

# Gravitational-Wave Tomography of the Moon: Constraining Lunar Structure with Calibrated Gravitational Waves

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The recent success of gravitational-wave (GW) astronomy together with renewed plans for lunar geophysical instrumentation has revived interest in using the Moon as a resonant detector for midfrequency (mHz–Hz) GWs. In realistic observational scenarios, the GW strain amplitude is expected to be constrained independently by networks of GW detectors, which motivates an inverse, *tomographic* question: to what extent can measurements of the Moon's seismic response to known GWs be used to infer its internal structure? In this Letter, we develop a first-principles, perturbative framework that maps spherically symmetric perturbations of the elastic and density structure to measurable changes in observables, especially GW-driven modal amplitudes of the Moon. The formalism combines (i) a normal-mode representation of the elastic response, (ii) first-order perturbation theory for eigenvalues and eigenfunctions, and (iii) a linearized observation model that links frequency and amplitude observables to model parameters (bulk and shear moduli, density, and interface locations) and their perturbations. We show that the estimation errors of the Moon's elastic parameters can be reduced by about an order of magnitude with observations of calibrated GWs.

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**Introduction**—Monitoring normal-mode excitations of a massive elastic body caused by gravitational waves (GWs) is an idea dating back to Weber's early proposals for a GW antenna [1]. This concept reemerged in recent years due to the renewed interest in the scientific utilization of the Moon. On the one hand, renewed lunar exploration initiatives and proposals for geophysical instrumentation on the Moon have greatly improved the feasibility of long-duration, low-noise seismic monitoring [2,3]. On the other hand, the concept of a future GW observatory on the Moon, including the Lunar Gravitational-Wave Antenna (LGWA) [4,5], Laser Interferometer Lunar Antenna [6,7], and related instrument studies [8,9], envisions a lunar observatory that exploits the Moon's exceptionally quiet seismic environment to probe the mHz–Hz GW band [10] complementary to both ground-based and space-based interferometers [11–14]. These developments motivate revisiting the Moon as a resonant GW detector and quantifying the

scientific returns possible from dedicated lunar seismometer networks.

The lunar response to GWs has been approached with a variety of methods. Theoretical efforts have revisited the lunar response using plane-wave models [15,16] and Green-tensor formalism with normal-mode summation [17,18] and have clarified the differences and relationships among various theories [19–21]. Complementary numerical efforts such as spectral-element models have started to explore detailed wave excitation and propagation in realistic lunar models [22,23]. In parallel, instrument studies for LGWA and mission-level feasibility assessments have been developed to investigate the detectability of GW-induced signals [5]. Collectively, these works provide the foundation to connect GW strains incident on the Moon to measurable seismic observables (modal amplitudes and eigenfrequency shifts) under increasingly realistic elastic models.

These forward-modeling advances naturally raise an inverse question: what aspects of the Moon's internal structure can be inferred from the GW-induced seismic response? In terrestrial geophysics, two complementary paradigms have been developed to tackle internal-structure inference. At long periods, the normal-mode perspective exploits high-precision frequency measurements to constrain large-scale structure [24], while at shorter periods, a propagating-wave perspective and full-waveform inversions

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target higher-resolution, three-dimensional heterogeneity [25,26]. Historically, normal-mode-based inversions primarily used eigenfrequencies and their splitting signatures to probe internal structure (and more recently, asphericity and lateral heterogeneity) [7,27,28].

For LGWA-like observational scenarios, however, an important practical difference arises: the GW source amplitude is likely to be constrained independently and with high precision by contemporaneous GW observatories [29–31], e.g., networks combining the Einstein Telescope [11], LISA [12], and LGWA. This multidetector capability turns mode *amplitude* observations, traditionally considered of limited value when source strength is *unknown*, into a potentially powerful, independent probe of internal structure. In other words, knowledge of the incident GW amplitude converts modal amplitudes into structural observables that complement frequency measurements and opens a new inverse problem: *GW tomography* of a planetary body.

We develop a largely analytical, linearized framework designed to enable forecasts and to guide numerical verification for GW-driven lunar tomography. The formalism combines (i) a normal-mode representation of the elastic response, (ii) first-order perturbation theory for eigenfrequencies and eigenfunctions under spherically symmetric parameter perturbations, and (iii) a linearized observation model that maps frequency and amplitude observables to model parameters (bulk and shear moduli, density, and interface locations) and their perturbations. From these elements, we construct explicit Jacobian kernels and embed them in a Fisher-matrix pipeline that quantifies parameter-estimation prospects under realistic noise and prior assumptions.

**Theoretical framework**—In this Letter, we model the Moon as a spherically symmetric elastic body. Self-gravity is omitted throughout this research but can be added back in principle. The structural model is defined by radial profiles of density  $\rho(r)$ , bulk modulus  $\kappa(r)$ , and shear modulus  $\mu(r)$ , as well as the radii of internal discontinuities  $d_i$ . The free oscillations are governed by the elastic eigenvalue problem,

$$V s_{nlm}^{\sigma} = \omega_k^2 T s_{nlm}^{\sigma}, \quad (1)$$

where  $\sigma \in \{S, T\}$  denotes spheroidal or toroidal branches, and  $s_{nlm}^{\sigma}(\mathbf{x})$  denotes the displacement eigenfunction for a mode  $k = (\sigma, n, l)$ .  $\omega_k$  is the eigenfrequency with  $(2l + 1)$ -fold degeneracy.  $V$  and  $T$  are potential and kinetic operators, respectively [32], and are determined by the baseline one-dimensional structure of the Moon.  $\omega_k$  and  $s_{nlm}^{\sigma}$  are, hence, also determined, and are usually calculated by a standard eigensolver such as MINEOS [33].

Under the excitation of a continuous gravitational wave with strain amplitude  $h_0$  and frequency  $\omega_g$ , the Moon acts as a resonant detector. Previous works show that under

spherical symmetry, spheroidal modes with  $l = 2$  are most strongly excited by GWs [17,34]. Based on the standard Green-function approach, the steady-state surface displacement for a specific mode  $n$  (shorthand for  $S_{n2}$ ) and for all modes (i.e., total surface response) are, respectively, given by [17–19]

$$\begin{aligned} \vec{\xi}_n(\mathbf{x}, \omega_g) &= h_0 \sum_{m=-2}^2 \frac{I_n f^m}{\omega_n^2 - \omega_g^2 + i\omega_n \omega_g / Q_n} \mathbf{s}_{n2m}^S(\mathbf{x}), \\ \vec{\xi}(\mathbf{x}, \omega_g) &= \sum_n \vec{\xi}_n(\mathbf{x}, \omega_g), \end{aligned} \quad (2)$$

where  $Q_n$  is the quality factor, and  $f^m$  is an  $\mathcal{O}(1)$  constant pattern function depending only on the GW wave vector and polarization [18]. The coupling strength is determined by the overlap integral  $I_n$ , which depends explicitly on the radial ( $U_n$ ) and horizontal ( $V_n$ ) scalar components of the eigenfunction and the shear modulus profile  $\mu(r)$  (see Supplemental Material [35]). At the resonant peak  $\omega_g = \omega_n$ , where the surface response is the strongest, the absolute amplitude is approximately  $h_0 I_n U_n(R) Q_n / \omega_n^2$  for the radial response and  $h_0 I_n V_n(R) Q_n / \omega_n^2$  for the horizontal response.

To quantify the constraints on the lunar interior parameters  $p \in \{\rho, \kappa, \mu, d\}$ , we next define a set of observables  $\mathbf{O}$  indexed by the mode number  $n$  and the observation type  $\alpha \in \{1, 2, 3\}$ : (i)  $\alpha = 1$ : eigenfrequency observables  $O_{1,n} = \omega_n^2$ ; (ii)  $\alpha = 2$ : radial amplitude observables at the resonance peak  $\omega_g = \omega_n$ :  $O_{2,n} = h_0 U_n(R) I_n \equiv A_n^r \simeq \xi_n^r \times (\omega_n^2 / Q_n)$ ; (iii)  $\alpha = 3$ : horizontal amplitude observables at the resonance peak  $O_{3,n} = h_0 V_n(R) I_n \equiv A_n^h \simeq \xi_n^h \times (\omega_n^2 / Q_n)$ .  $U_n(R)$  and  $V_n(R)$  in the definition correspond to the radial and horizontal components of  $\mathbf{s}_{n2m}^S(\mathbf{x})$  [Eq. (A1) in [35]]. We remove the  $(Q_n / \omega_n^2)$  structure in the amplitude response based on the following considerations: first, we assume that the quality factors  $Q_n$  are constrained independently (e.g., through spectral broadening measurements [27,36,37]) and treat them as fixed parameters; second, the measurement uncertainty of eigenfrequency  $\omega_n$  is assumed to be significantly smaller than the uncertainty of the displacement measurement, i.e.,  $\sigma_{\{2,3\},n} / O_{\{2,3\},n} = \sigma_A / A_n \simeq \sigma_{\xi} / \xi_n \gg \sigma_{\omega} / \omega_n$ . This is easily satisfied, as we will discuss later [before Eq. (6)]. Besides, the relative error in  $h_0$  is also negligible in this Letter (i.e.,  $\sigma_{\xi} / \xi_n \gg \sigma_h / h_0$ ), because for multiband calibration, the signal-to-noise ratio (SNR) should be much better than the SNR of the lunar GW detector itself [30,31]. We also omit the  $\mathcal{O}(1)$  constant factor  $f^m$  and the spherical harmonics  $[\mathbf{P}_{lm}$  and  $\mathbf{B}_{lm}$  in Eq. (A1) of [35]] in  $\mathbf{s}_{n2m}^S$ .

We then employ a perturbation formalism to evaluate the effect of structure variation. The sensitivity of the above observables to parameter perturbations  $\delta p(r)$  ( $p = \rho, \kappa, \mu, d_i$ ) is derived via first-order perturbation theory. Omitting the near-degeneracy effects (which

TABLE I. Baseline lunar model. IC = inner core, OC = outer core, M = mantle, C = crust, S = surface.

|    | $r(\text{km})$ | $\rho(\text{g/cm}^3)$ | $v_p(\text{km/s})$ | $v_s(\text{km/s})$ | $Q_p$ | $Q_s$ |
|----|----------------|-----------------------|--------------------|--------------------|-------|-------|
| IC | 0–352          | 7.76                  | 4.3                | 2.3                | 1000  | 1000  |
| OC | 353–480        | 3.4                   | 8.2                | 3.2                | 675   | 300   |
| M  | 481–1709       | 3.4                   | 7.9                | 4.5                | 3400  | 1500  |
| C  | 1709.1–1725    | 2.762                 | 5.5                | 3.3                | 6750  | 6750  |
| S  | 1725.1–1737.1  | 2.762                 | 3.2                | 1.8                | 6750  | 6750  |

can be incorporated via quasidegenerate perturbation theory), the shift in squared eigenfrequency is given by [32,38]

$$\delta\omega_k^2 = \langle \mathbf{s}_{km} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle. \quad (3)$$

The inner product used above [and in Eq. (4)] follows the standard geophysics definition in [32], and it can be calculated through Woodhouse perturbation kernels [see Eq. (A5) and related discussions in [35]]. Crucially, the perturbation also modifies the eigenfunctions, which in turn, alter the modes  $U_n$ ,  $V_n$  and the overlap integral  $I_n$ . The first-order correction to the eigenfunction projected onto an unperturbed mode  $k' \neq k$  is

$$\langle \mathbf{s}_{k'm'} | \delta \mathbf{s}_{km} \rangle = \frac{\langle \mathbf{s}_{k'm'} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle}{\omega_k^2 - \omega_{k'}^2}. \quad (4)$$

As a result, the final eigenfunction perturbation can be calculated as

$$\begin{aligned} \delta \mathbf{s}_{km} &= \sum_{k'm'} \frac{\langle \mathbf{s}_{k'm'} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle}{\omega_k^2 - \omega_{k'}^2} \mathbf{s}_{k'm'}, \\ \delta U_k &= \sum_{k'm'} \frac{\langle \mathbf{s}_{k'm'} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle}{\omega_k^2 - \omega_{k'}^2} U_{k'}, \\ \delta V_k &= \sum_{k'm'} \frac{\langle \mathbf{s}_{k'm'} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle}{\omega_k^2 - \omega_{k'}^2} V_{k'}. \end{aligned} \quad (5)$$

Using Eq. (5), we can analytically derive the Jacobian matrix elements  $G_{(\alpha,n),p} = \partial O_{\alpha,n} / \partial p$  (see the details in Supplemental Material [35]).

We assume a Gaussian noise model with uncorrelated uncertainties  $\sigma_{\alpha,n}$ . For nominal estimates, we parametrize the relative errors as  $\sigma_{1,n}/\omega_n \sim \varepsilon_\omega$  and  $\sigma_\xi/\xi_n \sim \varepsilon_\xi$ . While potentially optimistic for observations on the Moon, we adopt  $\varepsilon_\omega \in [10^{-5}, 10^{-3}]$  based on long-duration terrestrial seismic benchmarks [39]. For the relative error in the displacement measurement, we estimate  $\varepsilon_\xi \in [10^{-2}, 10^{-1}]$ , consistent with the proposed highly sensitive lunar GW detector [5] where SNRs  $\sim 100$  are achievable. Therefore, the condition of  $\sigma_{\{2,3\},n}/O_{\{2,3\},n} \equiv \varepsilon_A \simeq \varepsilon_\xi \gg \varepsilon_\omega$  is well satisfied [40]. Based on the standard procedure for a linear

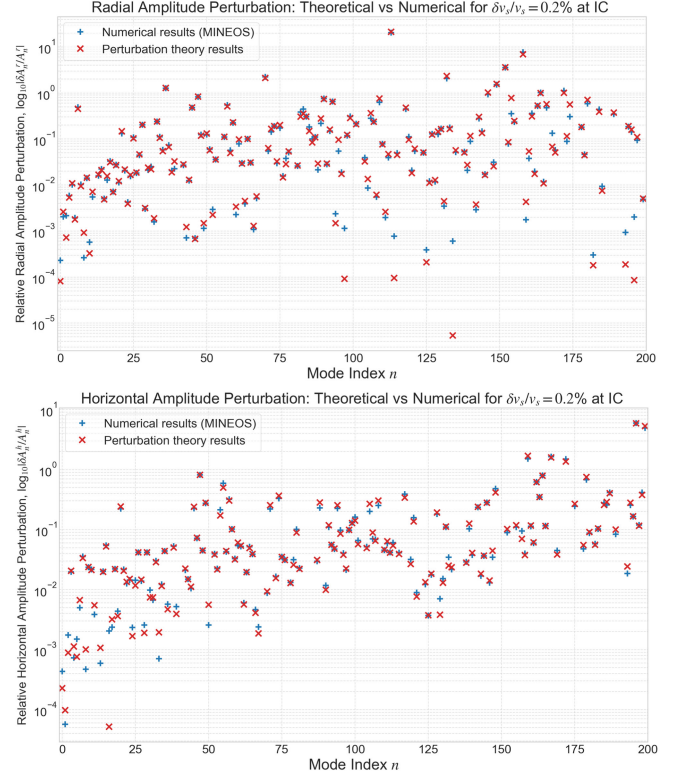


FIG. 1. MINEOS verifications for amplitude perturbation,  $\delta v_s/v_s = 0.2\%$  in the inner core. Only modes within the linear range (25% tolerance of deviation) are included.

model, we construct the Fisher information matrix [41] summing over all modes and observation types,

$$\mathcal{F}_{ij} = \sum_n \sum_{\alpha=1}^3 \frac{1}{\sigma_{\alpha,n}^2} \frac{\partial O_{\alpha,n}}{\partial p_i} \frac{\partial O_{\alpha,n}}{\partial p_j} + (\mathbf{C}_p^{-1})_{ij}, \quad (6)$$

with a covariance prior  $\mathbf{C}_p$  for the model parameters. We assume flat priors for the model parameters, i.e., vanishing prior information matrix,  $\mathbf{C}_p^{-1} \rightarrow 0$ . The posterior covariance matrix is approximated by the inverse of the Fisher matrix,  $\mathbf{C}_{\text{post}} \approx \mathcal{F}^{-1}$ . Consequently, the expected constraint on each model parameter  $p_i$  is quantified by its marginal posterior standard deviation,  $\sigma_{p_i} = \sqrt{(\mathbf{C}_{\text{post}})_{ii}}$ .

*Verification of perturbation theory*—In the following section, we present numerical evaluations of our analytical perturbation framework on a reference lunar model to demonstrate the resolving power of GW-induced seismic signals, especially the amplitudes. We use a standard normal-mode solver, MINEOS [33], to calculate the frequency and amplitude perturbations under small perturbations of the 1D model. We first introduce the baseline lunar model chosen for the practical calculation in this Letter, and then, show the results of linearity checks and comparisons with MINEOS.

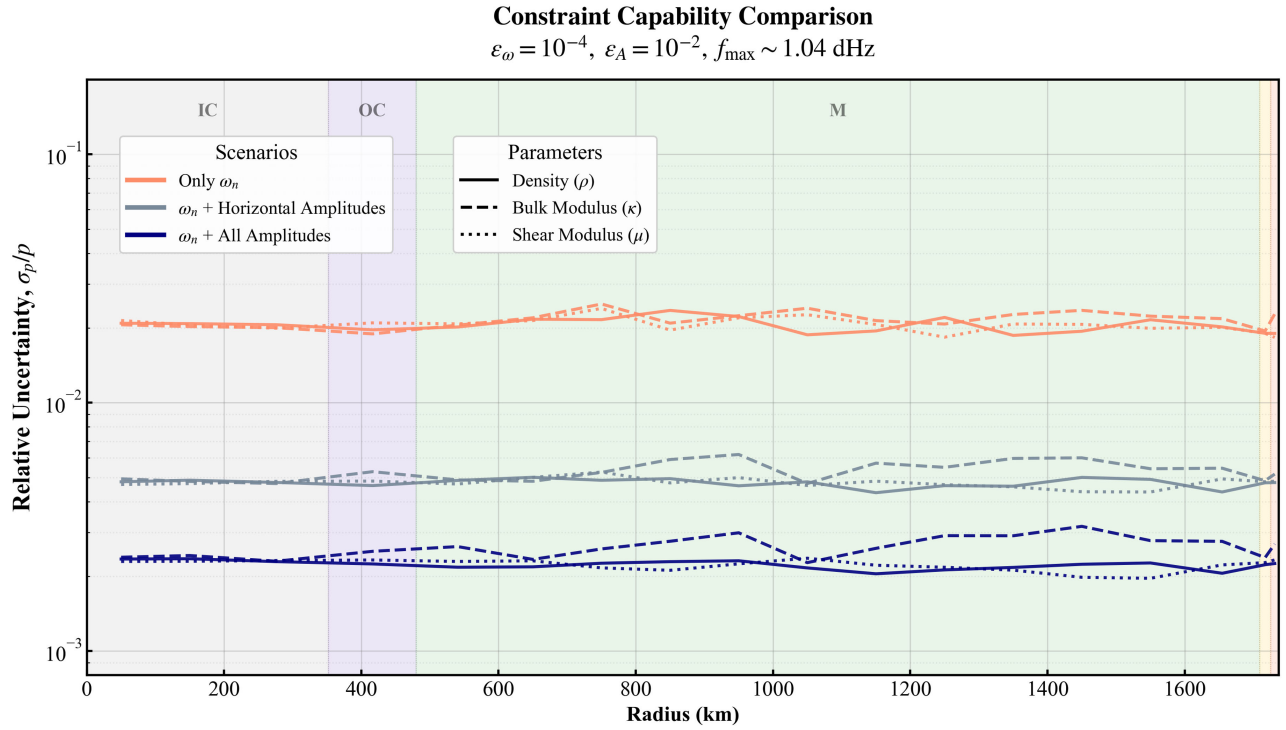


FIG. 2. Relative errors for  $\rho$ ,  $\kappa$ ,  $\mu$  with different combinations of observables. All modes with  $n \leq 150$  are considered as observables, while the perturbation summations are done for modes  $\leq 200$ .

We choose a simple but representative five-layer spherically symmetric model similar to the one we chose in our previous work [19,22], including inner core (“IC,” 0–352 km), outer core (“OC,” 353–480 km), mantle (“M,” 481–1709 km), crust (“C,” 1709.1–1725 km), and surface (“S,” 1725.1–1737.1 km). Within each layer, all physical parameters ( $v_p$ ,  $v_s$ ,  $\rho$ ,  $Q_p$ ,  $Q_s$  [42]) remain the same. Between two adjacent layers, the parameters are linearly interpolated. Original parameters of the baseline model are recorded in Table I, while the interpolated model can be found in “InterpolatedModel” in [43]. Radial refinement and interpolation of the initial model are intended solely to improve the accuracy and stability of the eigensolvers such as MINEOS. Once the refinement reaches a sufficient level, its finer details no longer affect the numerical results or the conclusions of this Letter [44].

We test the perturbation results from our theoretical formalism against those obtained with MINEOS. Comparisons are done mainly for amplitude observables ( $O_2$  and  $O_3$ ) rather than eigenfrequency observables ( $O_1$ ), because the latter have been widely investigated in past geophysics research. The result of a representative case, a shear-wave speed perturbation of 0.2% in the inner core, is plotted in Fig. 1. Other results are shown in Supplemental Material [35]. From these comparisons, we find that the theoretical formulation agrees remarkably well with the numerical results (MINEOS) across several orders of magnitude.

*Numerical example for GW tomography*—Using our lunar baseline model, we perform a Fisher analysis for a comprehensive set of parameters and observables. We consider three parameter types ( $\kappa$ ,  $\mu$ ,  $\rho$ ) across different piecewise radial layers (based on the preliminary model but with increased radial resolution around 100 km for most layers, as shown in Table A1 in [35,45]), and analyze a large suite of observables: eigenfrequencies, radial amplitudes, and horizontal amplitudes for  $0 \leq n \leq 150$  ( $f_{150} \simeq 103.8$  mHz) under various combinations. The perturbation summations ( $\sum_{n' \neq n}$ ) include all modes with  $n' \leq 200$  [46]. We fix  $\varepsilon_\omega = 10^{-4}$  and parametrize the uncertainty of the amplitude as  $\varepsilon_A = \alpha \varepsilon_\omega$ . We require  $\alpha \gg 1$  in this Letter.

Figure 2 shows results for  $\alpha = 100$  and three observable combinations: eigenfrequency only, frequency plus horizontal amplitude (the LGWA case), and frequency plus all amplitudes. We see that the relative uncertainties for the three observational scenarios are weakly dependent on radius, which is partly because we have only tens of layers (Table A1 in [35]) while employing hundreds of observables. The results indicate that even when the amplitude measurements are 2 orders of magnitude less precise than the frequency measurements (i.e.,  $\alpha = 100$ ), including the modal amplitudes in the dataset improves the model constraints by nearly an order of magnitude for all three parameters  $\rho$ ,  $\kappa$ ,  $\mu$ . This improvement reflects a much better sensitivity of the modal amplitudes, rather than

eigenfrequencies, to the model parameters, which can be mainly attributed to the  $[1/(\omega_k^2 - \omega_{k'}^2)]$  factor in Eq. (4).

*Conclusion and discussion*—In conclusion, we have presented a compact but operational framework for GW tomography of the Moon within the 1D (spherically symmetric) and linear approximation. The framework explicitly links GW-driven modal amplitudes and eigenfrequency shifts to small perturbations of interior structure through Woodhouse kernels and first-order perturbation theory. Taken together, the analytical formulation and the Fisher pipeline provide a practical recipe to translate instrument and observational assumptions into quantitative statements about which physical parameters (bulk modulus, shear modulus, density, and interface locations) are, in principle, recoverable from GW-driven seismic data.

We summarize our major conclusions and points for follow-up as follows. (i) Complementarity of frequency and amplitude measurements: Eigenfrequency shifts and GW-induced modal amplitudes respond to different combinations of model perturbations. Because these two types of observables are sensitive to different aspects of the eigenfunctions, jointly inverting frequency and amplitude measurements can substantially increase the effective degrees of freedom and reduce parameter covariances compared with using either observable alone. The quantitative benefit depends on the assumed observational precisions  $\epsilon_\omega$  and  $\epsilon_A$  ( $\epsilon_\xi$ ), the set of modes available for analysis, and the imposed priors. Practical diagnostics should be used to quantify these gains for any particular mission design. (ii) Utilizing GWs to study Solar System phenomena: An important conceptual point, perhaps the most novel implication of this Letter, is that GW-driven modal amplitudes, when the incident GW amplitude is calibrated by a detector network, become independent observables of the lunar interior structure. The tighter constraints on elastic parameters as a function of depth will greatly enhance our understanding of the lunar interior and might lead to breakthroughs in our understanding of the formation history of the Earth-Moon system [5,47]. (iii) Near degeneracy: Eigenmodes with similar eigenfrequencies naturally exhibit near-degeneracy effects, leading to greater errors or even failure of first-order perturbation theory. This problem can be overcome by applying a generalized version of the perturbation formalism: specifically, by using a larger subspace and solving the eigenvalue equation for a group of nearby eigenmodes [32,48]. (iv) Validity of the statistical and physical framework: In this proof-of-principle study, we employ a 1D linearized approach and the Fisher information matrix (FIM) to project parameter uncertainties. While the FIM can be a poor error predictor in regimes with low signal-to-noise ratios or severe parameter degeneracies [41], its application is mathematically well justified within our current framework. First, we adopt a relatively high expected SNR of around 100, which is achievable in future multiband GW observations. This ensures that the

statistical uncertainties remain small and the linearized-signal approximation remains fundamentally valid. Second, our simplified piecewise 1D structural model maintains a parameter dimensionality that is vastly outnumbered by the available observables (amplitudes at hundreds of resonance peaks across a broad frequency band). This heavily over-constrained setup naturally mitigates strong parameter correlations and yields a well-conditioned Fisher matrix, avoiding numerical instability and the severe amplification of errors during inversion. (v) Limitations and extensions: Nevertheless, future realistic inversions must address the physical and statistical limitations of the current framework. The 1D, linearized approach neglects lateral heterogeneity and higher-order effects that may be relevant for large perturbations or for modes strongly affected by asphericity and topography. Extensions to 3D tomography will require aspherical sensitivity kernels (which are already included in Woodhouse's formalism) and a substantial increase in parameter dimensionality. Furthermore, in practical observational scenarios, the number of measurable modes may be lower, and the actual SNR might not reach the ideal thresholds assumed here, potentially invalidating the linear approximation. To address weakly nonlinear effects and increased parameter dimensionality, analytical Jacobians must be substituted with iterative Gauss-Newton or full nonlinear Bayesian inference schemes that systematically incorporate prior errors in the baseline models. Practical inversions will also need to address uncertainties in damping (mode  $Q$  values) and explore joint inversions that combine GW-driven observables with independent lunar datasets (traditional seismic catalogs, tidal Love numbers, and geodetic constraints). Addressing these points will clarify the potential and limitations of GW tomography of the Moon under more realistic models and enable concrete recommendations for future lunar geophysical exploration.

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*Data availability*—The data that support the findings of this article are openly available [43].

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- [1] J. Weber, Detection and generation of gravitational waves, *Phys. Rev.* **117**, 306 (1960).

- [2] C. Wang, Y. Jia *et al.*, Scientific objectives and payload configuration of the Chang'E-7 mission, *Natl. Sci. Rev.* **11**, nwad329 (2023).
- [3] M. P. Panning, S. Kedar *et al.*, Farside seismic suite: Update on the status of the first seismic station on the farside of the moon, in *LPI Contributions*, LPI Contributions Vol. 3040 (Lunar and Planetary Institute, Houston, 2024), p. 2354.
- [4] J. Harms, F. Ambrosino, L. Angelini *et al.*, Lunar gravitational-wave antenna, *Astrophys. J.* **910**, 1 (2021).
- [5] P. Ajith, P. A. Seoane, M. Arca Sedda *et al.*, The lunar gravitational-wave antenna: Mission studies and science case, *J. Cosmol. Astropart. Phys.* **01** (2025) 108.
- [6] J. M. Trippe, K. Jani, V. Quetschke, R. A. Reed, B. D. Sierawski, G. Karsai, and P. Lognonné, Laser interferometer lunar antenna, in *LPI Contributions*, LPI Contributions Vol. 3063 (Lunar and Planetary Institute, Houston, 2024), p. 5043.
- [7] M. P. Panning, P. Lognonné, T. Creighton, J. Trippe, V. Quetschke, J. Majstorović, and K. Jani, Potential for lunar interior science by the gravitational-wave detector LILA, *arXiv:2509.15452*.
- [8] J. V. van Heijningen, H. J. M. ter Brake *et al.*, The payload of the lunar gravitational-wave antenna, *J. Appl. Phys.* **133**, 244501 (2023).
- [9] J. Li, F. Liu, Y. Pan, Z. Wang, M. Cao, M. Wang, F. Zhang, J. Zhang, and Z.-H. Zhu, Detecting gravitational wave with an interferometric seismometer array on lunar nearside, *Sci. China Phys. Mech. Astron.* **66**, 109513 (2023).
- [10] J. Harms, Seismic background limitation of lunar gravitational-wave detectors, *Phys. Rev. Lett.* **129**, 071102 (2022).
- [11] M. Maggiore, C. Van Den Broeck, N. Bartolo *et al.*, Science case for the Einstein telescope, *J. Cosmol. Astropart. Phys.* **03** (2020) 050.
- [12] M. Colpi, K. Danzmann *et al.*, LISA definition study report, *arXiv:2402.07571*.
- [13] J. Luo, L.-S. Chen, H.-Z. Duan *et al.*, TianQin: A spaceborne gravitational wave detector, *Classical Quantum Gravity* **33**, 035010 (2016).
- [14] Z. Luo, Y. Wang, Y. Wu, W. Hu, and G. Jin, The Taiji program: A concise overview, *Prog. Theor. Exp. Phys.* **2021**, 05A108 (2021).
- [15] F. J. Dyson, Seismic response of the Earth to a gravitational wave in the 1-Hz band, *Astrophys. J.* **156**, 529 (1969).
- [16] X. Bi and J. Harms, Response of the moon to gravitational waves, *Phys. Rev. D* **110**, 064025 (2024).
- [17] A. Ben-Menahem, Excitation of the Earth's eigenvibrations by gravitational radiation from astrophysical sources, *Nuovo Cimento C Geophys. Space Phys. C* **6**, 49 (1983).
- [18] J. Majstorović, S. Rosat, and Y. Rogister, Earth's spheroidal motion induced by a gravitational wave in flat spacetime, *Phys. Rev. D* **100**, 044048 (2019).
- [19] H. Yan, X. Chen, J. Zhang, F. Zhang, M. Wang, and L. Shao, Toward a consistent calculation of the lunar response to gravitational waves, *Phys. Rev. D* **109**, 064092 (2024).
- [20] E. Belgacem, M. Maggiore, and T. Moreau, Coupling elastic media to gravitational waves: An effective field theory approach, *J. Cosmol. Astropart. Phys.* **07** (2024) 028.
- [21] J. Majstorović, L. Vidal, and P. Lognonné, Modeling lunar response to gravitational waves using normal-mode approach and tidal forcing, *Phys. Rev. D* **111**, 044061 (2025).
- [22] L. Zhang, H. Yan, X. Chen, and J. Zhang, 2D numerical simulation of lunar response to gravitational waves using finite element method, *Phys. Rev. D* **111**, 063014 (2025).
- [23] L. Zhang, H. Yan, J. Zhang, and X. Chen, Numerical simulation of lunar response to gravitational waves and its 3D topographic effect using the spectral-element method, *Phys. Rev. D* **113**, 023031 (2026).
- [24] J. H. Woodhouse and A. M. Dziewonski, Mapping the upper mantle: Three dimensional modeling of Earth structure by inversion of seismic waveforms, *J. Geophys. Res.* **89**, 5953 (1984).
- [25] A. Tarantola, Inversion of seismic reflection data in the acoustic approximation, *Geophysics* **49**, 1259 (1984).
- [26] J. Tromp, C. Tape, and Q. Liu, Seismic tomography, adjoint methods, time reversal and banana-doughnut kernels, *Geophys. J. Int.* **160**, 195 (2005).
- [27] J. H. Woodhouse, The coupling and attenuation of nearly resonant multiplets in the Earth's free oscillation spectrum, *Geophys. J. Int.* **61**, 261 (1980).
- [28] A. Deuss, J. Ritsema, and H. van Heijst, Splitting function measurements for Earth's longest period normal modes using recent large earthquakes, *Geophys. Res. Lett.* **38**, L04303 (2011).
- [29] A. Sesana, Prospects for multiband gravitational-wave astronomy after GW150914, *Phys. Rev. Lett.* **116**, 231102 (2016).
- [30] S. Grimm and J. Harms, Multiband gravitational-wave parameter estimation: A study of future detectors, *Phys. Rev. D* **102**, 022007 (2020).
- [31] Y.-Y. Dong, J.-Y. Song, J.-F. Zhang, and X. Zhang, Prospects for joint multiband detection of intermediate-mass black holes by LGWA and the Einstein telescope, *J. Cosmol. Astropart. Phys.* **03** (2026) 046.
- [32] F. A. Dahlen and J. Tromp, *Theoretical Global Seismology* (Princeton University Press, Princeton, NJ, 1999).
- [33] G. Masters, J. H. Woodhouse, and G. Freeman, Mineos v1.0.2 [software], Computational Infrastructure for Geodynamics (2011), <https://geodynamics.org/cig>.
- [34] M. Bianchi, E. Coccia, C. N. Colacino, V. Fafone, and F. Fucito, Testing theories of gravity with a spherical gravitational wave detector, *Classical Quantum Gravity* **13**, 2865 (1996).
- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/3tcp-hqf9> for the theoretical details and numerical examinations.
- [36] R. V. Saylor and A. M. Dziewonski, Measurements and interpretation of normal mode attenuation, *Geophys. J. Int.* **53**, 559 (1978).
- [37] R. Widmer, G. Masters, and F. Gilbert, Spherically symmetric attenuation within the Earth from normal mode data, *Geophys. J. Int.* **104**, 541 (1991).
- [38] S. Chandrasekhar, A general variational principle governing the radial and the non-radial oscillations of gaseous masses, *Astrophys. J.* **139**, 664 (1964).
- [39] J. Park, T.-R. A. Song, J. Tromp *et al.*, Earth's free oscillations excited by the 26 December 2004 Sumatra-Andaman earthquake, *Science* **308**, 1139 (2005).
- [40] If  $\varepsilon_\omega$  is much larger than expected, our calculations in this work need to be slightly modified. However, the importance of amplitude measurement will be even greater.

- [41] M. Vallisneri, Use and abuse of the Fisher information matrix in the assessment of gravitational-wave parameter-estimation prospects, *Phys. Rev. D* **77**, 042001 (2008).
- [42] Note:  $\kappa = \rho(v_p^2 - \frac{4}{3}v_s^2)$ ,  $\mu = \rho v_s^2$ .
- [43] H. Yan, LunarResponse: Lunar Gravitational-wave Response Analysis (2026), [10.5281/zenodo.20309657](https://zenodo.org/record/20309657/files/1Dperturbation_folder), [1Dperturbation folder](#).
- [44] Previous work [19] has validated the appropriateness of this refinement-and-interpolation procedure.
- [45] Since we have a sufficient number of observables, far exceeding the number of model parameters, the specific layer thickness does not have significant influence on the results.
- [46] Contributions of higher modes decay rapidly as  $\omega_n^{-2}$ .
- [47] National Academies of Sciences, Engineering, and Medicine, *Origins, Worlds, and Life: Planetary Science and Astrobiology in the Next Decade* (The National Academies Press, Washington, DC, 2023).
- [48] A. Deuss and J. H. Woodhouse, Theoretical free-oscillation spectra: The importance of wide band coupling, *Geophys. J. Int.* **146**, 833 (2001).