

Self-Organized Hyperuniformity in a Minimal Model of Population Dynamics

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By generalizing a class of models recently introduced to account for protracted transients in biological systems, we identify a novel mechanism for hyperuniformity. In this model, competition of individuals over a shared resource serves as feedback that can asymptotically guide the population toward a critical steady state with a divergent individual lifetime. We show that, in its spatially extended form, this many-body model exhibits hyperuniform density fluctuations. Through explicit coarse-graining, we develop a hydrodynamic theory that conforms closely with the results of stochastic simulations. Unlike previous models for nonequilibrium hyperuniform states, our model does not exhibit conservation laws, even in the asymptotic regime. Instead, hyperuniformity arises from the divergence of the range of the resource-mediated interactions. These findings may find applications in engineering, cellular population dynamics, and ecology.

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Introduction—Over the past two decades, the phenomenon of hyperuniformity has drawn increasing attention across multiple contexts and disciplines [1–3]. It represents the remarkable property whereby a system of particles is statistically disordered at short length scales yet has vanishing density fluctuations at large scales, as in zero-temperature crystals [2]. Formally, the variance of the number of particles $N(\ell)$ in a given domain of size ℓ scales subextensively with volume, defying generic central limit theorem arguments. Mathematically, this translates to the condition

$$\frac{\text{Var}[N(\ell)]}{\langle N(\ell) \rangle} \sim \ell^{-\alpha}, \quad (1)$$

where the exponent $0 < \alpha \leq 1$ characterizes the degree of hyperuniformity [2]. Equivalently, hyperuniformity can be reformulated as the vanishing of the structure factor, $\lim_{q \rightarrow 0} S(q) = \lim_{\ell \rightarrow \infty} \text{Var}[N(\ell)] / \langle N(\ell) \rangle \stackrel{!}{=} 0$ [1], with

$$S(q) \equiv \frac{\langle \delta\rho(\mathbf{q})\delta\rho(-\mathbf{q}) \rangle}{\langle \rho \rangle}, \quad (2)$$

where $\delta\rho(\mathbf{q})$ denote density fluctuations around the mean $\langle \rho \rangle$ at Fourier mode $\mathbf{q}$ and S is a function of $q = |\mathbf{q}|$ from isotropy. Realizations of hyperuniformity have been established in several synthetic settings, including athermally jammed hard spheres [4,5], periodically driven emulsions [6,7], and active Quincke rollers [8]. More recently, biologically occurring instances of hyperuniformity have been identified, including the distribution of avian photoreceptors [9,10], the leaf vein network [11], and vegetation coverage in semiarid landscapes [12–14]. In a solid state setting, interest in hyperuniformity draws from their remarkable material properties, such as complete photonic band gaps [15–19] and superior mechanical [20–22] and acoustic [23,24] properties. In biological contexts, hyperuniformity may serve important functions, such as providing uniform coverage of cell types [9,10] or the optimization of resource acquisition [12,13] and resource distribution [11].

Finding generic mechanisms that give rise to hyperuniformity remains an outstanding challenge which, if met, can unlock the potential for novel functional materials and advance our understanding of the mechanisms driving hyperuniformity in biological systems. While equilibrium dynamics with short-ranged interactions cannot result in hyperuniformity [25], nonequilibrium systems, such as the examples listed above, can relax to a hyperuniform state [3]. Prominent models that capture such nonequilibrium hyperuniformity include those with an absorbing state phase transition at criticality [26–30], active and passive phase separating systems during spinodal decomposition

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[31], and various active fluid models [8,32–34]. In these contexts, steady-state hyperuniformity emerges when a control parameter is fine-tuned to a critical value for a phase transition. A unifying feature that underlies hyperuniformity in many of these systems is the effective emergence of center-of-mass conservation at criticality [26,28,31,34–39]. This is manifest in “superconservative” noise whose amplitude vanishes with wave number as q^2 , leading to vanishing large-scale fluctuations.

Here, we question whether other classes of systems can generate hyperuniformity. Notably, we find that hyperuniformity can arise in a biologically realistic setting without center-of-mass conservation and without parameter fine-tuning. Inspired by a recent study of plasma cell memory, we introduce a minimal population dynamics model based on programmed cell death [40–42]. Yet, we will show that this model captures the behavior of a broad class of systems. In this model, agents are (i) produced through a stochastic birth-type process and are (ii) eliminated (die) when the internal state of the agent transitions away from a “viable” regime. The viable regime is (iii) stabilized when the local concentration of a shared diffusive resource exceeds a critical threshold, analogous to the role of “survival factors” consumed by plasma cells in the bone marrow niche. This resource, depleted by the agents, acts as the control parameter of a dynamical bifurcation that stabilizes the viable state. Together, these three ingredients define a negative feedback loop that guides the system toward a steady state where the birth rate of agents is matched by their removal (death) rate and resource production matches its total consumption set by the population size (see Figs. 1 and S1 in Supplemental Material [43]). Remarkably, when the consumption rate of individuals is decreased (or the production rate is increased), the steady-state resource level tends to the critical threshold that stabilizes viability [44]. Mathematically, this model is a spatial extension of a

zero-dimensional framework used recently to study biological memory in the context of the transgenerational inheritance of gene silencing [42], the adaptive immune system [44], and cell cycle control [45]. Unlike classic models of self-organized criticality [46,47], the dynamics here do not exhibit large-scale avalanches over the parameter regime of interest. On the contrary, the system displays regular behavior over hydrodynamic length scales, allowing for systematic coarse-graining.

Within this dynamical framework, we find that, in the asymptotic limit of vanishing consumption rates, the population relaxes into a steady state with hyperuniform spatial correlations yet without total mass, let alone center-of-mass, conservation (as depicted in Figs. 2, 3, and S1 in Supplemental Material [43]). Crucially, the many-body system does not undergo a phase transition. Rather, feedback drives the steady-state resource level toward its critical threshold. As this limit is approached, the resource field mediates negative-feedback interactions over a diverging range, suppressing long-wavelength density fluctuations.

Resource competition model—Formally, immobile agents (labeled $i = 1, 2, \dots$) are introduced stochastically in d -dimensional space at random positions $\mathbf{x}_i$ at a mean rate-per-unit-space volume λ . In our minimal model, agents are immobile, yet endowing them with nonzero diffusivity does not change our findings qualitatively; see Supplemental Material [43]. The lifetime of an individual agent τ_i is set by local environmental conditions and, in general, could be governed by a high-dimensional dynamical system. Yet, in many contexts including the one considered here, the dynamics show a critical transition between finite and diverging lifespan. In the language of dynamical systems modeling, such a transition corresponds to the elimination of a stable fixed point from some effective (possibly very high-dimensional) state space, i.e., a saddle node (SN) bifurcation. Importantly, in the vicinity of the bifurcation, the dynamics is governed

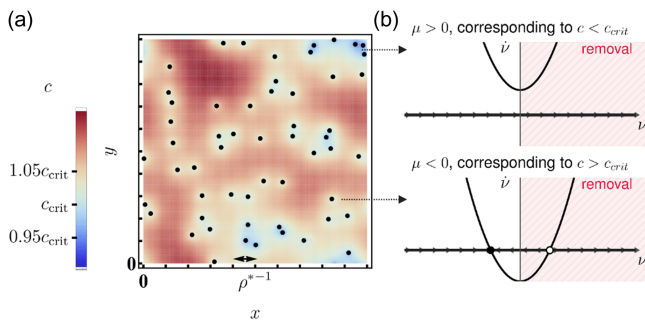


FIG. 1. (a) Snapshot of a two-dimensional realization of the population dynamics model [Eqs. (3)–(6)], where particles are marked in black dots and the resource field c is a color map displaying small variations around the critical value $c_{\text{crit}} = 1$. (b) Instantaneous flow of the internal dynamics (3) for two representative particles that are indicated by arrows. All parameters are set to unity except for $k = 7$, corresponding to $\mu^* \simeq 0.05$ (9).

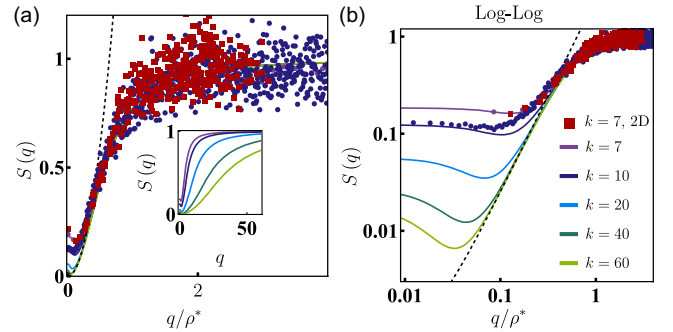


FIG. 2. Structure factor (2) at increasing values of k approaching criticality, with all other parameters set to unity. Solid lines are the theoretical prediction from the hydrodynamic theory [Eqs. (10) and (11)] (see Supplemental Material [43] for their explicit expressions). Symbols denote numerical simulations of Eqs. (3)–(6) in one dimension (circles) and two dimensions (2D, squares). The dashed black line is the asymptotics (12) at $k = 60$.

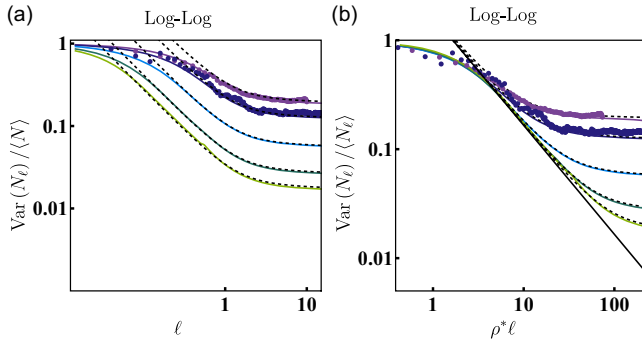


FIG. 3. Scaled number variance (1) color coded as in Fig. 2, showing good agreement with the large ℓ^{-1} asymptotics given by the first line in (13), here in a solid black line. Dashed black curves account for the nonperturbative corrections in the second line of (13), which become dominant beyond a diverging cross-over length scale $\sqrt{\ell_D \rho^*} \sim \mu^{*-1/2}$.

by progression along a one-dimensional center manifold [48,49], parameterized by an effective “viability” coordinate $\nu_i(t)$, which follows the SN normal form dynamics [50]

$$\dot{\nu}_i = \nu_i^2 + \mu_i. \quad (3)$$

Here, $\mu_i = \mu(\mathbf{x}_i, t)$ serves as the bifurcation control parameter. At fixed negative $\mu < 0$, the dynamics (3) supports a stable fixed point at $\nu = -\sqrt{|\mu|}$, where the agent remains “viable.” For $\mu > 0$, the fixed point is eliminated with ν monotonically increasing toward the positive half line $\nu > 0$, leading to removal. The precise value of the viability at which new arrivals are initiated, set here to $\nu_0 < 0$, does not affect the behavior of the system (see discussion in Supplemental Material [43]). The same is true for the viability threshold for “death,” set here to $\nu \geq 0$. In practice, small random fluctuations, not included here, would enable noisy escape from this fixed point, leading to a finite lifetime even at $\mu < 0$. While such fluctuations have a dramatic effect on the lifetime distribution [42], they have a subleading effect on density fluctuations that are at focus here, as shown in Supplemental Material [43]. Omitting them allows for analytical tractability, as we discuss below. The dynamics (3) is then coupled to a resource field $c(\mathbf{x}, t)$. We denote by c_{crit} the critical resource level below which the bifurcation ensues. Close to the bifurcation it is sufficient to consider a simple linear coupling [50],

$$\mu_i(\mathbf{x}, t) = c_{\text{crit}} - c(\mathbf{x}_i, t), \quad (4)$$

describing the elimination of the fixed point at $c < c_{\text{crit}}$. The resource is locally consumed by viable individuals, which is balanced by a fixed production rate, and is dispersed diffusively, as captured by the reaction-diffusion equation,

$$\dot{c} = p \left(1 - \frac{c\rho}{k} \right) + D \nabla^2 c, \quad (5)$$

where p is the production rate, k sets the ratio of production to consumption rates, and D is the diffusion coefficient. The number density field of surviving agents is given by

$$\rho(\mathbf{x}, t) = \sum_i \Theta(-\nu_i) \delta^d(\mathbf{x} - \mathbf{x}_i), \quad (6)$$

where the Heaviside theta-function $\Theta(\dots)$ selects for viable states and the d -dimensional Dirac delta-function $\delta^d(\dots)$ ensures that the consumption of resource is purely local. The coupled dynamics [Eqs. (3)–(6)] describe a negative feedback, which relaxes the system toward a dynamic steady state: when resource levels are elevated above the threshold level $c > c_{\text{crit}}$, new agents steadily accumulate, which according to (5) result in increased consumption and the down-regulation of resource levels. As the system admits a unique steady state, the initial conditions affect only the transient approach to stationarity. In the asymptotic limit of vanishing individuals’ consumption rate $k^{-1} \rightarrow 0$, the steady-state resource approaches the critical threshold $c \simeq c_{\text{crit}}$, as depicted in Fig. 1. Note, however, that the strict limit of $k^{-1} = 0$ is singular and does not define a regular steady state. As shown below, the particular choice of coupling to the resource [Eqs. (4) and (5)] also captures the generic case, where the minimal system of Eqs. (3)–(6) provides a universal description of critical tuning via resource competition.

A corresponding “mean-field” type of analysis can be found in [42] and is briefly recapped in the following. Remarkably, going beyond the mean-field description, we find that this dynamical steady state has hyperuniform density fluctuations, as illustrated in Figs. 2 and 3.

Mean-field steady state—At the dynamic steady state, resource production matches consumption, and the agent arrival rate matches their average removal rate. At the level of mean field, when the steady-state density and resource fields $\rho = \rho^*$ and $c = c^*$ are spatially homogeneous, these requirements are given, respectively, by

$$\rho^* = \frac{k}{c^*}, \quad \lambda = \frac{\rho^*}{\tau(\mu^*)}, \quad (7)$$

where λ is the average arrival rate of new agents and $\tau(\mu^*)$ is the mean lifetime of agents at a fixed resource level $c^* = c_{\text{crit}} - \mu^*$. The latter can be evaluated from Eq. (3) as

$$\tau(\mu^*) = \frac{\arctan(|\nu_0|/\sqrt{\mu^*})}{\sqrt{\mu^*}} \simeq \frac{\pi}{2\sqrt{\mu^*}}, \quad (8)$$

with $\mu^* = c_{\text{crit}} - c^*$ and where we have expanded close to criticality at small μ^* . The first equality in (7) comes from (5), while the second equality is simply a manifestation of Little’s law in queuing theory [51]. Combining Eqs. (7) and (8) provides the mean value of the control parameter to leading order at small μ^* , with

$$\mu^* \simeq \left(\frac{\pi \lambda c_{\text{crit}}}{2k} \right)^2. \quad (9)$$

This expression shows that the critical resource threshold $\mu^* \rightarrow 0$ is approached asymptotically either by decreasing the average arrival rate of agents λ or decreasing the rate of consumption k^{-1} . In this Letter, we analyze the latter limit where the system densifies $\rho^* \sim \mu^{*-1/2}$ [Eqs. (7) and (9)] and, as we explain below, the mean-field analysis becomes exact in any space dimension and explicit coarse-graining of the model becomes viable.

We are now in a position to evaluate the generality of our model [Eqs. (3)–(6)]. Little’s law (7) holds regardless of the particular choice for the dynamics of individuals (3) or of the resource (5); the mean density ρ^* (7) is expected to increase with decreasing particle consumption rate k^{-1} for a generic resource consumption coupling. Taken together, the critical threshold $c^* \rightarrow c_{\text{crit}}$ would be approached at small k^{-1} for a generic resource competition dynamics. As we show in the following, this asymptotic regime exhibits hyperuniform correlations, and these are found to be insensitive to nonlinearities in the model dynamics, Eqs. (4) and (5). Overall, our minimal model captures generic features of self-organized hyperuniformity via resource competition.

Fluctuating hydrodynamics beyond mean field—To analyze the hyperuniform behavior quantitatively, we turn to a coarse-grained description. This hydrodynamic limit depends crucially on the emergence of the diffusive length scale $\ell_D \equiv Dk/p \sim \mu^{*-1/2}$ prescribed by the reaction-diffusion dynamics, Eq. (5). It represents the length scale over which the resource field c varies in the presence of the δ -function like consumption “sinks” of the agents, Eq. (6). The hydrodynamic description emerges when this length scale is much larger than the average spacing between agents, $\ell_D \rho^* \gg 1$, and the density field (6) becomes smoothly varying over these scales. From Eq. (7), this condition is met when $Dk^2/p c_{\text{crit}} \gg 1$, which holds in our asymptotic limit of interest, where k becomes large, keeping all other parameters fixed. In this regime, nearby agents experience similar resource levels and will have similar lifetimes, allowing for their averaging over the hydrodynamic scale. Furthermore, at coarse grained scales, the noisy arrival rate of agents becomes deterministic with weak Gaussian white-noise corrections [52]. These two properties allow us to write down a local and time-varying balance equation for the density of agents, akin to Little’s law, that serves as a starting point for our coarse-graining procedure detailed in Supplemental Material [43]. Moreover, under weak noise, it is sufficient to consider linearized hydrodynamics for small deviations of the fields $\delta\rho = \rho(\mathbf{x}, t) - \rho^*$ and $\delta c = c(\mathbf{x}, t) - c^*$ around their mean-field values (7). A nontrivial step in the derivation is to evaluate the agent’s lifetime under time-varying resource levels. Crucially, this brings about a history dependence,

which spans the agent’s lifetime (8), resulting in a system of *delayed* partial differential equations,

$$\delta\dot{c} = -\frac{p}{c^*} \delta c - \frac{p}{\rho^*} \delta\rho + D\nabla^2 \delta c \quad (10)$$

$$\begin{aligned} \delta\dot{\rho} = & \frac{\lambda}{\mu^*} \left[\delta c - \sqrt{\mu^*} \int_0^{\tau(\mu^*)} dt' \delta c(\mathbf{x}, t - t') \sin(2\sqrt{\mu^*} t') \right] \\ & + \sqrt{\lambda} \{ \xi(\mathbf{x}, t) - \xi[\mathbf{x}, t - \tau(\mu^*)] \}, \end{aligned} \quad (11)$$

with the unit variance Gaussian white noise $\langle \xi(\mathbf{x}, t) \xi(\mathbf{y}, t') \rangle = \delta^d(\mathbf{x} - \mathbf{y}) \delta(t - t')$. Although the hydrodynamics [Eqs. (10) and (11)] are time nonlocal, they are linear and can be integrated to yield the structure factor and number variance in terms of explicit integral expressions derived in Supplemental Material [43], showing good agreement with the results of stochastic simulation of the microscopic dynamics (Figs. 2 and 3). Rescaling lengths by individuals’ spacing $q' = q/\rho^*$ and $\ell' = \ell\rho^*$, the structure factor approaches, nonuniformly, a limiting scaling form. Apart from a vanishing boundary layer around the origin, it has $\mathcal{O}(\mu^*)$ corrections with the small q' expansion,

$$S(q) \simeq \frac{\pi^2 D \lambda^2}{4p} q'^2 + \frac{\mu^*}{c^*}, \quad (12)$$

signaling the onset of class I hyperuniformity [2] ($S \sim q'^2$) for any spatial dimension (see Fig. 2). Correspondingly, the number variance (1) approaches, nonuniformly, a limiting scaling form. Together with the nonperturbative large ℓ' corrections, it is captured by a composite matched asymptotic expansion, which in $d = 1$ reads as

$$\begin{aligned} \frac{\text{Var}(N_\ell)}{\langle N_\ell \rangle} \simeq & C \sqrt{\frac{D \lambda^2}{p}} \frac{1}{\ell'} \\ & + \frac{\mu^*}{c^*} + \frac{2\sqrt{\mu^*} c^*}{\pi p} \left[1 + \frac{\sqrt{\ell_D \rho^*}}{2\ell'} \left(e^{\frac{2\ell'}{\sqrt{\ell_D \rho^*}}} - 1 \right) \right], \end{aligned} \quad (13)$$

with $C = 0.827\dots$ provided by an explicit integral expression in Supplemental Material [43]. Equation (13) shows a hyperuniform scaling with the class-I exponent $\alpha = 1$ [2] that persists over the range $1 \ll \ell' \ll \sqrt{\ell_D \rho^*} \sim \mu^{*-1/2}$. The nonperturbative corrections in the second line of Eq. (13) cut off the asymptotic decay past the crossover scale $\ell' \gtrsim \sqrt{\ell_D \rho^*}$, which diverges in the asymptotic limit $\mu^* \rightarrow 0$ (see Fig. 3). The same behavior holds in all space dimensions.

Analyzing hyperuniformity—To gain insight into the origin of hyperuniformity, we first consider the case where feedback is eliminated by holding the resource field fixed at some arbitrary profile $c^*(\mathbf{x}) < c_{\text{crit}}$, where, as shown in

Supplemental Material [43], the dynamics map *precisely* to a Poisson point process, with mean density $\langle \rho \rangle = \lambda \tau [\mu^*(\mathbf{x})]$. Such a point pattern has the nonhyperuniform number variance $\text{Var}(N)/\langle N \rangle = 1$, for all values of ℓ . Indeed, integrating the density equation (11) with $\delta c = 0$ and vanishing initial conditions we find [53]

$$\delta \rho(\mathbf{x}, t) = \sqrt{\lambda} \int_0^{\text{Min}[t, \tau(\mu^*)]} \xi(\mathbf{x}, t - t') dt', \quad (14)$$

with the uncorrelated variance $\langle \delta \rho(\mathbf{x}, t) \delta \rho(\mathbf{y}, t) \rangle = \langle \rho \rangle \delta^d(\mathbf{x} - \mathbf{y}) \text{Min}[t/\tau^*, 1]$, coinciding with that of a Poisson process as soon as $t > \tau^*$. Importantly, close to criticality, $c^* \sim c_{\text{crit}}$, it takes a divergently long time $\tau^* \sim \mu^{*-1/2}$ (8) for density fluctuations to build up and saturate the Poisson statistics. This slow evolution of the density fluctuations also holds when considering the hydrodynamics [Eqs. (10) and (11)] with feedback, allowing for adiabatic elimination of the resource field (10),

$$\delta c(\mathbf{x}', t) \simeq -\frac{c^*}{\rho^*} \int \frac{d^d \mathbf{y}'}{(\sqrt{\ell_D \rho^*})^d} G\left(\frac{|\mathbf{x}' - \mathbf{y}'|}{\sqrt{\ell_D \rho^*}}\right) \delta \rho(\mathbf{y}', t) \quad (15)$$

(see Supplemental Material [43]). Here, we rescaled lengths by individuals' spacing $\mathbf{x}' \equiv \mathbf{x} \rho^*$, and $G(\mathbf{x})$ is the Green's function of the screened Poisson equation $(\nabla^2 - 1)G = -\delta^d(\mathbf{x})$, which in $d = 1$ takes the pure exponential form $G = e^{-|x|}/2$ and is otherwise exponentially decaying in any space dimension. The approximation (15) produces the leading ℓ^{-1} term on the right side of the asymptotics (13) (see Supplemental Material [43] for details) and provides the mechanism for hyperuniformity: over the long individuals' lifetime, the resource has enough time to adiabatically relax over the screening length $\sqrt{\ell_D \rho^*} \sim \mu^{*-1/2}$. Inserting this approximation in the density dynamics (11), this signifies that the resource mediates negative feedback interactions over diverging scales. The divergence of the screening length suggests an instructive analogy with the Coulomb gas, which is hyperuniform by virtue of unscreened Poisson interactions [1]. However, the present dynamics remain nonconservative and time delayed, so this analogy does not amount to an exact mapping.

A Fourier space analysis, detailed in Supplemental Material [43], provides a complementary picture and the nonperturbative corrections in the second line of Eq. (13). While, in general, density fluctuations are established over the prolonged individual's lifetime (14), large wavelength excitations $q' \ll 1$ are dominated by much higher frequencies $\tau^{*-1} \ll \omega$, where the time-delay hydrodynamics [Eqs. (10) and (11)] can be approximated by a time-local form. Even so, as long as the wavelengths are not too large $\mu^{*1/4} \ll q'$, the relevant frequencies are not too high $\omega \ll 1$, and the adiabatic approximation (15) still holds. Put together, the dynamics of wave modes $\delta \rho(\mathbf{q}, t)$ with

$\mu^{*1/4} \ll q' \ll 1$ follows the equation

$$\partial_{t'} \delta \rho \simeq -\left(\frac{\pi^2 D \lambda^2}{4} q'^2 + \frac{\mu^*}{c^*}\right)^{-1} \delta \rho + \sqrt{2\rho^*} \xi(\mathbf{q}, t'), \quad (16)$$

where we used rescaled time $t' = t/\tau^*$, reproducing the asymptotics (12) given above. This demonstrates that the mechanism for hyperuniformity here is structurally distinct from conserved-noise scenarios: the noise, being nonconservative, remains of order $\sim q^0$ at small q , while hyperuniformity is produced by a restoring kernel that diverges as q^{-2} (16), mirroring the diverging length scales of the resource-mediated interactions (15). At the largest wave modes $q' \ll \mu^{*1/4}$, the dynamics is dominated by high temporal frequencies ω , and the adiabatic approximation (15) breaks down. Accounting for these “inertial” effects in the dynamics of the resource leads to the asymptotic corrections in the second line of (13), which are expected to arise generically whenever hyperuniform correlations are mediated via a diffusive resource field. We conclude by noting the role of the time-delay terms in (11) for hyperuniformity. These terms, which arise naturally in our model, cannot be ignored in evaluating the number variance decay (13). In particular, as shown in Supplemental Material [43], they are essential to dampen small wavelength fluctuations and guarantee a bounded number variance. Nevertheless, the asymptotic hyperuniform behavior (13) can be reproduced starting with an alternative time-local birth-death type model that we introduce in Supplemental Material [43], where one has to impose a singular death rate.

Outlook—Hyperuniformity has been observed across diverse biological systems, from tissues [9–11] to macroecological patterns [12,13], yet the mechanisms driving its emergence remain unclear. We showed that hyperuniformity naturally arises in a generic class of population dynamics models, where death processes are coupled through indirect interactions between agents. Our formulation is inspired by established mechanisms of cell dynamics in tissues [42], yet these are general and reflect the sharp increase in agent longevity near a bifurcation. The divergence in longevity and the associated hyperuniform correlations are approached asymptotically when resource consumption rates are decreased. This differs from previous nonequilibrium settings, where hyperuniformity is reached by fine-tuning a control parameter to a critical value for a many-body phase transition [26–30]. In our model, the microscopic dynamics support a critical resource threshold where an individual's lifetime diverges, but the coupled many-body system does not exhibit a phase transition between distinct collective states. Instead, the system can only asymptotically approach this threshold, where the interaction range of the negative feedback diverges and suppresses large-scale density fluctuations. Our results, therefore, show that hyperuniformity need not be a

signature of many-body criticality, and the present mechanism is complementary to, rather than belonging to, previously studied classes of nonequilibrium critical phenomena such as absorbing-state transitions. These results open a new avenue for mechanistic investigations into the origins of hyperuniformity in natural systems by identifying its underpinnings in nonconservative population dynamics.

More broadly, our dynamical framework highlights the possibility that many agents can collectively self-tune to critical states by interacting through a spatial field. Our analysis centered on the saddle-node bifurcation, a fundamental transition that captures discontinuous shifts in system dynamics. Other bifurcations have also been explored in biological contexts; for example, critical tuning to a Hopf bifurcation has been implicated in the mechanisms underlying hearing [54]. Extending such frameworks to include spatial dynamics, as we have done here, may reveal new emergent phenomena with potentially important physiological consequences [55].

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Data availability—The simulation code used to generate the numerical results in this work is publicly available [56].

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