

Factorization and Resummation for the Near-Side Energy-Energy Correlators

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By utilizing the dihadron fragmentation formalism, we extend the previous factorization of near-side energy-energy correlators (EECs) in the collinear limit and derive an all order resummation in the Fourier transform b_T -space. A perfect matching is obtained when we compare to the fixed-order results. We further demonstrate the resummation effects for the EECs in e^+e^- annihilation and show that they will significantly improve the theoretical predictions at small angles.

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Introduction—Energy-energy correlators (EECs) in e^+e^- annihilation [1–19] and hadronic collisions have played an important role in studying the strong interaction physics at the frontier with a significant progress made in recent years [20–76]. The EECs not only provide a great opportunity to explore the quantum chromodynamics (QCD), but may also shed light on the hadronization effects in high-energy processes, including e^+e^- annihilation, proton-proton, proton-nucleus, and nucleus-nucleus collisions; see a recent review in Ref. [69].

Formally, the EEC is defined as

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma}{d\cos\chi} = \frac{1}{\sigma_{\text{tot}}} \sum_{i,j} \int d\sigma^{ij} \frac{E_i E_j}{Q^2} \delta(\cos\chi - \hat{n}_i \cdot \hat{n}_j), \quad (1)$$

where $d\sigma^{ij}$ denotes the semi-inclusive differential cross section of two hadrons h_i and h_j , with energies E_i and E_j , and in the direction of the unit vectors $\hat{n}_i$ and $\hat{n}_j$, respectively. The total energy is $Q = \sum_i E_i$, and χ is the angle between the two directions. For convenience, we define the variable $z \equiv (1 - \cos\chi)/2$. For the EEC in e^+e^- annihilation, two special kinematics have been focused on in the studies: near-side with $z \rightarrow 0$ and away-side $z \rightarrow 1$.

In the near-side region, the EECs are dominated by collinear gluon radiations and a resummation of large logarithms can be further included [31,33,77]. Additional

contributions from power corrections also play important role in this region [35,53,54,77,78]. However, in the end-point region where z is very small, one has to go beyond the above formalism. This region is usually referred as the “free hadrons” region suggested by the experimental data [42]. A nonperturbative transverse momentum dependent (TMD) fragmentation function (FF) has been proposed to describe the distribution in this region [60,63].

In a recent paper [79], the classical and quantum scaling behaviors of the near-side EECs have been explored by connecting the dihadron FF [80–93] contributions to the light ray operator product expansion (OPE). Extending this and utilizing the dihadron FF contributions, in this Letter we investigate the factorization and resummation for the EECs, aiming at a full description of the angular distribution in the near-side region.

Our derivation also extends the previous factorization result for the EECs in the collinear limit [33]. Here, we focus on the kinematic region of small angles, where the distance between the two particles of (1) can be regarded as a transverse momentum difference in the transverse plane perpendicular to the jet axis. Therefore, a TMD description is a natural choice to perform the resummation and match to nonperturbative contributions [60,63]. Different from the arguments used in these earlier papers, here we show explicitly how the factorization works, and an all order resummation is derived as a result of the factorization and the associated evolution equations.

The near-side factorization is different from that for the away-side, although the TMD concept is applied in both cases. For the back-to-back region of the away-side EECs, the most important contributions come from soft gluon radiations that lead to Sudakov double logarithms to all orders. A resummation can be derived by solving the relevant Collins-Soper evolution equation [20–23] and has been included in the phenomenology studies of the EECs in

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e^+e^- annihilation [27,29,30,34,62]. On the other hand, in the near-side region, there is no soft gluon radiation, and the QCD evolution follows the exact DGLAP evolution [33]. As a consequence, there is no Sudakov-type of double logarithms; see below the explicit one-loop calculations. This highlights a unique opportunity to study the non-perturbative QCD which is relevant to particle hadronization but different from traditional TMD physics discussed in the literature [94].

Meanwhile, the dihadron fragmentation formalism has been extensively explored in hadron physics community to unveil novel nucleon property such as the nucleon's tensor charge [86–93]. In these studies, the associated FFs are usually parameterized and fitted to the experimental data. Our results in this Letter will help constrain these functions and provide an additional input for phenomenological studies.

Dihadron fragmentation in a jet—For any generic process in which two hadrons are produced, we have the following two contributions [83]:

$$\frac{d\sigma(h_1 h_2)}{dz_1 dz_2} = \sum_{ij} \hat{\sigma}_{ij} D_i^{h_1}(z_1) D_j^{h_2}(z_2) + \sum_i \hat{\sigma}_i ID_i^{h_1 h_2}(z_1, z_2), \quad (2)$$

where $D_i^{h_1}(z_1)$ represents the FF describing the probability for parton i to fragment into a hadron h_1 with a momentum fraction z_1 , and similarly for $D_j^{h_2}(z_2)$. On the other hand, the dihadron FF $ID_i^{h_1 h_2}(z_1, z_2)$ describes two hadrons h_1 and h_2 with momentum fractions z_1 and z_2 , respectively, coming from a single parton i . All these FFs and the partonic cross sections $\hat{\sigma}_{ij}$ and $\hat{\sigma}_i$ depend on the scales, which have been omitted for simplicity. In the above equation, the first term gives the two individual FFs from two partons (i and j) and the second gives the dihadron FF from a single parton i . The latter contribution becomes important when the two hadrons are close to each other, where a collinear divergence from the first term needs to be absorbed into the renormalization of the dihadron FF as well [83]. In perturbative calculations, this divergence has to be properly taken care of so that the final results are free of divergence.

In addition, the momentum conservation leads to the following identity for the dihadron FF [81,83],

$$\int dz_1 z_1 \sum_{h_1} ID_i^{h_1 h_2}(z_1, z_2) = (1 - z_2) D_i^{h_2}(z_2). \quad (3)$$

Applying the dihadron fragmentation contribution to the EEC inside a high energy jet, we define the EEC jet function,

$$\Gamma_i(\mu) = \sum_{h_1 h_2} \int dz_1 dz_2 z_1 z_2 ID_i^{h_1 h_2}(z_1, z_2, \mu), \quad (4)$$

which is directly connected to the EEC measurement in jets as we will show below. The EEC inside a jet is defined as [42]

$$\frac{d\langle \text{EEC}(\theta) \rangle}{d^2\theta} = \frac{1}{N_{jet}} \sum_{i \neq j \in J} \frac{E_i E_j}{E_J^2} \delta^{(2)}(\vec{\theta} - (\vec{\theta}_i - \vec{\theta}_j)),$$

where N_{jet} represents the number of jets. The sum is over all particles (i, j) with relative angle θ and the jet energy $E_J = \sum_i E_i$. Integrating over θ , we define the integrated EEC,

$$\langle \text{EEC} \rangle(E_J) = \frac{1}{N_{jet}} \sum_{jets J} \sum_{i \neq j \in J} \frac{E_i E_j}{E_J^2}. \quad (5)$$

Focusing on the collinear kinematics of two particle production and applying the similar factorization argument of Ref. [95], we can compute the above quantity in terms of the dihadron FF,

$$\begin{aligned} \langle \text{EEC} \rangle(E_J) &= \sum_{i, h_1, h_2} \int dx x^2 H_i(x, Q/\mu) \\ &\quad \times \int dz_1 dz_2 z_1 z_2 ID_i^{h_1 h_2}(z_1, z_2, \mu) \\ &= \sum_i \int dx x^2 H_i(x, Q/\mu) \Gamma_i(\mu), \end{aligned} \quad (6)$$

where H_i is the perturbative coefficient for parton i , carrying momentum fraction x of the jet. The above factorization is similar to that of a semi-inclusive observable in jet [95–101]. We take the leading order and collinear limit for the EEC in jet, while the first term of Eq. (2) will be included at higher orders, as we will show below through the QCD evolution. Here Q represents the hard momentum scale $Q = E_J$. At this order, the integrated EEC measures $\Gamma_q(\mu)$ and $\Gamma_g(\mu)$ for the quark and gluon jet, respectively. At higher order, there will be mixing between them, an important contribution to which comes from the evolution. The evolution of Γ_i can be simplified by applying the momentum sum rule in Eq. (3),

$$\Gamma_i(\mu) = 1 - \Gamma'_i(\mu), \quad (7)$$

where $\Gamma'_i(\mu)$ is related to the moments of the single hadron FFs

$$\Gamma'_i(\mu) = \sum_h \int dz z^2 D_i^h(z, \mu). \quad (8)$$

With the above equations, it is straightforward to derive the

evolution equations for Γ_i . Because of the mixing, it is appropriate to write down the evolution equation in the flavor-space matrix, and we have,

$$\frac{d}{d \ln \mu^2} \Gamma = -\Gamma \cdot \hat{\gamma}_T(N) + \mathbf{1} \cdot \hat{\gamma}_T(N), \quad (9)$$

where Γ represents (Γ_q, Γ_g) and $N = 3$. It is interesting to find out that the second term is the inhomogeneous contribution which comes from the first term in Eq. (2). The above evolution can be verified by applying the scale evolution for the dihadron FFs [83] in Eq. (6). The anomalous dimensions $\hat{\gamma}_T(N)$ are defined

$$\hat{\gamma}_T(N) \equiv - \int dx x^{N-1} \hat{P}_T(x), \quad (10)$$

with the timelike DGLAP splitting kernel $\hat{P}_T$ as a flavor-space matrix,

$$\hat{P}_T = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}. \quad (11)$$

In practice, we can solve the evolution equation for Γ' and make predictions for the integrated EEC, and we find that this gives a very good description of the integrated jet EECs of Eq. (5) at the LHC [102].

Infrared behavior at small angle and the factorization in Fourier Transform b_T -space—Now, we turn to the angular distribution of the EEC. It has been realized that the small-angle EEC depends on nonperturbative physics. Here, we study its infrared (IR) behavior by applying the EEC jet function constructed from the dihadron FFs and derive a factorization formalism. A similar idea has been suggested in Ref. [63]. Our framework is a complete answer to the small angle resummation.

Let us start with the unintegrated EEC jet function $\Gamma_i(\mu, q_\perp)$, which can be defined through the dihadron FFs, similar to that of Eq. (7). To do that, we keep the transverse momenta for the two hadrons $k_{1\perp}$ and $k_{2\perp}$, respectively, and define $\vec{q}_\perp = \vec{k}_{1\perp}/z_1 - \vec{k}_{2\perp}/z_2$. With this unintegrated EEC jet function, the angular distribution of EEC can be written as

$$\frac{d(\text{EEC}(\theta))(E_J)}{d^2\theta} = E_J^2 \sum_i \int dx x^4 H_i(x, Q/\mu) \Gamma_i(\mu, q_\perp), \quad (12)$$

where $q_\perp = x E_J \theta$. In the transverse plane perpendicular to the jet direction, the angular distribution between the two particles comes from their transverse momentum distribution. Therefore, an analogous factorization in terms of transverse-momentum-dependent distributions is plausible. In the following, we show that this is exactly the case.

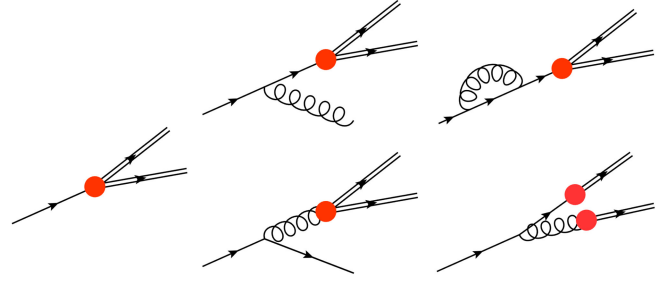


FIG. 1. Diagrammatic representations of the leading order (far left) and next-to-leading order contributions. The red blobs represent the fragmentation processes and the double lines represent the fragmented hadrons. The last diagram gives the inhomogeneous contribution where the two hadrons come from individual fragmentation of the daughter quark and gluon. All other diagrams are homogeneous contributions and they do not change the transverse momentum difference between the two final state particles.

In order to investigate the IR behavior for $\Gamma(\mu, q_\perp)$, we carry out a perturbative calculation; see, e.g., diagrams in Fig. 1. In our derivations, we take the leading logarithmic approximation (LLA). At the leading order, we have a simple $\square$ function,

$$\Gamma_q^{(0)}(\mu, q_\perp) = \Gamma_q(\mu) \delta^{(2)}(q_\perp), \quad (13)$$

for the quark part, where $\Gamma_q(\mu)$ represents the integrated distribution. At one-loop order, similar to the above integrated EEC jet function, it receives homogeneous and inhomogeneous contributions. The homogeneous term is also proportional to a $\square$ function of $q_\perp$,

$$\Gamma_q^{(1)}|_{\text{homo.}} = \delta^{(2)}(q_\perp) \left(-\frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu^2} \right) \Gamma_q(\mu) (-\gamma_{qq}^{(3)}), \quad (14)$$

where $\gamma_{qq}^{(3)}$ represents the $N = 3$ anomalous dimension; see Eq. (10). Similarly, we should have a gluon channel contribution of $\gamma_{gq}^{(3)}$.

The inhomogeneous term is the most important contribution, because it generates a nonzero $q_\perp$ (nonzero θ as well). At this order, it is straightforward to find out,

$$\Gamma_q^{(1)}|_{\text{inhomo.}} = \frac{\alpha_s C_F}{2\pi^2} \frac{1}{q_\perp^2} \frac{3}{2}, \quad (15)$$

where we have taken into account both qg and gq configurations from the quark splitting. At this order, it is easy to check that

$$\gamma_{qq}^{(3)} + \gamma_{gq}^{(3)} = \frac{\alpha_s}{2\pi} \frac{3}{2} C_F + \mathcal{O}(\alpha_s^2). \quad (16)$$

This leads to consistent evolution for the integrated EEC jet function after integrating over $q_\perp$.

Both equations of (14) and (15) contain IR divergences, and their total contributions lead to the IR behavior for the integrated EEC jet function $\Gamma_q(\mu)$. The resummation for a finite $q_\perp$ can be built on this important feature. In previous studies [33,80–82], an integral over $q_\perp$ (or z) was taken and the associated resummation was carried out with a differential respect to $q_\perp$ after all order resummation. In the following, we take a different path, following an analogous factorization argument as that for the TMD FFs [94].

The key element of our treatment is the Fourier transform to b_T space,

$$\tilde{\Gamma}_q(\mu; b_T) = \int d^2 q_\perp e^{iq_\perp \cdot b_T} \Gamma(\mu; q_\perp), \quad (17)$$

and factorization/resummation is performed in the b_T space. The final result for the $q_\perp$ (and θ) distribution is obtained by Fourier transforming back to $q_\perp$ space.

To illustrate the above point, let us start again a perturbative calculation in b_T space, and the leading order result for $\tilde{\Gamma}$ is simple,

$$\tilde{\Gamma}_q^{(0)}(\mu, b_T) = \Gamma_q(\mu), \quad (18)$$

and one loop homogeneous term takes the same form,

$$\tilde{\Gamma}_q^{(1)}|_{\text{homo.}} = \left(-\frac{1}{\epsilon} + \ln \frac{\mu^2}{\bar{\mu}^2} \right) \left[-\Gamma_q(\bar{\mu}) \gamma_{qq}^{(3)} - \Gamma_g(\bar{\mu}) \gamma_{gq}^{(3)} \right], \quad (19)$$

where we have included the contributions from both $q \rightarrow g$ and $q \rightarrow q$ channels. The inhomogeneous term comes again from the last diagram in Fig. 1. By Fourier transforming Eq. (15) to b_T space, we have

$$\tilde{\Gamma}_q^{(1)}|_{\text{inhomo.}} = \left(-\frac{1}{\epsilon} + \ln \frac{\mu_b^2}{\bar{\mu}^2} \right) (\gamma_{qq}^{(3)} + \gamma_{gq}^{(3)}), \quad (20)$$

where $\mu_b = 2e^{-\gamma_E}/b_T$ with γ_E the Euler constant. Adding them together, we have

$$\begin{aligned} \tilde{\Gamma}_q^{(1)}(\mu, b_T) = & \left(-\frac{1}{\epsilon} + \ln \frac{\mu_b^2}{\bar{\mu}^2} \right) \left[-\Gamma_q(\bar{\mu}) \gamma_{qq}^{(3)} - \Gamma_g(\bar{\mu}) \gamma_{gq}^{(3)} \right. \\ & \left. + \gamma_{qq}^{(3)} + \gamma_{gq}^{(3)} \right] - \ln \frac{\mu^2}{\mu_b^2} \left[\Gamma_q(\mu_b) \gamma_{qq}^{(3)} + \Gamma_g(\mu_b) \gamma_{gq}^{(3)} \right]. \end{aligned} \quad (21)$$

In the above equation, the first term contains IR divergence which will be renormalized by the evolution of $\Gamma_q(\mu)$ at one loop order. The above demonstrates that we can factorize the EECs at small angles in terms of the EEC jet function, which is constructed from the dihadron FFs.

All order resummation and compare to the fixed-order results—After renormalization of the EEC jet functions, from the above one-loop calculations, we obtain the

following result:

$$\tilde{\Gamma}_i^{(0+1)}(\mu, b_T) = \Gamma_i(\mu_b) - \ln \frac{\mu^2}{\mu_b^2} \gamma_{ji} \Gamma_j(\mu_b), \quad (22)$$

which applies to both quark and gluon cases, and the EEC jet functions on the right-hand side are renormalized ones. An all-order resummation can be derived by solving the scale evolution with respect to the scale μ . Following a similar method in [33], it can be solved as,

$$\begin{aligned} \tilde{\Gamma}_i(\mu, b_T) &= \Gamma_j(\mu_b) \text{P exp} \left[- \int_{\mu_b}^{\mu} d \ln \mu'^2 \gamma(\mu') \right]_{ji} \\ &= \Gamma_j(\mu_b) \text{P exp} \left[\int_{\alpha_S(\mu_b)}^{\alpha_S(\mu)} \frac{d\alpha_S}{\alpha_S} \frac{\gamma_0(\alpha_S)}{\beta_0(\alpha_S)} \right]_{ji}, \end{aligned} \quad (23)$$

at the LLA accuracy, where P implies a path-order matrix product in the integrating variable for the exponential term. The $\square$ function $\beta \equiv -d\alpha_S(\mu)/d \ln \mu^2$ has been defined, and can be expanded as $\beta = \sum_n a_S^{n+1} \beta_n$ and the anomalous dimension are expanded as $\gamma = \sum a_S^{n+1} \gamma_n$. In this work, we define the perturbative expansion parameter $a_S \equiv \alpha_S/(2\pi)$.

It is not surprising that the above evolution kernel is identical to that derived previously [33], because the collinear divergence is the same. The difference here is that we perform the resummation in the Fourier transform b_T space. It is interesting to note that the above result is similar to that suggested in [63]. In our derivation, the factorization and explicit calculation at one-loop order help identify the initial input for the evolution.

When Fourier transformed back to $q_\perp$ space, the above resummation formula will encounter the nonperturbative region. We follow the so-called b_* prescription [24], $b_T \rightarrow b_* = b_T / \sqrt{1 + b_T^2/b_{\text{max}}^2}$ where we choose $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$. With that, we will also apply a nonperturbative contribution. The final resummation result for $\tilde{\Gamma}$ reads as

$$\tilde{\Gamma}_{\text{res.}}(\mu, b_T) = \tilde{\Gamma}_{\text{perp}}(\mu, b_*) e^{-g_{q,g} b_T^2}, \quad (24)$$

where g_q and g_g are free parameters and will be fitted to experimental data. We note that different forms of the nonperturbative part [60,63] can be applied here.

The above resummation formula can be compared to the asymptotic behavior from the fixed-order calculations [32,33] and we find a perfect agreement; see the End Matter. This provides an important cross check for our results and leads to a matching between the resummation results and the fixed-order calculations when applying to the experimental data.

Now, we apply our LLA resummation formula to the unintegrated EEC in e^+e^- and demonstrate its importance in the small-angle region. With our resummation result, it can be written as

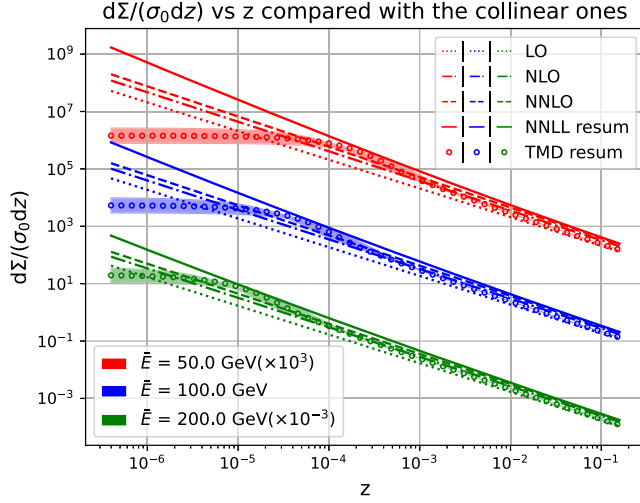


FIG. 2. EECs in e^+e^- annihilation, comparing our resummation results with those from fixed-order calculations [32] and the next-to-next-leading logarithmic (NNLL) resummation results from [33] at small angles for different energies. The colored bands around the resummation results are estimated theoretical uncertainties obtained by varying the nonperturbative parameters $g_{q,g}$ by a factor of 2. Note that the collinear calculations almost do not depend on the scale aside from that in the running coupling α_s , whereas the TMD-resummed results feature a transition scale that varies with the average parton energy $\bar{E}$.

$$\left. \frac{d\Sigma}{\sigma_0 dz} \right|_{z \ll 1} = 4\pi \bar{E}^2 \int \frac{d^2 b_T}{(2\pi)^2} e^{i\vec{x} \cdot \vec{E} b_T} H_q \tilde{r}_q^{\text{res.}}(\bar{E}, b_T), \quad (25)$$

where $\bar{E}$ is the average parent parton energy, which depends on the center-of-mass energy $\sqrt{s}$. Their relation will be further studied with experimental data in a future publication. In this analysis, we also set $x = 1$ to be consistent with the LLA; if needed, the full x dependence can be restored.

In Fig. 2, we show the small-angle EECs as functions of z for different energies, comparing the resummation results to the fixed-order calculations [32,33]. A number of important observations. First, at the moderate z , the resummation results match to the collinear calculations, and as expected, the agreement extends to smaller angles when we compare to the collinear resummation results from [33]. Second, our TMD type of resummation correctly predicts the turnover of the distributions at very small angles. An average momentum fraction carried by the quark [60] should be taken into account, and the turnover point will be shifted to the right. This mainly comes from the nonperturbative contribution in our formalism, see Eq. (24). In the numeric calculations, we assume the dominance of quark jets in e^+e^- annihilation and take the leading-order hard part $H_q = 1/2$. For the nonperturbative parameters, we take $g_q = g_g = 0.35$ which is consistent with the parametrization used in Ref. [60]. We also utilize the single-hadron FFs [103] at $\mu_0 = 2$ GeV and calculate $\Gamma'_q(\mu_0) = 0.246$ and $\Gamma'_g(\mu_0) = 0.176$ so that

$\Gamma_i(\mu_0) = 1 - \Gamma'_i(\mu_0) = (0.754, 0.824)$ set the initial condition of the evolution. Though we note that the agreement with the collinear calculations persists when varying these nonperturbative parameters, which affects mostly the plateau regions at very small angles. As shown by the colored bands that correspond to the estimated theoretical uncertainties obtained by varying the nonperturbative parameters $g_{q,g}$ by a factor of 2.

Conclusion—In summary, by utilizing the dihadron FF contributions to the EEC observables, we extend previous factorization and resummation formalism to the small-angle region. A TMD type of resummation formula was derived following the factorization argument and the evolution equations for the EEC jet functions. Explicit results have been shown at the one-loop order. Asymptotic expansion has also been checked against the fixed-order calculations, and a perfect agreement was found at the leading logarithmic level.

Comparing our resummation formula to the fixed-order calculations for the EECs in e^+e^- annihilation demonstrates the perfect matching between them. In particular, our resummation captures some important features of the collinear resummation results from Ref. [33]. In addition, our resummation results predict a turnover in the angular distribution at very small angles, which is consistent with previous calculations using the TMD FFs [60,63]. This type of turnover behavior has also been suggested by the recent ALEPH data [104]. We leave a detailed phenomenological study comparing our resummation formula and the experimental data in the near-side EEC from various collision systems to future work. This provides an important link between the “free hadron” phase and the collinear region in the EEC observables. Extension to higher-point energy-energy correlators shall follow as well.

The consistent treatment of the IR behavior derived in this Letter should be applied to the phenomenology of hadron physics as well. This will strengthen the theoretical prediction power in the associated studies to extract the nucleon property from the dihadron fragmentation processes [86–92]. For example, the Collins type of contribution has been computed in the large invariant mass region for the dihadron FF [90]. This should be part of the inhomogeneous contribution to the spin-dependent dihadron FF and a similar resummation should be applied there. We will come back to the above issues in a future publication.

Note added—After this paper was submitted, we noticed several other approaches have been reported to apply the dihadron fragmentation function formalism for the near-side EECs [105–107]. Different from these papers, we focus on the IR behavior of the EEC jet function and derive the factorization and resummation formalism based on a complete calculation at one-loop order with both homogeneous and inhomogeneous contributions.

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Data availability—The data that support the findings of this article are openly available [108].

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End Matter

In this End Matter, we briefly show that the resummation of EEC matches the collinear calculations in the leading logarithmic approximation (LLA). As shown in the main text, the EEC jet functions can be resummed in the b_T space as

$$\begin{aligned}\tilde{\Gamma}_i(\mu, b_T) &= \Gamma_j(\mu_b) \mathbf{P} \exp \left[- \int_{\mu_b}^{\mu} d \ln \mu'^2 \gamma(\mu') \right]_{ji} \\ &= \Gamma_j(\mu_b) \mathbf{P} \exp \left[\int_{\alpha_S(\mu_b)}^{\alpha_S(\mu)} \frac{d\alpha_S}{\alpha_S} \frac{\gamma_0(\alpha_S)}{\beta_0(\alpha_S)} \right]_{ji},\end{aligned}\quad (\text{A1})$$

in the LLA, where $\mathbf{P}$ implies a path-order matrix product in the integrating variable for the exponential term. β_0 and γ_0 are the leading order beta function and anomalous dimensions defined by $\beta = \sum_n a_S^{n+1} \beta_n$ and $\gamma = \sum a_S^{n+1} \gamma_n$, where $a_S \equiv \alpha_S/(2\pi)$.

Taking the LLA, we expand the $\tilde{\Gamma}_i(\mu, b_T)$ as a power series of $L \equiv a_S \ln(\mu^2/\mu_b^2)$ with the following leading log coefficients $A_{ji}^{(n)}$:

$$\tilde{\Gamma}_i(\mu, b_T) = \sum_j \Gamma_j(\mu_b) \sum_n L^n A_{ji}^{(n)}, \quad (\text{A2})$$

which can then be solved in matrix form as

$$\mathbf{A}^{(0)} = \mathbb{1}, \quad \mathbf{A}^{(1)} = -\boldsymbol{\gamma}_0, \quad (\text{A3})$$

$$\mathbf{A}^{(2)} = \frac{1}{2}(-\beta_0 \boldsymbol{\gamma}_0 + \boldsymbol{\gamma}_0^2), \quad (\text{A4})$$

$$\mathbf{A}^{(3)} = \frac{1}{6}(-2\beta_0^2 \boldsymbol{\gamma}_0 + 3\beta_0 \boldsymbol{\gamma}_0^2 - \boldsymbol{\gamma}_0^3), \quad (\text{A5})$$

where $\mathbb{1}$ stands for the identity matrix and all the powers of $\boldsymbol{\gamma}_0$ are matrix multiplications in flavor (q, g) space.

To compare it with the collinear calculation, we take the input $\boldsymbol{\Gamma} = (1, 1)$ and also put in the hard part $\mathbf{H}$ for the EEC in e^+e^- annihilation, noting that the leading order hard coefficients are $\mathbf{H}(x) = (1/2\delta(1-x), 0)$ and $\mathbf{H} \equiv \int dx x^4 \mathbf{H}(x) = (1/2, 0)$ at leading logarithmic accuracy. Then we have the following $\bar{\mathbf{A}}^{(n)}$ defined as $\bar{\mathbf{A}}^{(n)} \equiv \boldsymbol{\Gamma} \cdot \mathbf{A}^{(n)} \cdot \mathbf{H}$:

$$\bar{\mathbf{A}}^{(1)} = -\frac{3}{2} C_F = -2, \quad (\text{A6})$$

$$\bar{\mathbf{A}}^{(2)} = \frac{C_F(-428C_A + 375C_F + 53n_f)}{120} = -\frac{173}{30}, \quad (\text{A7})$$

$$\begin{aligned}\bar{\mathbf{A}}^{(3)} &= \frac{C_F}{10800} \left[96708C_A^2 - 2C_A(68000C_F + 16259n_f) \right. \\ &\quad \left. + 5(9375C_F^2 + 4619C_F n_f + 368n_f^2) \right] \\ &= -\frac{20317}{1350}.\end{aligned}\quad (\text{A8})$$

These results match perfectly with the leading logarithmic coefficients from collinear calculations [33], noting that a map in the form of

$$\frac{1}{z} \ln^n(z) \leftrightarrow \frac{(-1)^n}{n+1} \ln^{n+1} \left(\frac{\mu^2}{\mu_b^2} \right) \quad (\text{A9})$$

is needed, since $z \equiv (1 - \cos \chi)/2 \propto q_T^2$. Thus, a factor of $-1, 2, -3, \dots$ should be multiplied to the above coefficients when comparing with the collinear convention. Also, a mismatch factor due to the difference between the $a_S = \alpha_S/(2\pi)$ defined here and the $a_S = \alpha_S/(4\pi)$ in [33] has been implicitly taken into account.