

Bayesian Inferring Nucleon Gravitational Form Factors via Near-Threshold J/ψ Photoproduction

Yuxun Guo^{1,2,*}, Feng Yuan^{2,†} and Wenbin Zhao^{1,2,‡}

¹Physics Department, University of California, Berkeley, California 94720, USA

²Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA



(Received 31 January 2025; revised 7 August 2025; accepted 13 August 2025; published 10 September 2025)

With Bayesian inference, we investigate the impact of recent near-threshold J/ψ production measurements by the J/ψ 007 experiment and GlueX Collaboration on the extraction of proton gravitational form factors. We apply the generalized parton distribution framework at the next to leading order and demonstrate a stable expansion for the near-threshold kinematics. We find that the experimental constraints are in good agreement with the state-of-the-art lattice simulations, where negative $C_q(t)$ and $C_g(t)$ are strongly preferred. This highlights a great potential to extract them from future high-precision experiments.

DOI: 10.1103/3x7r-ythq

Introduction—Nucleon tomography encoded in the gravitational form factors (GFFs) has drawn rising interest in the past decade. GFFs hold the keys to understanding the fundamental structure, including the mass, spin, and mechanical properties, of the nucleon [1–7]. One of the most important questions is how to identify the gluonic components, which are challenging to access due to their elusive nature, i.e., gluons do not couple to electromagnetic probes directly. It was suggested long ago that heavy quarkonia can help to unveil the underlying dynamics of the gluon [8–13]. The near-threshold heavy quarkonium photoproduction,

$$\gamma + p \rightarrow J/\psi + p, \quad (1)$$

has been extensively investigated from both the experimental and theoretical sides to explore gluon GFFs in recent years [6,14–29]. Together with first-principles lattice calculations [30–33], these investigations provide a unique opportunity to uncover the fundamental nucleon structure.

One especially interesting feature of near-threshold kinematics is that the momentum transfer is large and the skewness parameter ξ approaches 1 in the heavy-quarkonia limit. Applying the QCD factorization in the generalized parton distribution (GPD) framework and utilizing this feature, it was found that the leading

contribution to the amplitude is from the lowest moments of GPDs [21,28,34–36]. This leading-moment dominance allows us to constrain the gluon GFFs through asymptotic expansion with large ξ [21,22,28]. In the GPD framework, the cross sections of near-threshold heavy quarkonium productions read [21]

$$\frac{d\sigma}{dt} = \frac{\alpha_{\text{EM}} e_Q^2}{4(W^2 - M_N^2)^2} \frac{(16\pi\alpha_S)^2}{3M_V^3} |\psi_{\text{NR}}(0)|^2 |G(t, \xi)|^2, \quad (2)$$

where t is the momentum transfer squared, W is the center of mass energy, and M_N and M_V represent the masses of the nucleon and heavy quarkonium, respectively. $\psi_{\text{NR}}(0)$ stands for the wave function of the heavy quarkonium at $r = 0$ in the nonrelativistic QCD (NRQCD) [37]. The hadronic matrix element $G(\xi, t)$ can be written as

$$|G(t, \xi)|^2 = (1 - \xi^2)(\mathcal{H} + \mathcal{E})^2 - 2\mathcal{E}(\mathcal{H} + \mathcal{E}) + \left(1 - \frac{t}{4M_N^2}\right)\mathcal{E}^2, \quad (3)$$

where $\mathcal{H} = \sum_{i=q,g} \mathcal{H}_i$ and $\mathcal{E} = \sum_{i=q,g} \mathcal{E}_i$ are the Compton-like form factors (CFFs) that can be factorized in terms of the quark and gluon GPDs, $H_{q/g}$ and $E_{q/g}$ [38–40]. For example, $\mathcal{H}_{q/g}(\xi, t)$ are written as

$$\mathcal{H}_{q/g}(\xi, t) \equiv \int_{-1}^1 dx C_{q/g}(x, \xi, \mu_f) H_{q/g}(x, \xi, t, \mu_f), \quad (4)$$

and similarly for $\mathcal{E}_{q/g}$. The Wilson coefficients $C_{q/g}$ have been computed up to next to leading order (NLO) [38–40]. At leading order (LO), only gluons contribute, and the leading-moment approximation leads to $\mathcal{H}(\xi, t) \approx 2[A_g(t) + 4\xi^2 C_g(t)]/\xi^2$ and $\mathcal{E}(\xi, t) \approx 2[B_g(t) - 4\xi^2 C_g(t)]/\xi^2$, with

*Contact author: yuxunguo@lbl.gov

†Contact author: fyuan@lbl.gov

‡Contact author: wenbinzhao@lbl.gov

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$A_g(t)$, $B_g(t)$, and $C_g(t)$ the gluon GFFs [2]. This provides an important constraint on the GFFs. To establish a solid case to extract the GFFs from these experiments, one has to go beyond the above LO picture. In this Letter, we will advance the theory developments in two aspects: apply the NLO calculations [38–40] in the threshold kinematics with large- ξ expansion and utilize the Bayesian inference method in the phenomenological analysis of the experimental data.

Taking into account the NLO corrections will not only help reduce the theoretical uncertainties but also lead to access to the quark GFFs because the quark GPDs will start to contribute at NLO. Therefore, at this order, we will be able to constrain both quark and gluon GFFs, and their total contribution to proton GFFs. In addition, to improve the accuracy of our analysis, we will employ the advanced Bayesian inference method. This offers a statistically rigorous and interpretable extraction, which will be extremely valuable since both quark and gluon GFFs enter at NLO and are strongly correlated in the analysis. Advanced computational methods, such as Bayesian inference, machine learning, or artificial intelligence, have been widely applied in nuclear physics (see, e.g., Refs. [41,42]) and in particular to constrain the nucleon properties related to the GPDs [43–46]. In our analysis, we will also consider the constraints from the lattice simulations [32]. These lattice results will constrain, in particular, the GFFs at small momentum transfer. The combined fit to the lattice results and experimental data give us the most accurate picture of the gluon GFFs.

We also notice that the underlying mechanism for near-threshold J/ψ photoproduction has been explored in different frameworks [47–51]. Future experiments at JLab and the electron-ion collider will help to identify the production mechanism.

GPD framework at NLO in QCD—In the GPD framework, the differential cross section of near-threshold heavy quarkonium photoproduction depends on the CFFs. As discussed in [28], the conformal expansion of CFFs provides better convergence in the large- ξ kinematics, which we will focus on in the following analysis. The gluonic CFFs (gCFFs) can be formally written in terms of the conformal moments of GPD as

$$\text{Re}\mathcal{H}_g(\xi, t) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \bar{C}_g^{(2n+1)} \bar{H}_g^{(2n+1)}(\xi, t), \quad (5)$$

which also applies to the quark CFF and the $\mathcal{E}_{q,g}$ CFFs. The basis $p_{q/g}^j(x, \xi)$ and its dual $c_{q/g}^j(x, \xi)$ of this conformal expansion with conformal spin j are orthonormal: $\int_{-1}^1 dx c_i^j(x, \xi) p_i^k(x, \xi) = (-1)^j \delta^{jk}$ for $i = q, g$ [52], such that the conformal moments of GPDs are

$$\bar{H}_{q/g}^{(2n+1)}(\xi, t) = \int_{-1}^1 dx H_{q/g}(x, \xi, t) c_{q/g}^{2n+1}(x, \xi), \quad (6)$$

and the corresponding Wilson coefficients are

$$\bar{C}_{q/g}^{(2n+1)} = \pm \int_{-1}^1 dx C_{q/g}(x, 1) p_{q/g}^{2n+1}(x, 1). \quad (7)$$

As the Wilson coefficient $C_{q/g}(x, \xi)$ depends solely on x/ξ up to an overall factor, its ξ dependence can be pulled out and ξ is set to 1 in the above expression. A conventional – sign appears in the prefactor for the quark. For heavy vector meson, only the singlet quark GPD $H_{\Sigma}(x, \xi, t) \equiv \sum_{q=u,d,\dots} H_q(x, \xi, t) - H_q(-x, \xi, t)$ contributes, and the corresponding Wilson coefficient $C_{\Sigma}(x, \xi)$ is normalized such that $\mathcal{H}_{\Sigma} = \sum_q \mathcal{H}_q$. In the following, the q will always refer to the singlet Σ .

The above integral relates the Wilson coefficients $C_{q/g}(x, \xi)$ to their moment space correspondence $\bar{C}_{q/g}^{(j)}$, which, however, does not always guarantee closed-form analytical expressions in terms of j , especially beyond LO. Here, we present the numerical results instead when analytical expressions are not available. For the leading moment with $j = 1$, they read

$$\bar{C}_g^{(1)} = \frac{5}{4} + \alpha_s \left[\bar{c}_g^1 - \frac{55}{16\pi} \log\left(\frac{m_c^2}{\mu_F^2}\right) \right] + \mathcal{O}(\alpha_s^2), \quad (8)$$

$$\bar{C}_q^{(1)} = 0 + \alpha_s \left[\bar{c}_q^1 + \frac{10}{9\pi} \log\left(\frac{m_c^2}{\mu_F^2}\right) \right] + \mathcal{O}(\alpha_s^2), \quad (9)$$

calculated with the NLO Wilson coefficients [38–40]. The quantities in front of the $\log(m_c^2/\mu_F^2)$ are nothing but the LO anomalous dimensions [53] and LO Wilson coefficients combined. On the other hand, the two genuine NLO scalars $\bar{c}_g^1$ and $\bar{c}_q^1$ are complicated, of which only the numerical values are presented here,

$$\bar{c}_g^1 \approx -0.369 \quad \text{and} \quad \bar{c}_q^1 \approx -0.891. \quad (10)$$

With these Wilson coefficients, the NLO correction in the leading-moment approximation can be included in the phenomenological analysis.

For clarity, we write the moments and Wilson coefficients as two-element vectors in the (q, g) space and the evolution operator $\mathcal{E}$, particularly, will be 2×2 matrices. Then the real part of CFFs can be written as

$$\text{Re}\mathcal{H} \approx \xi^{-2} \bar{\mathcal{C}}^{(1)}(\mu_F, m_c) \mathcal{E}^{(1)}(\mu_F, m_c) \bar{\mathbf{H}}^{(1)}(m_c), \quad (11)$$

with initial scale $\mu_0 = m_c$ and other dependence suppressed. We also define the evolved Wilson coefficients,

$$\bar{\mathcal{C}}^{\text{evo}} \equiv \alpha_s(\mu_F) \bar{\mathcal{C}}^{(1)}(\mu_F, m_c) \mathcal{E}^{(1)}(\mu_F, m_c), \quad (12)$$

including the LO α_s in the cross-section formula in Eq. (2), which also depends on $\mu_R (= \mu_F)$. Setting $\mu_F = m_c$

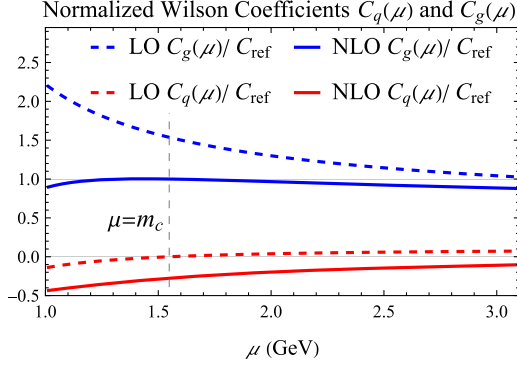


FIG. 1. The μ_F dependence of the LO and NLO quark and gluon Wilson coefficients $\bar{C}_q^{\text{evo}}(\mu_F)$ and $\bar{C}_g^{\text{evo}}(\mu_F)$, normalized by $C_{\text{ref}} = \bar{C}_g(\mu_c)$ at NLO. The dependence on μ_F gets much weaker when including NLO effects, implying a good perturbative convergence.

eliminates the evolution and resums the $\log(m_c^2/\mu_F^2)$ in the Wilson coefficients. Here, we can instead estimate the theoretical uncertainties beyond NLO by varying μ_F .

The μ_F dependence of the LO and NLO quark and gluon Wilson coefficients $\bar{C}_q^{\text{evo}}(\mu_F)$ and $\bar{C}_g^{\text{evo}}(\mu_F)$, normalized by $C_{\text{ref}} = \bar{C}_g(\mu_c)$ at NLO is shown in Fig. 1, utilizing the above Wilson coefficients and NLO GPD evolution code in [54,55]. While the scale dependence appears relatively strong at LO, it gets significantly reduced when including the NLO effects, suggesting a good perturbative convergence beyond NLO. Meanwhile, Fig. 1 also helps us to estimate the theoretical uncertainty due to perturbative corrections by varying the scale μ_F . In particular, we find that the uncertainty is reduced from 50% at LO to 10% at NLO for the gluon channel contribution with a scale variation from $\mu = 1$ GeV to $\mu = 2m_c$.

We would like to comment on two additional corrections that include the relativistic corrections from NRQCD and the large- ξ expansion in the near-threshold kinematics. As for the relativistic corrections, they are typically suppressed by $\mathcal{O}(v^2) \sim \mathcal{O}(\alpha_s^2)$, which are of order 10% around the scale of the charm mass [56]. This relativistic correction can be calculated in the NRQCD framework [57]. A major theoretical uncertainty comes from large- ξ expansion of the real part of quark and gluonic CFFs. Previously, we studied the asymptotic behavior of CFFs at LO for large ξ [28]. It gets more complicated at NLO as the Wilson coefficients now involve more singular structures. By explicitly calculating the contributions of each moments, we found that the asymptotic expansion remains valid at NLO, i.e., the contributions of higher moments decrease first and then diverge as n approaches infinity for large ξ close to 1. See Fig. S1 in Supplemental Material (SM) [58] for details. Then the truncation error can be estimated according to the supersymptotic approximation: the error of a truncated asymptotic series is given by the term of truncation.

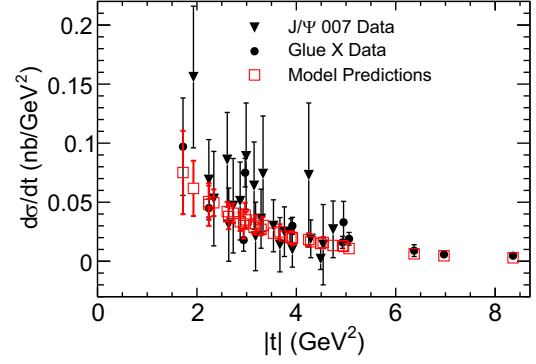


FIG. 2. The model predictions with the posterior distributions of the parameters fitting to experiments only with NLO accuracy. The experimental data is collected from the J/ψ 007 (triangle) [6] and GlueX (circle) [15] experiments across different beam energies, not shown for clarity, with a cut $\xi > 0.5$.

While the truncation errors appear larger at NLO, they are additionally suppressed by α_s . Thus we estimate the overall truncation error to be around 30% for $\xi \gtrsim 0.5$. Accordingly, we exclude data with $\xi < 0.5$ in this analysis to avoid regions with large theoretical uncertainties. We also note that consistent observations and estimation of uncertainties, including imaginary parts, are also discussed in [35].

Accordingly, we also remark that the current limitation of constraining the GFFs from experiments is mainly due to the lack of high-precision large- ξ (> 0.5) data. Existing GlueX [14,15] and J/ψ 007 [6] data in this region contain significantly large uncertainties; see Fig. 2. With future high-precision large- ξ data, proper modeling of the higher moments contributions to the real parts of quark and gluonic CFFs, as well as their imaginary parts, will be necessary to improve the theoretical uncertainties.

Phenomenological implications with Bayesian inference—Bayesian Inference is a general and systematic method to constrain the probability distribution of model parameters θ by comparing model calculations $\mathbf{y}(\theta)$ with experimental and lattice data $\mathbf{y}_{\text{data}}$. It provides the posterior distribution $\mathcal{P}(\theta|\mathbf{y}_{\text{data}})$ of model parameters according to Bayes' theorem: $\mathcal{P}(\theta|\mathbf{y}_{\text{data}}) \propto \mathcal{P}(\mathbf{y}_{\text{data}}|\theta)\mathcal{P}(\theta)$ with $\mathcal{P}(\mathbf{y}_{\text{data}}|\theta)$ the likelihood for model results with parameter θ to agree with the experimental data. We choose a multivariate normal distribution for the logarithm of the likelihood [61],

$$\ln[\mathcal{P}(\mathbf{y}_{\text{data}}|\theta)] = -\frac{1}{2}\Delta\mathbf{y}(\theta)^T \Sigma^{-1} \Delta\mathbf{y}(\theta) - \frac{1}{2}\ln[(2\pi)^n \det \Sigma], \quad (13)$$

where $\Delta\mathbf{y}(\theta) = \mathbf{y}(\theta) - \mathbf{y}_{\text{data}}$, n is the number of data points, and $\Sigma \equiv \Sigma_{\text{data}} + \Sigma_{\text{model}}$ is the $n \times n$ covariance matrix that encodes the uncertainties of data and models [62–64]. We include the covariance matrix of the lattice output [32] and assume a diagonal form for the covariance matrix of the experimental data: $\Sigma_{\text{exp}} = \text{diag}(\sigma_1^2, \dots, \sigma_n^2)$, combining the

TABLE I. Summary of parameters, their prior ranges, and the highest likelihood parameters fitting to experiment data with NLO accuracy, lattice GFFs, and their combination with $\mu = m_c$ and masses in GeV.

Parameters	Prior	Experiment Only	Lattice + Experiment	Lattice Only
$A_g(0)$	[0, 1]	0.40 (Fixed)	0.47	0.48
M_{A_g}	[0, 4]	1.95	1.27	1.23
$C_g(0)$	[-4, 4]	-0.12	-0.51	-0.46
M_{C_g}	[0, 4]	2.59	0.80	0.83
$A_q(0)$	[0, 1]	0.60 (Fixed)	0.53	0.53
M_{A_q}	[0, 4]	0.1	1.48	1.50
$C_q(0)$	[-4, 4]	0.14	-0.23	-0.27
M_{C_q}	[0, 4]	2.59	1.29	1.18

square of the statistical and systemic errors of each element, due to the lack of knowledge of the full covariance matrix [6,14,15].

In the analysis, we apply the leading conformal expansion of Eqs. (8), (9), and (11) and parametrize each of the GFFs $F_i(t)$ with two parameters $F_i(0)$ and M_{F_i} at $\mu = m_c$ as

$$F_i(t) = F_i(0)(1 - t/M_{F_i}^2)^{-p}, \quad (14)$$

with $F_i = \{A_q, A_g, C_q, C_g\}$. Here, we neglect the contributions from $B_g(t)$ and $B_{u+d+s}(t)$ form factors, which have been shown to be very small in various lattice simulations [32,65]. The $A_{q/g}(0)$ represent the total momentum fraction of quark and gluon and the dipole and tripole masses $M_{A_{q/g}}$ characterize their t dependence, which corresponds to their spatial distributions in the proton. Similar parametrizations also apply to $C_{q,g}(t)$, which are considered as the “last unknown global property” of the nucleon that carries its mechanic properties [1–7], in particular, as tensor monopole moments [66,67]. The physics interpretation of these form factors have attracted attention in the last few years [68–72]. We choose $p = 2$ (dipole) for the $A_{q,g}$ form factors and $p = 3$ (tripole) for the $C_{q,g}$ form factors based on their large- t asymptotic behaviors [23,24], though we note that the extracted GFFs virtually do not depend on the choice of parametrization form, such as values of p here. The cross sections are given by Eqs. (2) and (3) with a 40% theoretical uncertainty assigned to each quark or gluonic CFF as estimated above. The NRQCD matrix element is taken as $E|\psi_{\text{NR}}(0)|^2 = 1.0952/(4\pi)(\text{GeV})^3$ for the J/ψ determined from its leptonic decay width [73,74].

We employ the Monte Carlo Markov Chain method to efficiently explore the model parameter space [75,76], and perform the analysis in three setups: one with lattice GFFs only as the baseline for comparison, one with the threshold J/ψ photoproduction cross sections only, and another one

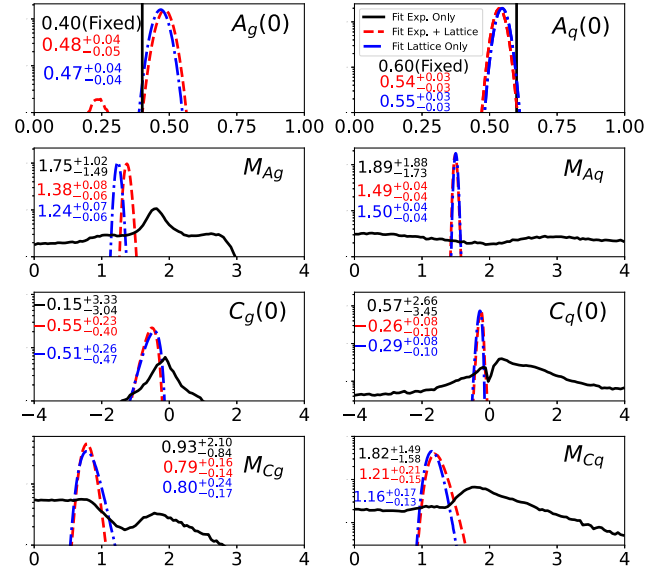


FIG. 3. Bayesian posterior distributions of the gluon (left) and quark (right) parameters by fitting to experiment data with NLO accuracy, lattice GFFs, and their combination, respectively, with $\mu = m_c$ and masses in GeV. The parameters’ median and 90% confidence intervals are shown. All y axes are in logarithmic scale.

combining the lattice GFFs and experiment data. The lattice data, including its covariance matrix and the experimental data included in the Bayesian analysis, are from [6,14,15,32]. Below, the key findings are summarized, with details in SM [58].

By fitting to lattice GFFs and experimental data simultaneously, we find that the two observables are generally compatible. In this fit, the lattice GFFs provide more stringent constraints on GFFs due to their much higher statistics. However, the lattice QCD results do not yet take full account of all sources of systematic error, which is likely why, e.g., the lattice GFFs give $A_g(0) \approx 0.47$, whereas the physical value is around 0.40 according to the CTEQ global parton distribution functions (PDFs) [77] at $\mu = m_c$. Therefore, we also consider the extraction with experimental data alone. Obviously, the quark and gluon GFFs that both enter the cross sections cannot be well-constrained with the limited amount of large- ξ threshold J/ψ production data. Here, we additionally consider the constraint $A_g(0) = 0.40 \pm 0.01$ at $\mu = m_c$ from CTEQ PDFs [77], and also impose the momentum sum rule that $A_g(0) + A_q(0) = 1$. Then, we are left with six parameters for the $A_{q,g}(t)$ and $C_{q,g}(t)$ form factors. These parameters are fitted to the 33 data points with $\xi > 0.5$, where a comparison between the experimental data and the prediction with the Bayesian posterior distributions of the model parameters fitting to experiment data only is shown in Fig. 2.

In Table I, we summarize the model parameters in all three fits, and present their Bayesian posterior distribution

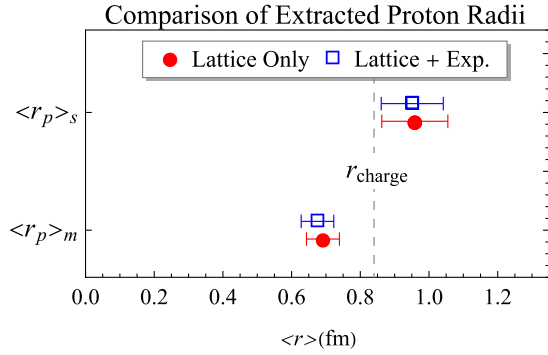


FIG. 4. The average proton mass and scalar radii $\langle r \rangle_{s,m} \equiv \sqrt{\langle r^2 \rangle_{s,m}}$ with parameters sampled from their posterior distributions. Lattice GFFs taken from [32].

in Fig. 3. Here, we emphasize the following findings. (1) Large gluonic dipole or tripole masses are eliminated by the experimental data with Bayesian posterior distributions consistent with the lattice results, as shown in Table I and Fig. 3, although with larger uncertainties due to the limited statistics. (2) Though the $C_{q,g}(0)$ form factors are constrained relatively not as well by the experiments alone compared to the lattice simulations, their Bayesian posterior distributions are found to peak at values statistically consistent with the lattice ones, benefitted from the fixed $A_{q,g}(0)$.

These illustrate the great accessibility of both the quark and gluon $C_{q,g}(t)$ form factors from such processes, even though the sensitivity to the quark only appears at NLO. Recent analyses of deeply virtual Compton scattering (DVCS) measurements [5,7] have demonstrated their sensitivity to the quark $C_q(t)$. A more precise determination of them with future high-precision data, including those from DVCS and near-threshold heavy quarkonium photoproduction, would be especially important and interesting. An intriguing aspect that can be derived from the GFFs are the proton mass and scalar radii defined through the 00 component and trace of the energy momentum tensor of the proton as [78]

$$\langle r^2 \rangle_m = 6 \frac{dA(t)}{dt} \Big|_{t=0} - 6 \frac{C(0)}{M_N^2}, \quad (15)$$

$$\langle r^2 \rangle_s = 6 \frac{dA(t)}{dt} \Big|_{t=0} - 18 \frac{C(0)}{M_N^2}, \quad (16)$$

respectively, where the contributions from $\tilde{C}_{q,g}(t)$ form factors have been canceled out between quark and gluon. The physics of these radii has attracted great interest in recent years [78–82]. In the following, we will present the constraints from our analysis.

In Fig. 4, we show the extracted proton mass and scalar radii from combined experimental data and lattice GFFs,

compared to the ones from solely the lattice GFFs. Although the experimental data provides extra constraints on the proton GFFs, the extracted proton radii appear to be rather close to the lattice ones. This is partially due to the limited statistics at large ξ to be improved with future data. Finally, we report a proton scalar radius of 0.95(09) fm, which is comparable to the charge radius and consistent with the one from trace anomaly form factors 0.89(10)(07) fm [33]. We would like to emphasize that the gluon C_g terms in Eqs. (15) and (16) are the dominant source of the final uncertainties for both mass and scalar radii, and hope that future data from experiment and lattice calculation can provide further constraint.

Conclusion—To conclude, we investigate the exclusive productions of heavy quarkonium near the threshold at NLO, which grants us the opportunity to constrain both the quark and gluon GFFs from such processes. We consider the NLO perturbative corrections of both quark and gluon Wilson coefficients as well as proper NLO GPD evolution and estimate the theoretical uncertainties.

With the above theoretical developments, we constrain the quark and gluon GFFs, combining the global CTEQ PDFs [77] and recent threshold J/ψ production data [6,14,15] at large $\xi > 0.5$ using Bayesian inference. We found that the data provide constraints on both quark and gluon $C_{q,g}(t)$ GFFs after fixing the momentum fraction $A_{q,g}(0)$ from global PDFs. We also study the corresponding proton mass and scalar radii with the extracted GFFs. This shows great potential to constrain the quark and gluon GFFs from such experiments. A combined analysis of both J/ψ production and DVCS data will be interesting to pursue as well. We plan to carry out this in the future.

Acknowledgments—We thank Yoshitaka Hatta for pointing out an error in $\tilde{C}_q^{(1)}$ of Eq. (9) in an earlier version of our paper. We thank Chun Shen and Hendrik Roch for useful discussions and correspondences. We also thank Dimitra Pefkou for providing data of lattice GFFs and useful discussions and comments. We thank Xiangdong Ji and Zein-Eddine Meziani for comments and suggestions. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Contract No. DE-AC02-05CH11231, under the umbrella of the Quark-Gluon Tomography (QGT) Topical Collaboration with Award DE-SC0023646, and the Saturated Glue (SURGE) Topical Theory Collaboration. W.B.Z. is also supported by NSF under Grant No. OAC-2004571 within the X-SCAPE Collaboration.

Data availability—Some of the data that support the findings of this Letter are openly available [1–3]. Data presented in the Figs. 1–4 to make comparisons are not publicly available upon publication because it is not

technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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