

Nondestructive Optomechanical Detection Scheme for Bose-Einstein Condensates

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We present a two-tone heterodyne optical readout scheme to extract unequal-time density correlations along an arbitrary stationary interaction path from a pancake-shaped Bose-Einstein condensate, using a modulated laser probe. Analyzing the measurement noise both from imprecision and backaction, we identify the standard quantum limit for the signal-extraction scheme, and examine how a class of two-mode squeezed initial states can be used to push beyond this limit. As an application, we show how the readout scheme can be used for an experimentally feasible realization of acceleration dependence of quantum-vacuum fluctuations in the system, including the analog spacetime circular motion Unruh effect. The scheme is adaptable beyond Bose-Einstein condensates, providing nondestructive access to unequal-time correlations in quantum fluids.

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Introduction—Bose-Einstein condensates (BECs) provide a powerful platform for exploring a range of intriguing quantum field theory processes, including information theoretical [1] and far-from-equilibrium phenomena such as quantum turbulence [2–5], first-order relativistic phase transitions [6–15], and curved spacetime dynamics that mimic the early universe [16–19] and black holes [20–22]. A key technical challenge lies in controlling the condensate, leading to innovations such as optical box traps [23,24] and multicomponent condensates [25]. A common objective across these developments is the advancement of novel detection schemes.

Currently, state-of-the-art detection methods are primarily destructive, capturing wave dynamics frozen in time [1,26]. Although this approach has been highly successful, enabling the extraction of statistical correlations and cumulants of density fluctuations between different points in the condensate, there remains strong motivation to develop improved, potentially nondestructive detection techniques [27]. A specific candidate technique was introduced in [28],

where a focused laser beam passes perpendicularly through a pancake-shaped BEC, sampling density fluctuations locally over time. The point of interaction between the probe beam and the condensate need not be static but can be placed in controlled motion. The probe beam thus records the local density fluctuations encountered along the interaction path, encoding the unequal space and time correlations and their dependence on the motion [28,29].

The measurement sensitivity in this detection technique is set by the competition between shot noise in the probe and radiation-pressure backaction onto the condensate. In the present Letter we go significantly beyond [28] by providing a complete end-to-end detection theory that controls the sensitivity: we derive the standard quantum limit (SQL) [30,31] for a two-tone heterodyne readout of a moving-point optical probe, include the full nonlinear Bogoliubov backaction structure, and show how two-mode squeezed input states enable sub-SQL sensitivity while preserving nondestructiveness. The scheme is adaptable beyond BEC, providing nondestructive access to unequal-time correlations in quantum fluids.

As an application, we apply our interferometric machinery to accelerated paths of the interaction point. We detail an experimentally feasible detection and readout scheme to extract unequal-time BEC correlations from the beam. We show that these correlations bear an imprint of the

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acceleration of the interaction-point path. This acceleration dependence renders the two-dimensional BEC a powerful platform for the quantum simulation of acceleration effects in quantum field theory, particularly the celebrated Unruh effect [32–35].

Phonon detectors—We describe a BEC of atomic mass m , confined to a plane (defined as $z = 0$) by a trapping potential. Upon integration in z , the BEC is described by a field $\Phi(t, \mathbf{x})$ governed by the two-dimensional Gross-Pitaevskii Lagrangian,

$$L_{\text{BEC}} = \int d^2\mathbf{x} \left[i\Phi^\dagger \partial_t \Phi - \frac{1}{2m} |\nabla \Phi|^2 + \frac{g_{2d}}{2} |\Phi|^4 \right], \quad (1)$$

where $\mathbf{x} = (x, y)$ and g_{2d} is the s -wave scattering strength. When we consider the real part of perturbations about a background condensate Φ_0 , these long-wavelength phonons evolve as a massless Klein-Gordon scalar field [20,36]. In terms of the physical BEC parameters, this corresponds to density fluctuations about a background density $\rho_0 = |\Phi_0|^2$, as detailed in Supplemental Material [37].

Long-wavelength phonons may be probed with a highly focused laser propagating transversely through the condensate parallel to the z direction; the effective refractive index of the BEC shifts the laser phase. As shown in the optical circuit diagram of Fig. 1, the beam probing the

condensate is prepared by sending monochromatic laser light of frequency ω_0 into an electro-optic modulator (EOM), and then filtering out the central frequency band. The resulting laser field $E(t)$ has two prominent modulation bands peaked at $\omega_\pm := \omega_0 \pm \Omega$ (with $\Omega \ll \omega_0$), which may be expressed as

$$E(t) = E_0(\omega_0)[a(t) + a^\dagger(t)], \quad (2a)$$

$$a(t) = a_+(t) + a_-(t), \quad (2b)$$

$$a_\pm(t) = [\alpha + \delta a_\pm(t)]e^{-i\omega_\pm t}, \quad (2c)$$

where $E_0(\omega_0)$ is a dimensionless real-valued prefactor evaluated at the central frequency ω_0 , $a_\pm$ are the mode operators for each modulation band, the full mode operator a satisfies $[a(t), a^\dagger(t')] = \delta(t - t')$, and, without loss of generality, the coherent amplitude α is assumed to be real. We note that our notation suppresses spatial dependence for brevity, despite the laser serving as a local probe.

The interaction between the laser and the BEC can be modelled as the interaction between a two-dimensional scalar field $\phi(t, \mathbf{x})$ —representing the density fluctuations in the BEC—and a one-dimensional scalar field $E(t)$ —representing the laser—that is localized on the path $\mathbf{x} = \mathbf{X}(t)$. By arranging ω_0 to coincide with an atomic resonance in the BEC, the phase shifts in the two modulation bands will be equal and opposite, as represented by the mode-operator transformation

$$a_\pm(t) \rightarrow e^{-i(\omega_\pm t \pm \psi_0)}(\alpha + \delta \tilde{a}_\pm(t)), \quad (3)$$

where ψ_0 is the time-independent phase shift due to the BEC bulk density. A tilde above an operator denotes that operator post interaction. These equal and oppositely detuned sidebands cancel out the zeroth-order Stark potential, reducing disturbance to the BEC due to stirring [28].

The first-order interaction $\delta \tilde{a}_\pm(t)$ in (3) is given by

$$\delta \tilde{a}_\pm(t) = \delta a_\pm(t) \mp \frac{1}{2} i \epsilon \alpha \phi(t), \quad (4)$$

where the $\mp$ sign originates from the opposite phase shifts, $\phi(t) := \phi(t, \mathbf{X}(t))$ is the field evaluated along the interaction path $\mathbf{X}(t)$, and, for sidebands detuned far from resonance, the interaction coupling is given by $\epsilon = 2|\hat{\alpha}_R|\omega_0\sqrt{m\rho_0}$, where $\hat{\alpha}_R$ is the real part of the atomic polarizability (see Appendix A). Physically, the field $\phi(t)$ corresponds to density fluctuations in the BEC sampled along the trajectory traced by the laser as it intersects the BEC.

The typical nondispersive frequency range of phonons in the BEC lies well below the modulation frequency Ω . We denote the boundary of this nondispersive range by Δ and write the field as

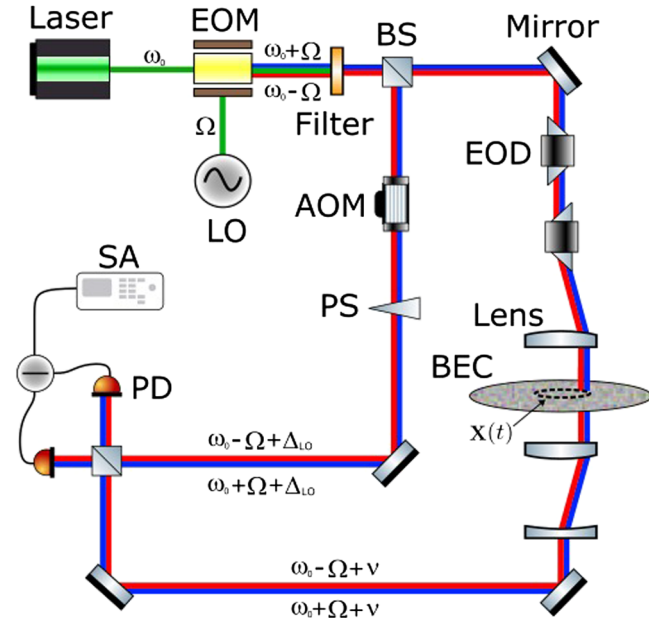


FIG. 1. Optical circuit diagram for sampling BEC density fluctuations along a stationary interaction point trajectory, using a modulated laser probe with two sidebands. The band centred at $\omega_+ = \omega_0 + \Omega$ ($\omega_- = \omega_0 - \Omega$) is shown in blue (red). Postinteraction, the modulated signal is heterodyned using a two-tone reference beam, in which an acousto-optic modulator has inserted a frequency shift Δ_{LO} . Beam splitter (BS); electro-optic deflector (EOD); local oscillator (LO); acousto-optic modulator (AOM); phase shifter (PS); photodiode (PD); spectrum analyzer (SA).

$$\phi(t) = \int_{-\Delta}^{\Delta} \frac{d\nu}{2\pi} e^{-i\nu t} D_{\nu}, \quad (5)$$

where $0 < \Delta \ll \Omega$, and D_{ν} are the annihilation (for $\nu > 0$) and creation (for $\nu < 0$) operators defined with respect to the frame comoving along the interaction path, such that $D_{-\nu} = D_{\nu}^{\dagger}$. For typical quasi-2D condensates, $\Delta/2\pi$ lies in the few-kHz range. We write the pre-interaction and post-interaction operators $\delta a_{\pm}(t)$ and $\delta \tilde{a}_{\pm}(t)$ similarly in terms of their Fourier transforms as $\delta a_{\pm}(t) = (2\pi)^{-1} \int_{-\Delta}^{\Delta} d\nu e^{-i\nu t} \delta a_{\pm}[\nu]$ and $\delta \tilde{a}_{\pm}(t) = (2\pi)^{-1} \int_{-\Delta}^{\Delta} d\nu e^{-i\nu t} \delta \tilde{a}_{\pm}[\nu]$.

Now, the laser-BEC interaction (4) can be described in terms of the frequency-space operators as a Bogoliubov transformation, and this transformation can be completed nonperturbatively to contain the backaction on the laser probe, quadratic in the coupling parameter ε [36]. As shown in Appendix B, the outcome is

$$\delta \tilde{a}_{\pm}[\nu] = \delta a_{\pm}[\nu] \pm \frac{i\mu D_{\nu}}{\sqrt{2}} \pm \frac{\mu^2}{4} \text{sgn}(\nu) \delta a_b[\nu], \quad (6)$$

where $\mu := -\varepsilon\alpha/\sqrt{2} < 0$ is a dimensionless coupling parameter and δa_b is the contribution from backaction, given by

$$\delta a_b[\nu] = \delta a_{-}[\nu] + \delta a_{-}[-\nu]^{\dagger} - \delta a_{+}[\nu] - \delta a_{+}[-\nu]^{\dagger}. \quad (7)$$

The signum function in (6) implies that backaction on the two sides of the modulation bands takes opposite signs. This may seem surprising; however, similar asymmetric backaction has been observed within optomechanical systems [38].

The term involving the BEC mode operator D_{ν} in (6) is the signal. This shows that the beam acts as a phonon detector. The last term in (6), representing backaction, shows that the laser also records the noise that it has injected into the BEC. The task of the detection scheme will be to separate the signal from the noise.

Detection scheme—We shall now present a detection scheme where the signal D_{ν} can be extracted from the postinteraction laser beam (6) despite the backaction noise contained in the last term and the shot noise contained in the first term. We assume from now on that the interaction point trajectory $X(t)$ is stationary (such as in uniform circular motion), and that the initial state of the BEC is stationary (such as a vacuum state or a thermal state). The quantity we wish to extract is the Fourier transform of the BEC unequal-time density two-point function along the path of the laser-BEC interaction point, given by

$$S_{\phi\phi}[\nu] = \int dt e^{-i\nu t} \langle \phi(t) \phi(0) \rangle. \quad (8)$$

Because of the stationarity, $S_{\phi\phi}[\nu]$ (8) is the BEC power spectral density (PSD). The interest of $S_{\phi\phi}[\nu]$ is that it is a

multiple of the transition rate of a pointlike two-state quantum system that moves along the path of the laser-BEC interaction point, had we such a quantum system at hand; ν is the system's energy gap, with $\nu > 0$ for excitations and $\nu < 0$ for deexcitations [28,34,39,40]. The optical detection scheme hence must be able to distinguish between positive and negative values of ν .

Inspired by optical demonstrations of mechanical sideband asymmetry [41], we consider the two-tone heterodyne scheme depicted in Fig. 1. Post-interaction, the signal riding on modulation bands centred at $\omega_{+} = \omega_0 + \Omega$ and $\omega_{-} = \omega_0 - \Omega$ is mixed with a reference beam whose frequency has been detuned by the amount Δ_{LO} , in the intermediate range $\Delta \ll \Delta_{\text{LO}} \ll 2\Omega$, using an acusto-optic modulator as shown in Fig. 1. The beamsplitter outputs are then converted into photocurrents via photodiodes and their difference is taken, yielding the difference-photocurrent $i(t)$.

The observed difference-photocurrent $i(t)$ is sent to a spectrum analyser. The PSD S_{ii} produced by the spectrum analyser is related to $S_{\phi\phi}[\nu]$ (8) by

$$S_{ii}[\Delta_{\text{LO}} - \nu] = \mu^2 (S_{\phi\phi}[\nu] + \mathcal{N}[\nu; \mu^2]), \quad (9)$$

where

$$\mathcal{N}[\nu; \mu^2] = \frac{1}{\mu^2} + \frac{3\mu^2}{8} + \text{sgn}(\nu), \quad (10)$$

as we show in Supplemental Material [37], building on Glauber's theory of photodetection [42] and the relation between photocurrent PSDs and electromagnetic correlators [43]. A measurement of S_{ii} hence yields $S_{\phi\phi}[\nu]$ over the full frequency band $\nu \in (-\Delta, \Delta)$.

In practice, two additional noise sources appear in the photocurrent: (i) relative intensity noise (RIN) of the probe and reference beams, and (ii) phase noise of the local oscillator. RIN enters primarily through common-mode fluctuations and is strongly suppressed by the balanced detection configuration shown in Fig. 1 (typically by 30–40 dB [44–46]). LO phase noise contributes an additive term to $S_{ii}[\omega]$ centred at the intermediate frequency Δ_{LO} ; its contribution scales inversely with the reference beam power, and is minimized by operating the LO at high power. For commercially available low-phase-noise lasers these terms remain well below the SQL noise floor across the detection bandwidth.

Noise minimization—The remaining task is to minimise the added noise term $\mathcal{N}[\nu; \mu^2]$ (10) that contributes to the measured PSD S_{ii} (9) on par with the signal $S_{\phi\phi}[\nu]$ that we wish to infer. The first term in $\mathcal{N}[\nu; \mu^2]$ (10) arises from shot noise, which represents sensor imprecision [38]. The second term arises from the backaction found in (6) and (7). The balance between the two terms depends on the parameter μ , which is proportional to the probe beam's

coherent amplitude and can be tuned by adjusting the beam's intensity. The minimum value of the noise is attained at $\mu^2 = 2\sqrt{2/3}$, where the coherent amplitude satisfies $\alpha^2 = \alpha_{\text{SQL}}^2 := 4\sqrt{2}/(\sqrt{3}\epsilon^2)$. This minimum is known as the standard quantum limit (SQL), which is the optimal balance between the shot-noise-dominated regime ($\alpha^2 < \alpha_{\text{SQL}}^2$) and the backaction-noise-dominated regime ($\alpha^2 > \alpha_{\text{SQL}}^2$). The added noise at the SQL is

$$\mathcal{N}_{\text{SQL}}[\nu] = \sqrt{\frac{3}{2}} + \text{sgn}(\nu). \quad (11)$$

To reduce the added noise *below* the SQL value (11), we replace the laser field's initial state by a two-mode squeezed state, with the real-valued squeezing parameter λ , where $\lambda = 0$ is the unsqueezed initial state considered above. Formula (10) is then replaced by

$$\mathcal{N}[\nu; \mu, \lambda] = \frac{1 + \frac{3}{2}\sinh^2\lambda}{\mu^2} + \frac{3\mu^2 e^{-2\lambda}}{8} + e^{-\lambda} \cosh \lambda \text{sgn}(\nu), \quad (12)$$

as shown in Supplemental Material [37]. Related demonstrations of squeezing-enhanced heterodyne or sideband detection include Li *et al.* [47] and references therein. Comparison with (10) shows that the squeezing has increased the shot noise, for either sign of λ , while the backaction noise has increased for $\lambda < 0$ but decreased for $\lambda > 0$. For given λ , the minimum value of the added noise is attained at

$$\mu^2 = \mu_\lambda^2 := 2e^\lambda \sqrt{\frac{2}{3} + \sinh^2\lambda}, \quad (13)$$

and the minimum value is

$$\mathcal{N}[\nu, \mu_\lambda, \lambda] = e^{-\lambda} \sqrt{\frac{3}{2} \left(1 + \frac{3}{2} \sinh^2\lambda \right)} + e^{-\lambda} \cosh \lambda \text{sgn}(\nu). \quad (14)$$

For $\nu > 0$, $\mathcal{N}[\nu, \mu_\lambda, \lambda]$ (14) is decreasing in λ , it is below the SQL value $\sqrt{3/2} + 1$ (11) for $\lambda > 0$, and as $\lambda \rightarrow \infty$ it approaches 5/4. For $\nu < 0$, $\mathcal{N}[\nu, \mu_\lambda, \lambda]$ is decreasing in λ for $\lambda < \lambda_0$ and increasing in λ for $\lambda > \lambda_0$, attaining its minimum at $\lambda_0 = \frac{1}{2} \ln(5 + 4\sqrt{10}) - \ln 3 \approx 0.3367$, with the minimum value $\frac{1}{6}(\sqrt{10} - 2) \approx 0.1937$, and approaching 1/4 as $\lambda \rightarrow \infty$. $\mathcal{N}[\nu, \mu_\lambda, \lambda]$ is below the SQL value $\sqrt{3/2} - 1$ (11) for $0 < \lambda < \lambda_1 = \frac{1}{2} \ln((5/23)(19 + 8\sqrt{6})) \approx 1.0635$.

Collecting, the added noise $\mathcal{N}[\nu, \mu_\lambda, \lambda]$ (14) can be brought below the SQL value (11) for both $\nu > 0$ and $\nu < 0$ by choosing the squeezing parameter to be in the interval $0 < \lambda < \lambda_1 \approx 1.0635$, when the dimensionless

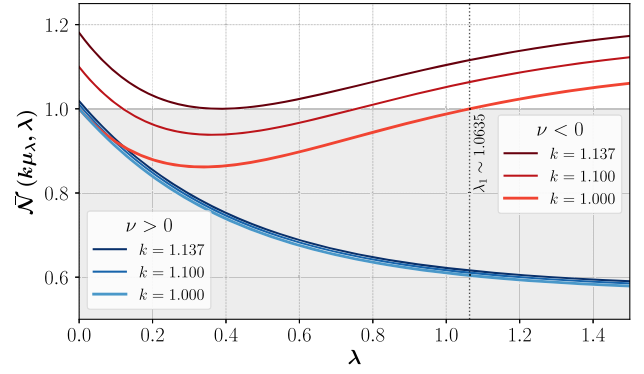


FIG. 2. Plots of the normalized added noise $\tilde{\mathcal{N}}$, defined as (12) divided by the SQL value (11), as a function of λ (squeezing parameter) and μ (laser-BEC coupling parameter), for $0 \leq \lambda \leq 1.5$ and $\mu = k\mu_\lambda$, where μ_λ (13) is the optimal value of μ . Three selected values of k are shown for both $\nu > 0$ and $\nu < 0$. The $k = 1$ curves are (14), and the ranges of λ where $\tilde{\mathcal{N}} < 1$ are as described in the text. For $k > 1$, the ranges of λ where $\tilde{\mathcal{N}} < 1$ are narrower: this range exists for $1 < k < 3^{-1/2}(2 + 2\sqrt{6} + \sqrt{19 + 8\sqrt{6}})^{1/2} \approx 2.091$ when $\nu > 0$, and for $1 < k < 3^{-1/4}(\sqrt{6} - 2 + \sqrt{13 - 4\sqrt{6}})^{1/2} \approx 1.137$ when $\nu < 0$. For $0 < k < 1$, replace k by $1/k$. The physical mechanism that makes the added noise more sensitive to μ for $\nu < 0$ than for $\nu > 0$ is the same as in the backaction cooling in the measurement of zero point mechanical oscillator in a cavity optomechanical system [38], as we show in Supplemental Material [37].

coupling parameter μ has the optimal value μ_λ (13). When μ deviates from the optimal value, the interval of λ where this happens becomes narrower, in a manner shown in Fig. 2. The squeezing range ($0 < \lambda < \lambda_1$) required for simultaneous sub-SQL performance for $\nu > 0$ and $\nu < 0$ corresponds to 6–9 dB of two-mode squeezing, well within the capabilities of fiber- and OPO-based sources [48].

We exemplify the achievable sensitivity of our detection scheme when operating near the SQL by estimating the threshold for resolving relative density fluctuations $\delta\rho/\rho_0$. This threshold is defined by a unit signal-to-noise ratio (SNR), $S_{\phi\phi}[\nu]/\mathcal{N}[\nu, \mu_\lambda, \lambda] = 1$. In Appendix A, we find that the field ϕ is related to BEC density fluctuations by $\phi = \delta\rho/(2\sqrt{m\rho_0})$. Using SI units and estimating $S_{\phi\phi} \approx \phi^2 \Delta t$, we find

$$\frac{\delta\rho}{\rho_0} \approx \sqrt{\frac{4mS_{\phi\phi}[\nu]}{\hbar\rho_0\Delta t}} = \sqrt{\frac{4m\mathcal{N}[\nu, \mu_\lambda, \lambda]}{\hbar\rho_0\Delta t}}. \quad (15)$$

Measuring ^{133}Cs BEC atoms for 10 ms with $\rho_0 = 10 \mu\text{m}^{-2}$ and $\nu < 0$ yields $\delta\rho/\rho_0 \approx 0.137$ when $\lambda = 1$. Remaining at the point of minimal added noise (14), the sensitivity of this scheme can be further improved either with squeezing, repeated experimental realizations, or potentially cavity enhancement [49].

Accelerated interaction paths—As discussed in the text surrounding (3) and (4), the effective relativistic field $\phi(t, \mathbf{x})$, corresponding to density fluctuations in the homogeneous BEC, evolves on an *analog* spacetime geometry determined by the bulk condensate Φ_0 [28], where the speed of sound in the BEC plays the role of the speed of light in a relativistic setting. It was further pointed out in [28] that a localized laser—as described above—measures these fluctuations in a manner consistent with the particle-detector model known in the literature as the Unruh-DeWitt detector.

With this in mind, we apply our detection scheme to stationary accelerated interaction paths, simulating the Unruh effect—the celebrated prediction in Minkowski spacetime quantum field theory that an accelerated observer and an inertial observer measure different quantum fluctuations [32–35]. Recall that from (8) onwards, we have assumed the interaction trajectory to be stationary [50,51]. While the best-known version of the Unruh effect is for uniform linear acceleration [32–34], versions of the effect exist for all types of uniform accelerations [52,53], including uniform circular motion, which has special experimental interest in that it stays in a finite spatial volume arbitrarily long. The Unruh effect exists also in analog spacetimes: the uniform circular motion case has been recently discussed in BECs [28,40] and in superfluid helium [54,55]. We shall now specialize to a circular path, and address the information that $S_{\phi\phi}[\nu]$ carries about the circular motion Unruh effect.

The quantum vacuum fluctuations as detected by linearly accelerating observers are thermally distributed, described by a temperature proportional to the acceleration. A subtlety with the circular motion Unruh effect is that it cannot be described by a single temperature parameter. It can, however, be associated with an effective, frequency-dependent temperature $T_{\text{eff}}(\nu)$ by writing

$$e^{\nu/T_{\text{eff}}(\nu)} := \frac{S_{\phi\phi}[-\nu]}{S_{\phi\phi}[\nu]}. \quad (16)$$

This amounts to matching the ratio of the excitation and deexcitation rates (8) along the accelerated path to Einstein’s detailed balance formula. For circular motion, T_{eff} is approximately constant and of the same order of magnitude as the predicted linear-acceleration Unruh temperature over most of the accessible parameter space [40,56].

In the experimental proposal [28] for realizing the circular motion Unruh effect, the physical system was a BEC comprised of ^{133}Cs atoms with a two-dimensional number density $\rho_0 = 10^3 \text{ } \mu\text{m}^{-2}$, and a laser of frequency $\omega_0/(2\pi) = 10^{14} \text{ Hz}$ with a beam width $r_0 = 3 \text{ } \mu\text{m}$. Upon fixing the BEC parameters, the chosen squeezing λ defines the power and detuning relations of the laser. Working at the point of minimal backaction, the sensitivity within the

phononic theory can only be improved by increasing the statistics or the total measurement time (analysis bandwidth) [28]. The nondestructiveness of the measurement beyond the phononic theory has to be ensured *a posteriori*; first a sufficiently long lifetime of the BEC to resolve phononic frequencies, given typical experimental heating and atom loss. Second a sufficiently small incoherent scattering rate $\Gamma_{\text{sc}} = 4\hat{\alpha}_I \bar{P}/(\pi r_0^2)$ connected to the imaginary part of the atomic polarizability $\hat{\alpha}_I$, which is beyond the presented phononic theory but completely defined by it. Here, $\bar{P} \approx 2\omega_0 \alpha^2$ is the laser power entering the BEC, averaged over modulation cycles. Working at the SQL with $\alpha = \alpha_{\text{SQL}}$, which corresponds to $\mu^2 = 2\sqrt{2/3}$, we find a photon scattering rate $\Gamma_{\text{sc}} \approx 0.0020 \text{ Hz}$, which is much smaller than unity. In the case of an initially squeezed state, we show in Supplemental Material [37] that $\Gamma_{\text{sc}} \approx 0.0031 \text{ Hz}$ when μ takes the optimal value (13) and λ is set to the minimum of the red curve in Fig. 2. As both scattering rates are considerably smaller than unity (non-destructive) and the typical frequencies of the phonons of interest, we conclude that the detection scheme presented in this Letter represents a promising and feasible approach for an experimental realization of the circular motion Unruh effect: the probe beam acts as an Unruh-DeWitt detector *beyond the standard quantum limit*.

Discussion—Traditionally, optomechanics deals with single- or few-mode mechanical systems, such as membranes or mirrors, and this approach has been extended to BECs and superfluids coupled to optical cavities [57–59]. In these systems, the optomechanical interaction typically involves a single collective excitation, such as the center-of-mass or breathing mode, which acts as the effective mechanical degree of freedom. By contrast, our approach expands the optomechanical toolbox to a quantum fluid probed with laser beams along arbitrary stationary spatial paths, enabling access to unequal-time density correlations in a multi-mode BEC environment.

By employing a two-tone heterodyne optical readout and systematically analyzing both imprecision and backaction noise, we provide a theoretical framework for extracting unequal-time correlation functions with sensitivity reaching, and potentially surpassing, the shot-noise limit of the probe field. This opens up investigations of fundamental properties of the smallest excitations of a quantum fluid, including the quantum vacuum state of the system and its observer dependence. Our detection scheme finds its optimal sensitivity by balancing the quantum imprints left by the probe beam against the disturbance it causes, allowing the wave dynamics to be recorded with maximal fidelity.

While our concrete implementation focuses on a BEC, with the experimentally realistic parameters provided in the text below (15) and in the last paragraph of the accelerated interaction paths section, the scheme is adaptable to other quantum fluids, placing nondestructive access to unequal-time correlations in quantum fluids in general reach.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] M. Tajik, I. Kukuljan, S. Sotiriadis, B. Rauer, T. Schweigler, F. Cataldini, J. Sabino, F. Møller, P. Schüttelkopf, S. Ji, D. Sels, E. Demler, and J. Schmiedmayer, Verification of the area law of mutual information in a quantum field simulator, *Nat. Phys.* **19**, 1022 (2023).
- [2] L. H. Dogra, G. Martirosyan, T. A. Hilker, J. A. P. Glidden, J. Etrych, A. Cao, C. Eigen, R. P. Smith, and Z. Hadzibabic, Universal equation of state for wave turbulence in a quantum gas, *Nature (London)* **620**, 521 (2023).
- [3] A. C. White, C. F. Barenghi, N. P. Proukakis, A. J. Youd, and D. H. Wacks, Nonclassical velocity statistics in a turbulent atomic Bose-Einstein condensate, *Phys. Rev. Lett.* **104**, 075301 (2010).
- [4] E. A. L. Henn, J. A. Seman, G. Roati, K. M. F. Magalhães, and V. S. Bagnato, Emergence of turbulence in an oscillating Bose-Einstein condensate, *Phys. Rev. Lett.* **103**, 045301 (2009).
- [5] N. G. Parker and C. S. Adams, Emergence and decay of turbulence in stirred atomic Bose-Einstein condensates, *Phys. Rev. Lett.* **95**, 145301 (2005).
- [6] B. Opanchuk, R. Polkinghorne, O. Fialko, J. Brand, and P. D. Drummond, Quantum simulations of the early universe, *Ann. Phys. (Berlin)* **525**, 866 (2013).
- [7] O. Fialko, B. Opanchuk, A. I. Sidorov, P. D. Drummond, and J. Brand, Fate of the false vacuum: Towards realization with ultra-cold atoms, *Europhys. Lett.* **110**, 56001 (2015).
- [8] O. Fialko, B. Opanchuk, A. I. Sidorov, P. D. Drummond, and J. Brand, The universe on a table top: Engineering quantum decay of a relativistic scalar field from a metastable vacuum, *J. Phys. B* **50**, 024003 (2017).
- [9] J. Braden, M. C. Johnson, H. V. Peiris, and S. Weinfurter, Towards the cold atom analog false vacuum, *J. High Energy Phys.* **07** (2018) 014.
- [10] T. P. Billam, R. Gregory, F. Michel, and I. G. Moss, Simulating seeded vacuum decay in a cold atom system, *Phys. Rev. D* **100**, 065016 (2019).
- [11] J. Braden, M. C. Johnson, H. V. Peiris, A. Pontzen, and S. Weinfurter, Nonlinear dynamics of the cold atom analog false vacuum, *J. High Energy Phys.* **10** (2019) 174.
- [12] K. L. Ng, B. Opanchuk, M. Thenabadu, M. Reid, and P. D. Drummond, Fate of the false vacuum: Finite temperature, entropy, and topological phase in quantum simulations of the early universe, *PRX Quantum* **2**, 010350 (2021).
- [13] A. C. Jenkins, J. Braden, H. V. Peiris, A. Pontzen, M. C. Johnson, and S. Weinfurter, Analog vacuum decay from vacuum initial conditions, *Phys. Rev. D* **109**, 023506 (2024).
- [14] A. Zenesini, A. Berti, R. Cominotti, C. Rogora, I. G. Moss, T. P. Billam, I. Carusotto, G. Lamporesi, A. Recati, and G. Ferrari, False vacuum decay via bubble formation in ferromagnetic superfluids, *Nat. Phys.* **20**, 558 (2024).
- [15] R. Cominotti, C. Baroni, C. Rogora, D. Andreoni, G. Guarda, G. Lamporesi, G. Ferrari, and A. Zenesini, Observation of temperature effects in false vacuum decay, *Phys. Rev. Lett.* **135**, 183401 (2025).
- [16] U. R. Fischer and R. Schützhold, Quantum simulation of cosmic inflation in two-component Bose-Einstein condensates, *Phys. Rev. A* **70**, 063615 (2004).
- [17] J.-C. Jaskula, G. B. Partridge, M. Bonneau, R. Lopes, J. Ruauadel, D. Boiron, and C. I. Westbrook, An acoustic analog to the dynamical Casimir effect in a Bose-Einstein condensate, *Phys. Rev. Lett.* **109**, 220401 (2012).
- [18] C. Viermann, M. Sparn, N. Liebster, M. Hans, E. Kath, Á. Parra-López, M. Tolosa-Simeón, N. Sánchez-Kuntz, T. Haas, H. Strobel, S. Floerchinger, and M. K. Oberthaler, Quantum field simulator for dynamics in curved spacetime, *Nature (London)* **611**, 260 (2022).
- [19] M. Tajik, M. Gluza, N. Sebe, P. Schüttelkopf, F. Cataldini, J. a. Sabino, F. Møller, S. Ji, S. Erne, G. Guarnieri, S. Sotiriadis, J. Eisert, and J. Schmiedmayer, Experimental observation of curved light-cones in a quantum field simulator, *Proc. Natl. Acad. Sci. U.S.A.* **120**, e2301287120 (2023).
- [20] L. J. Garay, J. R. Anglin, J. I. Cirac, and P. Zoller, Sonic analog of gravitational black holes in Bose-Einstein condensates, *Phys. Rev. Lett.* **85**, 4643 (2000).
- [21] O. Lahav, A. Itah, A. Blumkin, C. Gordon, S. Rinott, A. Zayats, and J. Steinhauer, Realization of a sonic black hole analog in a Bose-Einstein condensate, *Phys. Rev. Lett.* **105**, 240401 (2010).

- [22] J. Steinhauer, Observation of quantum Hawking radiation and its entanglement in an analogue black hole, *Nat. Phys.* **12**, 959 (2016).
- [23] N. Navon, R. P. Smith, and Z. Hadzibabic, Quantum gases in optical boxes, *Nat. Phys.* **17**, 1334 (2021).
- [24] A. L. Gaunt, T. F. Schmidutz, I. Gotlibovych, R. P. Smith, and Z. Hadzibabic, Bose-Einstein condensation of atoms in a uniform potential, *Phys. Rev. Lett.* **110**, 200406 (2013).
- [25] A. C. Jenkins, I. G. Moss, T. P. Billam, Z. Hadzibabic, H. V. Peiris, and A. Pontzen, Generalized cold-atom simulators for vacuum decay, *Phys. Rev. A* **110**, L031301 (2024).
- [26] T. Schweigler, V. Kasper, S. Erne, I. Mazets, B. Rauer, F. Cataldini, T. Langen, T. Gasenzer, J. Berges, and J. Schmiedmayer, On solving quantum many-body problems by experiment, *Nature (London)* **545**, 323 (2017).
- [27] J. E. Lye, J. J. Hope, and J. D. Close, Nondestructive dynamic detectors for Bose-Einstein condensates, *Phys. Rev. A* **67**, 043609 (2003).
- [28] C. Gooding, S. Biermann, S. Erne, J. Louko, W. G. Unruh, J. Schmiedmayer, and S. Weinfurter, Interferometric Unruh detectors for Bose-Einstein condensates, *Phys. Rev. Lett.* **125**, 213603 (2020).
- [29] P. O. Fedichev and U. R. Fischer, Observer dependence for the phonon content of the sound field living on the effective curved space-time background of a Bose-Einstein condensate, *Phys. Rev. D* **69**, 064021 (2004).
- [30] C. M. Caves, Quantum-mechanical radiation-pressure fluctuations in an interferometer, *Phys. Rev. Lett.* **45**, 75 (1980).
- [31] C. M. Caves, Quantum-mechanical noise in an interferometer, *Phys. Rev. D* **23**, 1693 (1981).
- [32] S. A. Fulling, Nonuniqueness of canonical field quantization in Riemannian space-time, *Phys. Rev. D* **7**, 2850 (1973).
- [33] P. C. W. Davies, Scalar production in Schwarzschild and Rindler metrics, *J. Phys. A* **8**, 609 (1975).
- [34] W. G. Unruh, Notes on black hole evaporation, *Phys. Rev. D* **14**, 870 (1976).
- [35] S. A. Fulling and G. E. A. Matsas, Unruh effect, *Scholarpedia* **9**, 31789 (2014).
- [36] W. G. Unruh, Black holes, acceleration temperature and low temperature analog experiments, *J. Low Temp. Phys.* **208**, 196 (2022).
- [37] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/4yfh-tm4f> for detail.
- [38] F. Y. Khalili, H. Miao, H. Yang, A. H. Safavi-Naeini, O. Painter, and Y. Chen, Quantum back-action in measurements of zero-point mechanical oscillations, *Phys. Rev. A* **86**, 033840 (2012).
- [39] B. S. DeWitt, Quantum gravity: The new synthesis, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979), pp. 680–745.
- [40] S. Biermann, S. Erne, C. Gooding, J. Louko, J. Schmiedmayer, W. G. Unruh, and S. Weinfurter, Unruh and analogue Unruh temperatures for circular motion in $3 + 1$ and $2 + 1$ dimensions, *Phys. Rev. D* **102**, 085006 (2020).
- [41] W. P. Bowen and G. J. Milburn, *Quantum Optomechanics* (CRC Press, Boca Raton, Florida, 2015).
- [42] R. J. Glauber, The quantum theory of optical coherence, *Phys. Rev.* **130**, 2529 (1963).
- [43] Z. Y. Ou, C. K. Hong, and L. Mandel, Coherence properties of squeezed light and the degree of squeezing, *J. Opt. Soc. Am. B* **4**, 1574 (1987).
- [44] H. P. Yuen and V. W. S. Chan, Noise in homodyne and heterodyne detection, *Opt. Lett.* **8**, 177 (1983).
- [45] J. Shapiro, Quantum noise and excess noise in optical homodyne and heterodyne receivers, *IEEE J. Quantum Electron.* **21**, 237 (1985).
- [46] M. Collett, R. Loudon, and C. Gardiner, Quantum theory of optical homodyne and heterodyne detection, *J. Mod. Opt.* **34**, 881 (1987).
- [47] Y.-q. Li, D. Guzun, and M. Xiao, Sub-shot-noise-limited optical heterodyne detection using an amplitude-squeezed local oscillator, *Phys. Rev. Lett.* **82**, 5225 (1999).
- [48] T. Eberle, V. Händchen, and R. Schnabel, Stable control of 10 db two-mode squeezed vacuum states of light, *Opt. Express* **21**, 11546 (2013).
- [49] O. Lueghamer, S. Nimmrichter, C. Conrad-Billroth, T. Juffmann, and M. Prüfer, Cavity-enhanced continuous-wave microscopy with potentially unstable cavity length, *Sci. Rep.* **15**, 27676 (2025).
- [50] J. R. Letaw, Stationary world lines and the vacuum excitation of noninertial detectors, *Phys. Rev. D* **23**, 1709 (1981).
- [51] C. R. D. Bunney, Stationary trajectories in Minkowski spacetimes, *J. Math. Phys. (N.Y.)* **65**, 052501 (2024).
- [52] J. R. Letaw and J. D. Pfautsch, The quantized scalar field in the stationary coordinate systems of flat space-time, *Phys. Rev. D* **24**, 1491 (1981).
- [53] M. Good, B. A. Juárez-Aubry, D. Moustos, and M. Temirkhan, Unruh-like effects: Effective temperatures along stationary worldlines, *J. High Energy Phys.* **06** (2020) 059.
- [54] C. R. D. Bunney and J. Louko, Circular motion analogue Unruh effect in a thermal bath: Robbing from the rich and giving to the poor, *Classical Quantum Gravity* **40**, 155001 (2023).
- [55] C. R. D. Bunney, V. S. Barroso, S. Biermann, A. Geelmuyden, C. Gooding, G. Ithier, X. Rojas, J. Louko, and S. Weinfurter, Third sound detectors in accelerated motion, *New J. Phys.* **26**, 065001 (2024).
- [56] W. G. Unruh, Acceleration radiation for orbiting electrons, *Phys. Rep.* **307**, 163 (1998).
- [57] N. Brahms, T. Botter, S. Schreppler, D. W. C. Brooks, and D. M. Stamper-Kurn, Optical detection of the quantization of collective atomic motion, *Phys. Rev. Lett.* **108**, 133601 (2012).
- [58] A. B. Shkarin, A. D. Kashkanova, C. D. Brown, S. Garcia, K. Ott, J. Reichel, and J. G. E. Harris, Quantum optomechanics in a liquid, *Phys. Rev. Lett.* **122**, 153601 (2019).
- [59] R. Gupta, P. Kumar, R. Kanamoto, M. Bhattacharya, and H. S. Dhar, Sensing atomic superfluid rotation beyond the standard quantum limit, *Phys. Rev. A* **110**, 053514 (2024).
- [60] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).

End Matter

Appendix A: Effective field theory—In this appendix, we use the effective field theory for the BEC field ϕ and the laser-BEC interaction to derive the interaction coupling ε appearing in the definition of the dimensionless effective coupling parameter $\mu = -\varepsilon\alpha/\sqrt{2}$.

We describe the $2d$ BEC in terms of the two-dimensional Gross-Pitaevskii field $\Phi(t, \mathbf{x})$, with an isotropic potential $V(r)$:

$$i\partial_t\Phi = -\frac{1}{2m}\nabla^2\Phi + V\Phi + g_{2d}\Phi|\Phi|^2. \quad (\text{A1})$$

Now consider small fluctuations Ψ of the field Φ about a background Φ_0 (i.e., $\Phi = \Phi_0 + \Psi$). Defining the background BEC density $\rho_0 = \Phi_0^2$ and assuming Φ_0 is static, real-valued, and extends to infinity in r , we can then take $V + g_{2d}\rho_0 = 0$ and express the linearized equations of motion for Ψ as

$$i\partial_t\Psi = -\frac{1}{2m}\nabla^2\Psi + g_{2d}\rho_0[\Psi + \Psi^\dagger], \quad (\text{A2})$$

and similarly for $\Psi^\dagger$. The corresponding Lagrangian is

$$L_\Psi = \int d^2x \left[i\Psi^\dagger \partial_t\Psi - \frac{1}{2m}|\nabla\Psi|^2 - \frac{g_{2d}\rho_0}{2}(\Psi + \Psi^\dagger)^2 \right]. \quad (\text{A3})$$

Writing the field Ψ in terms of real and imaginary parts ($\Psi = \Psi_R + i\Psi_I$), the dynamics can alternatively be expressed as

$$\partial_t\Psi_R - \frac{1}{2m}\nabla^2\Psi_I = 0 \quad (\text{A4})$$

and

$$\partial_t\Psi_I + \frac{1}{2m}\nabla^2\Psi_R - 2g_{2d}\rho_0\Psi_R = 0. \quad (\text{A5})$$

Combining these equations to eliminate Ψ_I and taking the long-wavelength limit then leads to a Klein-Gordon equation for Ψ_R , with propagation speed $c_s = \sqrt{g_{2d}\rho_0/m}$. From the corresponding Lagrangian

$$L_{\Psi_R} = \int d^2x \left[\frac{1}{c_s^2} \left(\frac{\partial_t\Psi_R}{\sqrt{m}} \right)^2 - \left(\frac{\nabla\Psi_R}{\sqrt{m}} \right)^2 \right], \quad (\text{A6})$$

we identify $\phi(t, \mathbf{x}) = \Psi_R(t, \mathbf{x})/\sqrt{m}$ as our effective $(2+1)$ -dimensional Klein-Gordon field.

The first-order laser-BEC interaction (4) is characterized by the interaction Lagrangian [28]

$$L_{\text{int}} = -\frac{A_0^2}{2}\hat{\alpha}_R\omega_0\delta\rho\partial_t\psi, \quad (\text{A7})$$

evaluated along the interaction trajectory, where $\delta\rho = 2\sqrt{\rho_0}\Psi_R$, $\hat{\alpha}_R$ is the real part of the polarizability, $\psi(t, z)$ is the effective laser phase, and $A_0 \sim \alpha$ is the laser amplitude (assumed to be unaltered by the interaction). The (complex) polarizability $\hat{\alpha} = \hat{\alpha}_R + i\hat{\alpha}_I$ is given explicitly by

$$\hat{\alpha}(\omega) = -\frac{24\pi^2}{\omega_r^3} \frac{1}{\delta_0} \left(1 - \frac{i}{\delta_0} \right), \quad (\text{A8})$$

where $\omega_r = \omega_0$ is the atomic resonance frequency and δ_0 is the detuning from resonance (in units of atomic half-linewidths). The laser phase obeys the effective Lagrangian

$$L_{\text{EM}} = \frac{A_0^2}{4} \int dz \left[\frac{1}{c_{\text{eff}}^2} (\partial_t\psi)^2 - (\partial_z\psi)^2 \right], \quad (\text{A9})$$

where c_{eff} is the effective speed of light. Subject to the interaction (A7), the input laser phase $\psi_0(t, z)$ becomes [28]

$$\psi = \psi_0 \pm \frac{\varepsilon}{2}\phi = \psi_0 \pm \frac{|\hat{\alpha}_R|\omega_0}{2}\delta\rho, \quad (\text{A10})$$

where the $\pm$ sign corresponds to the opposite detuning in each sideband. Hence, we can identify $\varepsilon = 2|\hat{\alpha}_R|\omega_0\sqrt{m\rho_0}$, from which the dimensionless effective coupling parameter μ is found to be

$$\mu = -\frac{\varepsilon\alpha}{\sqrt{2}} = -|\hat{\alpha}_R|\omega_0\sqrt{2m\rho_0}\alpha. \quad (\text{A11})$$

Appendix B: Backaction from the Bogoliubov transformation—In this appendix we give the Bogoliubov transformation through which the first-order interaction (4) leads to the postinteraction operators (6), including the crucial quadratic backaction term (7). We follow the procedure introduced in [36], adapting it to our notation.

In the notation of the main text, we introduce the new preinteraction mode operators

$$z_\nu = \frac{1}{\sqrt{2}}(\delta a_+[\nu] + \delta a_-[\nu]), \quad (\text{B1a})$$

$$Z_\nu = \frac{1}{\sqrt{2}}(\delta a_+[\nu] - \delta a_-[\nu]). \quad (\text{B1b})$$

We call z_ν the common-mode operator and Z_ν the difference-mode operator. The transformation (B1) can be inverted to give

$$\delta a_{\pm}[\nu] = \frac{1}{\sqrt{2}}(z_{\nu} \pm Z_{\nu}). \quad (\text{B2})$$

Next, as the operators D_{ν} in the BEC field mode decomposition (5) are annihilation operators for $\nu > 0$ and creation operators for $\nu < 0$, satisfying $D_{-\nu} = D_{\nu}^{\dagger}$, we from now on assume $\nu \in (0, \Delta)$ and write

$$Z_{\nu} = X_{\nu}, \quad Z_{-\nu} = Y_{\nu} \quad (\nu > 0). \quad (\text{B3})$$

The full set of independent preinteraction operators is then $(z_{\nu}, z_{-\nu}, X_{\nu}, Y_{\nu}, D_{\nu})$, with $\nu \in (0, \Delta)$.

For the corresponding postinteraction operators, (4) and (5) give

$$\tilde{z}_{\pm\nu} = z_{\pm\nu}, \quad \tilde{X}_{\nu} = X_{\nu} + i\mu D_{\nu}, \quad \tilde{Y}_{\nu} = Y_{\nu} + i\mu D_{\nu}^{\dagger}, \quad (\text{B4})$$

where $\mu = -\epsilon\alpha/\sqrt{2} < 0$ is the dimensionless laser-BEC coupling parameter. The signal from the BEC is hence carried entirely in the difference-mode operators X_{ν} and Y_{ν} , whereas the common-mode operators $z_{\pm\nu}$ remain unaffected by the interaction with the BEC.

We now complete the linear order transformation (B4) into a nonperturbative Bogoliubov transformation that includes quadratic terms in μ .

We introduce the triple $u(\nu) = (X_{\nu}, Y_{\nu}, D_{\nu})$, whose commutators are $[u_i(\nu), u_j^{\dagger}(\nu')] = \delta_{ij}2\pi\delta(\nu - \nu')$ and $[u_i(\nu), u_j(\nu')] = 0$, and a similar triple $\tilde{u}$ for the postinteraction operators, with similar commutators. We look for a nonperturbative Bogoliubov transformation

$$\tilde{u}_i(\nu) = \alpha_{ij}(\nu)u_j(\nu) + \beta_{ij}(\nu)u_j(\nu)^{\dagger}, \quad (\text{B5})$$

using the Einstein summation convention, such that the coefficients α_{ij} and β_{ij} depend on μ , and (B5) reproduces (B4) to linear order in μ .

We expand the Bogoliubov coefficients in (B5) in μ as $\alpha_{ij} = \delta_{ij} + \mu\alpha_{ij}^{(1)} + O(\mu^2)$ and $\beta_{ij} = \mu\beta_{ij}^{(1)} + O(\mu^2)$. Consistency with (B4) implies

$$\alpha^{(1)} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \beta^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{pmatrix}, \quad (\text{B6})$$

where $\alpha_{XD}^{(1)}$ and $\beta_{YD}^{(1)}$ are read off directly from (B4), and $\alpha_{DX}^{(1)}$ and $\beta_{DY}^{(1)}$ are then determined by the Bogoliubov identities that follow from the preservation of the commutation relations [60]. Writing $U = (u, u^{\dagger})$, and similarly for $\tilde{U}$, we then have

$$\tilde{U} = (I + \mu\gamma)U + O(\mu^2), \quad (\text{B7})$$

where U and $\tilde{U}$ are understood as column vectors, I is the 6×6 identity matrix, and

$$\gamma = \begin{pmatrix} \alpha^{(1)} & \beta^{(1)} \\ (\beta^{(1)})^* & (\alpha^{(1)})^* \end{pmatrix}. \quad (\text{B8})$$

We promote the linear order Bogoliubov transformation (B7) into the nonperturbative Bogoliubov transformation $\tilde{U} = \exp(\mu\gamma)U$. Geometrically, $\exp(\mu\gamma)$ is the one-parameter subgroup of Bogoliubov transformations that is determined uniquely by the infinitesimal generator γ , and it is thus the minimal nonperturbative completion of (B7). Further, as $\gamma^3 = 0$, the series for $\exp(\mu\gamma)$ terminates after the quadratic term: the geometric reason is that the Bogoliubov transformation matrices acting on U are in the matrix group $U(3, 3)$, and the subgroup $\exp(\mu\gamma)$ therein is of parabolic type, similar to null rotations in the Lorentz group.

Writing out the series for the matrix exponential, we hence have

$$\tilde{U} = \left(I + \mu\gamma + \frac{1}{2}\mu^2\gamma^2 \right)U, \quad (\text{B9})$$

as an exact relation. Writing (B9) out in terms of $(X_{\nu}, Y_{\nu}, D_{\nu})$, we then find

$$\tilde{X}_{\nu} = X_{\nu} \left(1 - \frac{\mu^2}{2} \right) + i\mu D_{\nu} - \frac{\mu^2}{2} Y_{\nu}^{\dagger}, \quad (\text{B10a})$$

$$\tilde{Y}_{\nu} = Y_{\nu} \left(1 + \frac{\mu^2}{2} \right) + i\mu D_{\nu}^{\dagger} + \frac{\mu^2}{2} X_{\nu}^{\dagger}, \quad (\text{B10b})$$

$$\tilde{D}_{\nu} = D_{\nu} + i\mu(X_{\nu} + Y_{\nu}^{\dagger}). \quad (\text{B10c})$$

The quadratic terms in (B10) show that the interaction amplifies the laser modes Y_{ν} , deamplifies the laser modes X_{ν} , and does neither to the BEC modes D_{ν} , despite the electromagnetic noise being injected into the BEC by the interaction.

Finally, the postinteraction operators in (6) and (7) in the main text are obtained from (B10a) and (B10b) via (B2) and (B3) and their tilded counterparts.

Appendix C: Backaction suppressed by squeezed laser fluctuations—In this appendix we identify the mechanism by which squeezing the initial state of the laser beam fluctuations suppresses the backaction on the BEC. This mechanism leads to the suppressed added noise formula (12) in the main text, as we show in Supplemental Material [37]. We follow the procedure introduced in [36], adapting it to our notation.

In the notation of Appendix B, we start with the pair of operators (X_{ν}, Y_{ν}) , where $0 < \nu < \Delta$, and define the new pair $(\hat{X}_{\nu}, \hat{Y}_{\nu})$ by the two-mode squeezing transformation

$$\hat{X}_{\nu} := \cosh(\lambda)X_{\nu} + \sinh(\lambda)Y_{\nu}^{\dagger}, \quad (\text{C1a})$$

$$\hat{Y}_\nu := \cosh(\lambda)Y_\nu + \sinh(\lambda)\hat{X}_\nu^\dagger, \quad (\text{C1b})$$

where $\lambda \in \mathbb{R}$ is the squeezing parameter. The nonvanishing commutators of the new operators are $[\hat{X}_\nu, \hat{X}_{\nu'}^\dagger] = [\hat{Y}_\nu, \hat{Y}_{\nu'}^\dagger] = 2\pi\delta(\nu - \nu')$. The inverse transformation is

$$X_\nu = \cosh(\lambda)\hat{X}_\nu - \sinh(\lambda)\hat{Y}_\nu^\dagger, \quad (\text{C2a})$$

$$Y_\nu = \cosh(\lambda)\hat{Y}_\nu - \sinh(\lambda)\hat{X}_\nu^\dagger. \quad (\text{C2b})$$

The crucial observation is now that the postinteraction BEC annihilation operator $\tilde{D}_\nu$ (B10c) can be written in terms of the new operators $\hat{X}_\nu$ and $\hat{Y}_\nu$ as

$$\tilde{D}_\nu = D_\nu + i\mu e^{-\lambda}(\hat{X}_\nu + \hat{Y}_\nu^\dagger). \quad (\text{C3})$$

This shows that if the laser beam fluctuations are initially prepared in the two-mode squeezed state $|\lambda\rangle$ that is annihilated by $\hat{X}_\nu$ and $\hat{Y}_\nu$, the phonon number expectation

value after the laser-BEC interaction is given by

$$\langle \tilde{D}_\nu^\dagger \tilde{D}_{\nu'} \rangle_\lambda = \langle D_\nu^\dagger D_{\nu'} \rangle + \mu^2 e^{-2\lambda} 2\pi\delta[\nu - \nu'], \quad (\text{C4})$$

where the subscript λ on the left-hand side denotes that the initial state of the laser fluctuations was $|\lambda\rangle$. The μ^2 term in (C4) is the backaction noise, and the factor $e^{-2\lambda}$ in this term shows that the noise is exponentially suppressed for large positive λ .

By Heisenberg's uncertainty principle, the exponential squeezing of $X_\nu + Y_\nu^\dagger = e^{-\lambda}(\hat{X}_\nu + \hat{Y}_\nu^\dagger)$ is concomitant with the exponential enhancement of the conjugate operator $-i(X_\nu - Y_\nu^\dagger) = -ie^\lambda(\hat{X}_\nu - \hat{Y}_\nu^\dagger)$. Preparing the laser fluctuations in the state $|\lambda\rangle$ hence suppresses the BEC noise but enhances the conjugate electromagnetic noise. We shall show in Supplemental Material [37] that the balance of this suppression and enhancement leads to the total added noise formula (12) in the main text, allowing the total added noise to be below the standard quantum limit.