

# Snap-Through Time of Arches Is Controlled by Slenderness and Imperfections

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Snap through occurs in elastic structures when a stable equilibrium configuration becomes unstable, resulting in rapid motion towards a new and distinct stable state. While static analyses of snap through are well documented, the dynamics of snap through remain underexplored, particularly in structures with natural curvature. Using a combination of finite element simulations and multiple-scales analysis, we show that the snap-through dynamics of an arch under a central point load are controlled by its slenderness and imperfections embedded in the system. As the slenderness increases, the snap-through dynamics slow down, and the mode of snap through changes from limit-point buckling to bifurcation buckling. When bifurcation buckling occurs, snap through is preceded by an extended period of oscillatory behavior. The duration of these pre-snap-through oscillations, and hence the snap-through time, is entirely controlled by imperfections in the system. Increasing the strength of imperfections dramatically reduces the snap-through time. Analytical expressions for the snap-through times are presented for limit-point and bifurcation buckling. Our Letter suggests that natural curvature and deliberately introduced imperfections can be used to tune the snap-through dynamics of new functional materials.

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Many biological and technological structures achieve functionality by moving between two distinct equilibrium configurations via snap-through instabilities as a load or stimulus is varied [1]. These snap-through instabilities occur when the original equilibrium configuration becomes unstable, at which point stored elastic energy is converted into kinetic energy, causing rapid deformation of the structure towards a new configuration. Snap-through instabilities allow plants and animals to achieve fast motions that would be impossible using muscular action [2,3] and have been harnessed to induce ultrafast motion in polymer gels [4], improve the response time of soft robots [5,6], enhance the controllability of smart materials [7], and carry out logic operations [8,9].

New metamaterials for energy harvesting [10,11], deployable structures [12], and mechanical computation [13] have been created by chaining together snap-through events and making use of elastic hysteresis [14], multistability [15],

and tailored imperfections [16]. The response time of sequentially buckling metamaterials is set by the speed of transition waves that propagate through the structure, which is linked to the snap-through time of individual elements. The snap-through time, defined as the time needed to snap between two equilibrium configurations, also limits the actuation rate of materials driven by electrical [17], magnetic [18], and optical [19] stimuli. Thus, a comprehensive understanding of the factors affecting the snap-through time is needed to fully exploit the enhanced functionalities of multistable structures.

Historically, the study of snap-through instabilities has focused on computing and characterising equilibria, e.g., through the energy landscape [1,20,21], with the aim of tuning the onset of instability. Analyses based on nonlinear dynamics have revealed that two modes of snap through can occur, termed “limit-point” and “bifurcation” buckling, originating from limit-point and pitchfork bifurcations in the system, respectively [20]. Recent interests have shifted toward understanding the dynamic transitions between stable equilibria [22–24].

Gomez *et al.* [22] showed that the delayed dynamics of snap through of a compressed beam with asymmetric boundary rotation is an elastic, rather than viscoelastic [2,25], bifurcation-induced phenomenon. By examining the same system but varying the symmetries in the boundary conditions, Radisson and Kanso [23,26] showed that a compressed beam can undergo both limit-point and

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bifurcation buckling, the latter of which is distinguished by an initial period where the beam oscillates before exponentially snapping through to its new configuration. Radisson and Kanso proposed a scaling law for the snap-through time associated with bifurcation buckling; however, their calculations of specific snap-through times required an *ad hoc* initial velocity, obtained from fitting to numerical simulations, suggesting the snap-through dynamics have yet to be fully elucidated.

In this Letter, we ask the question: how does the natural curvature of an arch impact its snap-through dynamics? The answer is surprisingly subtle and requires a complete understanding of the dynamics of bifurcation buckling, which we develop through a systematic multiple-scales analysis. The analysis reveals the nontrivial, yet critical, role that intrinsic and imposed imperfections play in bifurcation buckling.

To this end, we consider a slender and shallow arch with natural curvature  $1/R^*$ , cross-sectional area  $A^*$ , second moment of area  $I_x^*$ , opening angle  $2\Theta$ , density  $\rho^*$ , Young's modulus  $E^*$ , and sound speed  $c^* = \sqrt{E^*/\rho^*}$ , whose schematic is provided in Supplemental Material (SM) [27]. Stars denote dimensional quantities. The arch is acted on by a central load, which can trigger snap through [39].

The relevant dimensionless parameters are the slenderness  $\lambda = R^*\Theta^2 \sqrt{A^*/I_x^*}$ , which provides a measure of the arch curvature, and the load intensity  $P$ . By introducing the elastodynamic time scale  $t_e^* = R^*/c^*$  and radial and circumferential displacement scales  $R^*\Theta^2$  and  $R^*\Theta^3$ , respectively, the nondimensional equations for the arch are [27,40]

$$\frac{\partial^2 v}{\partial t^2} + \frac{1}{\lambda^2} \frac{\partial^4 v}{\partial \theta^4} - \frac{\partial}{\partial \theta} \left( \varepsilon_m \frac{\partial v}{\partial \theta} \right) - \varepsilon_m = \frac{P}{\lambda^2} \delta(\theta), \quad (1a)$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial \varepsilon_m}{\partial \theta}, \quad (1b)$$

where  $\theta = \theta^*/\Theta$  is the rescaled circumferential angle,  $t$  is time,  $\delta(\cdot)$  is the Dirac  $\delta$  function and  $\mathbf{u} = (v, w)$  is the displacement field with radial ( $v$ ) and circumferential ( $w$ ) components. The membrane strain is  $\varepsilon_m = \partial_\theta w - v + (1/2)(\partial_\theta v)^2$ . Hinged boundary conditions are applied.

By increasing the slenderness  $\lambda$ , the arch system under point indentation encompasses three regimes [27,41]: (i) for  $0 < \lambda < \lambda_c \simeq 3.9$ , the arch is monostable; (ii) for  $\lambda_c < \lambda < \lambda_{\text{hill}}^+ \simeq 10$ , the arch is bistable and undergoes limit-point buckling [Fig. 1(a)]; (iii) for  $\lambda_{\text{hill}}^+ < \lambda$ , the arch is bistable and undergoes bifurcation buckling [Fig. 1(b)]. When  $\lambda = \lambda_{\text{hill}}^+$ , the limit-point and pitchfork bifurcations occur at the same critical load, resulting in a hilltop bifurcation. A linear stability analysis shows [27] that the system has intrinsic symmetries that impact the shape of the arch during snap through. In the case of limit-point buckling, the

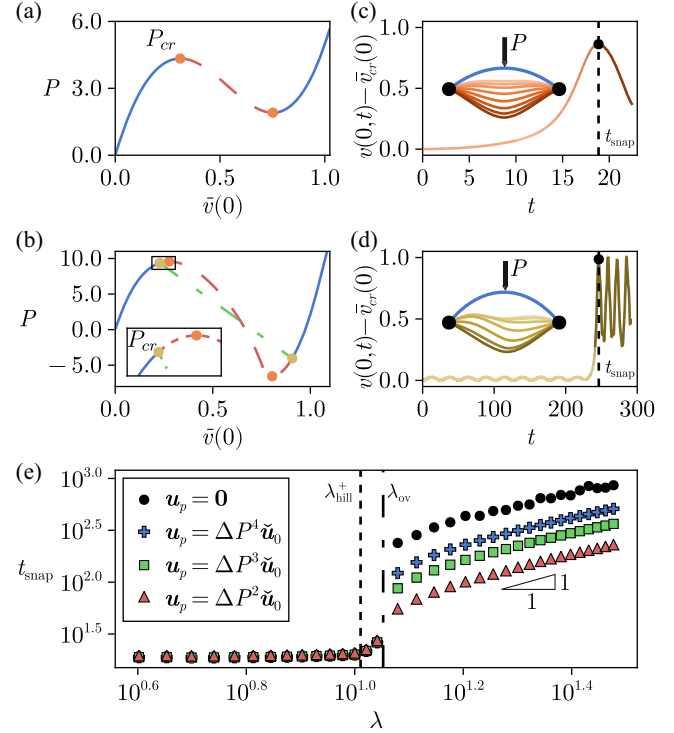


FIG. 1. Equilibrium load  $P$  vs midpoint displacement  $\bar{v}(0)$  curves for (a) limit-point ( $\lambda = 6$ ) and (b) bifurcation ( $\lambda = 12$ ) buckling. Solid, dashed, and dash-dot lines represent stable symmetric, unstable symmetric, and unstable asymmetric solutions, respectively. Arch midpoint displacements during (c) limit-point and (d) bifurcation buckling. The insets show the arch shape with color gradients denoting the passage of time; the initial shape is blue. (e) Snap-through time as a function of the slenderness  $\lambda$  for ideal initial conditions (circles) and imperfect initial conditions (crosses, squares and triangles) with  $\Delta P = 10^{-2}$ . Transitions between buckling dynamics occur at  $\lambda_{\text{hill}}^+$  and  $\lambda_{\text{ov}}$ .

critical buckling eigenmode  $\tilde{\mathbf{u}}_0$  with zero eigenvalue is symmetric about  $\theta = 0$  and satisfies  $(\tilde{\mathbf{v}}_0^S(-\theta), \tilde{\mathbf{w}}_0^S(-\theta)) = (\tilde{\mathbf{v}}_0^S(\theta), -\tilde{\mathbf{w}}_0^S(\theta))$ . For bifurcation buckling, the critical mode is antisymmetric and satisfies  $(\tilde{\mathbf{v}}_0^A(-\theta), \tilde{\mathbf{w}}_0^A(-\theta)) = (-\tilde{\mathbf{v}}_0^A(\theta), \tilde{\mathbf{w}}_0^A(\theta))$ . In both cases, the noncritical eigenmodes can be separated into symmetric modes  $\{\tilde{\mathbf{u}}_n^S\}$  and anti-symmetric modes  $\{\tilde{\mathbf{u}}_n^A\}$  with  $n \geq 1$ . The growth rates of the noncritical modes are imaginary, implying their linear response to the point load is oscillatory in time.

To study the nonlinear dynamics of snap through, we carry out numerical simulations [27] of Eqs. (1). We first quasistatically reach the bifurcation point by incrementing the applied load  $P$ . The displacement and load at the buckling threshold are denoted by  $\bar{\mathbf{u}}_{cr} = (\bar{v}_{cr}, \bar{w}_{cr})$  and  $P = P_{cr}(\lambda)$ , where  $\bar{\mathbf{u}}_{cr}$  is symmetric. We then perturb the system by increasing the load by a small amount  $\Delta P = (P - P_{cr})/P_{cr} \ll 1$ , forcing the system to dynamically leave the critical configuration, and solve Eqs. (1) with the initial conditions

$$\mathbf{u}(\theta, 0) = \bar{\mathbf{u}}_{cr}, \quad \frac{\partial \mathbf{u}}{\partial t}(\theta, 0) = \mathbf{0}. \quad (2)$$

During limit-point buckling, the midpoint of the arch monotonically increases until the everted configuration is reached and the arch remains symmetric [Fig. 1(c)]. However, during bifurcation buckling, there is an extended period where the arch midpoint oscillates around the symmetric, now unstable, equilibrium before snapping through [Fig. 1(d)] [27]. These small-scale oscillations are analogous to the pre-snap-through oscillations observed by Radisson and Kanso [23]. Numerical experiments show that the oscillation frequency is independent of  $\Delta P$  but strongly dependent on  $\lambda$  [27]. As  $\lambda \rightarrow \lambda_{\text{hill}}^+$ , the frequency tends to zero, showcasing a new type of critical slowing down that occurs when the system approaches the hilltop bifurcation. The arch remains symmetric during the oscillatory phase, becomes asymmetric during snap through, and returns to being symmetric as the new equilibrium configuration is approached [inset Fig. 1(d)]. By computing the snap-through time  $t_{\text{snap}}$  across a range of slenderness values  $\lambda$ , we find that the rate of snap through is vastly different for the two buckling modes. As shown in Fig. 1(e),  $t_{\text{snap}}$  is approximately independent of  $\lambda$  in the regime of limit-point buckling, and it becomes linearly dependent on  $\lambda$  in the regime of bifurcation buckling.

Unexpectedly, the snap-through times for bifurcation buckling have a weak noise-like variability that cannot be removed by refining the mesh, time step, or numerical method; see the black circles in Fig. 1(e) for  $\lambda > \lambda_{\text{hill}}^+$ . However, running simulations with a small imperfection introduced through the initial conditions,

$$\mathbf{u}(\theta, 0) = \bar{\mathbf{u}}_{cr}(\theta) + \Delta P \mathbf{u}_p(\theta), \quad \frac{\partial \mathbf{u}}{\partial t}(\theta, 0) = \mathbf{0}, \quad (3)$$

where  $\mathbf{u}_p$  is the arbitrary shape of the imperfection, eliminates the variability in the snap-through times; see the crosses, squares, and triangles in Fig. 1(e). Moreover, increasing the magnitude of the imperfection decreases the snap-through time. The snap-through times associated with the limit-point buckling are, instead, unaffected by the imperfection. The simulations are carried out by setting the imperfection to  $\mathbf{u}_p = \Delta P^\alpha \check{\mathbf{u}}_0$ , where  $\check{\mathbf{u}}_0$  is the critical buckling mode [27] and  $\alpha = 1, 2, 3$ .

The dependence of the snap-through time on  $\lambda$  can be estimated from a scaling analysis. Since snap through brings a curved arch close to its everted configuration, it is natural to scale the radial displacement as  $v^* \sim R^* \Theta^2$ . Following geometrical arguments [42,43], the membrane strain scales as  $\epsilon_m^* \sim v^* / \sqrt{R^* v^*}$ , so that the stretching energy density is  $\mathcal{E}_s^* \sim E^* (\epsilon_m^*)^2 \sim E^* \Theta^2$ . We expect the membrane strain to be independent of the system's slenderness, since stretching strain is related to changes in the length of the system's center line [44]. The strain associated

with the change in curvature is  $\epsilon_b^* \sim h^* / R^*$ , where  $h^* \sim (I_x^* / A^*)^{1/2}$  is the arch thickness. The bending energy density can be estimated as  $\mathcal{E}_b^* \sim E^* (\epsilon_b^*)^2 \sim E^* \Theta^4 / \lambda^2$ . The kinetic energy associated with snap through is given by  $\mathcal{E}_K^* \sim \rho^* (v^* / t_{\text{snap}}^*)^2$ . For  $\lambda$  large, we balance kinetic and bending energies to obtain  $t_{\text{snap}}^* \sim \lambda t_e^*$ . For smaller values of  $\lambda$ , we balance kinetic and stretching energy to find  $t_{\text{snap}}^* \sim t_e^*$ . While these arguments explain the empirical scaling laws found from numerical simulations [Fig. 1(e)], they do not capture the critical role that imperfections play in bifurcation buckling nor the slowing down that occurs as  $\Delta P$  tends to zero in both snap-through regimes [22].

To resolve the dynamics of bifurcation buckling, in which the arch oscillates on a fast  $O(1)$  time scale and snaps through on a slow, yet to be determined,  $O(\tau)$  time scale, we carry out a multiple-scales analysis [45] in the limit  $\Delta P \rightarrow 0$ . We consider general imperfections in the initial condition (3) that are written in terms of symmetric and antisymmetric eigenmodes,

$$\mathbf{u}_p(\theta) = p_0^A \check{\mathbf{u}}_0^A(\theta) + \sum_{n=1}^{\infty} [p_n^A \check{\mathbf{u}}_n^A(\theta) + p_n^S \check{\mathbf{u}}_n^S(\theta)], \quad (4)$$

where the  $p_n^A$  and  $p_n^S$  are Fourier-like coefficients characterizing the amplitude of the imperfection along the eigenmodes. The applied load is written as  $P = P_{cr}(1 + \Delta P)$ . In contrast to past works [23], which only resolve the dynamics of bifurcation buckling on the slow  $O(\tau)$  time-scale, our multiple-scales analysis simultaneously captures the  $O(1)$  and  $O(\tau)$  time scales, allowing the impact of fast oscillations on slow snap through to be determined. Moreover, our approach allows the slow time scale  $\tau$  to be rigorously derived without relying on *ad hoc* arguments or numerical considerations.

We find that oscillations play an essential role in triggering bifurcation buckling, which occurs across three distinct time regimes given by  $t = O(1)$ ,  $t = O(\Delta P^{-1/2})$ , and  $t = O(\Delta P^{-1/2} \ln(1/\Delta P))$ . A detailed analysis of these regimes can be found in SM [27]. When  $t = O(1)$ , nonlinear interactions between oscillatory symmetric modes and the antisymmetric critical mode lead to resonance, causing a monotonic growth of the critical mode. The amplitude of the critical mode remains smaller than the amplitudes of the symmetric modes. The amplitude of each symmetric mode is inversely proportional to the square of its natural frequency (in time). Thus, the behavior of the arch is dominated by the lowest-frequency symmetric mode, leading to the emergence of pre-snap-through oscillations. When  $t = O(\Delta P^{-1/2})$ , the sustained resonant interactions between oscillatory symmetric modes and the critical mode initiate snap through. However, the amplitude of the critical mode remains small and its motion is governed by a linear differential equation, resulting in exponential growth. Finally, when  $t = O(\Delta P^{-1/2} \ln(1/\Delta P))$ , the exponential

growth of the critical mode initiates nonlinear effects. Due to the growth of the critical mode, its amplitude becomes much greater than all other modes. The arch displacement is then approximately given by the superposition of the symmetric equilibrium displacement  $\bar{u}_{cr}$  and the antisymmetric critical mode  $\tilde{u}_0$ , resulting in an asymmetric shape during snap through.

In the final time regime of bifurcation buckling, the equation of motion for the amplitude of the critical mode is the normal form of a pitchfork bifurcation. The solutions of this nonlinear differential equation exhibit finite-time blow up. We associate the blowup time with the snap-through time of the arch [22]. By solving the differential equation and matching to the exponentially small solution from the  $O(\Delta P^{-1/2})$  time regime, we obtain a novel analytical expression for the snap-through time for bifurcation buckling,

$$t_{\text{snap}}^B = \frac{\lambda}{\sqrt{r_B \Delta P}} \ln \left( \frac{4}{p_0^A} \sqrt{\frac{2r_B}{q_B \Delta P}} \right), \quad (5)$$

where  $r_B$  and  $q_B$  are functions of  $\lambda$  that can be computed analytically [27]. As  $\lambda \rightarrow \infty$ , both  $r_B$  and  $q_B$  tend to constants, so  $t_{\text{snap}}^B = O(\lambda)$ , consistent with the scaling analysis and simulations. The appearance of the coefficient  $p_0^A$  in (5) quantifies how imperfections impact the snap-through time. As  $p_0^A$  decreases, the snap-through time logarithmically increases. Importantly, the snap-through time is only influenced by imperfections that have a component along the critical buckling mode, the magnitude of which is encoded in  $p_0^A$ .

The central point load leads to a symmetric actuation that only excites the symmetric eigenmodes of the system. The symmetric pre-snap-through oscillations are thus the leading-order response of the system on  $O(1)$  timescales. The critical mode, which is antisymmetric in bifurcation buckling, can only be excited through nonlinear interactions between itself and the symmetric modes. Therefore, if there are no imperfections, or if the imperfection does not have a component along the critical mode, e.g., the imperfection is symmetric, then the critical mode cannot be excited and it will never grow. In this case, the oscillations persist indefinitely and snap-through does not occur, despite the threshold of bifurcation buckling being exceeded. Indeed, taking  $p_0^A \rightarrow 0$  in (5) shows that the snap-through time blows up,  $t_{\text{snap}}^B \rightarrow \infty$ . When the oscillations persist, it can be shown that the oscillation period undergoes a slow modulation on an  $O(\Delta P^{-1})$  time scale. This is much greater than the time scale of snap through, which is why we find that the period of the oscillations does not depend on  $\Delta P$  in the numerical simulations.

Imperfection is what sets the finite snap-through time in the case of bifurcation buckling: increasing the size of the imperfection accelerates snap through, consistent with the

simulation results in Fig. 1(e). The *noisy* behavior that we highlighted earlier can thus be explained by the numerical discretization of the problem introducing small imperfections, but whose specific value cannot be controlled systematically for all values of the slenderness  $\lambda$ .

The interplay between imperfections and oscillations in bifurcation buckling has been discussed in the context of compressed beams with time-dependent loading, which exhibit precursor oscillations before the bifurcation threshold is crossed [24,46]. The loading rate was shown to control the nature of snap through and determine how imperfections are amplified through precursor oscillations to produce transient asymmetries. Our system is different in that the loading is static, and static loading causes the arch to oscillate about a globally unstable equilibrium, rather than dynamic loading causing oscillations about a stable equilibrium. In our case, imperfections are a necessary precondition for, and completely dictate the speed of, bifurcation buckling.

Imperfections do not only fix the snap-through time, but also affect the slowing down induced by the pitchfork bifurcation: the logarithmic contribution to the snap-through time in (5) represents an additional slowing down that has not been predicted in previous analyses of bifurcation buckling [23]. Plotting the rescaled snap-through time  $\Delta P^{1/2} \lambda^{-1} t_{\text{snap}}^B$  computed from numerical simulations as a function of  $\Delta P$  shows that the data do not lie on a horizontal line [Fig. 2(a)], highlighting the need to account

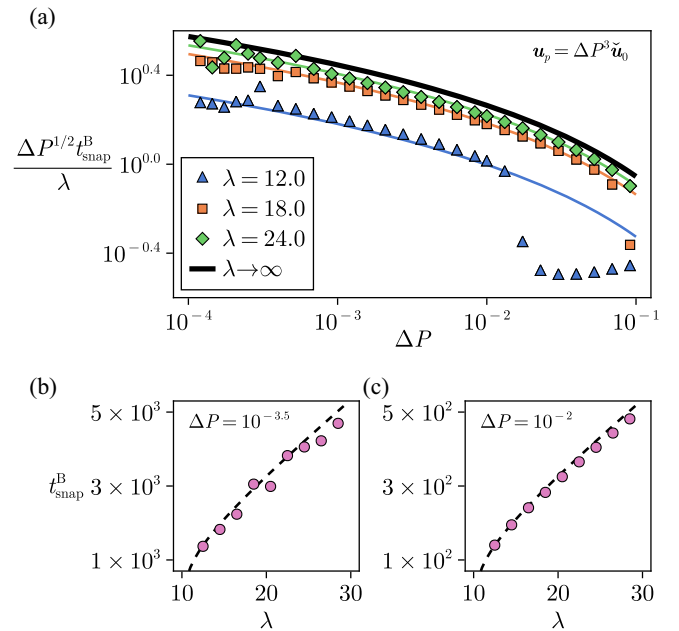


FIG. 2. (a) Rescaled snap-through times for bifurcation buckling as a function of  $\Delta P$  for different arch slendernesses  $\lambda$  and an applied imperfection of size  $\Delta P^3$ . Markers indicate times from finite-element simulations. Snap-through time as a function of  $\lambda$  for an applied imperfection that is (b) smaller and (c) greater than intrinsic imperfections. Solid and dashed curves are theoretical predictions.

for the logarithmic factor. Indeed, the analytical predictions from (5), shown as solid curves in Fig. 2(a) for selected values of  $\lambda$ , accurately capture the numerically computed snap-through times. Applied imperfections can only control the snap-through time if their effect is larger than intrinsic imperfections of the system, which in our case arise from numerical discretization. For small values of  $\Delta P$ , the applied imperfection, which has amplitude  $p_0^A = \Delta P^3$ , becomes smaller than intrinsic imperfections, and the numerical snap-through times in Fig. 2(b) show the same *noisy* behavior as when no imperfections are applied. For larger values of  $\Delta P$ , the applied imperfection is greater than the intrinsic imperfections, and the numerical snap-through times vary smoothly with controllable system parameters [Fig. 2(c)].

For large  $\lambda$ , the numerical data in Fig. 2(a) approach a universal curve, consistent with the prediction that  $t_{\text{snap}}^B \sim \lambda$ . The universal behavior is not followed when the slenderness is close to the threshold  $\lambda_{\text{hill}}^+ \simeq 10$ . In this case, the limit-point and pitchfork bifurcations occur for similar critical loads. Increasing the applied load by  $\Delta P$  can result in an “overshoot,” whereby limit-point buckling is triggered alongside bifurcation buckling [27], reducing the snap-through time. Alternatively, for a fixed  $\Delta P$ , there is a range of slenderness values  $\lambda_{\text{hill}}^+ < \lambda < \lambda_{\text{ov}}(\Delta P)$  for which overshoot occurs. The critical value  $\lambda_{\text{ov}}$ , rather than  $\lambda_{\text{hill}}^+$ , determines whether limit-point or bifurcation buckling occurs and hence controls the snap-through time [Fig. 1(e)].

The dynamics of bifurcation buckling do not depend on the nature of the imperfection. By replacing the imperfection in the initial condition (3) with a geometric imperfection, the same hierarchy of time regimes emerges [27]. The snap-through time is given by (5), but the parameter  $p_0^A$  now depends on the perturbation to the critical buckling configuration  $\mathbf{u}_{cr}$  induced by the imperfection.

There are no pre-snap-through oscillations during limit-point buckling. The multiple-scales analysis [27] shows that snap-through occurs across two time regimes. When  $t = O(1)$ , the applied load excites all of the symmetric modes, including the critical mode. The amplitude of the critical mode grows quadratically in time due to the load and quickly overtakes the oscillatory amplitudes of the noncritical modes, thus dominating the arch behavior. When  $t = O(\Delta P^{-1/4})$ , the amplitude of the critical mode is influenced by nonlinear interactions with itself, and its equation of motion is the normal form of a limit-point (or saddle-node) bifurcation. Solving this equation furnishes a snap-through time of

$$t_{\text{snap}}^{\text{LP}} = \left( \frac{3}{64\pi^2 r_{\text{LP}} q_{\text{LP}} \Delta P} \right)^{\frac{1}{4}} \Gamma\left(\frac{1}{4}\right)^2, \quad (6)$$

where  $\Gamma(\cdot)$  is the gamma function. We recover the slowing down  $t_{\text{snap}}^{\text{LP}} \sim \Delta P^{-1/4}$  induced by the limit-point

bifurcation [22,23], but we also capture the role of the slenderness through the functions  $r_{\text{LP}}(\lambda)$  and  $q_{\text{LP}}(\lambda)$  [27]. Although  $r_{\text{LP}}$  and  $q_{\text{LP}}$  depend on  $\lambda$ , the fourth root of their product remains approximately constant. Thus, the snap-through time (6) is approximately independent of  $\lambda$ , consistent with the numerical results in Fig. 1(e) and the scaling analysis. Importantly, the snap-through evolution during limit-point buckling is insensitive to applied imperfections.

Our analysis quantitatively characterizes the snap-through dynamics of an elastic arch under point indentation, revealing the role of slenderness as an internal parameter that discriminates symmetric from asymmetric evolution and controls snap-through time. For limit-point buckling, snap-through time is robust to both slenderness and applied imperfection size. In bifurcation buckling, increasing slenderness slows evolution and requires an asymmetric imperfection to produce a finite snap-through time, with the asymmetric mode growing via resonance with preceding symmetric oscillations. The previously proposed [23] scaling,  $t_{\text{snap}}^B \sim \Delta P^{-1/2}$ , must be amended with a logarithmic correction. These results extend beyond arches to any mechanical system with an equivalent bifurcation landscape.

Buckling is exploited in natural and artificial systems to enhance functionality. At small scales (e.g., in microelectronics [47] or fast actuators [48,49]) designing both stable equilibria and rapid dynamic transitions are equally important. Our framework offers a design tool to estimate morphing rates in slender systems where snap-through time is rate limiting and opens the doors to using deliberately introduced imperfections to precisely tune the snap-through times of new functional materials.

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*Data availability*—The data that support the findings of this article are openly available [27].

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