

Ergotropic Characterization of Continuous-Variable Entanglement

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Continuous-variable quantum thermodynamics in the Gaussian regime provides a promising framework for investigating the energetic role of quantum correlations, particularly in optical systems. In this Letter, we introduce an entropy-free criterion for entanglement detection in bipartite Gaussian states, rooted in a distinct thermodynamic quantity: ergotropy—the maximum extractable work via unitary operations. By defining the “relative ergotropic gap,” which quantifies the disparity between global and local ergotropy, we derive two independent analytical bounds that distinguish entangled from separable states. These bounds coincide for a broad class of quantum states, making the criterion both necessary and sufficient in such cases. Unlike entropy-based measures, our ergotropic approach captures fundamentally different aspects of quantum correlations and entanglement, particularly in mixed continuous-variable systems. We also extend our analysis beyond the Gaussian regime to certain non-Gaussian states and observe that Gaussian ergotropy continues to reflect thermodynamic signatures in entangled states, albeit with some limitations. These findings establish a direct operational link between entanglement and energy storage, offering an experimentally accessible approach to entanglement detection in continuous-variable optical platforms.

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Introduction—Quantum thermodynamics examines how intrinsically quantum resources shape energy and information processing in microscopic regimes [1–13]. With experimental advances enabling precise control over single quantum systems, addressing the energetics of nanoscale devices has become both timely and essential [14]. A key concept in this context is “ergotropy”—the maximum amount of energy that can be extracted from a quantum system through unitary evolution [15]. This makes ergotropy a fundamentally operational and physically measurable quantity, capturing distinct features of quantum states that evade entropy-based analysis [16–31].

Continuous-variable (CV) systems, particularly quantum optical platforms, provide a compelling setting for exploring these thermodynamic aspects. CV states are central to diverse quantum technologies, including quantum communication, sensing, and computation [32–34]. Among these,

Gaussian states and operations are particularly important due to their experimental feasibility [35], providing an attractive test bed for probing the energetic role of quantum correlations. Entanglement in Gaussian systems is typically assessed using entropic measures [36] or criteria such as the positive partial transpose condition [37,38]. However, these information-theoretic tools lack direct connections to energy observables and may overlook operational features of quantum correlations.

This motivates the development of energy-based entanglement witnesses that operate independently of abstract mathematical metrics. Recent work suggests that ergotropy offers a thermodynamically meaningful framework for revealing quantum correlations [20–31,39]. However, these ideas have primarily been explored in discrete-variable (DV) systems. Extending these concepts to CV systems is both challenging and essential, given their role in practical platforms like quantum batteries and optical processors [40].

In this Letter, we introduce the “relative ergotropic gap” (REG), which quantifies the disparity between extractable work from a global bipartite Gaussian state and its locally accessible components under Gaussian unitaries. We derive analytical bounds that distinguish separable from entangled states and show that REG is a necessary and sufficient

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indicator of entanglement in a wide class of Gaussian systems. Crucially, our analysis reveals that for mixed CV states, REG is fundamentally independent of entropic measures, demonstrating an unconventional approach to entanglement characterization. We further extend our energy-based framework to non-Gaussian states [41]. In particular, for photon-subtracted states, where the standard ergotropy is intractable due to their infinite-dimensional support, we use the Shchukin-Vogel criterion [42] to show that REG serves as a reliable entanglement witness in this class. These results highlight the utility of ergotropy as a robust witness of CV quantum correlations and open new directions for energy-driven quantum technologies.

Energy extraction in CV systems—In thermodynamics, the work extractable from a system is controlled by its internal energy and entropy. For adiabatic processes, work extraction depends solely on internal energy changes. In the quantum regime, the analogous process is unitary evolution, which preserves von Neumann entropy. Ergotropy is the maximum amount of work reversibly extractable [15,43–45]. For a state ρ with Hamiltonian H , it is defined as $\mathcal{E}(\rho) := E(\rho) - \min_U E(U\rho U^\dagger)$, where the minimization is over all unitaries U , and $E(X) := \text{Tr}(XH)$ denotes the system's energy in state X . This expression quantifies the energy difference between ρ and its passive counterpart ρ^p , the state with the same spectrum but minimal energy, beyond which no work can be extracted through unitaries. Ergotropy thus quantifies how far a state is from being passive, in purely energetic terms.

In systems with infinite-dimensional Hilbert spaces, the implementation of a generic unitary is practically challenging. However, Gaussian states and operations form the backbone of optical quantum technologies, and their simplified mathematical structure makes them a natural playground for quantum thermodynamics in CV systems [33,46–48]. Restricting to Gaussian unitaries leads indeed to the concept of “Gaussian ergotropy” [46,47], defined as

$$\mathcal{E}_G(\rho) := E(\rho) - \min_{U_G} E(U_G \rho U_G^\dagger) = E(\rho) - E(\rho_G^p), \quad (1)$$

where U_G denotes a Gaussian unitary and ρ_G^p is the corresponding “Gaussian passive state” that minimizes the system's energy under such operations. For an arbitrary state ρ , $\mathcal{E}_G(\rho) \leq \mathcal{E}(\rho)$, reaching the equality if ρ is itself a Gaussian state, because Gaussian unitaries are optimal; in such cases, we omit the subscript G and simply refer to ergotropy.

In composite or multimode CV systems, Gaussian ergotropy can be defined globally (via joint Gaussian unitaries U_G) or locally (via product unitaries $\otimes_i U_i$, where U_i acts on i th mode). Their difference defines the Gaussian ergotropic gap,

$$\Delta \mathcal{E}_G := \mathcal{E}_G^{\text{global}}(\rho) - \mathcal{E}_G^{\text{local}}(\rho) = E_{lp} - E_{gp}, \quad (2)$$

where E_{lp} and E_{gp} denote the energies of local and global Gaussian passive states. Operationally, this quantity captures the energetic advantage of performing correlated (global) operations over local ones. While the ergotropic gap has been studied in DV settings as a thermodynamic signature of quantum correlations [21–23,26], its extension to CV systems remains unexplored.

In phase space, an N -mode bosonic system is described by the quadrature operators $\hat{r} = (\hat{x}_1, \hat{p}_1, \dots, \hat{x}_N, \hat{p}_N)^T$, which satisfy the canonical commutation relations $[\hat{r}_j, \hat{r}_k] = i\Omega_{jk}$, with $\Omega = \bigoplus_N i\sigma_Y$ the symplectic form [33] and σ_Y the Pauli Y matrix. The energy of a Gaussian state depends on its covariance matrix σ , with entries $\sigma_{jk} = \langle \{\hat{r}_j, \hat{r}_k\} \rangle - 2\langle \hat{r}_j \rangle \langle \hat{r}_k \rangle$ and the system's Hamiltonian, which we assume in the form $\hat{H} = \sum_k \omega_k (\hat{x}_k^2 + \hat{p}_k^2)$. The mean energy of a Gaussian state ρ with covariance matrix σ is then

$$E(\rho) = \frac{1}{4} \sum_k \omega_k (\text{Tr}[\sigma_k] - 2), \quad (3)$$

where σ_k is the 2×2 covariance submatrix corresponding to mode k . Note that first moments also contribute to the energy of a Gaussian state, but since they can be removed through local unitary operations, they do not affect the ergotropic gap and thus can be disregarded in our analysis. For a two-mode Gaussian state, one can always bring the covariance matrix to the “standard form” σ_{sf} via local Gaussian unitary operations,

$$\sigma_{\text{sf}} = \begin{pmatrix} A & C \\ C & B \end{pmatrix}, \quad \text{with } A = a\mathbb{I}, \quad B = b\mathbb{I}, \quad C = \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix},$$

$$a \geq b \geq 1 \quad \text{and} \quad c_1 \geq |c_2|. \quad (4)$$

This form is particularly useful for computing both local and global passive energies. The local passive covariance matrix σ_{lp} is obtained via local Gaussian operations, while the global passive covariance matrix σ_{gp} is achieved via a symplectic transformation that diagonalizes σ_{sf} into its Williamson form: $\sigma_{gp} = \text{diag}(\nu_+, \nu_+, \nu_-, \nu_-)$, where $\nu_{\pm} \geq 1$ are the symplectic eigenvalues of σ_{sf} , i.e., the moduli of the eigenvalues of $i\Omega\sigma_{\text{sf}}$. Assuming $\omega_A \leq \omega_B$, the ergotropic gap for a two-mode Gaussian state ρ_G reads

$$\Delta \mathcal{E} := E_{lp} - E_{gp} = \frac{1}{2} [\omega_A(a - \nu_+) + \omega_B(b - \nu_-)], \quad (5)$$

where these local and global passive energies are computed directly from the covariance matrices.

Ergotropic gap versus correlations of Gaussian states—We begin by examining the interplay between quantum correlations and work extraction in *pure* two-mode Gaussian states.

Theorem 1—For pure two-mode Gaussian states ρ , the ergotropic gap $\Delta \mathcal{E}(\rho)$ vanishes if and only if ρ is separable.

Moreover, $\Delta\mathcal{E}(\rho)$ is a strictly increasing function of the quantum mutual information $I(\rho)$, and hence provides an equivalent quantification of entanglement for these states.

The proof is discussed in Appendix B of Supplemental Material [49]. We find that the ergotropic gap for pure two-mode Gaussian states is fully characterized by a single parameter—just like all standard entanglement measures—keeping them mutually dependent. This observation closely parallels the case of pure $2 \times d$ dimensional DV systems, where all entanglement measures are functionally related to the entanglement entropy [61,62]. The result in Theorem 1 readily extends to pure bipartite Gaussian states of an arbitrary number of modes, by virtue of their mode-wise decomposition into products of two-mode states via local unitaries [63]. In contrast, ergotropy-based entanglement measures in higher-dimensional or multipartite DV systems exhibit entropic independence [23,26], arising from the presence of multiple independent Schmidt coefficients. Interestingly, despite the infinite-dimensional nature of the Gaussian regime, this entropic independence remains absent, highlighting a fundamental structural distinction between Gaussian and general DV systems for pure states.

This leads to a compelling question: can the ergotropic gap also serve as a faithful entanglement measure for “mixed” states? Moreover, since it arises from closed-system dynamics, does it capture a distinct notion of “quantumness” beyond what is reflected by conventional information-theoretic quantifiers? Before addressing this, we examine whether $\Delta\mathcal{E}(\rho)$ satisfies basic criteria for a correlation measure [64]. In DV systems, the ergotropic gap has been shown to witness entanglement, but it can be nonzero even for product states [22]. Interestingly, our next result reveals a distinct behavior in the Gaussian setting.

Lemma 1—For general two-mode Gaussian states, $\Delta\mathcal{E}(\rho) > 0$ holds if and only if the state exhibits correlations.

The proof is Appendix D.1 of Supplemental Material [49]. This establishes that $\Delta\mathcal{E}(\rho)$ acts as a faithful indicator of correlations, being strictly positive if and only if the state is nonproduct. We now turn to the case of correlated mixed states, exploring whether this quantity can consistently reflect the strength of quantum correlations.

As a preliminary case study, we consider the well-known two-mode squeezed thermal state (TMS), obtained by applying a two-mode squeezing operation with parameter z to two bosonic modes of frequency ω , each initially in thermal equilibrium at temperature T . From the TMS covariance matrix, reported in Appendix C of Supplemental Material [49], we find that the ergotropic gap $\Delta\mathcal{E}_{\text{TMS}} = 2k\omega \sinh^2 r$ grows monotonically with temperature and diverges as $T \rightarrow \infty$. This is striking because thermal noise is generally expected to degrade quantum correlations. Indeed, the quantum mutual information (QMI) decreases with increasing temperature and remains bounded in the limit of infinite thermal noise: $\lim_{k \rightarrow \infty} I(\sigma_{\text{TMS}}) = 2 \log(\cosh(2r))$.

This behavior of $\Delta\mathcal{E}$ underscores a fundamental distinction between CV and DV systems. In DV systems, rising temperature leads the state toward a maximally mixed configuration—a global passive state—causing the ergotropic gap to vanish. In contrast, in CV systems, the unbounded energy spectrum allows the thermal population of arbitrarily high energy levels without necessarily rendering the state passive. Consequently, even minimal correlations in extremely energetic states can lead to large ergotropic gaps despite diminishing QMI. This shows that the ergotropic gap $\Delta\mathcal{E}$, in its “vanilla” form, is not a suitable candidate for quantifying (classical or quantum) correlations in mixed CV states.

To mitigate the divergence problem, we introduce the REG, defined as

$$\Delta\mathcal{E}_{\text{rel}} \equiv \frac{\Delta\mathcal{E}}{E_{gp}} = \frac{E_{lp} - E_{gp}}{E_{gp}}, \quad (6)$$

where E_{gp} denotes the energy of the global passive state. First, Lemma 1 guarantees that the REG is a faithful signature of (total) correlations. Second, as discussed in Appendix D.2 of Supplemental Material [49], the REG can also be regarded as a faithful witness of more general quantum correlations, like “discord.” Next, we investigate whether this quantity can capture the finer structure of quantum correlations, namely entanglement in the CV regime.

To this end, we first derive an explicit expression for $\Delta\mathcal{E}_{\text{rel}}$ for a general two-mode Gaussian state. We decompose Gaussian operations acting on an initial product state of two thermal modes with temperatures T_A and T_B in terms of typical parameters of quantum optical systems [65]. The resulting expression for the REG, as a function of the decomposition parameters, $\Delta\mathcal{E}_{\text{rel}} = f(k, \gamma, \alpha, \theta, z_1, z_2)$, is explicitly presented in Appendix D.1 of Supplemental Material [49]. Here, $k = [(k_A + k_B)/2]$ is the mean thermal fluctuation factor, $\gamma = [(k_A - k_B)/2]$ is the fluctuation difference, $\alpha = (\omega_B/\omega_A)$ is the frequency ratio between the two modes, z_A and z_B are the respective squeezing parameters, and θ represents the beam splitter angle applied to entangle the modes.

Having introduced the REG, we now reveal a key structural distinction with respect to QMI.

Lemma 2—The relative ergotropic gap $\Delta\mathcal{E}_{\text{rel}}$ is functionally independent of the quantum mutual information.

This independence, demonstrated in Appendix F.1 of Supplemental Material [49], highlights the intrinsic significance of REG and indicates that $\Delta\mathcal{E}_{\text{rel}}$ captures distinct features of quantum correlations not accounted by QMI alone. To further investigate this discrepancy, we compare entanglement witnesses based on QMI and REG, finding instances where certain entangled states are detected by the latter but not by the former, and vice versa (see Appendix F.2 of Supplemental Material [49]). Motivated

by this, we explore whether $\Delta\mathcal{E}_{\text{rel}}$ can discriminate between qualitatively distinct types of correlations, such as those in separable versus entangled states.

Theorem 2—Let ρ_G be a Gaussian state of two modes.

(a) If ρ_G is separable, then $\Delta\mathcal{E}_{\text{rel}}(\rho_G)$ is upper-bounded by

$$\Delta\mathcal{E}_{\text{rel}}(\rho_G) \leq \mathcal{B}_{\text{max}}^{\text{sep}}(k, \gamma, \alpha). \quad (7)$$

(b) If ρ_G is entangled, then $\Delta\mathcal{E}_{\text{rel}}(\rho_G)$ is lower-bounded by

$$\Delta\mathcal{E}_{\text{rel}}(\rho_G) > \mathcal{B}_{\text{min}}^{\text{ent}}(k, \gamma, \alpha). \quad (8)$$

The proof of the above theorem and explicit expressions for the bounds, together with a numerical illustration of the applicability of this result, are provided in Appendix E of Supplemental Material [49]. The two bounds are derived analytically employing the positive partial transpose (PPT) criterion [66] as a necessary and sufficient condition for separability in Gaussian two-mode systems. Notably, they depend exclusively on the mean fluctuation parameters of the modes, their fluctuation difference, and the ratio of their respective frequencies. Recall that Lemma 1 establishes $\Delta\mathcal{E}_{\text{rel}}$ as strictly positive for all correlated states—whether classical or quantum—whereas Theorem 2 provides a sharper distinction via two analytic bounds: Eq. (7) as a witness for entanglement, and Eq. (8) as a witness for separability. These bounds satisfy $\mathcal{B}_{\text{min}}^{\text{ent}}(k, \gamma, \alpha) \leq \mathcal{B}_{\text{max}}^{\text{sep}}(k, \gamma, \alpha)$, but they do not always coincide. As a result, $\Delta\mathcal{E}_{\text{rel}}$ is a useful qualifier but not a full entanglement quantifier [67], since states in the “coexistence region” [36,68] characterized by values of $\Delta\mathcal{E}_{\text{rel}}$ between the two bounds cannot be unequivocally classified. However, the bounds do coincide in a physically relevant case.

Corollary 1—Let ρ_G be a Gaussian two-mode state such that the same fluctuation factor characterizes both modes. Then

$$\Delta\mathcal{E}_{\text{rel}}(\rho_G) \leq \mathcal{B}_{\text{max}}^{\text{sep}} \quad (9)$$

is a necessary and sufficient condition for separability.

As a consequence of these results, the faithfulness of $\Delta\mathcal{E}_{\text{rel}}$ as an entanglement measure is determined by the distance between the bounds $\mathcal{B}_{\text{max}}^{\text{sep}}$ and $\mathcal{B}_{\text{min}}^{\text{ent}}$: the closer they are, the more accurately our criterion distinguishes between separable and entangled states. In the limiting case where the two bounds coincide, the condition based on the REG becomes equivalent to the PPT criterion. To illustrate this behavior, Fig. 1 shows the degree to which $\Delta\mathcal{E}_{\text{rel}}$ violates the separability bound for the parametric family of two-mode squeezed Gaussian states $\text{TMS}(k, z)$, for different values of the fluctuation difference γ .

Non-Gaussian states—Gaussian states are both experimentally and theoretically important. However, their ease of characterization also comes with fundamental limitations

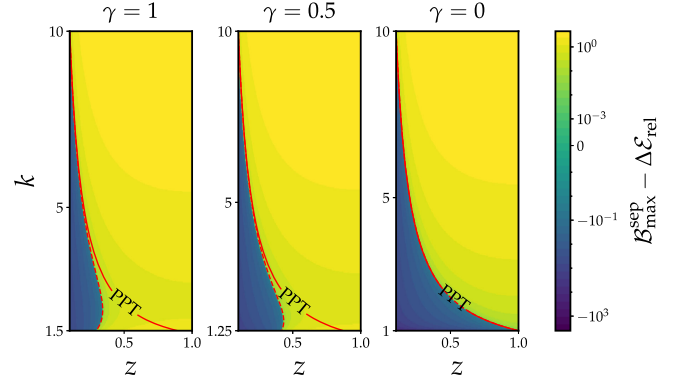


FIG. 1. Difference between $\mathcal{B}_{\text{max}}^{\text{sep}}$ and $\Delta\mathcal{E}_{\text{rel}}$ for the parametric family of Gaussian two-mode squeezed states, as a function of the squeezing parameter z and the mean fluctuations factor k . Values of the fluctuation difference γ have been set to $\{1, 0.5, 0\}$ (left to right), and the frequency ratio is set at $\alpha = 1$ for all cases. The dashed and solid red lines represent the boundaries between separable and entangled states according to our criterion [Eq. (7)] and the PPT condition, respectively. As predicted by Corollary 1, these boundaries coincide when $\gamma = 0$.

regarding their usefulness in quantum information tasks [69]. This motivates our exploration of the non-Gaussian regime [41] and the question of how far the relationship between the ergotrophic gap and quantum correlations can be extended. For generic non-Gaussian states, evaluating the ergotrophic gap becomes highly challenging and may even be intractable. By contrast, the Gaussian ergotrophic gap $\Delta\mathcal{E}_G$ [Eq. (2)]—together with the associated Gaussian REG (G-REG)—remains fully determined by the covariance matrix, thereby retaining computational accessibility. This observation suggests that while $\Delta\mathcal{E}$ provides the most general characterization, the Gaussian counterpart $\Delta\mathcal{E}_G$ may serve as a practical proxy in situations where the exact computation is prohibitive. In what follows, we investigate how $\Delta\mathcal{E}$ (where calculable), $\Delta\mathcal{E}_G$, and their relative forms can act as witnesses of entanglement in different families of non-Gaussian states.

By construction, the general results derived earlier (Theorems 1 and 2) apply directly to non-Gaussian states obtained from Gaussian states through local unitary operations. However, the separable bound established in Eq. (7) cannot be straightforwardly generalized to arbitrary non-Gaussian states. To illustrate these subtleties, we now examine a specific class of states where $\Delta\mathcal{E}_G$ becomes trivial and fails as an entanglement witness, while $\Delta\mathcal{E}$ retains its detecting power.

Lemma 3—Any symmetric superposition of bipartite Fock states, i.e., any state of the form $(1/\sqrt{2})(|nm\rangle + |mn\rangle)$ with $n, m \in \mathbb{N}$, is passive under Gaussian operations.

The proof, provided in Appendix G.1 of Supplemental Material [49], shows that the covariance matrix of such states coincides with that of a Gaussian-passive state. Consequently, $\Delta\mathcal{E}_G = 0$ even though the state is pure

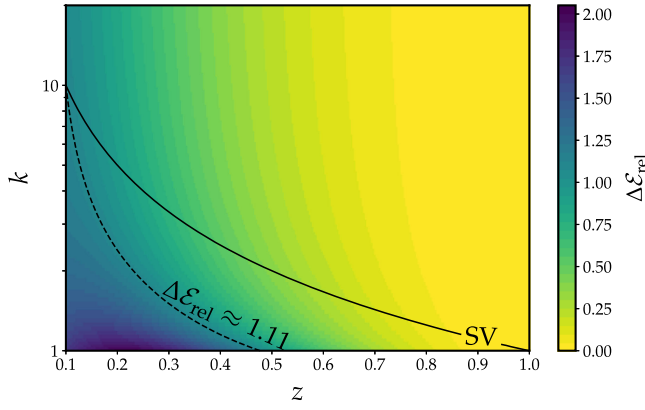


FIG. 2. Gaussian relative ergotropic gap of photon-subtracted two-mode squeezed states as a function of the fluctuation parameter k of the initial thermal state and the Gaussian squeezing parameter z . The region below the solid line is formed by entangled states as witnessed by the SV condition, while the region below the dashed line indicates entangled states witnessed by our G-REG-based criterion (Lemma 4).

and entangled. Notably, however, the entanglement in this class can still be faithfully witnessed through $\Delta\mathcal{E}$, when ergotropy is defined with respect to arbitrary unitary operations.

Interestingly, there exist classes of non-Gaussian states where one can restore Gaussian nonpassivity, leading to a nontrivial separable bound on $\Delta\mathcal{E}_G$. This is the case of the mixed bipartite Fock state $\rho = \lambda|\phi^+\rangle\langle\phi^+| + (1-\lambda)|\phi^-\rangle\langle\phi^-|$ with $n \in \mathbb{N}$, where $|\phi^\pm\rangle = 1/\sqrt{2}(|n, n\rangle \pm |n+1, n+1\rangle)$. For this example, ρ is separable if and only if $\Delta\mathcal{E}_G = 0$ (see proof in Appendix G.2 of Supplemental Material [49]). This example highlights that within certain non-Gaussian families, both $\Delta\mathcal{E}$ and $\Delta\mathcal{E}_G$ can serve as necessary and sufficient entanglement witnesses.

Importantly, for the above class of Fock states, $\Delta\mathcal{E}$ can be easily evaluated. In contrast, for genuinely infinite-dimensional non-Gaussian states, one must rely on $\Delta\mathcal{E}_G$ to circumvent both experimental and theoretical complications. This raises the question of whether nontrivial separability bounds for $\Delta\mathcal{E}_G$ can be established in such cases. Below, we provide an affirmative answer by considering an experimentally relevant family: photon-subtracted TMS states [41].

Figure 2 shows $\Delta\mathcal{E}_{\text{rel}}$ as a function of the squeezing and fluctuation parameters z and k of the initial Gaussian TMS state prior to photon subtraction. Here, G-REG ($\Delta\mathcal{E}_G$) is computed using Eqs. (3), (5), and (6), together with the covariance matrix derived in [70]. As a separability benchmark we employ the Shchukin-Vogel (SV) criterion [42], whose negativity suffices to certify entanglement. The maximum value of $\Delta\mathcal{E}_{\text{rel}}$ observed within the separable region (above the SV boundary) is then taken as a threshold, indicated by the dashed line in the figure.

Lemma 4—For the non-Gaussian family of photon-subtracted two-mode squeezed states, $\Delta\mathcal{E}_{\text{rel}} \gtrsim 1.11$ implies entanglement.

A detailed proof is given in Appendix G.3 of Supplemental Material [49]. We emphasize that the separability bounds we establish for non-Gaussian states are inherently class dependent: a bound derived for one family cannot, in general, be used as an entanglement witness for another. It therefore remains an open problem whether a universal nontrivial bound for all non-Gaussian separable states exists. If such a bound were found, G-REG could then serve as a fully general tool for entanglement witnessing in the non-Gaussian regime, independent of prior knowledge of the underlying state family.

Discussion—In this Letter, we characterize correlations in continuous-variable systems through thermodynamic processes. For bipartite Gaussian states, we demonstrate that correlation guarantees a strictly positive ergotropic gap, revealing an energy-based signature of correlations independent of entropy. Physically, this means that energy can be locked into correlations, making it inaccessible via local operations alone. For pure states, this yields an entanglement measure equivalent to the quantum mutual information. However, in mixed states, the ergotropic gap grows while correlations decrease due to the unbounded energy spectrum. To address this, we introduce the REG, a normalized quantity that is functionally independent of the quantum mutual information and more generally of entropic quantifiers. We derive analytical bounds on REG that distinguish entangled from separable states and serve as a necessary and sufficient separability criterion for a large class of states, paralleling the PPT test. Notably, when energy measurements are available [71], this could provide a more laboratory-friendly entanglement indicator than full state tomography. Moreover, we show that the REG may find broader applicability outside the Gaussian regime—an open direction for future study. Overall, our Letter establishes the REG as an energy-based, experimentally accessible witness of continuous-variable quantum correlations, bridging thermodynamics and information-theoretic perspectives, and offering a novel tool for entanglement detection and characterization in near-term quantum optical technologies.

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The views and opinions expressed in this Letter are the sole responsibility of the author(s).

Data availability—The data that support the findings of this article are openly available [72].

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