

Conformal Data for the O(3) Wilson-Fisher Conformal Field Theory from Fuzzy Sphere Realization of the Quantum Rotor Model

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We present a model for strongly interacting fermions with internal O(3) symmetry on the fuzzy sphere that (i) preserves the rotational symmetry of the fuzzy sphere and (ii) undergoes a quantum phase transition in the $(2 + 1)$ -dimensional O(3) Wilson-Fisher universality class. Using exact diagonalization and density matrix renormalization group, we locate the quantum critical point via conformal perturbation theory and obtain scaling dimensions from finite-size spectra. We identify 24 primary operators and determine some of their operator product expansion coefficients through first-order conformal perturbation theory. The results are benchmarked against conformal bootstrap and large quantum-number expansions and reveal a weakly irrelevant operator that plays a role in dimerized antiferromagnets. Our Letter provides a general framework for quantitatively accessing conformal data for $O(N)$ Wilson-Fisher conformal field theories.

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Introduction—Conformal field theories (CFTs) in $(d + 1)$ -dimensional spacetime $[(d + 1)D]$ are fully specified by their conformal data, scaling dimensions, and operator product expansion (OPE) coefficients, which govern universal critical behavior, and some measurable responses [1–7]. While $(1 + 1)D$ CFTs [2,8] enjoy remarkable theoretical completeness through Virasoro algebra [9] and integrability [10], the landscape of $(2 + 1)D$ CFTs remains largely unexplored, presenting rich opportunities for discovering new universal physics and testing fundamental principles of quantum field theory.

Standard approaches have limitations when it comes to accessing detailed information about operator spectra and OPE data of $(2 + 1)D$ CFTs, especially when they are non-Abelian. Large-scale Monte Carlo simulations [11] pin down critical exponents but have trouble resolving sub-leading operators or in extracting OPE data. Conformal bootstrap [12,13] studies of non-Abelian theories provide only a limited set of low-lying operators [14]. Perturbative analytics rely on resummations that lose accuracy for spinning operators [15–17]. Fuzzy-sphere regularization [18–29] now provides a fully symmetric Hamiltonian probe of spectra and correlators and even some OPE data. A prior

fuzzy sphere study of the $(2 + 1)D$ CFT with internal O(3) symmetry confirmed feasibility but identified only a limited number of operators [30].

Here, we introduce a fuzzy sphere realization (FZR) of a truncated quantum rotor model (TQRM) [7,18] that retains the full internal O(3) and spatial SO(3) symmetry of the $(2 + 1)$ -dimensional O(3) Wilson-Fisher (WF) universality class and is tractable to exact diagonalization (ED) and density matrix renormalization group (DMRG). Building on the well-known correspondence between O(3) quantum rotors and O(3) quantum criticality [7], the idea is to use a conformal map to identify a compactification of two-dimensional space with the unit two-sphere S^2 and to leverage the state-operator correspondence to extract conformal data. This is done by relating the finite-size gaps of the FZR of the TQRM to the scaling dimensions of the CFT operators. We identify 24 low-lying primary operators, with their scaling dimensions summarized in Tables I and II (Appendix A). Selected OPE coefficients are also determined. The full set of scaling dimensions and OPE values can be found in Supplemental Material [31]. We benchmark our results with bootstrap results [14] and large quantum-number expansion predictions [50,51], wherever they are available.

Quantum rotor model—A rigid rotor, a particle of mass μ constrained to move on the surface of a sphere of radius R , serves as the prototypical quantum model of rotational motion. Its energy is purely kinetic, arising from quantized angular momenta with moment of inertia $I = \mu R^2 > 0$. On a square lattice, an array of such rotors with nearest-neighbor ferromagnetic Heisenberg coupling $J > 0$ realizes

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TABLE I. Summary of selected primary operators. S labels the internal $\text{SO}(3)$ spin representation, L the $\text{SO}(3)$ angular momenta on the fuzzy sphere, and $\pm$ the $\mathbb{Z}_2$ parity (improper part) of the internal $\text{O}(3)$ symmetry, so that states are labeled by $S^\pm$ and L . The index I orders energy eigenstates within each symmetry sector. Scaling dimensions are extracted via ED and DMRG, and are compared to conformal bootstrap (CB); asterisks denote exact values, and parentheses indicate system size. The two bold-faced entries in the Δ (CB) column are used as input.

$S^\pm$	L	I	σ	$\Delta(\text{ED})$	$\Delta(\text{DMRG})$	$\Delta(\text{CB})$
1^-	0	1	σ	0.518 93 (12)	0.518 93 (28)	0.518 936 [14]
0^+	0	2	ε	1.561 89 (12)	1.617 81 (26)	1.594 88 [14]
0^+	0	4	ε'	3.772 79 (12)	3.801 77 (14)	3.766 80 [52]
0^+	2	1	$T_{\mu\nu}$	2.977 49 (12)	3.010 62 (20)	3*
1^+	1	1	j_μ	1.882 53 (12)	1.972 04 (26)	2*
2^+	0	1	$t_{(2)}$	1.254 07 (12)	1.237 67 (26)	1.209 54 [14]
4^+	0	1	$t_{(4)}$	3.268 58 (11)	3.164 59 (26)	2.990 56 [14]

an $\text{O}(3)$ magnetic transition when the dimensionless coupling IJ is tuned to quantum criticality [7].

The full two-dimensional interacting lattice Hamiltonian for $\text{O}(3)$ quantum rotors reads $\hat{H}_\infty := \hat{H}_{\text{kin}} + \hat{H}_{\text{int}}$,

$$\hat{H}_{\text{kin}} := \frac{1}{2I} \sum_{i=1}^N \hat{\mathbf{L}}_i^2, \quad \hat{H}_{\text{int}} := -J \sum_{\langle ij \rangle} \hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j, \quad (1a)$$

where the pair of operator-valued three-component vectors $(\hat{\mathbf{L}}_i, \hat{\mathbf{n}}_i)$ obey the equal-time $\text{O}(3)$ rotor algebra

$$\begin{aligned} [\hat{L}_i^a, \hat{L}_j^b] &:= i\hbar \epsilon^{abc} \hat{L}_i^c \delta_{ij}, & [\hat{n}_i^a, \hat{n}_j^b] &:= 0, \\ [\hat{L}_i^a, \hat{n}_j^b] &:= i\hbar \epsilon^{abc} \hat{n}_i^c \delta_{ij}, & a, b = 1, 2, 3 \equiv x, y, z, \end{aligned} \quad (1b)$$

with the local constraints

$$\hat{\mathbf{L}}_i^2 = \hbar^2 l(l+1) \hat{\mathbb{I}}, \quad \hat{\mathbf{n}}_i^2 = \hat{\mathbb{I}}. \quad (1c)$$

Here, $l \in \mathbb{Z}_{\geq 0}$ fixes a global representation of each quantum rotor, $\langle ij \rangle$ are nearest-neighbor sites on the square lattice with N the number of sites.

In the limit $IJ = \infty$, $\hat{H}_\infty$ simplifies to the classical $\text{O}(3)$ ferromagnetic Heisenberg model

$$\hat{H}_{\text{int}} = -J \sum_{\langle ij \rangle} \mathbf{n}_i \cdot \mathbf{n}_j, \quad \mathbf{n}_i^2 = 1, \quad (2)$$

with the ferromagnetic ground state $|\mathbf{n}_1, \dots, \mathbf{n}_N\rangle = |\mathbf{n}, \dots, \mathbf{n}\rangle$ whereby $\hat{\mathbf{n}}_i |\mathbf{n}_1, \dots, \mathbf{n}_N\rangle = \mathbf{n}_i |\mathbf{n}_1, \dots, \mathbf{n}_N\rangle$. In the limit $IJ = 0$, $\hat{H}_\infty$ simplifies to

$$\hat{H}_{\text{kin}} = \sum_{i=1}^N \sum_{l=0}^{\infty} \sum_{m_i=-l}^{+l} \frac{\hbar^2 l(l+1)}{2I} |l, m_i\rangle \langle l, m_i| \quad (3)$$

with $|l, m_i\rangle$ the eigenstate of $\hat{\mathbf{L}}_i^2$ and $\hat{\mathbf{L}}_i^z$ with eigenvalues $\hbar^2 l(l+1)$ and $\hbar m_i$, respectively. The ground state of $\hat{H}_{\text{kin}}$ is the singlet state with $l=0$. It is believed that, in the thermodynamic limit $N \uparrow \infty$, there exists an unstable quantum critical point when $IJ \approx 1$ that separates the long-range ferromagnetically ordered phase controlled by the fixed point Hamiltonian $\hat{H}_{\text{int}}$ from the paramagnetic phase controlled by the fixed point Hamiltonian $\hat{H}_{\text{kin}}$. This quantum critical point is believed to belong to the $(2+1)\text{D } \text{O}(3)$ WF universality class.

Numerical simulations of a lattice Hamiltonian require a Hilbert space of finite dimensionality. Therefore, we truncate each quantum rotor on the right-hand side of Eq. (3) at a maximum angular momenta $l_{\text{max}} = 1$, retaining all states $|l, m\rangle$ with $0 \leq l \leq l_{\text{max}}$ and $-l \leq m \leq l$. This procedure, by which $\hat{H}_\infty$ becomes $\hat{H}_{l_{\text{max}}}$, preserves the full $\text{O}(3)$ symmetry, since each l multiplet is kept intact [31]. Moreover, this procedure is not expected to change the topology of the phase diagram at vanishing temperature.

Fuzzy sphere many-body electronic Hamiltonian—To access conformal data, we map the quantum rotor model to a system of N fermions with internal degrees of freedom, moving on a sphere [53] and projected to the lowest Landau level (LLL). In this limit, the noncommuting coordinate operators generate the fuzzy sphere algebra [54], thereby providing a natural short-distance cutoff (the magnetic length) that preserves rotational symmetry.

We focus on the case where each fermion has four flavors and the system resides at quarter filling. At quarter filling, strong interactions enforce (on average) single occupancy per LLL orbital, analogous to the large- U limit of the Hubbard model.

The microscopic fuzzy sphere Hamiltonian that we shall numerically study is defined to be

$$\hat{H}_{\text{fzs}} := \hat{P}_{\text{LLL}} \left(u \hat{H}_{\text{Hub}} - v \hat{H}_{\text{Heis}} + h \hat{H}_{\text{trans}} \right) \hat{P}_{\text{LLL}}. \quad (4)$$

Here, $\hat{P}_{\text{LLL}}$ denotes the projector to the LLL. The dimensionless couplings $\delta u \equiv u - u_c$, $\delta v \equiv v - v_c$, $\delta h \equiv h - h_c \in \mathbb{R}$ measure the deviations away from a putative quantum critical point u_c, v_c, h_c in the $(2+1)\text{D } \text{O}(3)$ WF universality class. The Hamiltonian $\hat{H}_{\text{Hub}}$ is a short-range two-body repulsive $\text{U}(4)$ -flavor-symmetric density-density interaction between the fermions that implements the constraint that every LLL is singly occupied in the limit $u \uparrow \infty$, v, h fixed. The Hamiltonian $\hat{H}_{\text{Heis}}$ is a short-range two-body Heisenberg-like interaction that selects a ground state that breaks spontaneously the internal $\text{SO}(3)$ symmetry down to the subgroup $\text{SO}(2)$ in the limit $v \uparrow \infty$, $u, h = 0$ fixed. The Hamiltonian $\hat{H}_{\text{trans}}$ is a one-body term that breaks the $\text{U}(4)$ -flavor symmetry down to the subgroup $\text{U}(1) \times \text{U}(3)$ that selects a nondegenerate gapped singlet ground state in the limit $|h| \uparrow \infty$, u, v fixed. The information about the quantum rotor is encoded inside $\hat{H}_{\text{Heis}}$ and $\hat{H}_{\text{trans}}$.

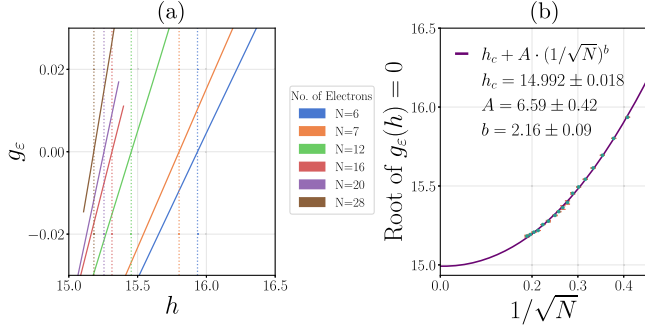


FIG. 1. Panel (a) shows the dependence on h of the relevant coupling $g_\epsilon(h)$ for a given $R \propto \sqrt{N}$. The critical point h_c is the root to $g_\epsilon(h) = 0$ after the thermodynamic limit $N \uparrow \infty$ has been taken. Panel (b) shows the dependence of the root to $g_\epsilon(h) = 0$ as a function of $1/\sqrt{N} \propto 1/R$. Fitting this dependence with an appropriate power law and using the first irrelevant anomalous dimension 0.7688 [52], we extract the critical value $h_c = 14.992 \pm 0.018$ and the critical exponent $1/\nu = 1.3908 \pm 0.0852$, which is in close agreement with the known value 1.406 [57].

The explicit representation of $\hat{H}_{\text{fzs}}$ can be found in Supplemental Material [31].

Symmetries—Hamiltonian (4) exhibits three symmetries: (a) spatial rotational symmetry $\text{SO}(3)$, arising from conservation of angular momenta with the quantum number L ; (b) internal $\text{SO}(3)$ spin symmetry with the quantum number S ; and (c) a $\mathbb{Z}_2$ symmetry corresponding to the conservation of the parity of the number of electrons occupying p orbitals in internal spin space. The quantum number for the $\text{SO}(3) \times \mathbb{Z}_2 \cong \text{O}(3)$ is denoted by $S^\pm$, where the superscript denotes the $\mathbb{Z}_2$ parity.

Accessing quantum criticality through the state-operator correspondence—First, we assume the existence of a quantum critical point $\delta u = \delta v = \delta h = 0$ at which $\hat{H}_{\text{fzs}}(R)$ exhibits the conformal invariance of the $(2+1)\text{D O}(3)$ WF universality class in the thermodynamic limit $R \propto \sqrt{N} \uparrow \infty$. If we denote eigenstates of $\hat{H}_{\text{fzs}}(R)$ by the label o (see Table I), the state-operator correspondence [3] implies that the eigenenergy $E_o(R)$ measured relative to that of the ground state $E_0(R)$ scales like

$$\delta E_o(R) \equiv E_o(R) - E_0(R) \sim \frac{c}{R} \Delta_o \quad (\hbar = 1), \quad (5)$$

in the limit $R \propto \sqrt{N} \uparrow \infty$. Here, the radius R of the fuzzy sphere is interpreted as the radius of compactification in the radial quantization of a CFT, Δ_o the scaling dimension of the CFT operator labeled by o and c the system dependent speed of “light” in the CFT.

If we switch on a small perturbation $\delta h = h - h_c \neq 0$ in $\hat{H}_{\text{fzs}}(R)$ holding $\delta u = \delta v = 0$ fixed, the energy shift to first order in perturbation theory is captured by adding the single relevant scalar operator ϵ (see Table I) with the

coupling $g_\epsilon(h)$, together with irrelevant operators to the $(2+1)\text{D O}(3)$ WF critical point. Motivated by first-order conformal perturbation theory (CPT) [55,56], we make the scaling ansatz

$$\delta E_o(R, h) = \frac{c}{R} \Delta_o(R) + g_\epsilon(h) f_{o\epsilon o}(R). \quad (6)$$

Here, $\Delta_o(R) = \Delta_o + \mathcal{O}(R^{-\omega})$ deviate from its value at quantum criticality due to the presence of the leading irrelevant operator perturbing the CFT with the scaling exponent $\omega > 0$. The value of h_c at the quantum critical point is identified by extrapolating the value of the root to $g_\epsilon(h) = 0$ to its limiting value when $R \uparrow \infty$. The interpretation of $f_{o\epsilon o}(R) = f_{o\epsilon o} + \mathcal{O}(R^{-\omega})$ in the limit $R \uparrow \infty$ is the following. (i) It is the (universal) OPE coefficient corresponding to fusing operators o and ϵ into o , if o labels a scalar primary field. (ii) It is proportional to $f_{o_p \epsilon o_p}(R)$ with the proportionality constant a known function of Δ_{o_p} and Δ_ϵ if o labels the descendant of a primary field labeled by o_p [55,56]. (iii) Finally, it is a combination of OPE coefficients [56] if o labels a field with nonvanishing conformal spin.

Our strategy to establish the existence of the $(2+1)\text{D O}(3)$ WF critical point from numerical diagonalization of $\hat{H}_{\text{fzs}}(R)$ proceeds in two steps. First, we choose two eigenstates (CFT operators) of $\hat{H}_{\text{fzs}}(R)$ labeled by o_1 and o_2 , for which estimates of Δ_{o_i} and $f_{o_i \epsilon o_i}$ with $i = 1, 2$ at the $(2+1)\text{D O}(3)$ WF critical point are known, to solve for c and $g_\epsilon(h)$ from the pair of equations (6) they generate. Second, for any eigenstate (CFT operator) of $\hat{H}_{\text{fzs}}(R)$ labeled by o , we extract $\Delta_o(R)$ by using the extracted value of c in Eq. (6) with $g_\epsilon(h) = 0$ and then extract $f_{o\epsilon o}$ from the dependence on R of Eq. (6) with $g_\epsilon(h) \neq 0$.

Results for c and h_c —To determine c and $g_\epsilon(h)$, we choose the pair of eigenstates labeled by $o_1 \equiv \sigma$ and $o_2 \equiv \partial_\mu \sigma$, where σ labels the lowest eigenenergy of $\hat{H}_{\text{fzs}}(R)$ with the quantum numbers $S = 1^-$ and $L = 0$, while $\partial_\mu \sigma$ labels the lowest eigenenergy of $\hat{H}_{\text{fzs}}(R)$ with the quantum numbers $S = 1^-$, $L = 1$. In CFT jargon, σ labels the most relevant primary field and $\partial_\mu \sigma$ is its first descendant. We insert in Eq. (6) $o_1 \equiv \sigma$ and $o_2 \equiv \partial_\mu \sigma$, the conformal bootstrap (CB) estimates $\Delta_\sigma = 0.518936$ from Table I and $\Delta_{\partial_\mu \sigma} = 1 + \Delta_\sigma = 1.518936$ together with the CB estimates for the OPE coefficient $f_{\sigma \epsilon \sigma} = 0.525$ and $f_{\partial_\mu \sigma \epsilon \partial_\mu \sigma} = f_{\sigma \epsilon \sigma} \mathcal{A}_{\sigma, \epsilon}$ with $\mathcal{A}_{\sigma, \epsilon} = 1 + [\Delta_\epsilon(\Delta_\epsilon - 3)]/(6\Delta_\sigma)$ and Δ_ϵ given by its CB value quoted in Table I. The factor $\mathcal{A}_{\sigma, \epsilon}$ accounts for the fact that the perturbation affects the first descendant differently from the primary [55,56]. Solving these equations, we obtain the speed of light $c \sim 0.0122$ and the coupling $g_\epsilon(h)$. Then, we solve for the root of $g_\epsilon(h) = 0$ for each value of N , to obtain the finite-size critical point as shown in Fig. 1(a). By extrapolating the root of $g_\epsilon(h) = 0$ to $N \uparrow \infty$ as shown in Fig. 1(b), we obtain the critical point of the system as $h_c = 14.992$.

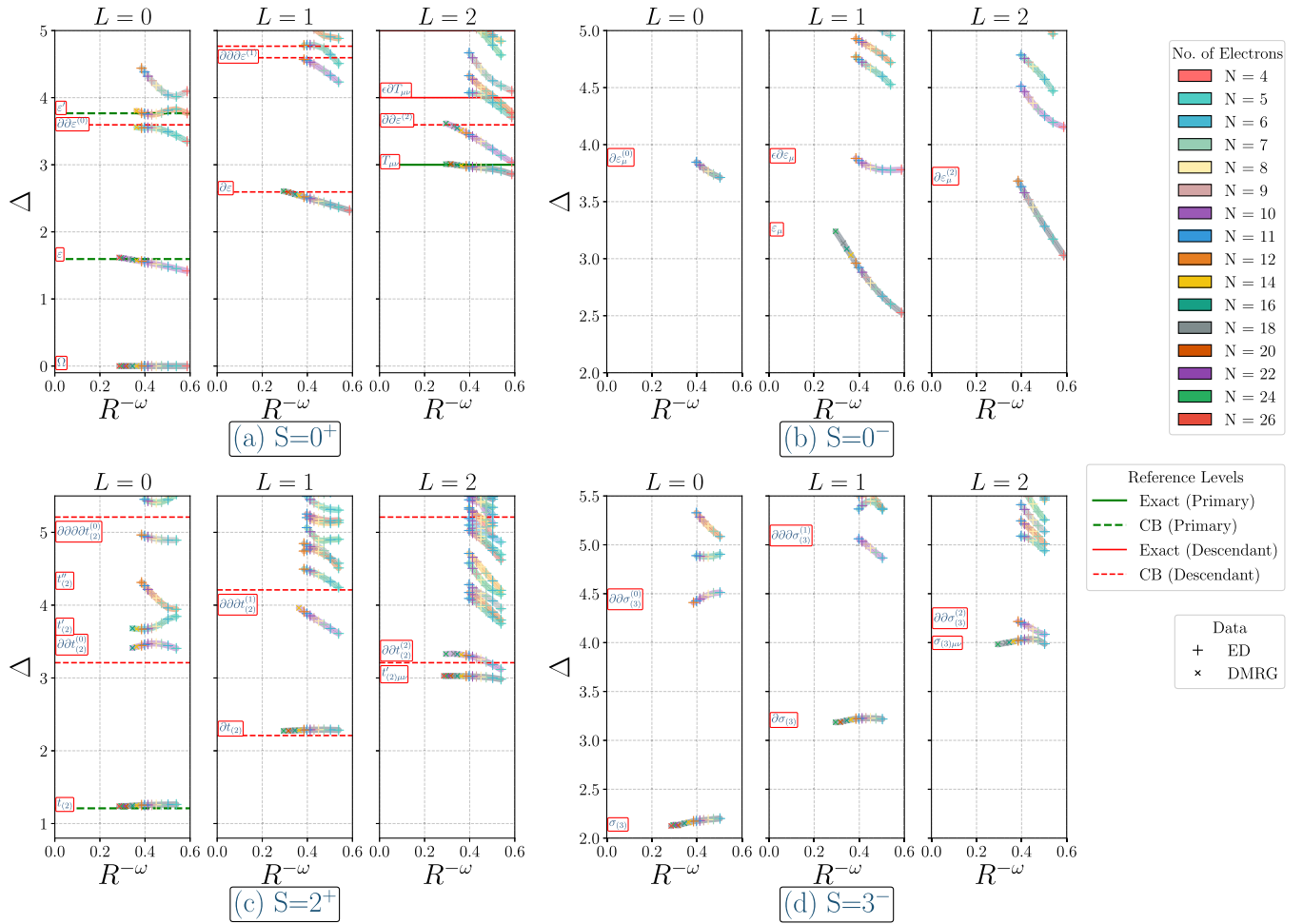


FIG. 2. Dependence on $R^{-\omega}$ of the dimensionless number $\Delta_o(R)$ defined by the scaling ansatz (6) with h chosen to be the root of $g_\varepsilon(h) = 0$ for each value of R . Here, the value $\omega \equiv \Delta_\varepsilon - 3 \approx 0.7668$ is deduced from the CB estimate for the scaling dimension Δ_ε of the leading irrelevant CFT perturbation (see Table I). The limiting value Δ_o of $\Delta_o(R)$ as $R \uparrow \infty$ is interpreted as the scaling dimension of the CFT operator labeled by o . The choice of the eigenstates labeled by o is organized by $S^\pm$ sectors across panels; and by angular momenta L within each panel. Dashed lines indicate conformal bootstrap predictions with the color code green for primary operators and red for descendants. Solid lines represent exact CFT values as applicable to the stress-energy tensor and the conserved Noether current (using the same color code). Operator names, such as ε and ε_μ , are indicated in red boxes. Only selected sectors are shown here; the complete spectra are available in Supplemental Material [31]. Panel (a) displays the $S = 0^+$ sector, panel (b) the $S = 1^+$ sector, panel (c) the $S = 1^-$ sector, and panel (d) the $S = 2^+$ sector. Note that, except for the σ and $\partial_\mu \sigma$ levels used to determine c and h_c , all the plotted values of $\Delta_o(R)$ as $R \uparrow \infty$ are parameter-free outputs.

Results for scaling dimensions—The dependence on $R^{3-\Delta_\varepsilon}$ of the dimensionless number $\Delta_o(R)$ defined by the scaling ansatz (6) with h chosen to be the root of $g_\varepsilon(h) = 0$ for each value of R is shown in Fig. 2 for a selection of eigenstates of $\hat{H}_{\text{fzs}}(R)$ obtained from ED and DMRG. Here, Δ_ε is the scaling dimension of the leading irrelevant perturbation, which is estimated to be 3.7668 by CB. Our estimates for the scaling dimensions Δ_o , obtained from the largest available ED and DMRG system sizes rather than from an explicit extrapolation to $R \uparrow \infty$, are reported in Table I when o labels a primary operator whose scaling dimension is known either from the conformal bootstrap or exactly, and in Table II (Appendix A),

otherwise. A systematic list for the values of the scaling dimensions for all symmetry sectors can be found in Supplemental Material [31].

Figure 2 reveals signatures of emergent conformal symmetry that includes the expected descendant structure, the stress energy tensor, and the conserved SO(3) Noether currents [18]. The CB value of the scaling dimension Δ_ε for the primary operator ε corresponding to the quantum numbers $S = 0^+$ and $L = 0$ is consistent with its ED lower bound and DMRG upper bound from Table I, respectively. The exact value of the scaling dimension $\Delta_{T_{\mu\nu}} = 3$ for the stress-energy tensor $T_{\mu\nu}$ corresponding to the quantum numbers $S = 0^+$ and $L = 2$ is consistent with the ED lower

bound and the DMRG upper bound from Table I. The exact value of the scaling dimension $\Delta_{j_\mu} = 2$ for the conserved SO(3) Noether current j_μ corresponding to the quantum numbers $S = 1^+$, $L = 1$, that reflects the internal SO(3) symmetry is also consistent with the ED and DMRG estimates from Table I. In addition, the CB predictions for the scaling dimensions of the rank-2 $t_{(2)}$ and rank-4 $t_{(4)}$ O(3) primaries are consistent with the ED and DMRG estimates from Table I. Finally, according to Fig. 2 and Table II (Appendix A), there is no CB prediction for the scaling dimension $\Delta_{\sigma_{(3)}} \approx 2.124$ where $\sigma_{(3)}$ corresponds to the quantum numbers $S = 3^-$ [rank 3 O(3) pseudotensor] and $L = 0$ [SO(3) scalar].

In addition, a variety of irrelevant primaries appear, including dangerously irrelevant [58] ones. Among these, the most significant are ϵ' , a scalar under both O(3) and SO(3) [see Fig. 2(a)], and ϵ_μ [17] a pseudoscalar under O(3) and a pseudovector under SO(3) [see Fig. 2(b)]. These operators govern leading corrections to scaling in lattice simulations [59,60]. Notably, ϵ_μ provides insight into a long-standing discrepancy in the critical exponents of columnar versus staggered dimerized antiferromagnets [59–62], once misattributed to different universality classes [63].

At last, the spectrum of scaling exponent that we found numerically is compared with those obtained from the large quantum-number or large- S expansion [see Refs. [31,50,51] and Appendix B], i.e., a Laurent expansion in powers of $1/S$ of the scaling dimensions of some low-lying primaries in the internal SO(3) symmetry sector labeled by the quantum number $S \gg 1$. Our data agree with this large S expansion, in particular for the low- L primaries. They are consistent with the existence of both a gapless, linearly dispersing phonon mode and of a dispersive gapped Goldstone mode.

Results for OPE coefficients—We extract $f_{\text{o eo}}(R)$ in the scaling ansatz (6) with the help of

$$f_{\text{o eo}}(R) = \left. \frac{\partial \delta E_o(R)}{\partial g_\epsilon(R)} \right|_{g_\epsilon(R)=0}. \quad (7)$$

In Fig. 3, we plot the dependence on $R^{-\omega}$ of $f_{\text{o eo}}(R)$ for some of these OPE coefficients and we compare their limiting values as $R \uparrow \infty$ to the available CB data. Other OPE coefficients are plotted and tabulated in Supplemental Material [31].

Discussion and outlook—We have mapped the square-lattice quantum rotor model, whose Hamiltonian exhibits a quantum critical point in the $(2 + 1)$ -dimensional O(3) WF universality class, onto the fuzzy sphere Hamiltonian (4) by placing interacting fermions with four internal flavors on the sphere. Our method remains computationally tractable using ED and DMRG even for a maximum of $N = 26$ interacting fermions and provides evidences for an emergent conformal symmetry consistent with the

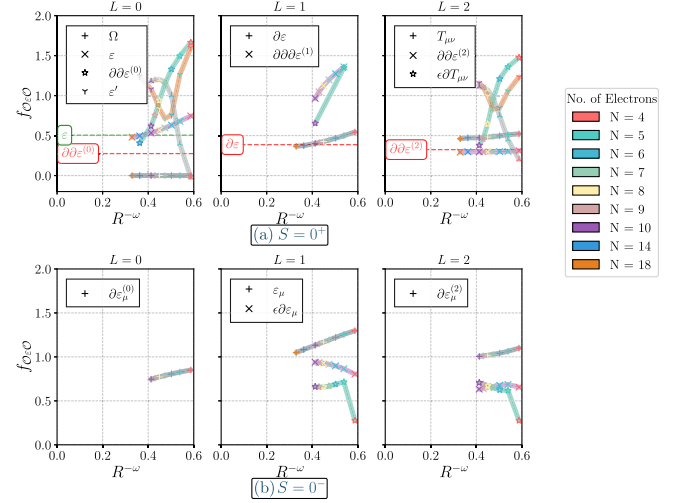


FIG. 3. Dependence on $R^{-\omega}$ with $\omega \equiv \Delta_{\epsilon'} - 3 \approx 0.7668$ of the coefficient $f_{\text{o eo}}(R) = f_{\text{o eo}} + \mathcal{O}(R^{-\omega})$ defined by Eq. (7) with o chosen in the $S = 0^\pm$ symmetry sectors. Solid lines denote CB estimates. Dashed red lines include CPT-derived descendant factors applied to the bootstrap data [55]. The states (CFT operators) selected by o include ϵ , ϵ_μ , $T_{\mu\nu}$.

$(2 + 1)$ -dimensional O(3) WF universality class. We identify 24 primary operators labeled by the index o in Tables I and II (Appendix A) and compute their scaling dimensions Δ_o as well as their diagonal OPE coefficients $f_{\text{o eo}}$. The values that we find agree with published ones and confirm predictions from the large- S expansion [50,51,64] (see Appendix B).

While analyzing the $(2 + 1)$ -dimensional O(3) WF CFT, we identified a weakly irrelevant operator ϵ_μ relevant to the criticality of staggered dimerized antiferromagnets. This operator transforms as a pseudoscalar under O(3) and a pseudovector under Lorentz [SO(3)] symmetry, originating from a cubic two-derivative term in the free theory, reminiscent of the topological θ term. This operator also admits a complementary description as a local bulk interaction in the soft-spin ϕ^4 framework [59], and its topological character under lattice regularization is not fully settled. Regardless of interpretation, its weak irrelevance generates anomalously large corrections to scaling in staggered dimerized antiferromagnets, supporting a single O(3) universality class with strong subleading corrections rather than a distinct universality class. Boundary CFT provides a natural setting to distinguish the two pictures, and our bulk identification of ϵ_μ makes this test quantitatively accessible.

Our framework naturally generalizes to the study of O(N) WF quantum criticality, with explicit Hamiltonian constructions provided in Supplemental Material [31]. Furthermore, it enables systematic exploration of models exhibiting richer internal symmetries, such as $O(n_1) \times O(n_2)$ [65], and, by introducing anisotropic terms that break O(n) symmetry down to a discrete hypercubic subgroup, the stable cubic universality class for $n \geq 3$.

The method is also compatible with boundary and defect modifications of the fuzzy sphere, opening access to boundary operator spectra and defect observables. More broadly, the 24 primaries and OPE coefficients reported here provide quantitative input for conformal bootstrap studies, control corrections to scaling in lattice simulations, and constrain universal features of experimental response functions near $O(3)$ quantum criticality.

Note added—Recently, we became aware of an independent related work [66].

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Data availability—The data that support the findings of this article are openly available [67], embargo periods may apply.

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- [1] A. M. Polyakov, JETP Lett. **12**, 381 (1970).
 - [2] P. Di Francesco, P. Mathieu, and D. Sénéchal, *Conformal Field Theory*, Graduate Texts in Contemporary Physics (Springer, New York, NY, 1997).
 - [3] S. Rychkov, *EPFL Lectures on Conformal Field Theory in $D \geq 3$ Dimensions*, SpringerBriefs in Physics (Springer, New York, 2016), [arXiv:1601.05000](https://arxiv.org/abs/1601.05000).
 - [4] J. Cardy, *Scaling and Renormalization in Statistical Physics*, Cambridge Lecture Notes in Physics (Cambridge University Press, 1996).
 - [5] N. Goldenfeld, *Lectures On Phase Transitions and the Renormalization Group* (CRC Press, Boca Raton, 2018).
 - [6] W. Witzczak-Krempa, P. Ghaemi, T. Senthil, and Y. B. Kim, *Phys. Rev. B* **86**, 245102 (2012).
 - [7] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, Cambridge, England, 2011).
 - [8] A. Belavin, A. Polyakov, and A. Zamolodchikov, *Nucl. Phys. B* **241**, 333 (1984).
 - [9] M. A. Virasoro, *Phys. Rev. D* **1**, 2933 (1970).
 - [10] M. Takahashi, *Thermodynamics of One-Dimensional Solvable Models* (Cambridge University Press, Cambridge, England, 1999).
 - [11] M. Hasenbusch, *Phys. Rev. B* **105**, 054428 (2022).
 - [12] S. El-Showk, M. F. Paulos, D. Poland, S. Rychkov, D. Simmons-Duffin, and A. Vichi, *Phys. Rev. D* **86**, 025022 (2012).
 - [13] F. Kos, D. Poland, D. Simmons-Duffin, and A. Vichi, *J. High Energy Phys.* **08** (2016) 036.
 - [14] S. M. Chester, W. Landry, J. Liu, D. Poland, D. Simmons-Duffin, N. Su, and A. Vichi, *Phys. Rev. D* **104**, 105013 (2021).
 - [15] S. E. Derkachov and A. N. Manashov, *Phys. Rev. Lett.* **79**, 1423 (1997).
 - [16] K. Lang and W. Rühl, *Nucl. Phys. B* **402**, 573 (1993).
 - [17] J. Henriksson, F. Herzog, S. R. Kousvos, and J. R. Nepveu, *Phys. Lett. B* **874**, 140235 (2026).
 - [18] W. Zhu, C. Han, E. Huffman, J. S. Hofmann, and Y.-C. He, *Phys. Rev. X* **13**, 021009 (2023).
 - [19] L. Hu, Y.-C. He, and W. Zhu, *Phys. Rev. Lett.* **131**, 031601 (2023).
 - [20] C. Han, L. Hu, W. Zhu, and Y.-C. He, *Phys. Rev. B* **108**, 235123 (2023).
 - [21] L. Hu, Y.-C. He, and W. Zhu, *Nat. Commun.* **15**, 9013 (2024).
 - [22] Z. Zhou, D. Gaiotto, Y.-C. He, and Y. Zou, *SciPost Phys.* **17**, 021 (2024).
 - [23] L. Hu, W. Zhu, and Y.-C. He, *Phys. Rev. B* **111**, 155151 (2025).
 - [24] G. Cuomo, Y.-C. He, and Z. Komargodski, *J. High Energy Phys.* **11** (2024) 061.
 - [25] Z. Zhou and Y. Zou, *SciPost Phys.* **18**, 031 (2025).
 - [26] G. Fardelli, A. L. Fitzpatrick, and E. Katz, *SciPost Phys.* **18**, 086 (2025).
 - [27] Z. Zhou, L. Hu, W. Zhu, and Y.-C. He, *Phys. Rev. X* **14**, 021044 (2024).
 - [28] Y.-C. He, [arXiv:2506.14904](https://arxiv.org/abs/2506.14904).
 - [29] J. Taylor, C. Voinea, Z. Papić, and R. Fan, *Phys. Rev. Lett.* **136**, 056503 (2026).
 - [30] C. Han, L. Hu, and W. Zhu, *Phys. Rev. B* **110**, 115113 (2024).
 - [31] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/4yfv-xbcj> for additional figures, derivations, and computational details, which includes Refs. [32–49].
 - [32] S. Hellerman, D. Orlando, S. Reffert, and M. Watanabe, *J. High Energy Phys.* **12** (2015) 071.
 - [33] L. Alvarez-Gaume, O. Loukas, D. Orlando, and S. Reffert, *J. High Energy Phys.* **04** (2017) 059.
 - [34] D. Banerjee, S. Chandrasekharan, and D. Orlando, *Phys. Rev. Lett.* **120**, 061603 (2018).
 - [35] D. Banerjee, S. Chandrasekharan, D. Orlando, and S. Reffert, *Phys. Rev. Lett.* **123**, 051603 (2019).
 - [36] L. Alvarez-Gaume, D. Orlando, and S. Reffert, *J. High Energy Phys.* **12** (2019) 142.
 - [37] M. Watanabe, *J. High Energy Phys.* **04** (2021) 264.
 - [38] S. Giombi and J. Hyman, *J. High Energy Phys.* **09** (2020) 184.
 - [39] L. Á. Gaumé, D. Orlando, and S. Reffert, *Phys. Rep.* **933**, 1 (2021).
 - [40] R. V. Lange, *Phys. Rev. Lett.* **14**, 3 (1965).
 - [41] H. Nielsen and S. Chadha, *Nucl. Phys. B* **105**, 445 (1976).
 - [42] H. Watanabe and T. c. v. Brauner, *Phys. Rev. D* **84**, 125013 (2011).
 - [43] H. Watanabe and T. c. v. Brauner, *Phys. Rev. D* **85**, 085010 (2012).
 - [44] A. Nicolis and F. Piazza, *Phys. Rev. Lett.* **110**, 011602 (2013).
 - [45] H. Watanabe, T. c. v. Brauner, and H. Murayama, *Phys. Rev. Lett.* **111**, 021601 (2013).
 - [46] M. Fishman, S. R. White, and E. M. Stoudenmire, *SciPost Phys. Codebases* **2022**, 4 (2022).

- [47] M. Fishman, S. R. White, and E. M. Stoudenmire, *SciPost Phys. Codebases* **2022**, 4 (2022).
- [48] S. R. White, *Phys. Rev. Lett.* **69**, 2863 (1992).
- [49] S. R. White, *Phys. Rev. B* **72**, 180403 (2005).
- [50] G. F. Cuomo, Ph.D. thesis, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland (2020), thèse no. 8397, Faculté des sciences de base, Laboratoire de physique théorique des particules, Programme doctoral en physique.
- [51] G. Cuomo, A. Esposito, E. Gendy, A. Khmelnitsky, A. Monin, and R. Rattazzi, *J. High Energy Phys.* **02** (2021) 068.
- [52] J. Henriksson, *Phys. Rep.* **1002**, 1 (2023).
- [53] F. D. M. Haldane, *Phys. Rev. Lett.* **51**, 605 (1983).
- [54] J. Madore, *Mod. Phys. Lett. A* **16**, 639 (2001).
- [55] B.-X. Lao and S. Rychkov, *SciPost Phys.* **15**, 243 (2023).
- [56] A. M. Läuchli, L. Herviou, P. H. Wilhelm, and S. Rychkov, *SciPost Phys.* **19**, 076 (2025).
- [57] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, and E. Vicari, *Phys. Rev. B* **65**, 144520 (2002).
- [58] D. J. Amit and L. Peliti, *Ann. Phys. (N.Y.)* **140**, 207 (1982).
- [59] L. Fritz, R. L. Doretto, S. Wessel, S. Wenzel, S. Burdin, and M. Vojta, *Phys. Rev. B* **83**, 174416 (2011).
- [60] N. Ma, P. Weinberg, H. Shao, W. Guo, D.-X. Yao, and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 117202 (2018).
- [61] M. T. Kao, D. J. Tan, and F. J. Jiang, *arXiv:1202.1057*.
- [62] S. Yasuda and S. Todo, *Phys. Rev. E* **88**, 061301 (2013).
- [63] S. Wenzel, L. Bogacz, and W. Janke, *Phys. Rev. Lett.* **101**, 127202 (2008).
- [64] A. Monin, D. Pirtskhalava, R. Rattazzi, and F. K. Seibold, *J. High Energy Phys.* **06** (2017) 011.
- [65] D. R. Nelson, J. M. Kosterlitz, and M. E. Fisher, *Phys. Rev. Lett.* **33**, 813 (1974).
- [66] W. Guo, Z. Zhou, T.-C. Wei, and Y.-C. He, *arXiv:2512.02234*.
- [67] A. Dey, L. Herviou, C. Mudry, and A. M. Läuchli. Data for the article “Conformal data for the O(3) Wilson-Fisher conformal field theory from fuzzy sphere realization of the quantum rotor model,” Zenodo, version v1 (2026), [10.5281/zenodo.19889910](https://zenodo.org/record/19889910).

End Matter

Appendix A: Primary operators—Table II presents the primary operators whose scaling dimensions we have computed, for which conformal bootstrap results are not currently available.

Appendix B: Large- S expansion—The large- S expansion provides a controlled semiclassical framework for computing the scaling dimensions of local operators in a CFT with internal O(3) symmetry when these local operators transform according to the $2S + 1$ -dimensional irreducible representation of O(3). In the sector of the CFT with $S \gg 1$, some scaling dimensions can be associated with the dispersions of two weakly interacting fields associated to the explicit symmetry breaking of the internal symmetry SO(3) down to SO(2). One is massless and called a phonon mode. The other is gapful and called a gapped Goldstone mode.

Three key predictions from the large- S expansion are the following when spacetime is three dimensional.

Lowest scalar primaries: The scaling dimension of the lowest scalar ($L = 0$) operator in each $S \gg 1$ sector obeys the semiclassical expansion

$$\Delta_0(S^{\kappa(S)}, L = 0) = \alpha S^{3/2} + \beta S^{1/2} - 0.093\,7256 + \gamma S^{-1/2} + \mathcal{O}(1/S), \quad (\text{B1})$$

where $\kappa(S) = (-1)^S$ determines the parity. The Wilson coefficients $\alpha = 0.310\,76$, $\beta = 0.298\,18$, and $\gamma = 0.003\,70$ are obtained by fitting the lowest level in the sectors 1^- , 2^+ , 4^+ to bootstrap data for σ , $t_{(2)}$, and $t_{(4)}$.

Gapped Goldstone modes: The scaling dimensions of the state with spin $S \gg 1$, parity $\kappa(S + 1) = -\kappa(S)$, and nonvanishing angular momenta $L \geq 1$ obeys the

semiclassical expansion

$$\Delta(S^{\kappa(S+1)}, L) = \Delta_0(S^{\kappa(S)}, L = 0) + \mu(S) + \chi \frac{L(L+1)}{2\mu(S)} + \mathcal{O}\left(\frac{L^4}{\mu^3}\right), \quad (\text{B2a})$$

where

$$\mu(S) := \frac{\partial \Delta_0(S, L = 0)}{\partial S} \quad (\text{B2b})$$

is the chemical potential that explicitly breaks the internal continuous symmetry from SO(3) to SO(2). The term $L(L+1)/(2\mu)$ encodes the dispersion of the gapped Goldstone boson. The value of the Wilson coefficient χ is obtained by fitting the lowest scaling dimension in the sector 1^+ to the value 2 of the scaling dimension of the Noether current j_μ .

Gapless phonon modes: The scaling dimension of the state with the quantum numbers $S \gg 1$ for the spin, parity $\kappa(S) = (-1)^S$, and the angular momenta $L = 0, 1, \dots$, obeys the semiclassical expansion

$$\Delta_0(S^{\kappa(S)}, L) = \Delta_0(S^{\kappa(S)}, L = 0) + \sum_L n_L \omega_L, \quad (\text{B3a})$$

where

$$\omega_L = \sqrt{L(L+1)}/2, \quad (\text{B3b})$$

is the energy of the phonon with angular momenta L and the integer n_L is its occupancy. The value $L = 1$

TABLE II. Selected primary operators with the same conventions as in Table I.

S	L	I	o	$\Delta(\text{ED})$	$\Delta(\text{DMRG})$
0^-	1	1	ε_μ	2.960 67 (12)	3.2396 (18)
1^+	1	3	ϕ_μ	3.708 28 (12)	3.757 03 (18)
1^-	1	2	σ_μ	2.870 55 (12)	3.043 73 (26)
1^-	2	2	$\sigma_{\mu\nu}$	3.3557 (12)	3.5937 (24)
2^+	0	3	$t'_{(2)}$	3.671 28 (12)	3.681 21 (16)
2^+	0	4	$t''_{(2)}$	4.312 61 (12)	...
2^+	2	1	$t'_{(2)\mu\nu}$	3.025 38 (12)	3.028 25 (26)
2^-	1	1	$t_{(2)\mu}$	2.689 70 (12)	2.781 87 (26)
2^-	2	1	$t'_{(2)\mu\nu}$	3.294 89 (12)	3.5341 (24)
3^+	1	1	$\chi_{(3)\mu}$	3.687 69 (12)	3.750 82 (26)
3^+	2	1	$\chi_{(3)\mu\nu}$	4.223 26 (12)	...
3^-	0	1	$\sigma_{(3)}$	2.173 10 (12)	2.124 86 (26)
3^-	0	3		4.888 95 (11)	...
3^-	2	1	$\sigma_{(3)\mu\nu}$	4.022 21 (12)	3.9825 (24)
4^+	0	3		6.253 69 (11)	...
4^+	2	1	$t_{(4)\mu\nu}$	5.177 57 (11)	...
4^-	1	1		4.851 39 (11)	4.871 82 (26)
4^-	2	1		5.297 24 (11)	...

corresponds to a phonon descendant, while the values $L \geq 2$ correspond to phonon primaries.

The agreement between these theoretical predictions and fuzzy sphere numerical results (shown in Fig. 4) provides strong evidence for the presence of both gapped Goldstone modes and gapless phonon modes in the $(2+1)$ -dimensional $O(3)$ WF CFT, validating the large- S expansion framework.

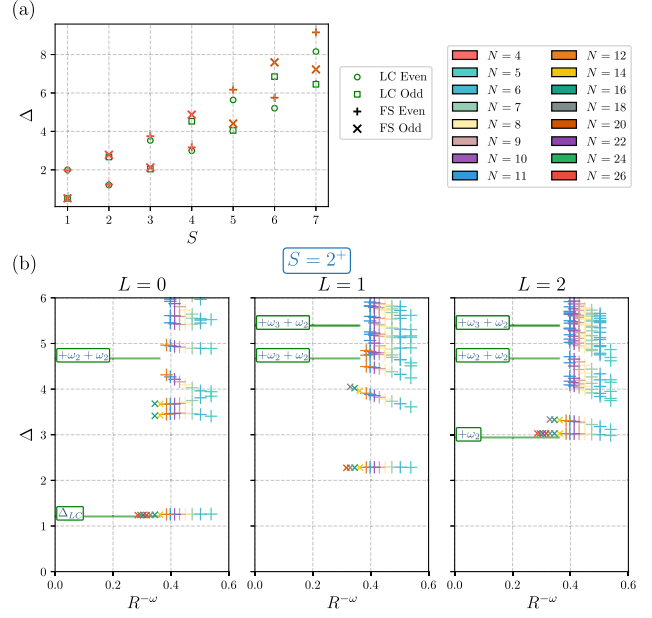


FIG. 4. (a) For each quantum number $S = 1, 2, \dots$, two scaling dimensions are reported. The lowest one corresponds to the CFT state in the symmetry sector with the spin quantum number $S^{\kappa(S)} = 1^-, 2^+, \dots$, with $\kappa(S) = (-1)^S$ for the internal symmetry group $O(3) = \mathbb{Z}_2 \times SO(3)$ and the angular momenta $L = 0$. The larger one corresponds to the CFT state with the quantum numbers $S^{\kappa(S+1)} = 1^+, 2^-, \dots$, and $L = 1$. The scaling dimensions obtained from ED and DMRG of $\hat{H}_{\text{fzs}}(R)$ and reported as $+$ ($\times$) for $\kappa(S) = +(-)$. For fuzzy sphere results, number of electrons is 26 (DMRG) for $S = 1, 2, 3, 4$, and 12 (ED) for $S = 5, 6, 7$. The scaling dimensions obtained from fitting the large- S expansions (B1) and (B2) with bootstrap data are reported as $\circ$ ($\square$) for $\kappa(S) = +(-)$. (b) Scaling dimensions as a function of $R^{-\omega}$ with $\omega \approx 0.7668$ for phonon primaries and their descendants obtained from ED of $\hat{H}_{\text{fzs}}(R)$ in the symmetry sector $S = 2^+$. The green lines report the scaling dimensions obtained from fitting the large- S expansion (B3) with bootstrap data.