

Structure and Classification of Matrix Product Quantum Channels

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We develop a framework for matrix product quantum channels, a one-dimensional tensor-network description of completely positive, trace-preserving maps. We focus on translation-invariant channels, generated by a single repeated tensor, that admit a local purification. We show that their purifying isometry can always be implemented by a constant-depth brickwork quantum circuit, implying that such channels generate only short-range correlations when applied on product states. In contrast to the unitary setting, where one-dimensional quantum cellular automata (in one-to-one correspondence with matrix product unitaries) carry a nontrivial index, we prove that all locally purified channels belong to a single phase, that is, they can be continuously deformed into one another. Then, we extend the framework to a broader class of translation-invariant channels capable of generating long-range entanglement and show that these remain deterministically implementable in constant depth using two rounds of measurements and feedforward.

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Introduction—Tensor-network methods, particularly matrix product states (MPS) and matrix product density operators (MPDOs), have become indispensable tools in the study of one-dimensional quantum many-body systems. MPS efficiently approximate ground states of 1D local and gapped Hamiltonians [1,2] and provide a framework to classify 1D gapped phases of matter [3,4]. MPDOs can accurately represent thermal states of local 1D Hamiltonians [5,6], but also appear as boundary states of 2D gapped systems [7,8]. Yet, a complete description of physical processes requires going beyond states and operators to encompass quantum channels, i.e., completely positive trace-preserving maps governing open-system dynamics. Representing these channels in matrix-product form is important for several reasons. First, they preserve the area-law and, thus, naturally arise in classifying mixed-state phases [9–16]. Second, they can encode symmetry actions on matrix product density operators, similar to how matrix product operator (MPO) symmetries function for MPS [17–19]. Finally, they offer a compact tensor-network description for correlated noise models beyond low-weight

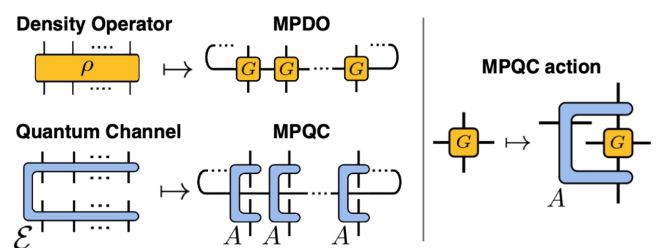


FIG. 1. We analyze the structure, classification, and physical implementation of matrix product quantum channels (MPQCs) composed of a repeated tensor A . These act on matrix product density operators (MPDOs), preserving their matrix-product form.

errors on quantum devices, which can be used as a variational *Ansatz* for noise tomography [20–22].

Despite their relevance, a systematic analysis of the correlation structure and a classification of matrix product quantum channels (MPQCs) remain lacking. What has been thoroughly analyzed is the unitary case, namely matrix product unitaries (MPUs) [23–29]. In the homogeneous setting, where the same tensor is used at every site, MPUs are equivalent to 1D quantum cellular automata (QCA) [23]—unitaries with a strict, finite light cone [30]. This equivalence has several strong consequences. First, it reveals that the correlations homogeneous MPUs can generate are heavily constrained. Second, it implies a classification of homogeneous MPUs, as a direct consequence of the classification of QCA via index theory [31]. Third, it solves the problem of the physical implementation of MPUs—a *a priori* nontrivial since individual tensors do not directly correspond to physical processes—implying a

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low-depth quantum circuit for their physical implementation [23,32], a fact that can be extended for more general MPUs [33]. These properties, derived from homogeneity, raise the question of whether similar structural constraints and classification principles apply to quantum channels.

Here, we develop a systematic framework for MPQCs, focusing on those modeling processes that originate from an environment with an underlying matrix-product structure, which we refer to as locally purifiable (LP) channels. We show that imposing homogeneity restricts LP channels to generating only short-range correlations, a consequence of the fact that they all admit a depth-two circuit representation as isometric gates. Remarkably, we find that all homogeneous LP channels belong to a single equivalence class, connected through continuous deformations. We further examine a class of translation-invariant channels capable of producing long-range correlations and show how these can still be implemented efficiently as quantum circuits. Finally, we present a construction that shows how such channels can be represented in terms of simpler building blocks, namely MPUs and MPS.

Matrix product quantum channels—To define an MPQC, we start from a rank-six tensor A , graphically represented as

$$A_{mn}^{ij,op} = \text{[Diagram: A rank-6 tensor A with two horizontal bonds labeled m and n, and two vertical bonds labeled i and j. The top vertical bond is labeled o and the bottom vertical bond is labeled p.]} \in \mathbb{C}, \quad (1)$$

see Fig. 1. The indices $i, j = 1, \dots, d_{\text{in}}$ label the input physical space, $o, p = 1, \dots, d_{\text{out}}$ the output physical space, and $m, n = 1, \dots, D$ the auxiliary (bond) space. The integer D is the bond dimension of the MPQC. Hence, each $A^{ij,op}$ can be represented as a $D \times D$ matrix. The tensor A generates a homogeneous matrix product quantum channel (hMPQC) if the superoperator

$$\text{MPQC}_N^A: \mathcal{L}(\mathbb{C}^{d_{\text{in}}})^{\otimes N} \rightarrow \mathcal{L}(\mathbb{C}^{d_{\text{out}}})^{\otimes N}, \quad (2)$$

where $\mathcal{L}(X)$ is the linear space of operators acting on X , transforming density matrices as

$$\rho'_{o_1, \dots, o_N, p_1, \dots, p_N} = \sum_{\substack{i_1, \dots, i_N \\ j_1, \dots, j_N}} \text{Tr} \left[\prod_{k=1}^N A_{k, j_k, o_k, p_k}^{i_k, j_k} \right] \rho_{i_1, \dots, i_N, j_1, \dots, j_N}, \quad (3)$$

is completely positive (CP) and trace preserving (TP) for all $N \geq 1$. The above construction repeats a fixed tensor A along the chain with periodic boundary conditions—hence, homogeneous—and defines the hMPQC class [34]. We use a stylized font throughout to denote different channel classes.

Homogeneous MPDOs, and, thus, also the corresponding MPS, can be expressed in the hMPQC form by setting $d_{\text{in}} = 1$, i.e., by trivializing the input of the channel. From the matrix-product form of the coefficients in Eq. (3), it follows that a hMPQC acting on an MPDO can increase the bond dimension by at most a multiplicative factor D (but decrease it arbitrarily).

As with MPS, one can more generally allow arbitrary site-dependent tensors A_k with varying bond dimensions D_k . In fact, any quantum channel can be represented as an MPQC in this general form by successive singular value decompositions after vectorizing, following the standard MPS procedure [35,36]. However, for a generic channel, the bond dimension might grow exponentially with system size N , rendering the matrix-product representation useless. Therefore, we define the class MPQC, specified over increasing system sizes N , consisting of only channels admitting a representation with a bond dimension uniformly bounded by a constant independent of N . Henceforth, we focus on the homogeneous setting, where this property holds by definition, and discuss the general case in the Supplemental Material [37].

CP and TP, needed for the superoperator MPQC_N^A to define a proper quantum channel, can be equivalently formulated in terms of the corresponding Choi-Jamiołkowski state $C_N^A \equiv (\text{MPQC}_N^A \otimes \mathbb{I}_{\text{in}})(|\Phi^+\rangle\langle\Phi^+|)$

$$(i) C_N^A \geq 0, \quad (ii) \text{Tr}_{\text{out}}(C_N^A) = \mathbb{I}_{\text{in}} \quad \forall N, \quad (4)$$

where $|\Phi^+\rangle \equiv (\sum_{i=1}^{d_{\text{in}}} |i\rangle|i\rangle)^{\otimes N}$ is an (unnormalized) Bell state. However, verifying the first condition, i.e., whether the matrix product operator C_N^A is positive semidefinite for all system sizes, is undecidable in the general case [56]. Moreover, we want to focus on physically relevant channels, i.e., noise models arising from an environment with a matrix-product structure. Consequently, we focus on a subclass of MPQCs in which CP and TP can be explicitly imposed and verified.

Locally purified quantum channels—Now, we introduce the homogeneous locally purified (hLP) subclass of hMPQC. Within this class, CP is automatic and TP can be characterized on the level of the single tensor.

Definition 1 [Local purification]—An MPQC tensor A admits a local purification if there exists another tensor $\mathcal{V}$ such that

$$\text{[Diagram: Tensor A with two horizontal bonds and two vertical bonds]} = \text{[Diagram: Two vertical bonds connected by a box labeled V]} , \quad \text{[Diagram: Box labeled V]} = \left(\text{[Diagram: Box labeled V with a horizontal line through it]} \right)^*. \quad (5)$$

An hMPQC belongs to hLP if its tensor admits a local purification [57].

Informally, this condition guarantees the existence of a local Kraus decomposition at each physical site. More precisely, admitting a local purification generates, by construction, a CP map; this can be directly verified via the Choi-Jamiołkowski state, which takes the form $C_A^N = W^\dagger W$, and, thus, is positive semidefinite [37]. The Stinespring dilation of the resulting CP map is the MPO $V_N: (\mathbb{C}^{d_{\text{in}}})^{\otimes N} \rightarrow (\mathbb{C}^{d_{\text{out}}\chi})^{\otimes N}$, defined by

$$V_N \equiv \text{---} \left(\text{---} \text{---} \right) \text{---} \quad (6)$$

Here, χ is the dimension of the local purification space, denoted in gray. TP, which needs to be imposed separately, amounts to the condition $V_N^\dagger V_N = \mathbb{I}_{d_{\text{in}}}^{\otimes N}$, i.e., that V_N is a homogeneous matrix product isometry (hMPI). The resulting hMPQC is then

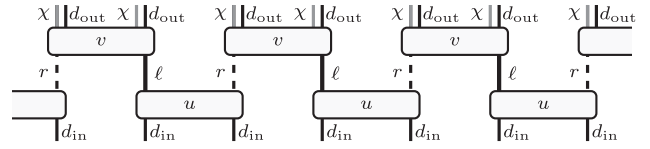
$$\text{MPQC}_N^\mathcal{V} = \text{Tr}_{\text{pur}}[V_N(\cdot)V_N^\dagger]. \quad (7)$$

Physically, an hMPI acting on an MPS can increase the entanglement entropy across any bipartition by at most a constant amount. Consequently, an hLP channel maps any MPDO whose purifying MPS obeys an area law to one that also obeys it. In fact, locally purified channels are the most general quantum channels with this property, once area-law preservation is required to be stable under ancillary extensions [37].

Structure of hMPIs—Now, we will show that imposing homogeneity and the existence of a local purification has very strong implications for the correlations that the resulting channel can generate. To this end, we will fully characterize their purifying hMPI. This question parallels the characterization of homogeneous matrix product unitaries (MPUs) in Refs. [23,24], where they were shown to coincide with QCA in 1D, that is, unitaries possessing a strict light cone [30,58]. Here, we extend this framework to hMPIs.

For the correlation structure to emerge, short-range entanglement must be coarse-grained away. On the tensor level, this is done by blocking, i.e., grouping q consecutive tensors $\mathcal{V}$ into a single effective tensor $\mathcal{V}_q$ (see, e.g., Ref. [59]). Throughout, when we block, q will be independent of the system size N so that blocking eliminates only short-range, but not long-range, correlations.

Theorem 1—Any hMPI can be written as a depth-two brick wall quantum circuit of isometric gates u, v , satisfying $u^\dagger u = \mathbb{I}_{d_{\text{in}}}$ and $v^\dagger v = \mathbb{I}_{\ell r}$, after blocking at most D^4 times:



The isometries $u: \mathbb{C}^{d_{\text{in}}} \mapsto \mathbb{C}^{\ell r}$ and $v: \mathbb{C}^{\ell r} \mapsto \mathbb{C}^{\chi^2 d_{\text{out}}}$ satisfy $d_{\text{in}}^2 \leq r\ell \leq d_{\text{out}}\chi^2$.

As a result, the correlations that a hMPI—and, thus, also a hLP channel—can generate are bounded by a strict light cone because of the depth-two brick-wall circuit structure [60]. Concretely, it means that, after blocking, the reduced state ρ_{AB} of the global pure state $V_N|0\rangle^{\otimes N}$ over any regions A, B , separated by more than four sites, factorizes: $\rho_{AB} = \rho_A \otimes \rho_B$ if $\text{dist}(A, B) > 4$. This clearly generalizes to other input states with a finite correlation range [62].

Physically, Theorem 1 also directly implies that the channels in hLP have low circuit complexity. More precisely, the isometries in Theorem 1 can be physically implemented via local unitary circuits and ancillas with only a constant depth, depending only on the bond dimension but not the system size. This also includes, e.g., left and right lattice translations by one site, known as shift unitaries [31,63]. Another direct consequence of Theorem 1 is that, if there exists two hLP channels mapping the MPDOs $\rho_N \rightleftharpoons \sigma_N$, then the two states are necessarily in the same mixed-state phase [9–16]. In the homogeneous setting, this amounts to compact transformations over a single tensor, despite the channel having a brickwork structure.

Classification of hLP channels—A landmark result for one-dimensional QCAs is their classification by a positive-rational-valued index [31], which quantifies the net transport of quantum information along the chain. The index labels disconnected sectors of the QCA space: two QCAs share the same index if and only if they can be continuously deformed into one another while preserving the QCA property. Since homogeneous MPUs coincide with translation-invariant 1D QCAs [23], this classification can be formulated directly at the level of a single MPU tensor: two QCAs have the same index if and only if their MPU tensors are continuously deformable within the space of MPU-generating tensors [23].

In the same spirit, we now define an equivalence relation for hLP channels. As in the case of MPUs, this is defined by continuous deformations at the level of the single tensor generating the MPI, i.e., the channel dilation.

Definition 2 [Equivalence]—Let $\mathcal{E}_0$ and $\mathcal{E}_1$ be channels in hLP with identical input and output dimensions d_{in} and d_{out} . They are equivalent if there exists a continuous map of hMPI-generating tensors

$$[0, 1] \ni s \mapsto \mathcal{V}(s) = \text{---} \text{---} \text{---} \quad D, \chi \text{ const.}, \quad (8)$$

with associated MPI $V_N(s)$, such that

$$\mathcal{E}_i = \text{Tr}_{\text{pur}}[V_N(i)(\cdot)V_N(i)^\dagger], \quad i \in \{0, 1\} \quad \forall N. \quad (9)$$

In essence, two channels are equivalent if their purifying isometries can be continuously deformed into one another, allowing enlargement of the (possibly redundant) purification space, such that the endpoints reproduce the respective channels after tracing out the environment. Our result is

Theorem 2—Any two channels in hLP with matching input and output dimensions are equivalent.

For the proof, we embed the two MPI tensors into a common enlarged purification space, arranged so that at the initial point of the path, one tensor acts over the input while the other acts on a fixed ancilla qubit and is effectively discarded. The interpolation is then implemented by a continuous local unitary rotation that exchanges their roles, smoothly transferring the action from one tensor to the other. The result of the theorem further extends to arbitrary input and output dimensions once equivalence is relaxed to allow for ancillary inputs that are traced out and ancillary outputs prepared in fixed product states [37].

At first sight, this seems in tension with the QCA classification, since the shift and the identity, viewed as unitaries, have distinct indices and are, therefore, inequivalent. The resolution is that, now, we regard them as channels: only their action after tracing out a purification space is fixed, not their specific unitary realization. This additional freedom permits continuous deformations that are forbidden in the strictly unitary setting. In this sense, the purification space plays the role of ancillas in QCA theory, whose inclusion is known to trivialize any QCA [32,64].

Appearance of long-range entanglement—As shown before, hMPIs do not produce long-range entanglement. Theorem 1 raises the question of what physically forbids long-range correlations. We will show that the key ingredient is the exact isometry condition imposed for all N . The situation parallels the MPS setting, where normalized hMPS are necessarily short-range entangled, while relaxing the normalization condition by a constant restores long-range correlated states such as the GHZ state [4]. Now, we show that the analogous relaxation applies to MPIs.

Example 1—Consider the quantum channel $\mathcal{E}(\cdot) = \text{Tr}(\cdot)|\text{GHZ}\rangle\langle\text{GHZ}|$ acting on N qubits. For all N , it admits a purifying MPI with $D_{\text{MPI}} = \chi = 2$

$$V_N = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \cdots \\ \text{---} \bullet \text{---} \bullet \text{---} \cdots \end{array} \right),$$

where $\begin{array}{c} \text{---} \bullet \text{---} \end{array} = \begin{array}{c} \text{---} \bullet \text{---} \end{array} = 1$ or vanishing, otherwise, is the local tensor of an unnormalized GHZ state. The resulting channel generates long-range entanglement due to the correlations present in the GHZ state.

This appears to be in tension with Theorem 1, which states that hMPIs cannot generate long-range entanglement. The discrepancy arises because this example does not fit within the scope of the theorem, but merely because of the $1/\sqrt{2}$ constant: the homogeneous MPI constructed from

the tensor $\mathcal{V} = \begin{array}{c} \text{---} \bullet \text{---} \end{array}$ is an isometry only after normalizing

with an N -independent constant, given that $V_N^\dagger V_N = 2\mathbb{1}_{d_{\text{in}}}^{\otimes N}$. Since this constant can have important physical implications (short-range versus long-range entanglement, as the example shows), this motivates relaxing the strict isometry requirement $V_N^\dagger V_N = \mathbb{1}_{d_{\text{in}}}^{\otimes N}$ of hMPI to

$$V_N^\dagger V_N = c\mathbb{1}_{d_{\text{in}}}^{\otimes N} \quad \forall N \geq 1, \quad (10)$$

for some N -independent constant $c > 0$. Notice that the constant cannot be incorporated into the generating tensor $\mathcal{V}$ of V_N without breaking homogeneity. We call the resulting class satisfying the relaxed condition scaled homogeneous MPI or sMPI, which we study as the minimal possible extension exhibiting features absent in the homogeneous framework.

We emphasize that the resulting isometries are valid physical objects; the only nuance is that they admit a homogeneous matrix-product representation only after an appropriate normalization. However, since Theorem 1 no longer applies, their quantum circuit complexity and the structure of the entanglement they generate are not immediately clear. We address both questions by explicitly characterizing their structure.

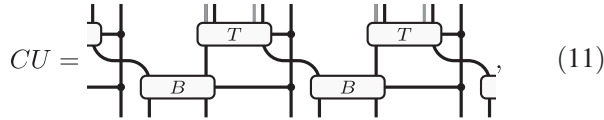
Physical implementation of sMPIs—Now, we address how to efficiently implement isometries in sMPI. Recall that, although the tensor network representation of sMPIs gives a compact mathematical description, the individual tensors do not directly relate to physical operations. Thus, now, we consider the problem of how to turn these tensor-network isometries into quantum circuits composed of geometrically local unitary gates. In this setting, due to the existence of long-range entanglement as in Example 1, it follows that necessarily at least an $\Omega(N)$ -depth circuit is necessary to prepare these isometries. The same holds when allowing local ancillas initialized in a product state, which are eventually discarded.

Despite these fundamental limitations, now, we show how to deterministically implement all isometries in sMPI, and, thus, also the resulting MPQCs, in constant (i.e., N -independent) depth, by allowing, along with the unitary gates, local product-basis measurements and nonlocal classical communication (feedforward), in the spirit of Refs. [29,65–76]. That is, we allow conditioning subsequent unitary gates on classical measurement outcomes, but forbid any postselection. Our result is

Theorem 3—Any sMPI can be deterministically implemented by a constant-depth circuit of local unitary gates,

assisted by two rounds of local measurements, nonlocal classical communication and local unitary corrections, as well as $O(N)$ local ancilla qubits.

The result provides an explicit protocol for implementation, given the single-site tensor that generates the sMPI. The steps, explained in detail in [37], are (i) Prepare a state of the form $|\text{GHZ}\rangle = (1/\sqrt{c}) \sum_{j=0}^{g-1} m_j |j\rangle_A^{\otimes N}$ on an ancillary system consisting of N g -dimensional qudits distributed across the system ($g \leq D$). This can be achieved deterministically with local operations in constant depth with a single round of measurements and feedforward [77], (ii) Feed this to the control space of a depth-two control-brickwork circuit of the form



where B and T are unitary gates for all values of the control. (iii) Measure all ancilla qudits in parallel and record the outcomes. Discard the ancillas, which factorize after the measurements. (iv) Apply a phase unitary at a single site, which depends on all classical outcomes of the previous measurement step.

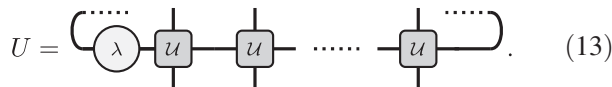
The key technical result we obtain, which allows us to design the protocol—specifically the circuit of (ii)—and guarantee its validity and overcome the need for postselection, is a complete characterization of the structure of all sMPIs. More precisely, we show that they all decompose as

$$V_N = \frac{1}{\sqrt{c}} \sum_{j=0}^{g-1} m_j V_{j,N} \quad (12)$$

where all V_j are hMPIs which are orthogonal, i.e., $V_{i,N}^\dagger V_{j,N} = \delta_{ij} \mathbb{1}_{d_{\text{in}}}^{\otimes N}$, $g \leq D$ and the integer constants m_j satisfy $\sum_j m_j^2 = c$ (see Ref. [37] for details).

sMPIs from MPUs—Any hMPI can be viewed as a hMPU acting on a fixed input MPS (a product state): the isometric gates in Theorem 1 can be extended to unitaries, whose action on a fixed product-state input reproduces the original isometries. Likewise, Example 1 can be viewed as a product MPU—namely, the product of swaps—of which half of the input legs are contracted with a GHZ state.

This naturally raises the question of whether every sMPI admits a representation in terms of simpler components, that is, as a hMPU acting on an input MPS. We show that the answer is, indeed, positive, with the corresponding MPU having homogeneous bulk but also a boundary, graphically



The need for a boundary can be expected from the case of sMPUs that create long-range correlations, e.g., $U = (1/\sqrt{2})(1^{\otimes N} + iX^{\otimes N})$ [27].

Theorem 4—Any normalized sMPI V on an N -qudit system can be represented as an MPU with homogeneous bulk and a boundary, acting over a $|\text{GHZ}\rangle = (1/\sqrt{c}) \sum_{j=0}^{g-1} m_j |j\rangle^{\otimes N}$ input state.

The construction follows Theorem 3 up to step (iii), where it diverges due to the requirement that the overall operation be unitary, as we seek an MPU and, thus, cannot directly employ measurements. Instead, the idea is to use an amplitude amplification method, which deterministically yields the target isometry by iteratively suppressing undesired components using unitary operations alone [33,78] (see Ref. [37] for details).

Outlook—We have developed the theory of MPQCs as a tensor-network representation of one-dimensional quantum channels and the physically motivated LP subclass, where the environment admits a locality structure. For homogeneous channels, we proved a circuit decomposition into depth-two isometries after blocking, introduced a classification, and established the existence of a unique equivalence class. Finally, we identified a translation-invariant class with long-range entanglement and provided an efficient constant-depth implementation protocol using measurements and feedforward. We have also established how this class connects to elementary components, namely, MPUs and MPS.

The present work suggests several future directions. First, the focus was on translationally invariant channels, but nontranslationally invariant MPQCs could be relevant as an *Ansatz* for describing correlated noise beyond the low-weight regime [20–22]. Theorem 1 shows that tomography schemes based on locally purified matrix-product *Ansätze* and translational invariance will fail to reconstruct long-range correlated noise, motivating the development of more expressive channel classes such as the sLP introduced here. From the many-body perspective, one could study MPDO symmetries arising from homogeneous MPQC, systematically extending the results of Ref. [61], similar to how MPO symmetries function for MPS [17–19]. hLP may also be relevant for describing edge dynamics in dissipative 2D Floquet systems. This parallels how causality-preserving unitaries, which are equivalent to 1D MPUs, determine edge behavior in the closed setting [79,80]. Moreover, local quantum channels naturally arise in methods for classifying mixed-state phases [9–16]. In that context, it would be relevant to extend the structure result to MPIs allowing for normalization constants dependent on the system size. Finally, it would be interesting to investigate if the strict light cone of hMPIs, and, thus, also the classification of the resulting channels, extends to higher spatial dimensions.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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