}^{\infty} \frac{R_n^{E,O}(\lambda)}{n + S - \alpha}, \\ R_n^{E,O}(\lambda) &= \frac{\lambda^n}{2n!} [(-1)^n \pm e^{-2\lambda}], \end{aligned} \quad (15)$$

with $R_n(\lambda) = R_n^E(\lambda) + R_n^O(\lambda)$. The residues obey the sum rules

$$\sum_{n=0}^{\infty} R_n^E(\lambda) = e^{-\lambda}, \quad \sum_{n=0}^{\infty} R_n^O(\lambda) = 0, \quad (16)$$

which determine the large- $|S - \alpha|$ behavior of the two sectors,

$$G_E(\alpha, \lambda) \sim \frac{e^{-\lambda}}{S - \alpha}, \quad G_O(\alpha, \lambda) \sim \frac{\lambda e^{-\lambda}}{(S - \alpha)^2}, \quad (17)$$

since the vanishing residue sum of the odd projection cancels its leading power.

Meson-baryon form factors—The even and odd projections of the Regge spectral generator determine the meson and baryon form factors, $F_M(t, \lambda)$ and $F_B(t, \lambda)$, up to an overall normalization,

$$F_{M,B}(t, \lambda) = \frac{G_{E,O}[\alpha(t), \lambda]}{G_{E,O}[\alpha(0), \lambda]}, \quad (18)$$

where $F_{M,B}(0, \lambda) = 1$. In the Regge limit, $\alpha(t) \sim \alpha' t$ at large spacelike t , the asymptotic forms (17) reproduce the leading constituent counting rules [2,3],

$$F_M \sim \frac{1}{\alpha' |t|}, \quad F_B \sim \frac{1}{(\alpha' |t|)^2}, \quad (19)$$

through the explicit residue sum rules (16). This behavior is also consistent with the Phragmén-Lindelöf theorem, which relates the asymptotic limits in the spacelike and timelike regions, $F(t \rightarrow -\infty) \sim F(s \rightarrow +\infty)$, up to a phase [31].

We determine the mean parton excitation above the valence configuration, λ , from the measured pion charge radius $\langle r_\pi^2 \rangle$ and $F_M(t, \lambda)$, using the vector meson Regge trajectory (1) for $S = 1$ and the spectral values $\kappa = 0.534$ GeV and $\alpha(0)_\rho = 0.483$ from [16]. We obtain $\lambda = 0.87 \pm 0.07$ from the Particle Data Group value $\langle r_\pi^2 \rangle = 0.434 \pm 0.005$ fm² [32]. Having fixed λ , we compute the spacelike $Q^2 = -t \geq 0$ dependence of the pion form factor as a parameter-free prediction, shown in Fig. 1.

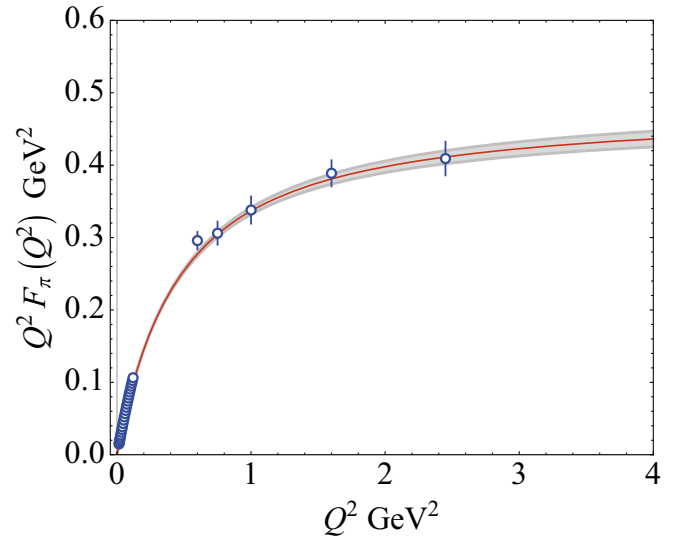


FIG. 1. Experimental data for the pion electromagnetic form factor in the spacelike ($Q^2 \geq 0$) region are compared with the model prediction (red curve) for $\lambda = 0.87 \pm 0.07$, with the uncertainty band in grey. Data from NA7 [33] and JLab [34,35].

The compact expression of the form factors (18) can be represented as a sum of products of poles by using the spectral relation (8). We find

$$F_M(t, \lambda) = \frac{1}{N_M} \sum_{n=0}^{\infty} \frac{\lambda^{2n}}{(\alpha')^{2n+1}} \prod_{k=0}^{2n} \frac{1}{M_k^2 - t}, \quad (20)$$

$$F_B(t, \lambda) = \frac{1}{N_B} \sum_{n=0}^{\infty} \frac{\lambda^{2n+1}}{(\alpha')^{2n+2}} \prod_{k=0}^{2n+1} \frac{1}{M_k^2 - t}, \quad (21)$$

where N_M and N_B are normalization factors. In practice, truncating the sum to a few terms already yields a result nearly indistinguishable from the compact expression (18). The expanded form is useful to extend the model to the timelike domain, $t > 0$, where resonance widths and the phase space of the opening decay channels require phenomenological input or a specific model computation.

The expanded form is also useful for comparison with previous holographic results. From the meson form factor (20) we obtain the twist coefficients C_τ in (9),

$$C_2 = 0.83, \quad C_4 = 0.16, \quad C_6 = 0.008, \quad C_8 \simeq 2 \times 10^{-4}, \quad (22)$$

showing the rapid convergence of the twist expansion. The coefficients, previously treated as fitted parameters [10], are here determined to all orders, and the fast convergence of the truncated expansion in [10] follows as a structural consequence of the spectral generator.

The coupling of the dressed probe to the orbital components of the target, critical for the nucleon Pauli form factor and the neutron sector [10], is not determined in the present Letter. A detailed analysis of the nucleon form factors, which requires the internal spin-flavor and orbital structure of the nucleon, is left for future work.

Coherent Poisson sums for parton distributions—Parton distributions in holographic light-front QCD are given by [20,21]

$$h_\tau(\alpha, x) = w(x)^{S-\alpha-1} w'(x) (1-w(x))^{\tau-2}, \quad (23)$$

where $\int_0^1 h_\tau(\alpha, x) dx = B(\tau-1, S-\alpha)$ independently of the specific form of $w(x)$. The function $w(x)$ is assumed to be identical for quarks and gluons and is constrained by imposing the QCD inclusive counting rules at $x \rightarrow 1$, namely $h_\tau \sim (1-x)^{2\tau-3}$ [7,36], as well as Regge behavior at small x , $h_\tau \sim x^{S-\alpha-1}$. This leads to the conditions $w'(1) = 0$ and $w(x) \sim x$ as $x \rightarrow 0$ [20]. A natural question is how this structure is modified by the coherent Poisson summation over the full tower of twist amplitudes.

To examine this point we define, in analogy with the spectral generator, the even and odd sums of the twist tower of parton distributions, K_E and K_O ,

$$K_E(\alpha, x, \lambda) = \sum_{\substack{\tau=2 \\ \tau \text{ even}}}^{\infty} \frac{\lambda^{\tau-2} e^{-\lambda}}{(\tau-2)!} h_\tau(\alpha, x) \\ = e^{-\lambda} w(x)^{S-\alpha-1} w'(x) \cosh(\lambda[1-w(x)]), \quad (24)$$

$$K_O(\alpha, x, \lambda) = \sum_{\substack{\tau=3 \\ \tau \text{ odd}}}^{\infty} \frac{\lambda^{\tau-2} e^{-\lambda}}{(\tau-2)!} h_\tau(\alpha, x) \\ = e^{-\lambda} w(x)^{S-\alpha-1} w'(x) \sinh(\lambda[1-w(x)]), \quad (25)$$

with $\int_0^1 K_{E,O}(\alpha, x, \lambda) dx = G_{E,O}(\alpha, \lambda)$.

Near $x = 1$, the conditions $w(1) = 1$ and $w'(1) = 0$ give the model-independent expansion $1 - w(x) = -\frac{1}{2} w''(1)(1-x)^2$, with $w''(1) < 0$, and $w'(x) = -w''(1) \times (1-x)$. Expanding the hyperbolic kernels at the endpoint, the projections (24) and (25) reduce to their minimal-twist components,

$$K_E \sim e^{-\lambda}(1-x), \quad K_O \sim \lambda e^{-\lambda}(1-x)^3, \quad (26)$$

reproducing the inclusive counting rules $(1-x)^{2\tau_{\min}-3}$ for $\tau_{\min} = 2$ and 3 [7,36], with the Poisson weights $e^{-\lambda}$ and $\lambda e^{-\lambda}$ of the valence $q\bar{q}$ and qqq configurations. At small x , where $w(x) \sim x$, both projections retain the Regge behavior $\sim x^{S-\alpha-1}$.

The physical parton distributions are obtained by normalizing the projected even-odd coherent Poisson sums at $t = 0$,

$$\mathcal{H}_{M,B}(x, t, \lambda) = \frac{K_{E,O}[\alpha(t), x, \lambda]}{G_{E,O}[\alpha(0), \lambda]}, \quad (27)$$

with $\int_0^1 \mathcal{H}_{M,B}(x, 0, \lambda) dx = 1$. At finite momentum transfer the x integral reproduces the form factors (18), $\int_0^1 \mathcal{H}_{M,B}(x, t, \lambda) dx = F_{M,B}(t, \lambda)$, since the projected distributions and form factors derive from the same spectral structure.

Concluding remarks—The structural analysis presented here organizes the full twist tower of exclusive amplitudes into a single generating function, whose physical consequences—the Regge spectrum, the meson-baryon projection, the constituent counting rules, the form factors, and parton distributions—follow from the universal spectral characteristics of the probe and the meson-baryon nature of the target. The pion form factor provides a quantitative test of this framework.

The spectral generator $G(\alpha, \lambda)$ is the Poisson-weighted coherent sum of the infinite tower of twist amplitudes, evaluated in closed form as a meromorphic function of the Regge trajectory α . Its Mittag-Leffler expansion exhibits simple poles at $\alpha = S + n$, corresponding to the radial excitations of the exchanged trajectory, with positions independent of λ .

The even- and odd-twist projections of the Poisson generator yield the meson and baryon form factors and

parton distributions, reflecting the $q\bar{q}$ and qqq valence content of the two hadron sectors. Both projections share the Regge poles of the common exchanged trajectory, and the constituent counting rules follow from the residue sum rules of the spectral generator: $\sum_n R_n^E = e^{-\lambda}$ fixes the leading meson power, while the vanishing sum $\sum_n R_n^O = 0$ produces the additional baryon suppression. The exclusive counting rules in t and the inclusive counting rules at $x \rightarrow 1$ are both governed by identical Poisson weights of the minimal Fock configuration of each sector, with the single parameter λ fixed by the pion charge radius. The extension to the nucleon sector, which involves its internal spin-flavor and orbital structure, is deferred to future work. The analytic framework introduced in the present Letter can be extended to other exclusive processes including form factors and parton distributions of hadrons with heavy and light quarks.

The present framework connects two historically distinct strands: the duality program of the pre-QCD era, in which global spectral information was used to constrain local amplitudes [23,24,28], and the constituent counting rules of perturbative QCD [2,3], which determine the local short-distance behavior. In the spectral generator both aspects reside in a single analytic function: its global residue structure, summed over the full nonperturbative tower, determines the local power-law behavior through exact sum rules. The coherent hard exclusive amplitude is thereby determined within this framework by the confinement spectrum itself.

Exact analytic Regge spectral sums have recently been obtained in QCD(1+1) [37] using nonperturbative methods developed in [38]. A corresponding derivation from first-principles QCD in (3+1) dimensions remains an outstanding challenge.

Acknowledgments—The authors are grateful to Tanja Horn for her guidance regarding the JLab pion form factor experimental data.

Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] G. P. Lepage and S. J. Brodsky, Exclusive processes in perturbative quantum chromodynamics, *Phys. Rev. D* **22**, 2157 (1980).
 - [2] S. J. Brodsky and G. R. Farrar, Scaling laws at large transverse momentum, *Phys. Rev. Lett.* **31**, 1153 (1973).
 - [3] V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Automodelity in strong interactions, *Lett. Nuovo Cimento* **5S2**, 907 (1972).
 - [4] J. Polchinski and M. J. Strassler, Hard scattering and gauge/string duality, *Phys. Rev. Lett.* **88**, 031601 (2002).

- [5] S. J. Brodsky and G. F. de Téramond, Hadronic spectra and light-front wave functions in holographic QCD, *Phys. Rev. Lett.* **96**, 201601 (2006).
- [6] G. F. de Téramond and S. J. Brodsky, Light-front holography: A first approximation to QCD, *Phys. Rev. Lett.* **102**, 081601 (2009).
- [7] S. D. Drell and T.-M. Yan, Connection of elastic electromagnetic nucleon form factors at large Q^2 and deep inelastic structure functions near threshold, *Phys. Rev. Lett.* **24**, 181 (1970).
- [8] G. B. West, Phenomenological model for the electromagnetic structure of the proton, *Phys. Rev. Lett.* **24**, 1206 (1970).
- [9] G. F. de Téramond, Limits of applicability of holographic dual descriptions to QCD: Virtuality and coherence, *Proc. Sci. QCDEX2025* (2026) 040 [arXiv:2512.23663], <https://pos.sissa.it/508/040/pdf>.
- [10] S. J. Brodsky, G. F. de Téramond, H. G. Dosch, and J. Erlich, Light-front holographic QCD and emerging confinement, *Phys. Rep.* **584**, 1 (2015).
- [11] V. de Alfaro, S. Fubini, and G. Furlan, Conformal invariance in quantum mechanics, *Nuovo Cimento A* **34**, 569 (1976).
- [12] S. Fubini and E. Rabinovici, Superconformal quantum mechanics, *Nucl. Phys.* **B245**, 17 (1984).
- [13] H. G. Dosch, G. F. de Téramond, and S. J. Brodsky, Superconformal baryon-meson symmetry and light-front holographic QCD, *Phys. Rev. D* **91**, 085016 (2015).
- [14] G. F. de Téramond, H. G. Dosch, and S. J. Brodsky, Baryon spectrum from superconformal quantum mechanics and its light-front holographic embedding, *Phys. Rev. D* **91**, 045040 (2015).
- [15] H. G. Dosch, G. F. de Téramond, and S. J. Brodsky, Holographic light-front QCD, *J. Subatomic Part. Cosmol.* **5**, 100339 (2026).
- [16] R. S. Sufian, T. Liu, G. F. de Téramond, H. G. Dosch, S. J. Brodsky, A. Deur, M. T. Islam, and B.-Q. Ma, Nonperturbative strange-quark sea from lattice QCD, light-front holography, and meson-baryon fluctuation models, *Phys. Rev. D* **98**, 114004 (2018).
- [17] J. Polchinski and M. J. Strassler, Deep inelastic scattering and gauge/string duality, *J. High Energy Phys.* **05** (2003) 012.
- [18] H. R. Grigoryan and A. V. Radyushkin, Structure of vector mesons in a holographic model with linear confinement, *Phys. Rev. D* **76**, 095007 (2007).
- [19] L. Zou and H. G. Dosch, A very practical guide to light front holographic QCD, *arXiv:1801.00607*.
- [20] G. F. de Téramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, and A. Deur (HLFHS Collaboration), Universality of generalized parton distributions in light-front holographic QCD, *Phys. Rev. Lett.* **120**, 182001 (2018).
- [21] G. F. de Téramond, H. G. Dosch, T. Liu, R. S. Sufian, S. J. Brodsky, and A. Deur (HLFHS Collaboration), Gluon matter distribution in the proton and pion from extended holographic light-front QCD, *Phys. Rev. D* **104**, 114005 (2021).
- [22] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, Linear confinement and AdS/QCD, *Phys. Rev. D* **74**, 015005 (2006).

- [23] R. Dolen, D. Horn, and C. Schmid, Finite-energy sum rules and their application to πN charge exchange, *Phys. Rev.* **166**, 1768 (1968).
- [24] G. Veneziano, Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories, *Nuovo Cimento A* **57**, 190 (1968).
- [25] M. Ademollo and E. Del Giudice, Nonstrong amplitudes in a Veneziano-type model, *Nuovo Cimento A* **63**, 639 (1969).
- [26] P. V. Landshoff and J. C. Polkinghorne, The scaling law for deep inelastic scattering in a new Veneziano-like amplitude, *Nucl. Phys.* **B19**, 432 (1970).
- [27] I. Bender, H. G. Dosch, and H. J. Rothe, Fixed poles in photoproduction of charged pions, *Nuovo Cimento A* **62**, 1026 (1969).
- [28] I. Bender, H. J. Rothe, H. G. Dosch, and V. F. Mueller, Duality and fixed poles in pion photoproduction, *Lett. Nuovo Cimento* **3S1**, 625 (1970).
- [29] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Quantum chromodynamics and other field theories on the light cone, *Phys. Rep.* **301**, 299 (1998).
- [30] S. J. Brodsky and G. F. de Teramond, AdS/QCD, light-front holography, and sublimated gluons, *Proc. Sci. QCD-TNT-II2011* (2011) 008 [arXiv:1112.4212], <https://pos.sissa.it/136/008/pdf>.
- [31] S. Pacetti, R. Baldini Ferroli, and E. Tomasi-Gustafsson, Proton electromagnetic form factors: Basic notions, present achievements and future perspectives, *Phys. Rep.* **550–551**, 1 (2015).
- [32] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [33] S. R. Amendolia *et al.* (NA7 Collaboration), A measurement of the space-like pion electromagnetic form factor, *Nucl. Phys.* **B277**, 168 (1986).
- [34] T. Horn *et al.*, Scaling study of the pion electroproduction cross sections, *Phys. Rev. C* **78**, 058201 (2008).
- [35] G. M. Huber *et al.* (Jefferson Lab Collaboration), Charged pion form factor between $Q^2 = 0.60 \text{ GeV}^2$ and 2.45 GeV^2 II. Determination of, and results for, the pion form factor, *Phys. Rev. C* **78**, 045203 (2008).
- [36] R. Blankenbecler and S. J. Brodsky, Unified description of inclusive and exclusive reactions at all momentum transfers, *Phys. Rev. D* **10**, 2973 (1974).
- [37] A. Artemev, A. Litvinov, and P. Meshcheriakov, QCD₂'t Hooft model: Two-flavor mesons spectrum, *Phys. Rev. D* **111**, 125001 (2025).
- [38] V. A. Fateev, S. L. Lukyanov, and A. B. Zamolodchikov, On mass spectrum in 't Hooft's 2D model of mesons, *J. Phys. A* **42**, 304012 (2009).
- [39] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (Dover Publications, New York, 1965).

End Matter

Appendix: Some useful results—

Integral representations: Two integral representations are used in the derivations below. The Euler beta function $B(u, v) = \Gamma(u)\Gamma(v)/\Gamma(u+v)$ admits the representation

$$B(u, v) = \int_0^1 t^{u-1} (1-t)^{v-1} dt, \quad (\text{A1})$$

with $\Re(u) > 0$, $\Re(v) > 0$. The confluent hypergeometric function ${}_1F_1(a; b; z)$, defined by the series

$${}_1F_1(a; b; z) = \frac{\Gamma(b)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(b+n)} \frac{z^n}{n!}, \quad (\text{A2})$$

admits the integral representation

$${}_1F_1(a; b; z) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 e^{zt} t^{a-1} (1-t)^{b-a-1} dt, \quad (\text{A3})$$

valid for $\Re(b) > \Re(a) > 0$.

Poisson generating function: Consider the Poisson generating function $G(u, s)$ for the beta function sequence $f_n(u) = B(n+1, u)$,

$$G(u, s) = \sum_{n=0}^{\infty} \frac{s^n e^{-s}}{n!} B(n+1, u), \quad (\text{A4})$$

where s is the mean of the Poisson probability distribution $P_n(s) = s^n e^{-s}/n!$, with $\langle n \rangle = s$.

Using the integral representation (A1) and interchanging the sum and integral, we obtain

$$G(u, s) = e^{-s} \int_0^1 (1-t)^{u-1} e^{st} dt. \quad (\text{A5})$$

Applying (A3) with $a = 1$, $b = u+1$, and $z = s$ then gives

$$G(u, s) = \frac{e^{-s}}{u} {}_1F_1(1; u+1; s). \quad (\text{A6})$$

Mittag-Leffler expansion: The pole expansion of $G(u, s)$ follows from Kummer's transformation [39],

$$e^{-z} {}_1F_1(a; b; z) = {}_1F_1(b-a; b; -z). \quad (\text{A7})$$

Applying (A7) with $a = 1$, $b = u+1$, $z = \pm s$, and expanding the right-hand side with the series (A2), we find

$$\frac{1}{u} {}_1F_1(1; u+1; \pm s) = e^{\pm s} \sum_{n=0}^{\infty} \frac{(\mp s)^n}{n!(u+n)}. \quad (\text{A8})$$

The Poisson generator (A6) then admits the Mittag-Leffler expansion

$$G(u, s) = \sum_{n=0}^{\infty} \frac{R_n(s)}{u+n}, \quad R_n(s) = \frac{(-s)^n}{n!}, \quad (\text{A9})$$

with simple poles at $u = -n$ ($n = 0, 1, 2, \dots$).

Even-odd projection: The Poisson generator $G(u, s)$ admits a decomposition into even and odd infinite sums while preserving its analytic properties. The even sum

$$G_E(u, s) = \sum_{n=0}^{\infty} \frac{s^{2n} e^{-s}}{(2n)!} B(2n+1, u) \quad (\text{A10})$$

is evaluated using (A1) and the series expansion of the hyperbolic cosine, giving

$$G_E(u, s) = e^{-s} \int_0^1 (1-t)^{u-1} \cosh(st) dt. \quad (\text{A11})$$

Using $\cosh(st) = \frac{1}{2}(e^{st} + e^{-st})$ and (A5), we obtain

$$G_E(u, s) = \frac{e^{-s}}{2u} [{}_1F_1(1; u+1; s) + {}_1F_1(1; u+1; -s)]. \quad (\text{A12})$$

The odd sum is obtained by the same steps with the hyperbolic sine,

$$\begin{aligned} G_O(u, s) &= \sum_{n=0}^{\infty} \frac{s^{2n+1} e^{-s}}{(2n+1)!} B(2n+2, u) \\ &= e^{-s} \int_0^1 (1-t)^{u-1} \sinh(st) dt \\ &= \frac{e^{-s}}{2u} [{}_1F_1(1; u+1; s) - {}_1F_1(1; u+1; -s)], \end{aligned} \quad (\text{A13})$$

with $G(u, s) = G_E(u, s) + G_O(u, s)$ exactly.

Residues of the even-odd projections: The Mittag-Leffler expansions of the even-odd projections follow from (A8)

$$\begin{aligned} G_{E,O}(u, s) &= \sum_{n=0}^{\infty} \frac{R_n^{E,O}(s)}{u+n}, \\ R_n^{E,O}(s) &= \frac{s^n}{2n!} [(-1)^n \pm e^{-2s}], \end{aligned} \quad (\text{A14})$$

with $R_n^E + R_n^O = (-s)^n/n!$, the residues of the full generator. Both projections thus contain the complete pole ladder $u = -n$. The residues obey the sum rules

$$\sum_{n=0}^{\infty} R_n^E(s) = e^{-s}, \quad \sum_{n=0}^{\infty} R_n^O(s) = 0, \quad (\text{A15})$$

which control the large- u behavior. Expanding (A14) for large u : $uG(u) \simeq \sum_n R_n - (1/u) \sum_n nR_n + \dots$, one obtains the asymptotic expressions

$$G_E(u, s) \sim \frac{e^{-s}}{u}, \quad G_O(u, s) \sim \frac{e^{-s}s}{u^2}. \quad (\text{A16})$$

The vanishing residue sum of the odd projection cancels its leading power, and the first moment, $\sum_n nR_n^O = -se^{-s}$, fixes the coefficient of the subleading term. For the even projection the first moment vanishes, $\sum_n nR_n^E = 0$, thus corrections to the leading behavior begin only at order $1/u^3$.

The same asymptotic behavior follows independently from the large- u expansion

$${}_1F_1(1; u+1; \pm s) = 1 \pm \frac{s}{u+1} + \mathcal{O}\left(\frac{1}{u^2}\right), \quad (\text{A17})$$

providing a consistency check of the sum rules (A15).

Identifying $u = S - \alpha > 0$ and $s = \lambda$ reproduces the expressions used in the main text.