

AdS $\times$ S Mellin Bootstrap, Hidden 10D Symmetry and Five-Point Kaluza-Klein Functions in $\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory

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We propose an AdS $\times$ S factorization formula at the level of the generating function for correlators with arbitrary Kaluza-Klein configurations and implement it in the supergravity limit of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. By incorporating this mechanism into Mellin space bootstrap, together with an observed Z_2 symmetry under AdS $\leftrightarrow$ S, we manage to simultaneously work out unified formulas both for all five-point half-BPS correlators and for all four-point correlators with one superdescendant. This AdS $\times$ S bootstrap method is directly applicable to generic multipoint computation at tree level.

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Introduction—Analytic bootstrap has recently boosted the study of four-point correlators of half-BPS operators at various Kaluza-Klein (KK) levels in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) in the strong coupling region, dual to type IIB superstring in AdS₅ $\times$ S⁵ [1–20] (see recent reviews [21,22]). This development reveals an interesting connection between interactions among these operators and a hidden 10D conformal symmetry [23]. Correspondingly, a viewpoint has gradually emerged that all four-point correlators involving arbitrary KK configurations can be elegantly packaged into a generating function in position space [24–26] or equivalently into an AdS $\times$ S Mellin amplitude [19,20,24,27–30].

In order to understand whether this hidden higher-dimensional structure is a general feature of the model, it is natural to inspect its potential existence in multipoint correlators. Despite the recent promising progress in a

cousin model in AdS₅ $\times$ S³ [28,33–36], the multipoint computation in SYM is still restricted to relatively low KK charges [37–39]. The main obstacles here are the fast-growing size of the space of internal symmetry structures and of the field contents in the operator product expansion (OPE) due to maximal supersymmetry, which render the original bootstrap strategy impractical. In this Letter, we overcome these difficulties by formulating a factorization approach in AdS $\times$ S Mellin space that simultaneously incorporates all KK contributions. This framework is then integrated into the bootstrap machinery, allowing one to directly construct an ansatz for the AdS $\times$ S Mellin amplitudes. The solution follows from this new form of factorization, together with a newly observed Z_2 symmetry in AdS $\leftrightarrow$ S, as well as known constraints from the Drukker-Plefka twist [38–40]. We demonstrate the effectiveness of this strategy with an explicit five-point computation, in which the generating function for all KK correlators is fully determined.

Setup—The KK modes we study are the half-BPS operators

$$\mathcal{O}_p(X, T) \propto \text{tr}\{\phi(X) \cdot T\}^p + (\text{multitrace terms}), \quad (1)$$

with protected dimension $\Delta = p$. Here we describe the spacetime coordinates by an SO(4,2) null vector X in the embedding space (e.g., [41]), and the R-symmetry indices in ϕ are contracted with an SO(6) null vector T , realizing the rank- p symmetric traceless representation. Through AdS/CFT correspondence, they correspond to

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scalar fields in anti-de Sitter (AdS) space at KK level p and are commonly identified as single-particle states of supergravitons [2]. In the large N limit, all multitrace terms in the operator are suppressed, and we adopt the normalization from [25] such that

$$\langle \mathcal{O}_p(X_1, T_1) \mathcal{O}_p(X_2, T_2) \rangle = \mathcal{N}_\mathcal{O} \left(\frac{T_{ij}}{X_{ij}} \right)^p, \mathcal{N}_\mathcal{O} = \frac{1}{p}. \quad (2)$$

Here, $X_{ij} = -X_i \cdot X_j$ and $T_{ij} = T_i \cdot T_j$. Under a suitable parametrization, X_{ij} reduces to the ordinary four-dimensional distance x_{ij}^2 .

Instead of studying specific KK correlators

$$G_{\{p_i\}}(X_{ij}, T_{ij}) = \langle \mathcal{O}_{p_1}(X_1, T_1) \cdots \mathcal{O}_{p_n}(X_n, T_n) \rangle \quad (3)$$

individually, we can construct a master operator for half-BPS operators by summing over the KK level [25]

$$\mathcal{O}(X, T) = \sum_{p=2}^{\infty} \mathcal{O}_p(X, T), \quad (4)$$

and consider their correlator

$$\mathcal{G}_n = \langle \mathcal{O}(X_1, T_1) \cdots \mathcal{O}(X_n, T_n) \rangle = \sum_{p_1, \dots, p_n=2}^{\infty} G_{\{p_i\}}, \quad (5)$$

which acts as a generating function for all KK correlators. This approach offers significant advantage, enabling the simultaneous treatment of all KK levels and revealing previous hidden symmetries. To extract correlators with specific KK levels, we can simply expand the generating function in T_{ij} and pick up terms with the correct weights.

Within the supergravity approximation, the operator spectrum simplifies considerably. Nonprotected single-trace operators decouple due to their large anomalous dimensions, thereby streamlining our analysis. Consequently, the OPE of two half-BPS operators contains six types of single-trace operators, as enumerated in Table I.

These operators are labeled by their conformal dimension Δ and spin J , as well as the “sphere dimension” R and “sphere spin” S , related to the usual SU(4) Dynkin labels as $[S, R - S, S]$. This terminology will become clear later. We also define the twists $\tau = \Delta - J$ and $\sigma = R + S$ for later convenience. All six operators reside in the same

TABLE I. Quantum numbers of single-trace operators contributing to the OPE of two half-BPS operators.

	$\mathcal{O}_p$	$\mathcal{J}_p$	$\mathcal{T}_p$	$\mathcal{A}_p$	$\mathcal{C}_p$	$\mathcal{F}_p$
(Δ, J)	$(p, 0)$	$(p+1, 1)$	$(p+2, 2)$	$(p+2, 0)$	$(p+3, 1)$	$(p+4, 0)$
(R, S)	$(p, 0)$	$(p-1, 1)$	$(p-2, 0)$	$(p-2, 2)$	$(p-3, 1)$	$(p-4, 0)$

superconformal multiplet $\mathbb{O}_p$. They are related by the action of supercharges, schematically as

$$\mathcal{O}_p \xrightarrow{\mathcal{Q}\bar{\mathcal{Q}}} \mathcal{J}_p \xrightarrow{\mathcal{Q}\bar{\mathcal{Q}}} \mathcal{T}_p \xrightarrow{\mathcal{Q}\bar{\mathcal{Q}}} \mathcal{C}_p \xrightarrow{\mathcal{Q}\bar{\mathcal{Q}}} \mathcal{F}_p. \quad (6)$$

AdS $\times$ S amplitudes—The computation of the generating function $\mathcal{G}_n$ is most naturally addressed in AdS $\times$ S Mellin space [27]. In AdS $\times$ S Mellin space, we transform both X_{ij} and T_{ij} into the AdS and sphere Mellin variables γ_{ij} and n_{ij} (see Supplemental Material [42])

$$G_{\{p_i\}}(X_{ij}, T_{ij}) = \oint' \mathcal{M}_{\{p_i\}}(\gamma_{ij}, n_{ij}) \prod_{i < j} \frac{T_{ij}^{n_{ij}} \Gamma(\gamma_{ij})}{n_{ij}! X_{ij}^{\gamma_{ij}}}. \quad (7)$$

These Mellin variables are symmetric in their two indices and obey the constraints

$$\forall i: \sum_{j \neq i} \gamma_{ij} = \tau_i = p_i, \quad \sum_{j \neq i} n_{ij} = \sigma_i = p_i. \quad (8)$$

In the sum-integral, we sum over any set of independent n_{ij} at all integers [47] and integrate the independent γ_{ij} parallel to the imaginary axis as follows:

$$\oint' = \sum_{n_{ij}}' \int_{-i\infty}^{i\infty} \prod_{\gamma_{ij}}' \frac{d\gamma_{ij}}{2\pi i}, \quad (9)$$

and the prime indicates that γ_{ij} and n_{ij} satisfy (8).

If the amplitude is analytic in p_i (as we always assume in the following), we can further eliminate the explicit p_i dependence by replacing p_i with n_{ij} using (8), and obtain the AdS $\times$ S amplitude of the generating function

$$\mathcal{G}_n(X_{ij}, T_{ij}) = \oint' \mathcal{M}_n(\gamma_{ij}, n_{ij}) \prod_{i < j} \frac{T_{ij}^{n_{ij}} \Gamma(\gamma_{ij})}{n_{ij}! X_{ij}^{\gamma_{ij}}}. \quad (10)$$

This expression looks very similar to the previous one, but after the sum over p_i , the Mellin variables satisfy fewer constraints (hence, we drop the prime)

$$\forall i: \sum_{j \neq i} \gamma_{ij} = \sum_{j \neq i} n_{ij}. \quad (11)$$

These constrain γ_{ij} , and all n_{ij} are now independent and summed over integers.

In AdS $\times$ S space, the well-known four-point function for arbitrary KK modes can be expressed as [27]

$$\mathcal{M}_4 = \widehat{R}_{1234} \circ \widetilde{\mathcal{M}}_4, \quad (12)$$

where $\widehat{R}_{1234} \circ$ denotes the composite action of the difference operators $\hat{\gamma}_{ij}, \hat{n}_{ij}$ [1,2]

$$\begin{aligned} \hat{R}_{1234} = & (\hat{n}_{12}\hat{n}_{34}\hat{\gamma}_{14}\hat{\gamma}_{23} - \hat{n}_{14}\hat{n}_{23}\hat{\gamma}_{12}\hat{\gamma}_{34})(\hat{n}_{12}\hat{n}_{34}\hat{\gamma}_{13}\hat{\gamma}_{24} \\ & - \hat{n}_{13}\hat{n}_{24}\hat{\gamma}_{12}\hat{\gamma}_{34}) + (2 \leftrightarrow 3) + (2 \leftrightarrow 4), \end{aligned} \quad (13)$$

where

$$\begin{aligned} \hat{\gamma}_{ij} \circ F(\gamma_{ij}, n_{ij}) &= \gamma_{ij} F(\gamma_{ij} + 1, n_{ij}), \\ \hat{n}_{ij} \circ F(\gamma_{ij}, n_{ij}) &= n_{ij} F(\gamma_{ij}, n_{ij} - 1). \end{aligned} \quad (14)$$

In position space, $\hat{R}_{1234}$ corresponds to the superconformal invariant R_{1234} appearing in the superconformal Ward identity [48]

$$\begin{aligned} R_{1234} = & (T_{12}T_{34}X_{14}X_{23} - T_{14}T_{23}X_{12}X_{34})(T_{12}T_{34}X_{13}X_{24} \\ & - T_{13}T_{24}X_{12}X_{34}) + (2 \leftrightarrow 3) + (2 \leftrightarrow 4). \end{aligned} \quad (15)$$

The reduced amplitude $\widetilde{\mathcal{M}}_4$ is a simple rational function

$$\widetilde{\mathcal{M}}_4 = \frac{-1}{(\rho_{12} - 1)(\rho_{13} - 1)(\rho_{14} - 1)}, \quad (16)$$

which depends only on the combination $\rho_{ij} = \gamma_{ij} - n_{ij}$. This is how the hidden 10D symmetry manifests in $\text{AdS} \times S$ space: for reduced amplitudes, we can change the variables to ρ_{ij}, n_{ij} in (10) and perform the sum over all n_{ij} , which leads to an integral over independent ρ_{ij} [see Eq. (35) in [24]]

$$\sum \widetilde{\mathcal{M}}(\rho_{ij}) \prod_{i < j} \frac{T_{ij}^{n_{ij}} \Gamma(\rho_{ij} + n_{ij})}{n_{ij}! X_{ij}^{\rho_{ij} + n_{ij}}} = \int \widetilde{\mathcal{M}}(\rho_{ij}) \prod_{i < j} \frac{\Gamma(\rho_{ij})}{\widetilde{X}_{ij}^{\rho_{ij}}}, \quad (17)$$

which depends only on the 10D distance $\widetilde{X}_{ij} = X_{ij} - T_{ij}$.

Factorization in $\text{AdS} \times S$ space—In ordinary Mellin space, factorization of amplitudes [43] is a powerful tool. The Mellin amplitude $M(\gamma_{ij})$ is a function of AdS variables γ_{ij} , and the exchange of single-trace operators manifests as a series of poles [49]. The corresponding residues factorize into products of two subamplitudes [43,50]. For example, when a scalar operator of twist τ is exchanged, these poles take the form

$$M(\gamma_{ij}) \sim \frac{-2\Gamma(\tau)m!}{(\tau - 1)_m} \frac{M_{L,m} \times M_{R,m}}{\gamma_{LR} - \tau - 2m}, \quad (18)$$

$$M_{L,m} = \frac{1}{m!} (\widehat{\gamma}_{LL})^m \circ M_L(\gamma_{ab}), \quad (19)$$

and similarly for $M_{R,m}$. Here $(A)_B = \Gamma(A+B)/\Gamma(A)$, $\gamma_{LR} = \sum_{a \in L, b \in R} \gamma_{ab}$, $\widehat{\gamma}_{LL} = \sum_{a,b \in L, a < b} \widehat{\gamma}_{ab}$, L and R denote the sets of external operators in the two subamplitudes, respectively, and $m = 0, 1, \dots$ labels the level of conformal descendants.

This factorization property originates from the OPE. It can be derived by solving the Casimir equations in Mellin space

$$\widehat{C}_2 M(\gamma_{ij}) = [\Delta(\Delta - 4) + J(J + 2)]M(\gamma_{ij}), \quad (20)$$

where $\widehat{C}_2$ denotes the Casimir operator acting on the left-hand side operators. Since this is a group-theoretical result, and the R-symmetry group $\text{SO}(6)$ is simply a Wick rotation of the conformal group $\text{SO}(4,2)$, there exists an $\text{SO}(6)$ analog of the factorization formula for the “sphere amplitude.” The solutions of the spherical Casimir equations are related to the conformal solutions through

$$(\Delta, J, \tau, m, \gamma_{ij}) \leftrightarrow (-R, S, -\sigma, r, -n_{ij}), \quad (21)$$

$$M|_{\gamma_{ij} \rightarrow \gamma_{ij} \pm 1} \leftrightarrow M|_{n_{ij} \rightarrow n_{ij} \mp 1}, \quad (22)$$

up to an overall normalization that can be easily determined. This explains our earlier names for R, S . For an exchange of the sphere scalar $[0, R, 0]$, the factorization formula follows directly from (18) via Wick rotation

$$M(n_{ab})|_{n_{LR} = \sigma - 2r} = \frac{(-1)^r r!}{\sigma! [\sigma + 1]_r} M_{L,r} \times M_{R,r}, \quad (23)$$

$$M_{L,r} = \frac{1}{r!} (\widehat{n}_{LL})^r \circ M_L(n_{ab}), \quad (24)$$

with $[A]_B \equiv A!/(A - B)!$.

The Mellin and sphere factorization formulas can be combined into the $\text{AdS} \times S$ factorization formula (see Supplemental Material [42]). For a conformal and sphere scalar $\mathcal{X}$ (say $\mathcal{O}, \mathcal{F}$)

$$\mathcal{M}(\gamma_{ij}, n_{ij}) \sim \sum_{m+r=k} A_{(\tau, m|\sigma, r)} \frac{\mathcal{M}_{L,m,r} \times \mathcal{M}_{R,m,r}}{\rho_{LR} - (2\delta + 2k)}, \quad (25)$$

where $2\delta = \tau - \sigma$, and

$$A_{(\tau, m|\sigma, r)} = \frac{1}{\mathcal{N}_{\mathcal{X}}} \frac{-2\Gamma(\tau)m!}{(\tau - 1)_m} \frac{(-1)^r r!}{\sigma! [\sigma + 1]_r}, \quad (26)$$

$$\mathcal{M}_{L,m,r} = \frac{(\widehat{\gamma}_{LL})^m}{m!} \frac{(\widehat{n}_{LL})^r}{r!} \circ \mathcal{M}_L(\gamma_{ab}, n_{ab}), \quad (27)$$

with $\mathcal{N}_{\mathcal{X}}$ the normalization factor in the two-point function. One important remark is that δ does not depend on the KK level. For example, all $\mathcal{O}_p$ satisfy $2\delta = p - p \equiv 0$. Therefore, the entire KK family contributes to the same poles and follows the same factorization formula (25). Moreover, substituting τ and σ back with Mellin variables

$$A_{(\tau, m|\sigma, r)} \Big|_{\substack{\tau \rightarrow \gamma_{LR} - 2m \\ \sigma \rightarrow n_{LR} + 2r}} \quad (28)$$

makes (25) completely independent of KK levels. This allows us to perform a bootstrap analysis for the generating function, and the factorization on *infinitely many* KK modes reduces to the factorization on merely *six* master operators.

AdS ↔ S Z₂ symmetry—Before discussing the bootstrap computation, we note a remarkable Z₂ symmetry that interchanges variables in AdS and sphere. This symmetry manifests in the kinematic variables in the embedding formalism (X and T) and the AdS × S formalism (γ and n). Moreover, it appears in the spectrum of single-trace operators, where taking

$$(\Delta, J) \leftrightarrow (-R, S), p \leftrightarrow -p \quad (29)$$

in Table I maps each single-trace operator to itself, except $\mathcal{T}_p$ and $\mathcal{A}_p$ are interchanged. This symmetry also holds at the correlator level [51]

$$\mathcal{M}_4(\gamma_{ij}, n_{ij}) = \mathcal{M}_4(-n_{ij}, -\gamma_{ij}). \quad (30)$$

Since it exchanges $p \leftrightarrow -p$, the Z₂ symmetry is invisible for a particular KK correlator, but observable only when considering the entire family or the generating function of the correlators [52].

More importantly, the symmetry persists for correlators involving superdescendants $\mathcal{X} = \mathcal{J}, \mathcal{T}, \mathcal{A}, \mathcal{C}, \mathcal{F}$. This can be proven using the superspace method, where superdescendants are extracted from the superoperator $\mathcal{O}_p$ via a differential operator

$$\mathcal{X}_p = \mathcal{D}^{(\mathcal{X}_p)} \mathcal{O}_p|_{q=\lambda=0}, \quad (31)$$

and the operator $\mathcal{D}^{(\mathcal{X}_p)}$ turns out to be Z₂ symmetric (see Supplemental Material [42]). It is worth noting that, since both the AdS × S factorization formula and all subamplitudes (including those with superdescendants) respect the Z₂ symmetry, for arbitrary n -point functions we expect to have

$$\mathcal{M}_n(\gamma_{ij}, n_{ij}) = (-)^n \mathcal{M}_n(-n_{ij}, -\gamma_{ij}), \quad (32)$$

where $(-)^n$ comes from the normalization $\mathcal{N}_\mathcal{O}$.

Four-point function—We first apply the AdS × S factorization formula on the four-point amplitude $\mathcal{M}_4$ and see what we can learn from the computation. In the s-channel, $\rho_{LR} = -2\rho_{12}$, and we have the following poles:

$$\mathcal{M}_4 \sim \frac{\mathcal{Q}_0(\gamma_{ij}, n_{ij})}{\rho_{12}} + \frac{\mathcal{Q}_1(\gamma_{ij}, n_{ij})}{\rho_{12} + 1}, \quad (33)$$

where $\mathcal{Q}_0$ and $\mathcal{Q}_1$ can be obtained from (12). Both $\mathcal{O}$ and $\mathcal{J}$ have $\delta = 0$ and contribute to the leading residue $\mathcal{Q}_0$, and we can extract the subamplitudes $\mathcal{M}_{3,\mathcal{O}}$, $\mathcal{M}_{3,\mathcal{J}}$ of correlators $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle$, $\langle \mathcal{J}\mathcal{O}\mathcal{O} \rangle$ from it. For instance,

$$\mathcal{M}_{3,\mathcal{O}} = n_{12}n_{1I}n_{2I}, \quad (34)$$

where I labels the exchanged operator [53]. One can recover the well-known three-point function result by

substituting $\mathcal{M}_{3,\mathcal{O}}$ back in (7). Note that $\mathcal{M}_4$ only contains two poles $\rho_{12} = 0, -1$, but the gluing of the three-point amplitudes $\mathcal{M}_{3,\mathcal{X}}$ according to (25) produces higher-descendant poles $\rho_{12} = -k$ for $k \geq 2$. By requiring the residue at $k = 1$ matches $\mathcal{Q}_1$ and the residues for all $k \geq 2$ vanish after summing over the contribution from all six master operators, we extract all other superdescendant amplitudes $\mathcal{M}_{3,\mathcal{X}}$ with $\mathcal{X} = \mathcal{T}, \mathcal{A}, \mathcal{C}, \mathcal{F}$ (see Supplemental Material [42]). The delicate cancellation of the higher-descendant poles between different superdescendants can be regarded as a consequence of supersymmetry. Moreover, the coefficient (28) introduces spurious poles of γ_{LR} and n_{LR} (instead of ρ_{LR}) for individual master exchanges. In principle, these spurious poles could spoil the hidden 10D symmetry in $\mathcal{M}_4$. However, in the end they all cancel and result in a rational function with only ρ poles. We can say, in a rough sense, that supersymmetry leads to hidden 10D symmetry in $\mathcal{M}_4$. In the following bootstrap of five-point functions, we will flip the logic around, using hidden 10D symmetry as an input to bootstrap $\mathcal{M}_5$, as well as all the unknown four-point subamplitudes involving superdescendants.

Bootstrapping five-point function—Now we bootstrap the five-point AdS × S amplitude $\mathcal{M}_5$, together with all four-point superdescendant amplitudes $\mathcal{M}_{4,\mathcal{X}}$ (the amplitude of $\langle \mathcal{X}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$, $\mathcal{X} = \mathcal{J}, \mathcal{T}, \mathcal{A}, \mathcal{C}, \mathcal{F}$), in *one bootstrap computation*. The strategy is as follows: (1) Hidden 10D symmetry: We first write down an ansatz satisfying (a weaker version of) hidden 10D symmetry, in the sense that there are only truncated ρ poles in the amplitudes [54]

$$\mathcal{M}_5 = \sum_{k_1, k_2=0} \frac{P^{(k_1, k_2)}(\gamma_{ij}, n_{ij})}{(\rho_{12} + k_1)(\rho_{45} + k_2)} + \sum_{k_3=0} \frac{P^{(k_3)}(\gamma_{ij}, n_{ij})}{\rho_{12} + k_3} + P(n_{ij}) + (\text{perms}), \quad (35)$$

$$\mathcal{M}_{4,\mathcal{X}} = \sum_{k_4=0}^{\delta+1} \frac{Q_{\mathcal{X}}^{(k_4)}(\gamma_{ij}, n_{ij})}{\rho_{12} + k_4} + Q_{\mathcal{X}}(\gamma_{ij}, n_{ij}) + (\text{perms}), \quad (36)$$

where P and Q 's are polynomials of γ_{ij} and n_{ij} . When $\mathcal{X}$ has nonzero spin, $\mathcal{M}_{4,\mathcal{X}}$ will have spinning components, but we suppress their labels above. These components are not independent and satisfy a “gauge invariance” condition (see Supplemental Material [42]).

In the ansatz, $P^{(k_1, k_2)}$ and $Q_{\mathcal{X}}^{(k_4)}$ depend quadratically on γ , while $P^{(k_3)}$ and $Q_{\mathcal{X}}$ are linear in γ and P independent of γ . This power counting comes from the flat space limit of conventional Mellin amplitudes [38]. For the total power of γ and n , the amplitudes should scale like $\mathcal{M}_5 \sim \lambda^7$ and $\mathcal{M}_{4,\mathcal{X}} \sim \lambda^{5,6,6,8,9}$ for $\mathcal{X} = \mathcal{J}, \mathcal{T}, \mathcal{A}, \mathcal{C}, \mathcal{F}$ when $\gamma, n \sim \lambda \rightarrow \infty$. The power counting for $\mathcal{M}_5$ is based on the factorization of $\mathcal{O}$ in $\mathcal{M}_5 \supset \mathcal{M}_{3,\mathcal{O}} \times \mathcal{M}_{4,\mathcal{O}}/\rho_{LR} \sim \lambda^{3+5-1}$, and those for $\mathcal{M}_{4,\mathcal{X}}$ are to ensure

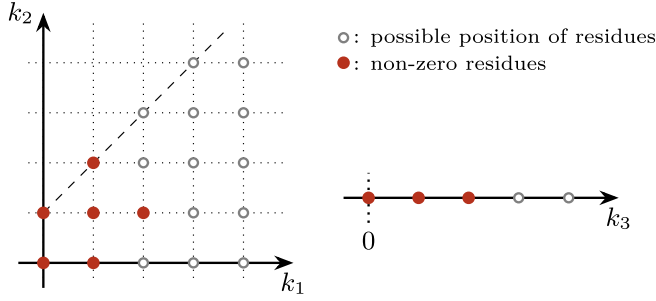


FIG. 1. The position of nonzero residues in the five-point amplitude ansatz.

all these subamplitudes give rise to λ^7 in $\mathcal{M}_5$ using the factorization formula.

The truncation of ρ poles in $\mathcal{M}_{4,\mathcal{X}}$ follows also from considerations of the factorization on $\mathcal{O}$. When gluing $\mathcal{M}_{3,\mathcal{O}}$ and $\mathcal{M}_{4,\mathcal{O}}$ in the channel (12)-(345), poles in $\mathcal{M}_{4,\mathcal{O}}$ give rise to double poles in $\mathcal{M}_5$ [see Eq. (39)], located at the spots in Fig. 1, inside the region under the dashed line $k_2 = k_1 + 1$. The truncation of poles in $\mathcal{M}_{4,\mathcal{X}}$ ensures that double poles due to the exchange of $\mathcal{X}$ are also located inside the same region. We do not truncate poles in the ansatz of $\mathcal{M}_5$ by hand [55], but they turn out to be truncated automatically after the bootstrap computation.

In principle, one can write down a larger ansatz with higher-degree polynomials and more poles. The bootstrap computation justifies our minimal ansatz above, which already yields a consistent solution that passes various checks. (2) Crossing symmetry: The ansatz should satisfy crossing symmetry. This means that $\mathcal{M}_5$ should be invariant under S_5 , and $\mathcal{M}_{4,\mathcal{X}}$ should be invariant under the S_3 symmetry among the three superprimaries $\mathcal{O}$. This is largely accounted by the “+ (perms)” term in the ansatz but still imposes some additional constraints on the polynomials when it maps the poles into the same channel. For example, $(1,2) \leftrightarrow (4,5)$ leaves the channel $\rho_{12}\rho_{45}$ unchanged, and requires that

$$P^{(k_1,k_2)}(\gamma_{ij}, n_{ij}) = P^{(k_2,k_1)}(\gamma_{ij}, n_{ij})|_{(1,2) \leftrightarrow (4,5)}. \quad (37)$$

There are similar constraints on other P and Q ’s, but we refrain from writing them down explicitly. (3) Z_2 symmetry: In addition, we impose the AdS $\leftrightarrow$ S symmetry on $\mathcal{M}_{4,\mathcal{X}}$. It takes the following form on the scalar operator $\mathcal{F}$:

$$\mathcal{M}_{4,\mathcal{F}}(\gamma_{ij}, n_{ij}) = \mathcal{M}_{4,\mathcal{F}}(-n_{ij}, -\gamma_{ij}), \quad (38)$$

and those with spinning operators are presented in Supplemental Material [42]. This Z_2 symmetry halves the unknown coefficients in the ansatz of $\mathcal{M}_{4,\mathcal{J}/\mathcal{C}/\mathcal{F}}$, and reduces the ansatz of $\mathcal{M}_{4,\mathcal{A}}$ to that of $\mathcal{M}_{4,\mathcal{T}}$ completely. (4) AdS $\times$ S factorization: Using the AdS $\times$ S factorization formula, we can relate the polynomials in $\mathcal{M}_5$ and $\mathcal{M}_{4,\mathcal{X}}$ by

$$P^{(k_1,k_2)} = \sum_{\mathcal{X}} \text{Fac}_{\mathcal{X},k_1} [\mathcal{M}_{3,\mathcal{X}} \times \tilde{Q}_{\mathcal{X}}^{(k_2)}], \quad (39)$$

$$P^{(k_3)} = \sum_{\mathcal{X}} \text{Fac}_{\mathcal{X},k_3} [\mathcal{M}_{3,\mathcal{X}} \times \tilde{Q}_{\mathcal{X}}], \quad (40)$$

where $\text{Fac}_{\mathcal{X},k}$ stands for the factorization formula for the residue of $\mathcal{X}$ exchange at pole $\rho_{12} = -k$. $\mathcal{X}$ runs over all six master operators, and $\tilde{Q} \equiv Q|_{(1,2,3,4) \rightarrow (1,3,4,5)}$ is the relabeling of points in the subamplitudes. Since P ’s are polynomials, these two equations implicitly require that there are no spurious poles on the rhs after applying the factorization formula. The absence of spurious poles turns out to be a very strong constraint, which solves a large part of unknowns at this stage.

It is worth noting that, after imposing the factorization formula, all the superdescendant amplitudes $\mathcal{M}_{4,\mathcal{X}}$ are completely fixed, without even using supersymmetry. Also, the poles in $\mathcal{M}_5$ turn out to be truncated at $k \leq 2$ (we checked the result up to $k = 5$). The nonzero residues are marked red in Fig. 1. (5) Drukker-Plefka twist: There remains a regular term $P(n_{ij})$ to be determined in $\mathcal{M}_5$. This term is not probed by the factorization, and we fix it by the Drukker-Plefka (DP) twist [40] (note that this twist also respects Z_2 symmetry)

$$G_{\{p_i\}}(X_{ij}, T_{ij})|_{T_{ij} \rightarrow X_{ij}} = \text{const}. \quad (41)$$

We can impose this condition in AdS $\times$ S space. Note that $\mathcal{M}_5(\rho_{ij} + n_{ij}, n_{ij})$ is a polynomial with respect to n_{ij} . We can always rewrite any single-variable polynomial of n_{ij} with a different basis (the factorial powers)

$$[n_{ij}]_c \equiv n_{ij}(n_{ij} - 1) \cdots (n_{ij} - c + 1), \quad (42)$$

which leads to the following decomposition with some integer $c_{ij,k}$:

$$\mathcal{M}(\rho_{ij} + n_{ij}, n_{ij}) \equiv \sum_k \mathcal{M}^k(\rho_{ij}) \prod_{i < j} [n_{ij}]_{c_{ij,k}}. \quad (43)$$

The Drukker-Plefka twist requires the amplitude to vanish upon replacing

$$[n_{ij}]_{c_{ij,k}} \rightarrow \left(\frac{a_i a_j}{1 - a_i a_j} \right)^{c_{ij,k}} (\rho_{ij})_{c_{ij,k}}, \quad (44)$$

with a_i some auxiliary variables (see Supplemental Material [42]). Imposing the AdS $\times$ S DP twist completely determines $\mathcal{M}_5$.

Result—The full expression for $\mathcal{M}_5$ is too lengthy to include in the main text. We therefore present selected components in Supplemental Material [42] and provide the complete result as an auxiliary file. Our results pass several nontrivial consistency checks: $\mathcal{M}_5$ reduces to the known

result M_{pp222} in [38], and the amplitudes $\mathcal{M}_{4,\mathcal{X}}$ for $\mathcal{X} = \mathcal{J}, \mathcal{T}, \mathcal{A}$ agree with results independently derived using the superspace method (see Supplemental Material [42]).

Furthermore, we confirm that $\mathcal{M}_5$ can be recast in a factorized form that generalizes (12) to five points

$$\mathcal{M}_5(\gamma_{ij}, n_{ij}) = \sum_k \widehat{R}^{(k)} \circ \widetilde{\mathcal{M}}_5^{(k)}(\rho_{ij}) + (\text{perms}), \quad (45)$$

where $\widetilde{\mathcal{M}}_5^{(k)}$ are simple rational functions with five distinct poles

$$\widetilde{\mathcal{M}}_5^{(k)}(\rho_{ij}) = \prod_{(\text{number of poles})=5} \frac{1}{\rho_{ij} - 1}, \quad (46)$$

and $R^{(k)}$ are (the bosonic component of) some superconformal invariants constructed by superspace method (see Supplemental Material [42]), manifesting supersymmetry in $\mathcal{M}_5$. Notably, we did not impose supersymmetry as a bootstrap constraint (except for the DP twist), but it emerges as a result from hidden 10D symmetry. However, the representation (45) we found is not unique. The ambiguities arise from certain superconformal invariants that sum up to 0 in (45). Note that, some of these zero functions in Mellin space are actually nonzero in position space due to the contour pinching issue [2], and the corresponding ambiguities can be fixed in position space. We leave it for future work.

Outlook—The obvious next target is computing complete six- and higher-point amplitudes for half-BPS operators, where our method directly applies and gives all KK correlators at given points once and for all. In principle, this method can be applied recursively: we expect that bootstrapping $\mathcal{M}_n$ would generate $\mathcal{M}_{n-1,\mathcal{X}}$ as byproducts, and these $\mathcal{M}_{n-1,\mathcal{X}}$ can then be fed into the factorization constraints of $\mathcal{M}_{n+1}$

$$\mathcal{M}_{n+1} \sim \sum_{\mathcal{X}} \mathcal{M}_{4,\mathcal{X}} \times \mathcal{M}_{n-1,\mathcal{X}} \sim \sum_{\mathcal{X}} \mathcal{M}_{3,\mathcal{X}} \times \mathcal{M}_{n,\mathcal{X}} \quad (47)$$

to bootstrap $\mathcal{M}_{n+1}$ itself as well as $\mathcal{M}_{n,\mathcal{X}}$. Another exciting direction is to further incorporate supersymmetry in this formalism and combine the six master operators into a super master operator $\mathbb{O}$, as was studied in [26] at weak coupling. This operator would be the only object we need to consider in the factorization of tree-level functions and may help to reveal the relation between supersymmetry and hidden 10D symmetry in $\mathcal{N} = 4$ SYM at strong coupling.

Multitrace operators naturally appear when we consider the OPE in higher-point correlators. However, very little is known about the spectrum and correlators of these multitrace operators (see Refs. [56–62] for some recent progress). By analyzing the OPE data in our five-point function, we can extract the triple-trace anomalous

dimensions for arbitrary KK modes, which would be crucial to the study of multiparticle bound states in AdS and might reveal how hidden 10D symmetry encodes in multitrace spectrum.

So far, the vast majority of results for more than four points have been limited to the supergravity approximation; thus, it would be interesting to study stringy and loop corrections to our result [63], and also investigate at the weak coupling region [64,65]. This could give us hints of different representations for the amplitude.

A key ingredient in establishing the double-copy structure of four-point amplitudes in AdS [66] was the identification of a universal supersymmetric prefactor. In this Letter, we proposed an extension of this prefactor to five-point functions. It would be interesting to explore whether a similar double-copy structure exists at five points, both in the half-BPS sector studied here and for gluon correlators in models such as $\text{AdS}_5 \times S^3$ [28].

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