

Measurement-Driven Quantum Advantages in Shallow Circuits

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Quantum advantage schemes probe the boundary between classically simulatable and classically intractable quantum dynamics. We explore the impact of midcircuit measurements on the computational power of quantum circuits. To this effect, we focus on quantum sampling and introduce a constant-depth measurement-driven approach for efficiently sampling from a broad class of commuting diagonal quantum circuits and associated structured phase states, previously requiring polynomial-depth unitary circuits. By interleaving midcircuit measurements with feedforward in randomized “fan-out staircases,” our dynamical circuits bypass Lieb-Robinson light-cone constraints, enabling global entanglement with flexible auxiliary qubit usage on bounded-degree lattices (e.g., two-dimensional grids). The generated phase states exhibit random-matrix statistics and anticoncentration comparable to fully random architectures. We further demonstrate measurement-driven feature maps that distinguish phases of an extended Su-Schrieffer-Heeger model from random eigenstates in a quantum machine-learning benchmark (reservoir computing). Technologically, our results harness midcircuit measurements to realize quantum advantages on bounded-degree hardware with a favorable topology. Conceptually, they provide complexity-theoretic support for quantum speedups by midcircuit measurements.

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A leading paradigm for demonstrating quantum advantage is quantum sampling: implementing a quantum evolution and sampling from its output distribution that is believed to be classically intractable under standard assumptions [1]. Notable examples include boson sampling [2–6] and random circuit sampling [7–17]. Paradigmatically, these tasks are central to clarifying the precise fine print of quantum advantage, but they also expose a core tension: the deep, highly connected circuits demanded conflict with the constraints of noisy, locally connected hardware. This tension motivates the pursuit of hardware-efficient routes to quantum advantage.

Instantaneous-quantum-polynomial-time (IQP) circuits—commuting diagonal circuits whose output probabilities are complex-temperature partition functions—form a central model [9–13,18,19]. The argument for their classical sampling hardness connects the sampling task to the #P-hard problem of estimating single output probabilities, as an efficient sampler would imply an efficient estimator (leading to an unlikely collapse of the polynomial hierarchy). This connection requires the distribution to exhibit

anticoncentration, where probabilities are not dominated by a few outcomes, precluding trivial classical strategies based on guessing dominant bit strings. Worst-to-average reductions extend the hardness from contrived instances to typical random circuits used in experiments [1]. Random, long-range IQP circuits at polynomial depth satisfy these criteria, providing strong evidence for their hardness [11,12]. That said, experimentally realizing and witnessing an IQP sampling advantage faces two key challenges: limited hardware connectivity makes the long-range interactions needed for hardness depth expensive [20], and noise that grows with depth renders noisy IQP circuits—even with additional controlled-NOT (CX) layers—classically simulable beyond a constant depth under natural noise models [21,22].

In this Letter, we suggest a different approach using midcircuit measurements, which not only overcomes the above challenges but is also independently interesting. Basically, in our Letter, we show that constant-depth quantum circuits are substantially more powerful computationally when equipped with midcircuit measurements compared to the unitary setting. This regime of noisy intermediate-scale quantum (NISQ) devices with midcircuit measurements has been dubbed “NISQ+” [23]: We present a NISQ+ and NISQ computational separation under plausible assumptions, highlighting the computational power of measurements along the way. To put this into context, dynamic circuits with measurements and feedforward have shown great potential in implementing certain deep quantum circuits and preparing highly entangled states [24–38].

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By conditioning operations on measurement outcomes, these circuits create nonlocal “shortcuts” that enable the rapid generation of complex, long-range entanglement at minimal depth [37–41]. Inspired by these advances, we introduce a measurement-driven fan-out staircase architecture that implements the dense, long-range interactions required for IQP computational hardness at constant depth, even on simple two-dimensional (2D) hardware. Importantly, we show that the output distributions from these shallow circuits retain key signatures of complexity (such as anticoncentration), providing strong evidence for their classical hardness, while significantly relaxing the experimental requirements of depth and connectivity.

Beyond establishing a new route to quantum sampling advantages, our measurement-driven technique provides strongly entangled feature maps for quantum machine learning. We demonstrate its effectiveness in quantum reservoir computing, where a fixed quantum evolution acts as a feature map, while only a simple classical readout is trained [42,43]. Even under a realistic noise model, our measurement-based reservoir—which leverages global entanglement created at constant depth—accurately classifies distinct topological phases, surpassing conventional local reservoirs.

The upshot of the present Letter is as follows. Midcircuit measurements are known to enable the preparation of symmetry-protected topological states [30,31] and long-range entangled states [26,44] that elude constant-depth unitary circuits. They also underpin quantum advantage in certain random shallow Clifford architectures [33]. Extending these insights has proved difficult. Here we provide some of the clearest algorithmic-level evidence that measurements with real-time feedforward enhance shallow quantum algorithms. We do so by compressing dense IQP circuits to constant depth and demonstrating robust anti-concentration, noise resilience, and a provable expressivity advantage in a quantum machine-learning benchmark.

Measurement-prepared Hamiltonian phase states—The core of our protocol is a constant-depth, measurement-driven method for implementing the dense, many-body Pauli-Z rotations that constitute computationally hard IQP circuits. While one could conventionally generate such a term by conjugating a single-qubit rotation with a deep unitary CX circuit, this requires polynomial depth on locally connected hardware.

Our measurement-driven fan-out staircase (FS) blocks—layered circuits of fan-out gates that build global correlations—achieve this same goal in constant depth. The mechanism of a block involves three conceptual steps. First, we locally entangle system qubits with nearby auxiliary qubits using a short pattern of nearest-neighbor CX gates. Next, we measure the auxiliaries in the X basis. An X -basis outcome $m_k = 1$ is equivalent to having applied a Pauli-Z operator to that auxiliary qubit just before its measurement. The resulting Pauli-Z correction string on the

system qubits is determined by tracking how this “hypothetical” Z operator is transformed under conjugation by the full entangling unitary. This transformation—calculated by iteratively applying commutation rules [e.g., $CX(c, t)Z_t = Z_c Z_t CX(c, t)$] through the circuit’s layers—establishes a linear map from the measurement-outcome vector $\mathbf{m}$ to the correction string, captured by a binary transfer matrix $\mathcal{T}$. The exact FS construction is characterized by a depth parameter $\mathfrak{D}$ and connectivity parameters $\mathbf{r}_1, \mathbf{r}_2$, as detailed in the End Matter (Algorithm 1) and Supplemental Material [45] and illustrated in Fig. 1(a).

By interleaving $\mathfrak{R}$ layers of FS blocks and single- Z rotations, the full circuit is

$$C_{\text{MD}} = \bigotimes_{j=1}^n e^{i\vartheta_{\mathfrak{q}+1,j}Z_j} \left(\prod_{i=1}^{\mathfrak{R}} \text{FS}_i \bigotimes_{j=1}^n e^{i\vartheta_{i,j}Z_j} \right), \quad (1)$$

We absorb the Pauli-frame corrections by classically updating the subsequent rotation angles via $\hat{\vartheta}_{i+1,j} := \vartheta_{i+1,j} + (\pi/2)(\mathcal{T}^{(i)}\mathbf{m}^{(i)})_j \pmod{2\pi}$, as depicted in Fig. 1(c). Since an FS block conjugates single-qubit Z ’s into Z

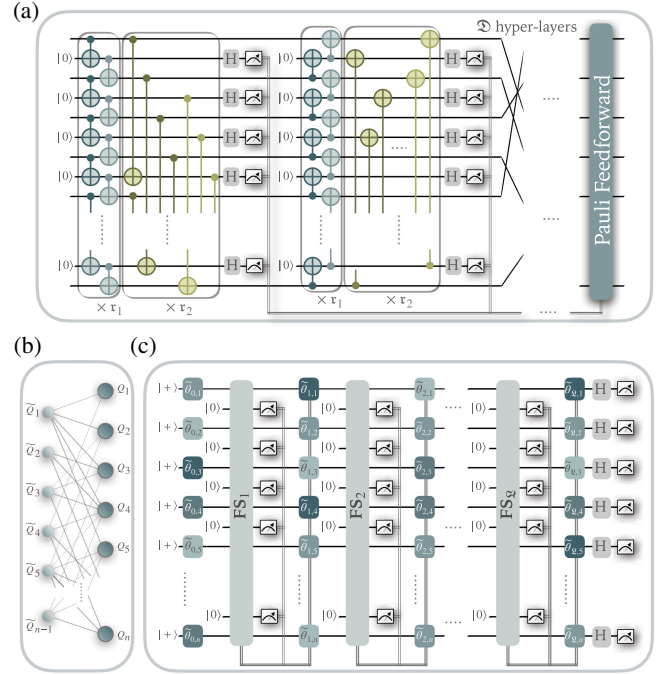


FIG. 1. Schematic of measurement-based fan-out staircases for dense IQP sampling. (a) Multiloop fan-out staircases (alternating-direction ladders) generated by random dynamic circuits with measurement and feedforward. (b) Conceptual bipartite (system-auxiliary) coupling graph. Our geometrically local, bounded-degree 2D layouts are subgraphs of this schematic. (c) Constant-depth implementation of dense IQP sampling via interleaved phase rotations and randomized fan-out staircases, where measurement outcomes inform subsequent corrections. Staircase construction and transfer-matrix feedforward are detailed in the End Matter.

strings, the constant-depth protocol effectively realizes an IQP circuit $\mathcal{C}_{\mathbf{A},\boldsymbol{\theta}} = \exp(i \sum_{i=1}^s \theta_i \otimes_{j=1}^n Z^{\mathbf{A}_{ij}})$ and prepares the Hamiltonian phase-state vectors $|\psi_{\mathbf{A},\boldsymbol{\theta}}\rangle = \mathcal{C}_{\mathbf{A},\boldsymbol{\theta}}|+\rangle^{\otimes n}$. These are defined by a binary matrix $\mathbf{A} \in \{0,1\}^{s \times n}$, hereafter termed the “architecture matrix,” which our protocol constructs row by row from the Pauli-Z string generated by FS blocks, generating random IQP architectures exhibiting anticoncentration.

Proposition 1: Anticoncentration—Fully random architectures $\mathbf{A} \in \mathbb{Z}_2^{s \times n}$ achieve anticoncentration with $s \in \mathcal{O}(n)$.

Restricting the angles leads to our first main result.

Theorem 1: Measurement-driven short IQP circuits—Fix $k \geq 1$. For angles $\boldsymbol{\theta} \in \{\ell\pi/2^k\}_{\ell=0}^{2^k-1}$, the measurement-driven circuit $\mathcal{C}_{\text{MD}}$ implements a dense k -local IQP circuit.

Here, a “ k -local” IQP circuit consists of commuting Z interactions involving at most k qubits (i.e., weight $\leq k$). We call it dense if an n -independent constant fraction of admissible Z strings appear with $\Theta(1)$ angles. In particular, the diagonal gates $e^{i\pi Z/8}$ and $e^{i\pi ZZ/4}$ arise from the synthesis with $k = 3$ and $k = 2$, respectively, using a constant number of FS blocks on bounded-degree 2D layouts. To quantitatively evaluate the statistical randomness of binary circuit architectures, we propose the following criterion:

Criterion 1— $\mathbf{A} \in \mathbb{Z}_2^{s \times n}$ is statistically random if (i) the eigenvalue distribution of the standardized covariance matrix $(2\mathbf{A} - 1)(2\mathbf{A} - 1)^T/s$ matches Marchenko-Pastur law [51]; (ii) row and column Hamming weights and pairwise distances follow binomial distributions; and (iii) $\geq 90\%$ of random submatrices exhibit ranks within 2 of full rank over the Galois field of order 2.

Remark 1—Our randomness diagnostics probe typicality only—analogue to statistical validation in boson and random circuit sampling—and do not imply a worst-to-average reduction [1]. Such a reduction would require distribution-specific random self-reducibility, which we leave open.

These conditions guarantee spectral, statistical, and algebraic universality [51–53]. Using Criterion 1, we numerically compare measurement-driven randomized FS against random CX circuits without auxiliary qubits under both 2D grid and all-to-all connectivity. In the 2D configuration, system and auxiliary qubits alternate, with random directed Hamiltonian paths generated according to Algorithm 2 in the End Matter, respecting local constraints [Fig. 2(a)]. Figure 2(b) shows eigenvalue distributions from measurement-driven FS closely follow the Marchenko-Pastur law, unlike random local CX circuits at comparable depths. Numerical results [Figs. 2(c) and 2(d)] indicate that circuits without auxiliary qubits require polynomial (2D grid) or logarithmic (all-to-all) depths to satisfy Criterion 1. In contrast, measurement-driven FSs meet this criterion at constant depth, with parameters $\mathbf{r}_1 = \mathbf{r}_2 = 1$, and $\mathfrak{D} = 2$ for all-to-all connectivity and $\mathfrak{D} = 4$ for 2D grid connectivity, even in the large-scale regime. Subsequent IQP simulations employ the 2D grid connectivity, interleaving

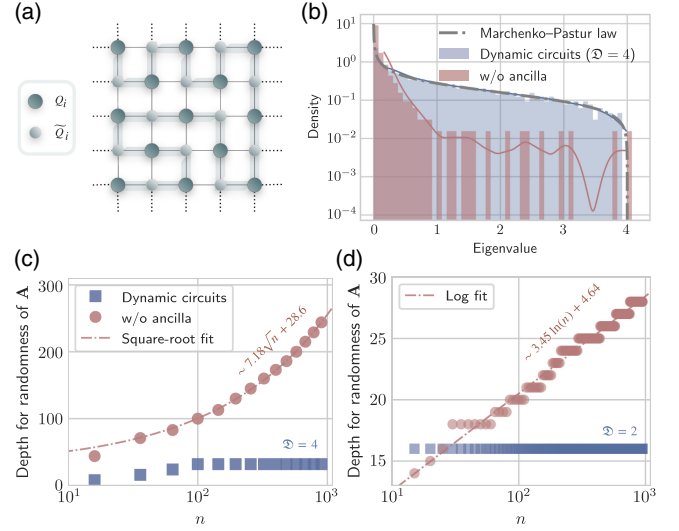


FIG. 2. (a) Checkerboard layout of alternating system and auxiliary qubits on a 2D square lattice, with a sampled Hamiltonian path covering all qubits. (b) Comparison of eigenvalue distributions from randomized fan-out staircases with the theoretical Marchenko-Pastur law for a representative 41×41 -qubit system. (c),(d) Minimal circuit depths required to satisfy Criterion 1, contrasting measurement-driven fan-out staircases (blue squares) with circuits without auxiliary qubits (red circles), for (c) 2D grid and (d) all-to-all connectivity.

single-qubit Z rotations with two FS layers (each with $\mathfrak{D} = 2$).

Quantum circuit cost quantifies the minimal resources required to implement a given unitary operation by integrating contributions from local gate interactions, where the entanglement entropy generated across suitable 1D bipartitions provides a fundamental lower bound for this cost [54,55]. We extend this to 2D lattices by considering horizontal (ℓ_x) and vertical (ℓ_y) cuts, yielding the summed entanglement entropy

$$\xi(\psi) := \frac{1}{\eta} \left(\sum_{\ell_x} S(\psi, \ell_x) + \sum_{\ell_y} S(\psi, \ell_y) \right), \quad (2)$$

where η is a geometry-dependent constant. ξ provides a lower bound for the geometrically local circuit cost required to prepare a given state from a product in two dimensions (see Supplemental Material [45]). We benchmark against ξ_{lin} , the average for output states of deep, linear-depth random CX circuits ($\sim 6n$). Although such constant-depth circuits on 2D lattices are not expected to achieve genuine volume-law entropy across each cut—particularly for larger systems—Fig. 3(a) shows that measurement-driven IQP circuits nonetheless achieve $\xi/\xi_{\text{lin}} \approx 1$, matching the entangling power of linear-depth CX circuits. In contrast, conventional depth-matched IQP circuits without auxiliary systems fall significantly short.

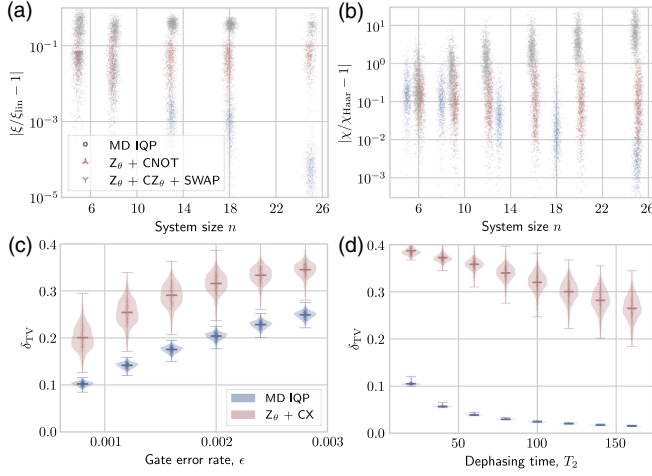


FIG. 3. Comparison of measurement-driven IQP sampling (blue) with standard IQP sampling (red, gray) at matched constant two-qubit depth on a 2D nearest-neighbor grid. (a) Relative entanglement-based circuit cost ξ/ξ_{lin} for fixed-depth circuits $\mathfrak{I} = 2$, $\mathfrak{D} = 2$ versus system size. (b) Collision probability ratio χ/χ_{Haar} versus system size. (c) Total variation distance between noisy and ideal output distributions under gate depolarizing errors. (d) Total variation distance under pure dephasing noise assuming fixed gate duration.

The collision probability, $\chi := \mathbb{E}_C[\sum_x p_C(x)^2]$, quantifies quantum circuit anticoncentration. For sufficiently deep Haar-random circuits, the collision probability approaches $\chi_{\text{Haar}} = 2/(2^n + 1)$. Anticoncentration requires the ratio χ/χ_{Haar} to remain bounded by a small constant [56], a condition underpinning complexity arguments and benchmarks like cross-entropy benchmarking [7,57]. Figure 3(b) compares collision probabilities between measurement-driven IQP circuits and standard IQP circuits without auxiliary qubits at comparable depths. Unlike standard IQP circuits, measurement-driven IQP circuits exhibit a decreasing collision probability ratio with increasing system size, indicating robust anticoncentration at constant depth and minimal sensitivity to connectivity constraints.

Building upon (i) Proposition 1, (ii) our 2D numerics, and (iii) Lemma 8 of Ref. [58], which establishes $\#P$ hardness for depth- $\mathcal{O}(n)$ random circuits over $\{e^{i\pi Z/8}, e^{i\pi ZZ/4}, \text{SWAP}\}$, we make the following observation. On bounded-degree 2D layouts with an interleaved system-auxiliary placement, our constant two-qubit depth protocol implements a subclass of dense k -local diagonal layers, including the gates $e^{i\pi Z/8}$ and $e^{i\pi ZZ/4}$ central to that hardness result (Theorem 1). Since the SWAP permutations also required by the full construction in Ref. [58] are realizable in our model via measurement-mediated routing without added unitary depth, our scheme therefore realizes a subset of instances from this known hard circuit family at constant depth. This serves as strong motivation—not a formal worst-to-average reduction [8,11,15]—for the more permissive conjecture that follows.

Conjecture 1: Average-case hardness—For a broad class of connected bounded-degree bipartite lattices in dimension $d \geq 2$, there exists a constant two-qubit depth, measurement-driven IQP family—realized via randomized fan-out staircases on n qubits with at most $\mathcal{O}(n \log n)$ auxiliaries and independent identically distributed (i.i.d.) phases in $[0, 2\pi)$ —whose output distributions anticoncentrate and for which, on a non-negligible fraction of instances, at least one fixed output probability is $\#P$ -hard to approximate within a constant multiplicative factor.

Remark 2—Our numerics suggest $\mathcal{O}(n)$ auxiliaries often suffice on 2D grids; the $\mathcal{O}(n \log n)$ budget in the conjecture is a conservative high-probability allowance to guarantee strong pseudorandomness across a wider range of bounded-degree layouts (e.g., by modestly widening fan-out targets or layering additional auxiliary resources), at constant two-qubit depth.

Implication—Assuming Conjecture 1 and noncollapse of the polynomial hierarchy, the standard Stockmeyer route implies that approximate sampling from this family is classically intractable up to inverse-polynomial total variation error [1,59].

Measurement-driven IQP circuits exhibit notably enhanced noise resilience compared to conventional IQP circuits, which typically become classically simulable under constant gate noise beyond certain depth thresholds [21,22]. By compressing a family of polynomial-depth circuits into constant depth, measurement-driven circuits evade these simulation methods typically applicable to deeper noisy circuits.

To quantify this advantage, we numerically evaluate the total variation distance $\delta_{\text{TV}}(p, \tilde{p}) := \sum_x |p(x) - \tilde{p}(x)|/2$ between the ideal output distribution $p(x)$ and the noisy distribution $\tilde{p}(x)$ for standard and measurement-driven IQP circuits on a 6×6 2D grid, assuming ideal measurements. We first analyze gate depolarizing noise at varying error rates [Fig. 3(c)]. Measurement-driven circuits consistently yield lower δ_{TV} compared to standard IQP circuits, primarily due to efficient long-range interactions without additional SWAP gates. Moreover, measurement-driven circuits exhibit narrower δ_{TV} distributions, as local errors propagate into correlated global errors through feedforward corrections.

In a second analysis, we omit gate errors and focus exclusively on decoherence characterized by the dephasing coherence time T_2 (assuming an infinite T_1). The total variation distance δ_{TV} is expected to saturate exponentially with circuit duration Δt according to $\delta_{\text{TV}}(\Delta t) \approx \delta_{\text{TV},\infty}(1 - e^{-\kappa \Delta t})$, where $\delta_{\text{TV},\infty}$ is the asymptotic distance between the ideal and uniform distributions, and κ is an effective dephasing rate. Numerical results (assuming 200 ns per CX layer) in Fig. 3(d) show that depth compression in measurement-driven circuits significantly reduces δ_{TV} , demonstrating substantial mitigation of decoherence effects.

Measurement-based quantum reservoir computing (RC)—Reservoir computing (RC) embeds inputs using a fixed dynamical

system while training only a simple classical readout [60,61]. In quantum RC, the reservoir is a fixed quantum evolution whose intrinsic dynamics generate rich features, with measurements inducing nonlinearity and entanglement enabling long-range mixing [42,43]. In our scheme, interleaving single-qubit gates with FS and FS[†] forms a constant-depth reservoir where midcircuit measurements and Pauli-frame feedforward create input-dependent nonlinear feature maps.

We benchmark our measurement-based quantum reservoir to an extended bosonic Su-Schrieffer-Heeger chain, $\mathcal{H} = \sum_{\mu \in \{x,y,z\}} \gamma_{\mu} (J \sum_{i=1}^{n/2} \sigma_{2i-1}^{\mu} \sigma_{2i}^{\mu} + J' \sum_{i=1}^{n/2-1} \sigma_{2i}^{\mu} \sigma_{2i+1}^{\mu})$, where $\gamma_{\mu} = 1 + (\delta - 1)\delta_{\mu,z}$. This model features three distinct phases—trivial, topological, and symmetry broken—depending on the values of J , J' , and δ [62–64]. For each phase, we sample 500 eigenstates uniformly from the lowest 20 levels, apply weak local perturbations, and evolve under the candidate reservoirs. Only local spin observables $\langle \sigma_i^z \rangle$ are measured for each reservoir (8192 shots), with a simulated symmetric per-qubit readout error of 0.5%.

We compare four reservoirs: (i) a random anisotropic Heisenberg reservoir, $\mathcal{H}_H = \sum_{\langle i,j \rangle} \sum_{\mu} J_{i,j}^{\mu} \sigma_i^{\mu} \sigma_j^{\mu}$, where $\mu \in \{x, y, z\}$ labels spin components; (ii) a random transverse-field Ising reservoir, $\mathcal{H}_{\text{TFI}} = \sum_{\langle i,j \rangle} J_{i,j} \sigma_i^z \sigma_j^z + \sum_i h_i \sigma_i^x$; (iii) a random transverse-field XY reservoir $\mathcal{H}_{\text{XY}} = \sum_{\langle i,j \rangle} (J_{i,j}^x \sigma_i^x \sigma_j^x + J_{i,j}^y \sigma_i^y \sigma_j^y) + \sum_i h_i \sigma_i^z$; and (iv) a measurement-based multibody XY reservoir $\mathcal{H}_A = \sum_{i=1}^s (c_i^x \prod_{j \in A_i} \sigma_j^x + c_i^y \prod_{j \in A_i} \sigma_j^y)$, with all coupling coefficients $J_{i,j}^{x,y,z}$, h_i , and $c_i^{x,y,z}$ drawn i.i.d. from $\mathcal{N}(0, 1/2)$, $s = 16$, and A encodes the measurement-enabled architecture. All reservoirs use 16 system qubits on 2D-square and hexagonal lattices and undergo ten identical Floquet cycles generated by their Hamiltonian terms. The classification of phases is performed using a support vector machine with a radial basis function kernel [65,66].

The classification accuracy versus Floquet cycle for the reservoirs is shown in Fig. 4, with each violin distribution obtained from 50 independent random realizations of A and coupling strengths. In contrast to local reservoirs (i)–(iii), the measurement-driven multibody reservoir rapidly reaches high accuracy at short times. However, removing the feedforward adaptivity leads to a marked drop in performance. From an expressivity standpoint, local-Hamiltonian reservoirs with constant Floquet cycles cannot approximate certain measurement-based feature maps even within inverse-polynomial error, a distinction formalized in Theorem 2 (proof and additional benchmarks in Supplemental Material [45]).

Theorem 2: Expressivity separation for local Hamiltonian reservoirs—Let G be a connected bounded-degree graph with n qubits and fix a Floquet period T . There exist encoded input state vectors $|\psi_0\rangle, |\psi_1\rangle$, a two-outcome observable O , and a measurement-driven Floquet reservoir $\mathcal{R}_{\text{MD}}^{(T)}$ such that

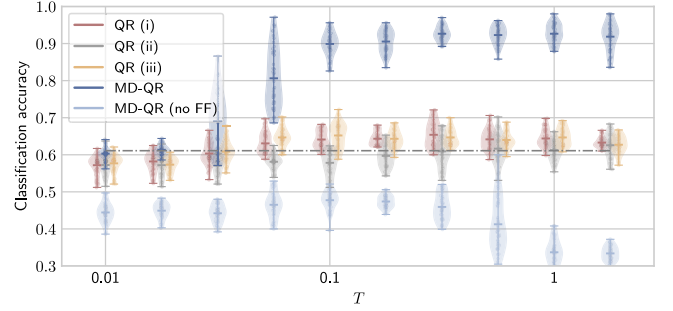


FIG. 4. Phase-classification accuracy versus Floquet cycle for different reservoirs. Each violin distribution is obtained from 50 random realizations of architectures and coupling strengths. The measurement-driven multibody XY reservoir achieves high accuracy at short times, outperforming local reservoirs (i)–(iii), while removing feedforward (FF) adaptivity significantly reduces performance. QR denotes quantum reservoir; MD-QR denotes measurement-driven quantum reservoir.

$$\left| \text{Tr}[\mathcal{O} \mathcal{R}_{\text{MD}}^{(T)}(|\psi_0\rangle)] - \text{Tr}[\mathcal{O} \mathcal{R}_{\text{MD}}^{(T)}(|\psi_1\rangle)] \right| = \Omega(1), \quad (3)$$

while any nearest-neighbor Hamiltonian-based Floquet evolution on the same graph G , with the same total evolution time, achieves a separation at most exponentially small in n .

Discussion and outlook—Our measurement-driven framework incorporates extensive interactions between system and auxiliary qubits to broaden the scope of dynamic quantum circuits. In contrast to depth-heavy random-unitary experiments [7], our dynamic-circuit scheme achieves anticongestion at constant two-qubit depth via midcircuit measurements with modest auxiliary overhead. Beyond sampling, its associated Hamiltonian phase-state preparations can enhance cryptographic protocols [67]. An extension to fermion sampling is also promising, as midcircuit measurements can introduce the non-Gaussian effects crucial for computational hardness [14]. Furthermore, our measurement-based feature maps can advance quantum machine-learning applications by enabling constant-depth global operations that capture data structures inaccessible to local unitary kernels [68–70].

The primitives our protocol requires—midcircuit measurements, active reset, and low-latency feedforward—have been demonstrated on leading platforms. Superconducting processors enable real-time conditional logic via parity measurements and fast (sub- μs –few- μs) resets [71–75]. Trapped-ion systems, benefiting from long coherence times, support conditional operations in modular quantum CCD architectures and through networked feedforward [29,76–78], while neutral-atom arrays have achieved site-selective conditional branching [79,80]. These milestones confirm the availability of the necessary NISQ+ primitives. Our classical postprocessing, a sparse Pauli-frame parity update, adds negligible latency overhead compared to typical gate times and coherence windows.

Nevertheless, midcircuit measurement errors (below 1% in leading systems) and reset infidelity, measurement-induced crosstalk, and latency-induced idle dephasing remain practical constraints; these effects have been directly characterized across platforms [81–83]. In our constant-depth realization, each hyperlayer executes a global Pauli-frame parity update, so performance is set by the effective midcircuit error rate and the feedback delay rather than additional quantum depth. Consistent with Figs. 3 and 4, we expect a graceful degradation under realistic error levels. With sub- μ s feedback and percent-level readout now available, near-term demonstrations on moderate-size dynamic-circuit hardware appear feasible, while scalability will hinge on continued improvements in measurement fidelity and latency.

Regarding quantum error correction, our shallow-circuit techniques offer a promising route to enhance quantum coding protocols based on low-depth random circuits [84]. Our approach generalizes dynamic-circuit methods for preparing toric code ground states with long-range topological order [29,85] and may enable efficient realization of broader classes of quantum low-density parity-check codes [86], thereby providing a viable blueprint for scalable and fault-tolerant quantum-information processing.

Conclusion—We introduced an explicit, measurement-based scheme achieving dense IQP sampling at constant depth via randomly generated fan-out staircases, bypassing conventional light-cone constraints to enable global entanglement, robust anticoncentration, and high noise resilience. Additionally, we demonstrated that measurement-based circuits effectively serve as quantum reservoirs, efficiently distinguishing topological phases from random eigenstates. Overall, our results highlight that midcircuit measurements and feedforward can significantly enhance quantum algorithm performance at shallow depth—alleviating connectivity constraints and opening pathways for NISQ+ quantum advantages as next steps to be taken toward achieving fault-tolerant application-scale [17] quantum computers.

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Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

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End Matter

Protocol details: Measurement-driven fan-out staircases—Throughout, we arrange system and auxiliary qubits in an alternating pattern and build randomized fan-out staircase blocks by interleaving short entangling patterns with midcircuit measurements; the detailed scheme is given in Algorithm 1. Theoretically, an FS block maps a single-qubit Z phase into a many-body Z rotation via conjugation, $\text{FS}_i e^{i\theta_j Z_j} \text{FS}_i^\dagger = \exp(i\theta_j \otimes_{\ell=1}^n Z_\ell^{A_{i,\ell}})$. For state preparation from $|+\rangle^{\otimes n}$, this simplifies: $\text{FS}_i^\dagger$ acts trivially. In each FS ladder (forward-backward along a chosen path) we apply nearest-neighbor CXs between systems and auxiliaries, measure all auxiliaries in the X basis, and absorb the induced Pauli- Z string on the systems via Pauli-frame feedforward using the binary transfer matrix $\mathcal{T}$. We denote the forward-backward ladder at round j by $\text{FL}_j^{(\uparrow/\downarrow)}$ [optionally, $\text{FL}_j^{(\uparrow)} := \prod_{i=1}^{n-1} \text{fan-out}(Q_{\ell_i}^{(j)} \rightarrow T_{\ell_i}^{(j,\uparrow)})$] and a staircase with $2\mathfrak{D}$ measurement rounds by $\text{FS} := \prod_{j=0}^{\mathfrak{D}-1} (\text{FL}_{\mathfrak{D}-j}^{(\downarrow)} \text{FL}_{\mathfrak{D}-j}^{(\uparrow)})$. A staircase with $2\mathfrak{D}$ measurement rounds thus has constant two-qubit depth $4\mathfrak{D}$ (independent of n), while the total

two-qubit gate count is $\mathcal{O}(\mathfrak{D}n)$. In this Letter, we focus on the minimal-auxiliary setting with $(n-1)$ auxiliaries; however, adding auxiliary-auxiliary couplings extends the scheme to richer interaction patterns without changing the constant-depth scheduling. This dynamic construction circumvents Lieb-Robinson light-cone limits and rapidly generates long-range entanglement on bounded-degree layouts.

Proposition 2: Measurement-driven fan-out staircases—Implementing the randomized fan-out staircase circuits described above without auxiliary qubits necessitates a circuit depth of at least $\Omega(\log n)$ under all-to-all connectivity. A matching logarithmic depth can be achieved in the simplest scenario, where each fan-out targets a fixed contiguous set of qubits (via Ref. [87]).

Proof—Implementing randomized fan-out staircases without auxiliary qubits demands each input qubit’s information to influence $\mathcal{O}(n)$ qubits to achieve global entanglement. Local two-qubit gates impose constraints on information propagation, allowing at most an exponential growth in the range of influence per layer. Thus, after d layers, an input qubit can influence at most $\mathcal{O}(2^d)$ qubits. To ensure global entanglement [influencing $\mathcal{O}(n)$ qubits], the following

$$2^d \geq \mathcal{O}(n) \Rightarrow d = \Omega(\log n) \quad (\text{A1})$$

Algorithm 1. Measurement-based randomized fan-out staircases.

```

1: Input: System qubits  $\{Q_i\}_{i=1}^n$ , auxiliary qubits  $\{\tilde{Q}_j\}_{j=1}^{n-1}$ ,
   constants  $\mathbf{r}_1, \mathbf{r}_2$ , and number of paths  $\mathfrak{D}$ .
2: for  $d = 1, \dots, \mathfrak{D}$  do ▷ Iterate over  $\mathfrak{D}$  directed paths
3:   Select a random directed qubit path  $\{Q_{\ell_i}\}_{i=1}^n, \{\tilde{Q}_{\ell_j}\}_{j=1}^{n-1}$ .
4:   for  $r = 1, \dots, \mathbf{r}_1$  do
5:     Apply  $\text{CX}(Q_{\ell_i}, \tilde{Q}_{\ell_i})$  and  $\text{CX}(\tilde{Q}_{\ell_i}, Q_{\ell_{i+1}})$  sequentially
       along the path.
6:   end for
7:   for  $r = 1, \dots, \mathbf{r}_2$  do
8:     for each system qubit  $Q_{\ell_i}$  do
9:       Randomly select an auxiliary qubit  $\tilde{Q}_{\ell_j}$ ,
          $0 \leq j - i < n/\mathbf{r}_2$ , without replacement;
         apply  $\text{CX}(Q_{\ell_i}, \tilde{Q}_{\ell_j})$ .
10:    end for
11:    for each auxiliary qubit  $\tilde{Q}_{\ell_j}$  acted on do
12:      Randomly select system qubit  $Q_{\ell_k}$ ,
         $\mathbf{r}_1 < k - j < n/\mathbf{r}_2$ , without replacement;
        apply  $\text{CX}(\tilde{Q}_{\ell_j}, Q_{\ell_k})$ .
13:    end for
14:  end for ▷ Forward fan-out ladder
15:  Measure auxiliary qubits in  $X$  basis; store outcomes.
16:  Reverse indexing order:  $Q_{\ell_i} \rightarrow Q_{\ell_{n+1-i}}, \tilde{Q}_{\ell_j} \rightarrow \tilde{Q}_{\ell_{n-j}}$ .
17:  Repeat steps 4–15 using reversed indices.
18: end for ▷ Backward fan-out ladder
19: Feedforward: Compute transfer matrices  $\{\mathcal{T}^{(i)}\}_{i=1}^{2\mathfrak{D}}$  and
   apply corresponding Pauli- $Z$  corrections to system qubits.
20: Output: A  $2\mathfrak{D}$ -layer fan-out staircase circuit comprising
    $\mathcal{O}(\mathfrak{D}n)$  fan-out gates, each targeting  $\mathcal{O}(\mathbf{r}_1 + 4\mathbf{r}_2)$  qubits.
    
```

must hold. Hence, the logarithmic depth lower bound is fundamental and unavoidable.

In the special scenario where each fan-out gate in the fan-out ladder targets a fixed, contiguous set of qubits—specifically, for each control qubit Q_{ℓ_i} , the target set is $T_{\ell_i} = \{Q_{\ell_{i+1}}, Q_{\ell_{i+2}}, \dots, Q_{\ell_{i+k}}\}$ for a finite integer k —the corresponding CX circuit can be represented as a matrix $\mathbf{M} \in \text{GL}(n, 2)$ [general linear group (GL)] with entries equal to 1 only on the diagonal and within a band of width k around the diagonal. Under these conditions, the original $\mathcal{O}(n)$ -depth fan-out ladder circuit can be efficiently compiled into an equivalent $\mathcal{O}(\log n)$ -depth circuit without auxiliary qubits, as follows:

Initially, apply sequential CX gates between neighboring qubits: explicitly, for $i = 1, 2, \dots, n-1$, apply $\text{CX}(Q_{\ell_i}, Q_{\ell_{i+1}})$. Subsequently, apply CX gates between qubits separated by distance k in reverse order: specifically, for $i = n-k, n-k-1, \dots, 1$, apply $\text{CX}(Q_{\ell_i}, Q_{\ell_{i+k}})$. This procedure yields an equivalent circuit to the original contiguous fan-out ladder. This compiled circuit consists of $(k+1)$ sequential CX ladders, each compilable into $\mathcal{O}(\log n)$ depth without auxiliary qubits, as demonstrated in Ref. [87]. Consequently, the complete fan-out ladder can be realized as a CX circuit of depth $\mathcal{O}(k \log n)$. ■

Reference [30] shows that other FSs realized by constant-depth dynamic circuits also compile to the logarithmic depth

bound on all-to-all hardware, albeit with more complex constructions.

Each of the $2\mathfrak{D}$ fan-out ladders involves midcircuit measurements yielding binary outcomes $\{m_j\}_{j=1}^{n-1}$. A measurement outcome $m_j = 1$ is equivalent to the scenario of applying a Pauli-Z operator to the $\tilde{Q}_j$ before its Hadamard measurement, effectively converting the outcome to 0. Using propagation relations $CX(Q_i, Q_j)Z_j = Z_i Z_j CX(Q_i, Q_j)$, these hypothetical Z operations translate into feedforward corrections exclusively on system qubits. The necessary corrections after measurements can be succinctly expressed using an $n \times (n-1)$ binary transfer matrix $\mathcal{T}$ as

$$\bigotimes_i Z^{(\mathcal{T}\mathbf{m})_i}, \quad \text{with} \quad (\mathcal{T}\mathbf{m})_i := \sum_j \mathcal{T}_{i,j} m_j \pmod{2}. \quad (\text{A2})$$

Computing $\mathcal{T}$ across all $2\mathfrak{D}$ rounds has complexity $\mathcal{O}[\mathfrak{D}(\mathbf{r}_1 + \mathbf{r}_2)n^2]$. Rather than immediate corrections, we propagate all corrections to circuit end using relations

$$\begin{aligned} & \text{fan-out}(Q_i \rightarrow T_i) Z_{Q_i} \\ &= \begin{cases} Z_{Q_i} Z_{Q_j} \text{fan-out}(Q_i \rightarrow T_i), & Q_j \in T_i, \\ Z_{Q_j} \text{fan-out}(Q_i \rightarrow T_i), & Q_j \notin T_i, \end{cases} \end{aligned} \quad (\text{A3})$$

to propagate all corrections to the end of the staircase circuit. An illustrative example follows, with further details and examples provided in Supplemental Material [45].

Example 1: Transfer matrix for a CX ladder [38]—Consider n system qubits $\{Q_i\}$ and $n-1$ auxiliary qubits $\{\tilde{Q}_i\}$ arranged linearly. Apply two nearest-neighbor layers: first $CX(Q_i, \tilde{Q}_i)$ for $i = 1, \dots, n-1$, then $CX(\tilde{Q}_i, Q_{i+1})$ for $i = 1, \dots, n-1$. Measure all $\tilde{Q}_i$ in the X basis with outcomes $m_i \in \{0, 1\}$. Using $CX(c, t)Z_t = Z_c Z_t CX(c, t)$ and $Z|0\rangle = |0\rangle$, a putative $Z_{\tilde{Q}_i}$ backpropagates to $Z_{Q_1} Z_{Q_2} \cdots Z_{Q_i}$, so the system-side Pauli-frame bits are

$$z_j = \bigoplus_{k=j}^{n-1} m_k, \quad \text{for } j = 1, \dots, n-1 \quad (\text{A4})$$

and $z_n = 0$. Equivalently, $\mathbf{z} = \mathcal{T}\mathbf{m} \pmod{2}$ with $\mathcal{T} \in \{0, 1\}^{n \times (n-1)}$ being the upper-triangular matrix (suffix parity). For $n = 4$,

$$\mathcal{T} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{A5})$$

Algorithm 2. Random directed Hamiltonian path on a 2D grid.

-
- 1: **Input:** Grid dimensions (width, height), start qubit (x_0, y_0) , and number of rerouting iterations count.
 - 2: **Initialization:**
 - (i) For each qubit (x, y) , assign an initial zigzag direction:
 - (a) If y is even, set $\text{dir}(x, y) \leftarrow \text{WEST}$, except for $x = 0$, set $\text{dir}(0, y) \leftarrow \text{NORTH}$.
 - (b) If y is odd, set $\text{dir}(x, y) \leftarrow \text{EAST}$, except for $x = \text{width} - 1$, set $\text{dir}(x, y) \leftarrow \text{NORTH}$.
 - (ii) Set $\text{dir}(x_0, y_0) \leftarrow \text{NOWHERE}$, ensuring the start qubit has no incoming edge.
 - 3: **for** $i = 1, \dots, \text{count}$ **do** ▷ Iterative reconfiguration
 - 4: **Split:** Select a 2×2 subsquare whose two diagonals carry opposite directions. Let one diagonal be (p, q) ; follow stored directions from p until q is reached, marking the square as a loop. If this fails, switch to the other diagonal (p', q') and retry.
 - 5: **Modify₁:** Flip (p, q) from vertical $(\{\text{NORTH}, \text{SOUTH}\})$ to horizontal $(\{\text{EAST}, \text{WEST}\})$, or vice versa.
 - 6: **Mend:** Choose a diagonal pair (u, v) with exactly one end point inside the loop; if none exists, set $(u, v) \leftarrow (p, q)$.
 - 7: **Modify₂:** Apply the same flip to (u, v) to merge the loop into the global path.
 - 8: **end for**
 - 9: **Extract path:**
 - (i) Compute the in-degree of each qubit.
 - (ii) Select any qubit with in-degree 0 and follow its edges to obtain a single chain covering all qubits.
 - 10: **Output:** A directed Hamiltonian path that visits each qubit exactly once.
-

On a 2D grid, the exponential number of Hamiltonian paths allows us to generate a distinct, effectively deep, constant-depth IQP circuit for each sampling round. We generate these paths as detailed in Algorithm 2 (e.g., Fig. 5), using 2000 rerouting iterations in our simulations.

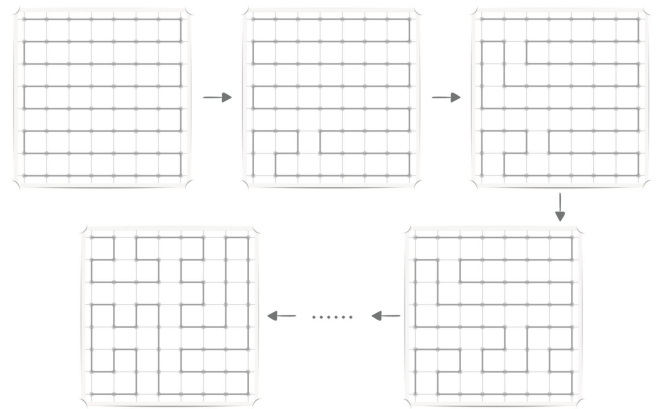


FIG. 5. An example of the split-and-mend procedure for constructing a random directed Hamiltonian path on a 2D grid.