

Continuous Family of Conformal Field Theories and Exactly Marginal Operators

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Does a conformal manifold imply the existence of exactly marginal operators? We answer this question affirmatively under the assumption that there exists a conformal interface with certain properties connecting nearby conformal field theories. We show that the exactly marginal operator that connects the conformal field theories can be reconstructed from the interface displacement operator. Our construction is model independent and based on the general principles of conformal symmetry.

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Introduction and summary—If exactly marginal operators exist in a conformal field theory (CFT), they generate a conformal manifold, which parametrizes a continuous family of CFTs [1]. One might naturally ask if the converse also holds: Does a conformal manifold require the existence of exactly marginal operators? In this Letter, we address this question by deriving the existence of exactly marginal operators under certain assumptions.

The existence of exactly marginal operators has important implications for the swampland program, particularly for the distance conjecture [2]. The earliest distance conjecture may have been stated by Albert Einstein in his autobiographical notes, in which he claimed that there are no free parameters in the basic equations of physics. This was reformulated as Conjecture 0 in [2] as “the moduli space of a consistent quantum gravity is parametrized by inequivalent expectation values of massless scalar fields.” The existence of exactly marginal operators for a conformal manifold allows us to prove Conjecture 0 for gravitational theories in anti-de Sitter (AdS) space as follows [3]: If there is a continuous family of gravitational theories in AdS, then

there must be a corresponding family of CFTs. If we can show that each CFT parameter is associated with an exactly marginal operator, the AdS/CFT dictionary would tell us that the exactly marginal operator corresponds to a massless scalar field with a flat potential, and the asymptotic behavior of this field determines the parameters. Therefore, every parameter in quantum gravity in AdS is related to a value of a dynamical field, as claimed in Conjecture 0 [4].

We note an interesting analogy with the absence of global symmetry in quantum gravity in AdS, which was proven in [9,10] using the splittability of symmetry generators [11] in CFT. For continuous symmetry, the splittability is the consequence of the well-known fact that the symmetry generators are expressed as exponentiated integrals of the Noether currents over a Cauchy surface. If we regard a continuous deformation of CFT as (-1) -form global symmetry, the existence of the corresponding exactly marginal operator is the splittability of the symmetry. In this way, the absence of free parameters and the absence of global symmetry in gravitational theories in AdS are unified and understood in a single framework as consequences of the splittability of symmetry generators.

In what follows, we show that exactly marginal operators exist, under the assumption of the existence of a conformal interface between nearby CFTs that reduces to the identity when the CFTs coincide. The argument is constructive and model independent. We focus on two-dimensional CFTs for simplicity, but the proof can be generalized to higher dimensions and will be reported in [12].

To motivate our proposal, consider a theory with an exactly marginal operator ϕ , which we can use to deform

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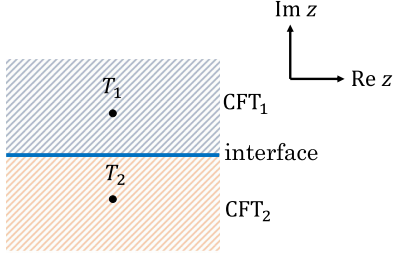


FIG. 1. Interface CFT. We denote the stress tensors of the CFTs on the upper and bottom sides of the interface by T_1 and T_2 , respectively.

the CFT in the upper half-plane as (see Fig. 1 for our choice of convention)

$$S_0 \rightarrow S_0 + \delta\lambda \int_{\text{Im } z > 0} d^2z \phi(z, \bar{z}), \quad (1)$$

where S_0 is the original action of the CFT. At the leading order in $\delta\lambda$, this defines a conformal interface satisfying the gluing condition at $\text{Im } z = 0$ [13–15]:

$$T_1 - \bar{T}_1 = T_2 - \bar{T}_2, \quad (2)$$

where $T_{1,2}$ ($\bar{T}_{1,2}$) are the holomorphic (antiholomorphic) stress tensors on the upper and bottom half-planes. Furthermore, it follows from the Ward identity that

$$T_1(x + i0) - T_2(x - i0) = \pi\delta\lambda\phi(x) + O(\delta\lambda^2), \quad (3)$$

where we have written $z = x + iy$. Now we consider the displacement operator associated with this conformal interface [16]:

$$D \equiv T_1 + \bar{T}_1 - T_2 - \bar{T}_2 = 2(T_1 - T_2). \quad (4)$$

The two-point function of the displacement operator has the following form:

$$\langle D(z)D(w) \rangle = \frac{N_D}{(z-w)^4}, \quad (5)$$

where N_D can be computed using (2) and (3) as [17]

$$N_D = (2\pi\delta\lambda)^2 + O(\delta\lambda^3). \quad (6)$$

This shows that one can extract the marginal operator $\phi(x)$ from (see Fig. 2)

$$\phi = \lim_{\delta\lambda \rightarrow 0} \hat{D}, \quad (7)$$

where $\hat{D}$ is the normalized displacement operator:

$$\hat{D} \equiv D/\sqrt{N_D}. \quad (8)$$

In the rest of this Letter, we show that one can reverse this logic; assuming the existence of a conformal interface

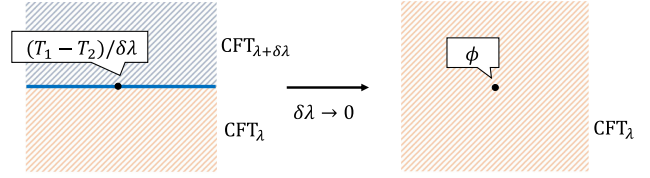


FIG. 2. A rough sketch of our construction of the exactly marginal operator ϕ . We parametrize the conformal manifold by λ . As $\delta\lambda \rightarrow 0$, the two CFTs on both sides coincide, the interface vanishes, and what remains is the bulk local operator. We show that this local operator is actually the exactly marginal operator.

between nearby CFTs that reduces to the identity when the CFTs coincide, we provide a simple proof for the existence of an exactly marginal operator. It is desirable to prove the result without assuming the existence of such an interface. We are actively investigating this and hope to present details in an extended version forthcoming [12].

Displacement operator as exactly marginal operator—We define a conformal manifold as a collection of CFT data which depend continuously on a set of parameters. For every choice of parameters, the stress tensor exists and generates two copies of the Virasoro algebra. Our proof further assumes that (1) for any pair of CFTs that are close to each other on the conformal manifold, there is a conformal interface that becomes trivial (i.e., the identity interface) when the two CFTs coincide; (2) correlators involving unit normalized defect operators have finite limits as the interface disappears (*smoothness condition*) [18]. The consequence of these two assumptions is that defect operators become local operators of the CFT in the limit, unless they decouple (see Fig. 2) [19]. This is true, in particular, for the (normalized) displacement operator $\hat{D}$, which exists on every conformal interface. For the reasons explained later, we also make the additional technical assumption that (3) the bulk CFT has no operators with conformal weights of $(1/2, 1/2)$ that are neutral under all the global symmetries preserved by the interface. Note that this third assumption excludes Liouville theory from our discussion [20].

On the other hand, we do not need to assume that the central charges on both sides ($c_{L,R}$) are the same. Furthermore, chiral and antichiral central charges are allowed to differ, while the condition $c_1 - \bar{c}_1 = c_2 - \bar{c}_2$ is necessary for the existence of a conformal interface [16]. In other words, the central charge can vary on the conformal manifold, but $c - \bar{c}$ cannot [21].

Our proof proceeds by showing the following three facts: (1) $\lim_{N_D \rightarrow 0} \hat{D}$ is an interface primary operator of conformal dimension 2. (2) $\lim_{N_D \rightarrow 0} \hat{D}$ is, in fact, a bulk primary operator of conformal dimension $(1, 1)$. (3) The three-point function of $\lim_{N_D \rightarrow 0} \hat{D}$ vanishes.

Here, an interface primary operator refers to a primary operator localized on the interface, associated with a highest weight state under the single Virasoro symmetry that remains unbroken in the interface CFT. Interface

quasiprimaries are defined analogously with respect to the $sl(2, \mathbb{R})$ algebra preserved by the defect. Note that, unlike bulk scaling operators, which are labeled by conformal weights $(h, \bar{h})$ under the bulk symmetry $\text{Vir} \otimes \overline{\text{Vir}}$, interface scaling operators are characterized by a single eigenvalue corresponding to the remaining Virasoro symmetry.

Let us show that the displacement operator $\hat{D}$ becomes an interface primary of weight 2 in the limit where the two CFTs coincide. When evaluated on the interface, the stress tensors reorganize in two interface primaries $(W, \bar{W})$ and a quasiprimary $(T_{\text{tot}} = T_1 + \bar{T}_2)$; see Ref. [22]. Notice that the gluing condition implies $T_{\text{tot}} = \bar{T}_{\text{tot}}$. In particular, the displacement operator D can be shown to be an interface quasiprimary. Indeed, one can check that

$$\begin{aligned} (T_1(z) + \bar{T}_2(z))\hat{D}(w) &= \frac{(c_1 - c_2)/\sqrt{N_D}}{(z-w)^4} \\ &+ \left(\frac{2}{(z-w)^2} + \frac{1}{z-w} \partial \right) \\ &\times \hat{D}(w) + O(1), \end{aligned} \quad (9)$$

where z and w lie on the real axis and $O(1)$ denotes terms that are nonsingular at $z = w$. Equation (9) is obtained by making use of the interface boundary condition (2) as well as of the fact that $T_1(z)\bar{T}_2(w)$ and $\bar{T}_1(z)T_2(w)$ have no singularities at $z = w$. This absence of singularity follows from the cluster decomposition principle [16].

In the limit $N_D \rightarrow 0$, the singular part of (9) vanishes, thanks to the averaged null energy conditions, which enforce the bound $N_D \geq 2|c_1 - c_2|$ [23,24]. We obtain the operator product expansion (OPE)

$$\begin{aligned} (T(z) + \bar{T}(z))\hat{D}(w) \\ = \frac{2}{(z-w)^2}\hat{D}(w) + \frac{1}{(z-w)}\partial\hat{D}(w) + O(1). \end{aligned} \quad (10)$$

Hence, $\lim_{N_D \rightarrow 0} \hat{D}$ is indeed an interface primary of weight 2. It is worth emphasizing that this expression shows that $\hat{D}(w)$ does not decouple from the theory.

Given that $\hat{D}$ is an interface operator of conformal dimension 2, this constrains the possible bulk local operators that can emerge in the $N_D \rightarrow 0$ limit. In the following, we examine each candidate and show that the only viable possibility is a primary operator with conformal dimensions $(1, 1)$. We broadly distinguish two cases: a highest weight state under the conformal symmetry $\text{Vir} \otimes \overline{\text{Vir}}$ (i.e., a primary state) and those that can be obtained by acting with the stress tensor modes L_{-n} on primary states (i.e., descendant states).

Primary fields: Suppose we have a bulk primary field $\phi(z, \bar{z})$ of conformal weights $(h, \bar{h})$. The OPE between a primary operator and the stress tensor is given by

$$\begin{aligned} T(z)\phi(w, \bar{w}) &= \frac{h}{(z-w)^2}\phi(w, \bar{w}) + \frac{\partial\phi(w, \bar{w})}{(z-w)} + O(1), \\ \bar{T}(\bar{z})\phi(w, \bar{w}) &= \frac{\bar{h}}{(\bar{z}-\bar{w})^2}\phi(w, \bar{w}) + \frac{\bar{\partial}\phi(w, \bar{w})}{(\bar{z}-\bar{w})} + O(1). \end{aligned} \quad (11)$$

Restricting z and w to the real axis and taking the difference between the two expressions, we obtain

$$(T(z) - \bar{T}(z))\phi(w) = \frac{h - \bar{h}}{(z-w)^2}\phi(w) + \frac{\partial - \bar{\partial}}{z-w}\phi(w) + O(1). \quad (12)$$

By symmetrizing with respect to z and w , we obtain

$$\begin{aligned} (T(z) - \bar{T}(z))\phi(w) + (T(w) - \bar{T}(w))\phi(z) \\ = \frac{h - \bar{h}}{(z-w)^2}(\phi(z) + \phi(w)) + O(1). \end{aligned} \quad (13)$$

Recalling that ϕ is constructed from $T_1(w) - T_2(w)$, this expression implies that the OPE between $T_1(z) - \bar{T}_1(z)$ and $T_1(w) - T_2(w)$ contains symmetric singular terms, unless $h - \bar{h} = 0$. On the other hand, the interface boundary conditions on the energy-momentum tensor imply that the singular parts of the OPE are antisymmetric in z and w :

$$\begin{aligned} (T_1(z) - \bar{T}_1(z))(T_1(w) - T_2(w)) \\ = T_1(z)\bar{T}_1(w) - \bar{T}_1(z)T_1(w) + O(1), \end{aligned} \quad (14)$$

where we used the gluing condition (2) and the fact that $T_1(z)\bar{T}_2(w)$ and $\bar{T}_1(z)T_2(w)$ do not have singularities at $z = w$. Note that $T_1(z)\bar{T}_1(w)$ is singular because of the gluing condition. Consequently, we must have $h = \bar{h}$. Since $h + \bar{h} = 2$, the bulk primary field ϕ that can appear in $\hat{D}$ must be of conformal weights $(1, 1)$, i.e., marginal.

Descendant fields, $L_{-2}|0\rangle$ or $\bar{L}_{-2}|0\rangle$: The local operator corresponding to $L_{-2}|0\rangle$ is the energy-momentum tensor. It cannot appear by itself in $\hat{D}(z)$, since the OPE of $T(z) + \bar{T}(\bar{z})$ with $T(z)$ contains a $(z-w)^{-4}$ singularity due to the conformal anomaly, which contradicts the expression given in (10). However, if we instead consider the combination $\bar{c}T(z) - c\bar{T}(z)$, such a higher-order pole may cancel on the real axis.

Nevertheless, this possibility can be excluded as follows. On the one hand, we have

$$(T(z) - \bar{T}(z))(\bar{c}T(w) - c\bar{T}(w)) = \frac{c\bar{c}}{(z-w)^4} + \dots, \quad (15)$$

where z and w are restricted to the real axis. On the other hand, as shown in (14), the OPE singularities between $T(z) - \bar{T}(z)$ and $\hat{D}(w)$ must be antisymmetric under the exchange of z and w . Since there are no other operators that

can produce a $(z - w)^{-4}$ singularity to cancel the conformal anomaly term in (15), we conclude that $\bar{c}T(z) - c\bar{T}(z)$ cannot appear in $\hat{D}(z)$.

Descendant fields, $L_{-1}|1, 0\rangle$ or $\bar{L}_{-1}|0, 1\rangle$: In this case, the CFT must possess either a holomorphic current J or an antiholomorphic current $\bar{J}$. Suppose that $\hat{D}(z)$ contains a linear combination of ∂J and $\bar{\partial}\bar{J}$. Since we have

$$\begin{aligned} & (T(z) + \bar{T}(z))\partial J(w) \\ &= \frac{2J(w)}{(z-w)^3} + \frac{2\partial J(w)}{(z-w)^2} + \frac{\partial^2 J(w)}{z-w} + O(1), \\ & (T(z) + \bar{T}(z))\bar{\partial}\bar{J}(w) \\ &= \frac{2\bar{J}(w)}{(z-w)^3} + \frac{2\bar{\partial}\bar{J}(w)}{(z-w)^2} + \frac{\bar{\partial}^2\bar{J}(w)}{z-w} + O(1), \end{aligned} \quad (16)$$

in order to satisfy (10), we need to consider the combination $(\partial J - \alpha\bar{\partial}\bar{J})$ for some constant α and impose the totally reflective boundary condition $J(z) = \alpha\bar{J}(\bar{z})$. However, this boundary condition is not compatible with full conformal invariance, namely, with preserving the full symmetry $\text{Vir} \otimes \overline{\text{Vir}}$ without the interface. Hence, it cannot arise when $N_D = 0$, where the interface is absent. Therefore, this possibility must be excluded.

Descendant fields, $L_{-1}|1/2, 1/2\rangle$ or $\bar{L}_{-1}|1/2, 1/2\rangle$: Suppose there exists a bulk primary field ρ of conformal weight $(1/2, 1/2)$. Then, we have

$$\begin{aligned} (T(z) + \bar{T}(z))\rho(w, \bar{w}) &= \left(\frac{1/2}{(z-w)^2} + \frac{1/2}{(z-\bar{w})^2} \right) \rho(w, \bar{w}) \\ &+ \left(\frac{\partial}{z-w} + \frac{\bar{\partial}}{z-\bar{w}} \right) \rho(w, \bar{w}) + O(1), \end{aligned} \quad (17)$$

for z on the real axis. It then follows that

$$\begin{aligned} & (T(z) + \bar{T}(z))\partial\rho(w, \bar{w}) \\ &= \frac{\rho(w, \bar{w})}{(z-w)^3} + \frac{2\partial\rho(w, \bar{w})}{(z-w)^2} + \frac{(\partial+\bar{\partial})\partial\rho(w, \bar{w})}{z-w} + O(1), \\ & (T(z) + \bar{T}(z))\bar{\partial}\rho(w, \bar{w}) \\ &= \frac{\rho(w, \bar{w})}{(z-w)^3} + \frac{2\bar{\partial}\rho(w, \bar{w})}{(z-w)^2} + \frac{(\partial+\bar{\partial})\bar{\partial}\rho(w, \bar{w})}{z-w} + O(1). \end{aligned} \quad (18)$$

It implies that the combination $(\partial - \bar{\partial})\rho$ satisfies (10).

In summary, we have shown that the conformal symmetry alone guarantees that the limit of $\hat{D}$ is a linear combination:

$$\lim_{N_D \rightarrow 0} \hat{D} = \phi + (\partial - \bar{\partial})\rho, \quad (19)$$

where ϕ and ρ are dimension $(1, 1)$ and $(\frac{1}{2}, \frac{1}{2})$ primaries, respectively. Both ϕ and ρ must be neutral under any global symmetry that preserves the interface since $\hat{D}$ is. Since we assumed that the theory does not contain neutral $(\frac{1}{2}, \frac{1}{2})$ primaries, this establishes the existence of a marginal operator (since $\rho = 0$ and, therefore, ϕ is necessarily present). We will elaborate more on this point in the next section.

Let us now verify that the three-point function of ϕ vanishes, as expected from exact marginality. Given a pair of quasiprimary operators O_1 and O_2 , with the same spin $h_1 - \bar{h}_1 = h_2 - \bar{h}_2$, on either side of any conformal interface, their correlators with and without the displacement are constrained as follows [26]:

$$\langle O_1(z, \bar{z}) O_2(\bar{z}, z) \rangle = \frac{a_{12}}{(2 \text{Im } z)^{\Delta_1 + \Delta_2}}, \quad (20)$$

$$\langle O_1(z, \bar{z}) O_2(\bar{z}, z) \hat{D}(w) \rangle = \frac{(\Delta_1 - \Delta_2) a_{12} / \sqrt{N_D}}{(2 \text{Im } z)^{\Delta_1 + \Delta_2 - 2} |z - w|^4}, \quad (21)$$

where $\Delta = h + \bar{h}$, $w \in \mathbb{R}$, a_{12} is a constant which, at $N_D = 0$, is fixed by normalization, i.e., $a_{12} = 1$, and it should be noted that O_1 and O_2 are placed at symmetric positions with respect to the interface. Having established the existence of an operator of weight 2 at a generic point of the conformal manifold, we select $O_1 = O_2 = \phi$, so that the rhs of (21) vanishes identically for any value of N_D , while (20) does not, and, in particular, $\lim_{N_D \rightarrow 0} a_{12} = 1$. Since the rhs of (21) reduces to the three-point function of ϕ at $N_D = 0$, we conclude that

$$\langle \phi(z) \phi(w) \phi(u) \rangle = 0, \quad (22)$$

establishing the marginality of ϕ at the leading order.

More generally, let $O_1 = O_{\lambda+\delta\lambda}$ and $O_2 = O_\lambda$ denote the same quasiprimary operator at two nearby points on the conformal manifold. Then we define $\delta\Delta_O \equiv \Delta_1 - \Delta_2$. Using $a_{12} \rightarrow 1$ in (20) and $\hat{D} \rightarrow \phi$ as $N_D \rightarrow 0$, (21) reduces to the bulk three-point function with ϕ , and we obtain

$$\delta\Delta_O = \sqrt{N_D} c_{O\phi} + o(\sqrt{N_D}). \quad (23)$$

This is precisely the expected first-order relation between the variation of scaling dimensions and the OPE coefficient with the exactly marginal operator in conformal perturbation theory. This agreement provides an additional justification for identifying $\hat{D}$ with an exactly marginal operator.

Interface conformal manifold—Why did we need to assume the absence of neutral $(\frac{1}{2}, \frac{1}{2})$ primaries? Let us first clarify what ρ actually represents in (19). If a CFT contains a weight-one scalar operator, one can produce a line defect by integrating it with an infinitesimal coefficient. If,

furthermore, $c_{\rho\rho\rho} = 0$, together with the required higher order conditions, the line defect is part of a conformal manifold. As a concrete example, the Ising CFT admits a defect conformal manifold generated by the following interface exactly marginal deformation [14]:

$$S_{\text{defect}} = \gamma \int dx \varepsilon, \quad (24)$$

where ε is the energy density operator with conformal dimensions $(1/2, 1/2)$. The displacement operator is

$$D = \frac{\gamma}{2} (\partial - \bar{\partial}) \varepsilon. \quad (25)$$

This is precisely of the same form as the second term in (19). Vice versa, given a family of interfaces connected to the identity, whose displacement obeys (19) with $\phi = 0$, one can prove that $c_{\rho\rho\rho} = 0$ —see Supplemental Material [25]. Furthermore, it is easy to check that, if $\lim_{N_D \rightarrow 0} \hat{D} = (\partial - \bar{\partial})\rho$, the lhs of (21) vanishes in the limit; therefore, no scaling dimension changes at leading order in N_D . More generally, symmetry-neutral exactly marginal deformation on the interface can arise from the breaking of a *phantom symmetry* [27]: a symmetry that emerges when one interprets the interface between CFT_1 and CFT_2 as a boundary in $\text{CFT}_1 \otimes \overline{\text{CFT}_2}$ using the folding trick. In such cases, a *phantom current* with conformal dimension $(1,0)$ in the folded theory induces an interface exactly marginal operator with dimension $(1/2, 1/2)$ in the unfolded theory. This is precisely the case in the example of the Ising CFT, where the phantom current is given by $j = \psi_1 \psi_2$. Here, $\psi_{1/2}$ represents the fermion field on the upper or bottom side of the interface, and the energy density operator in the Ising CFT can be expressed in terms of these fermions as $\varepsilon_{1/2} = i\psi_{1/2} \bar{\psi}_{1/2}$.

The explicit examples discussed above describe interfaces between identical CFTs rather than interfaces connecting different points on a conformal manifold as considered in the main text. However, our leading-order analysis does not rule out the possibility that an interface could connect theories at distinct points on a conformal manifold while still obeying $\lim_{N_D \rightarrow 0} \hat{D} = (\partial - \bar{\partial})\rho$. Consider, for instance, the case where $D \sim \tilde{\phi} + (\partial - \bar{\partial})\rho$ with $\langle \tilde{\phi} \tilde{\phi} \rangle = O(N_D^{1+\epsilon})$ with $\epsilon > 0$. Equation (19) would be obeyed with $\phi = 0$. This example shows that additional input is needed to treat conformal manifolds containing protected dimension-1 operators within our framework [28].

Examples—Let us illustrate our results with concrete examples: the compact free boson, whose action is given by

$$S = \frac{1}{4\pi} \int d^2 z \partial X \bar{\partial} X, \quad X \sim X + 2\pi R, \quad (26)$$

where R represents the compactification radius. The marginal operator in this CFT is $\phi = \partial X \bar{\partial} X$. The exactly marginal deformation leads to the shift in the

compactification radius, i.e., $R \rightarrow R' \equiv R e^{\pi \delta \lambda}$. The interface with a CFT deformed by $\delta \lambda$ can be described by the following gluing matrix [15]:

$$\begin{pmatrix} \partial X_1 \\ \bar{\partial} X_2 \end{pmatrix} = S \begin{pmatrix} \bar{\partial} X_1 \\ \partial X_2 \end{pmatrix}, \quad S = - \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \quad (27)$$

where we define $\tan \theta = e^{\pi \delta \lambda}$. This gluing condition allows us to express the stress tensor on the bottom side of the interface in terms of the free boson fields on the upper side:

$$\begin{aligned} T_2 &= \frac{1}{2} \partial X_2 \partial X_2 = \frac{1}{2} \partial X_1 \partial X_1 - \pi \delta \lambda \partial X_1 \bar{\partial} X_1, \\ \bar{T}_2 &= \frac{1}{2} \bar{\partial} X_2 \bar{\partial} X_2 = \frac{1}{2} \bar{\partial} X_1 \bar{\partial} X_1 - \pi \delta \lambda \partial X_1 \bar{\partial} X_1. \end{aligned} \quad (28)$$

This expression leads to the following displacement operator:

$$\lim_{\delta \lambda \rightarrow 0} \hat{D} = \partial X \bar{\partial} X. \quad (29)$$

This coincides precisely with the exactly marginal operator $\phi = \partial X \bar{\partial} X$.

Another example is the supersymmetric sigma model whose target space is a Calabi-Yau manifold. For each exactly marginal deformation, there is a primary field ρ of dimensions $(1/2, 1/2)$ and an exactly marginal operator ϕ , which is a superconformal descendant of ρ . However, since ρ carries nonzero R charges with respect to the $\mathcal{N} = (2, 2)$ superconformal algebra, it cannot appear on the right-hand side of (19). For a generic Calabi-Yau manifold, there are no neutral $(1/2, 1/2)$ primaries. Therefore, the limit of $\hat{D}$ can be identified with the exactly marginal operator ϕ .

Comments—In the Introduction, we noted the interesting analogy between the absence of free parameters and the absence of global symmetry in AdS gravitational theories, both of which are consequences of the splittability of symmetry generators. The absence of free parameters has been quantified as distance conjectures 1 and 2 [2] and formulated in AdS [29], and some of these conjectures have been proven [30,31]. The absence of global symmetry has been quantified as the weak gravity conjecture [32] and formulated in AdS [10,33–35], and some of these have been proven [36,37]. Since both are formulated in terms of inequalities, it would be interesting to determine whether they are analogous and can be understood within a single framework.

Our result is conditional on assumption 1. At present, we do not know a general theorem guaranteeing the existence of conformal interfaces approaching the identity between arbitrary nearby points on a conformal manifold, nor are we aware of explicit counterexamples. In the examples we know, including the free boson example, the assumption is satisfied. Clarifying its general validity is an interesting open problem, which we hope to return to in [12].

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- [19] If the interface in the limit is not trivial but topological and noninvertible, defect operators may become operators with a topological line attached.
- [20] For arbitrary values of the coupling constant, Liouville theory has a symmetry-neutral $(1/2, 1/2)$ operator, hence violating our assumption 3. This operator may not correspond to a normalizable state in Liouville theory, but our current argument is insensitive to such a normalizability issue.
- [21] From modular T invariance,

$$h - \bar{h} - \frac{c - \bar{c}}{24} \in \mathbb{Z},$$

while the single-valuedness of correlation functions implies the quantization of spin, $h - \bar{h} \in \mathbb{Z}/2$. Combining these relations, we conclude that $c - \bar{c}$ must be quantized. This, in turn, means that $c - \bar{c}$ is invariant under continuous deformations.

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