

Edge of Safety: Charge-Charge Correlation in the Back-to-Back Limit

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We investigate the charge-charge correlation (QQC) in electron-positron annihilation as a probe of charge dynamics in quantum chromodynamics. While generally divergent beyond leading order, we show that the QQC is infrared and collinear safe in the back-to-back limit, a property that we dub leading-power safety. This enables an analytic perturbative treatment that bypasses the reliance on nonperturbative track or fragmentation functions. Using soft-collinear effective field theory, we derive a factorization theorem and determine its logarithmic behavior analytically up to four loops in QCD, uncovering remarkable connections with the energy-energy correlation (EEC). Prior to this Letter, the behavior of the QQC beyond leading order was entirely unknown; we now establish its resummation to next-to-next-to-next-to-next-to-leading logarithmic (N⁴LL) accuracy, placing it among the observables with the highest perturbative precision alongside the EEC. Our results are validated through a numerical analysis using Event2, exhibiting excellent agreement with the predicted singular terms. This Letter establishes the QQC as a novel, calculable probe of charge dynamics and unveils a new window into the inner workings of non-Abelian gauge theories.

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Introduction—Understanding the internal structure of hadronic events in quantum chromodynamics (QCD) has long been a central objective in high-energy physics. Among the most powerful probes of this structure are event-shape observables and correlation functions, which can reveal the detailed flow of quantum numbers such as energy and charge.

In this Letter, we investigate the charge-charge correlation (QQC) observable, defined in electron-positron annihilation as the pairwise angular correlation between the charges of all final-state particles. The QQC provides a direct measure of how electric charge is distributed across an event and how it correlates between widely separated regions of phase space. Specifically, the QQC can be defined as (cf. Fig. 1)

$$\text{QQC}(\chi) = \sum_{a,b} \int d\sigma_{e^+e^- \rightarrow a,b+X} Q_a Q_b \delta(\cos\chi - \vec{n}_a \cdot \vec{n}_b), \quad (1)$$

where Q_i is the electric charge of particle i , the sum runs over all final-state particles, $\hat{n}_a$ represents the lightlike direction of the parton a . Formally, the charge can be defined as the light transform of the electromagnetic current [1–9]:

$$Q(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_{-\infty}^{\infty} dt n^i J_i(t, r\vec{n}). \quad (2)$$

Therefore, the QQC too can be defined in terms of a double insertion of such charge flow operators as

$$\text{QQC}(\chi) = 8\pi^2 \int d^4x e^{iqx} \langle J(x) Q(\vec{n}_a) Q(\vec{n}_b) J(0) \rangle_W, \quad (3)$$

with $\vec{n}_a \cdot \vec{n}_b = \cos\chi$, and q the total momentum of the hadronic state. Consequently, the QQC is related to the four-point Wightman correlation function of the electromagnetic current operator, providing a connection to fundamental operator structures within the theory and highlights its potential sensitivity to the intricate charge dynamics of the underlying gauge theory. This connection places the QQC within a broader program aimed at understanding collider observables through the language of correlation functions, a perspective that has recently gained prominence across a range of theoretical studies [7–30]; see Ref. [31] for a recent review. Despite its conceptual appeal, the QQC has received limited theoretical attention because it is known to be infrared and collinear (IRC) unsafe beyond leading order (LO) in perturbative QCD [32]. This has historically precluded precision

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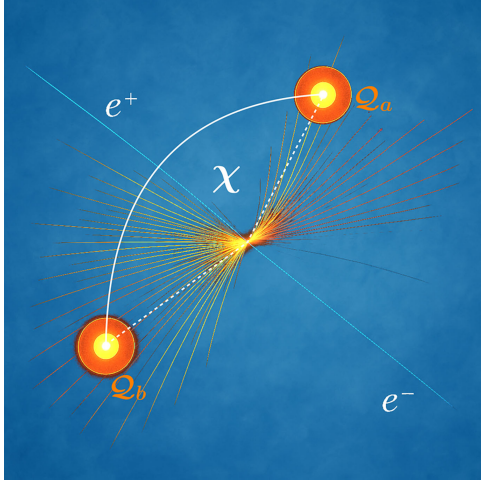


FIG. 1. Graphical illustration of the charge-charge correlation in electron-positron annihilation.

predictions for the QQC, necessitating reliance on nonperturbative models of hadronization via track or fragmentation functions; see, e.g., Refs. [33–35].

In this Letter, we demonstrate that the QQC becomes IRC safe, and thus perturbatively calculable, in the back-to-back kinematic limit, where $\chi \rightarrow \pi$, corresponding to the $z \equiv (1 - \cos \chi)/2 \rightarrow 1$ regime. We will refer to this property as *leading power (LP) safety*. This observation is used to derive a state of the art prediction for this observable at lepton colliders, enabling high-precision phenomenological studies of charge dynamics in QCD.

The first source of IRC unsafety of the QQC in QCD arises—The first source arises when the resulting $q\bar{q}$ pair propagates along one of the two streams, and are correlated with particles belonging to the opposite stream. This configuration is protected by charge conservation, that ensures that the resulting contribution to the QQC vanishes upon summing over the q and the $\bar{q}$. The second source of IRC unsafety is due to a soft gluon with a wide angle with respect to the large energy flow that splits into a $q\bar{q}$ pair that fly into opposite directions. While this configuration is not protected by charge conservation, we observe that the corresponding squared amplitude is exponentially suppressed in the back-to-back limit, that is, when the rapidity separation $\Delta\eta_{q\bar{q}}$ between the $q\bar{q}$ pair is large. The squared amplitude for the splitting in Fig. 2 scales as [36] (cf. also Ref. [37])

$$|\mathcal{M}_{q\bar{q}}|^2 \sim (8\pi\mu^{2\epsilon}\alpha_s)^2 C_F \frac{|\vec{k}_{T,q} - \vec{k}_{T,\bar{q}}|^2}{|\vec{k}_{T,q}|^3 |\vec{k}_{T,\bar{q}}|^3} e^{-3|\Delta\eta_{q\bar{q}}|}, \quad (4)$$

with $\vec{k}_{T,i}$ denoting the transverse momentum with respect to the direction of the energetic particles. This observation crucially implies that the divergence is suppressed in the back-to-back limit by powers of $(1 - z)$, hence allowing for

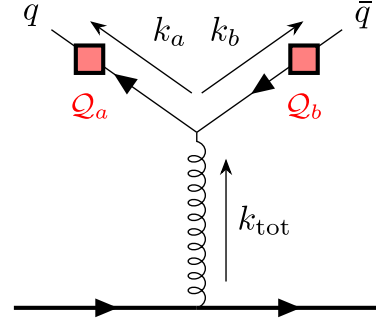


FIG. 2. Charge correlation of a correlated $q\bar{q}$ soft pair.

the application of perturbative methods to the calculation of QQC in this regime. Using the framework of soft-collinear effective theory (SCET) [38–41] we now establish for the first time a factorization theorem in QCD for the leading power asymptotic of QQC in this limit.

Factorization theorem in SCET—Our theoretical framework starts from the description of dihadron production in the limit of small transverse momentum q_T of the virtual photon in the back-to-back frame of the two hadrons. The factorization for this regime was established by Collins and Soper [42–44] and reads,

$$\frac{d\sigma_{e^+e^- \rightarrow h_1 h_2 + X}}{dx_1 dx_2 d^2\vec{q}_T} = \sum_q \sigma_0^q H_{q\bar{q}}(Q) \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \times D_{h_1/q}(x_1, b_T) D_{h_2/\bar{q}}(x_2, b_T) S_q(b_T), \quad (5)$$

up to power corrections in $\mathcal{O}(q_T^2/Q^2)$. Here σ_0^q denotes the Born cross section for the production of a single quark flavor q . In the above expression, a summation over all quark flavors q is understood. The hard function $H_{q\bar{q}}$ captures the virtual corrections to the tree-level process $e^+e^- \rightarrow q\bar{q}$, and it is known to N⁴LO in QCD [45,46]. The function $D_{h/q}$ represents the transverse-momentum-dependent fragmentation function (TMDFF), describing the probability for a quark q to hadronize into a hadron h . The soft function S_q accounts for the net transverse momentum generated by soft-gluon radiation in the event. It is known to N³LO [47] and it coincides with the soft function employed in q_T resummation for pp collisions [48], as it is independent of the orientation of the associated Wilson lines [49,50].

We can then use Eq. (5) to write the QQC in the back-to-back limit as

$$\frac{d\sigma_{\text{QQC}}^{e^+e^-}}{dz} \stackrel{z \rightarrow 1}{=} \int_0^1 dx_a \int_0^1 dx_b \int d^2\vec{q}_T \delta\left(1 - z - \frac{\vec{q}_T^2}{Q^2}\right) \times \sum_{h_a, h_b} Q_{h_a} Q_{h_b} \frac{d\sigma_{e^+e^- \rightarrow h_a h_b + X}}{dx_a dx_b d^2\vec{q}_T}, \quad (6)$$

up to power corrections of $\mathcal{O}((1 - z)^0)$. In the form of Eq. (6), the QQC still involves the nonperturbative TMDFF

$D_{h_a/q}$. In the following, we prove that Eq. (6) is actually independent of them, in line with the expectation for an IRC safe observable. We start by noting that for perturbative values of q_T , we can derive an operator product expansion (OPE) for the TMDFF [44,51]. At leading power in $\mathcal{O}(b_T^2 \Lambda_{\text{QCD}}^2)$ we can match the TMDFF onto the longitudinal FF $d_{h/q}$ as

$$D_{h/q}(x, b_T) = \sum_i \int_x^1 \frac{dt}{t} \mathcal{K}_{qi}(t, b_T) d_{h/i}\left(\frac{x}{t}\right), \quad (7)$$

where $\mathcal{K}_{qi}$ is a perturbative matching kernel and the sum runs over all parton flavors i including gluons. These matching kernels are well-known objects, and have been calculated up to N^3LO in Refs. [52,53]. Taking into account the integral over the energies of the tagged hadron and its charge weight from (6) we can factorize the dependence on the longitudinal FF, obtaining

$$\begin{aligned} J_q^{\text{QQC}} &\equiv \sum_{h_a} \int_0^1 dx_a Q_{h_a} D_{h_a/q}(x_a, b_T) \\ &\times \sum_{h_a, i} \int_0^1 dx_a \int_{x_a}^1 Q_{h_a} \frac{dz_a}{z_a} \mathcal{K}_{qi}(z_a, b_T) d_{h_a/i}\left(\frac{x_a}{z_a}\right) \quad (8) \\ &= \sum_i \int_0^1 dz_a \mathcal{K}_{qi}(z_a, b_T) \left[\sum_{h_a} \int_0^1 d\tau_a Q_{h_a} d_{h_a/i}(\tau) \right] \\ &\times \sum_i \int_0^1 dz_a \mathcal{K}_{qi}(z_a, b_T) Q_i, \quad (9) \end{aligned}$$

where we used the charge sum rules for quark fragmentation functions [54]. Therefore, we have demonstrated that leading power asymptotic for the QQC does not depend on nonperturbative objects. We can thus rewrite (6) in terms of purely perturbative objects obtaining our factorization formula for QQC in the back-to-back limit

$$\begin{aligned} \text{QQC}(z) &= \sum_q \frac{\sigma_0^q}{2} H_{q\bar{q}}(Q, \mu) \int_0^\infty d(b_T Q)^2 J_0(b_T Q \sqrt{\bar{z}}) \\ &\times J_q^{\text{QQC}}\left(b_T, \mu, \frac{b_T Q}{v}\right) J_{\bar{q}}^{\text{QQC}}\left(b_T, \mu, v b_T Q\right) \\ &\times [1 + \mathcal{O}(\bar{z})], \quad (10) \end{aligned}$$

where $\bar{z} \equiv 1 - z$, $H_{q\bar{q}}$ is the same hard function as in Eq. (5), $J_0(x)$ is the 0th Bessel function of the first kind, and J_q^{QQC} are the QQC jet functions (8), that appear in this context for the first time. In Eq. (10) μ denotes the renormalization scale and v denotes the rapidity regularization scale in the pure regularization scheme [55,56] in which the soft function is equal to one. While the result in Eq. (8) is given in the pure regularization scheme for

simplicity, the scheme dependence always cancels in the product of the jet and soft functions, yielding a scheme-independent physical prediction.

The QQC jet function—The existence of the QQC jet function (8) is far from trivial. Indeed, each of the renormalized matching kernels $\mathcal{K}_{ij}(x, \vec{b}_T)$ entering in its definition diverge when integrated over the whole range of energies $x \in [0, 1]$. However, the integral defining J^{QQC} is well defined. To show this we make use of the sign difference in the charge when summing over quarks and antiquarks to obtain

$$J_q^{\text{QQC}}(b_T) = \sum_{q'} Q_{q'} \int_0^1 dx [\mathcal{K}_{qq'}(x, b_T) - \mathcal{K}_{q\bar{q}'}(x, b_T)], \quad (11)$$

where q' runs over all flavors (including q), but excludes the gluon and the antiquarks (or the quarks in case q is an antiquark). Nontrivially, the $x \rightarrow 0$ limit is regulated by the universality of the Regge limit of quark-antiquark matching kernels, i.e., the fact that

$$\lim_{x \rightarrow 0} [\mathcal{K}_{qq'}(x, b_T) - \mathcal{K}_{q\bar{q}'}(x, b_T)] = \mathcal{O}(x^0), \quad (12)$$

which has been explicitly shown to three loops in Refs. [52,53]. On the other hand, for $x \rightarrow 1$ Eq. (11) is finite due to the standard UV regularization in SCET of the matching kernels. This ensures that the QQC jet function is well defined at any order in perturbation theory. Using Eq. (11) and the result for the TMDFF calculated in [52,53], we obtain the QQC jet function to N^3LO and we make them available in Ref. [57]. The anomalous dimension of QQC jet function is completely dictated by the anomalous dimension of the hard function by the consistency of the factorization theorem. Moreover, since J^{QQC} represents the only difference in the factorization theorem compared to the EEC SCET factorization in the back-to-back limit [49,56,58] (see also Refs. [42,59,60]) the consistency of both factorizations implies the ratio of the QQC and EEC jet functions, $\mathcal{J}_q^{\text{QQC}}/\mathcal{J}_q^{\text{EEC}}$, obeys

$$\mu \frac{d}{d\mu} \ln \left[J_q^{\text{QQC}}\left(b_T, \mu, \frac{b_T Q}{v}\right) / J_q^{\text{EEC}}\left(b_T, \mu, \frac{b_T Q}{v}\right) \right] = 0, \quad (13)$$

as well as

$$v \frac{d}{dv} \ln \left[J_q^{\text{QQC}}\left(b_T, \mu, \frac{b_T Q}{v}\right) / J_q^{\text{EEC}}\left(b_T, \mu, \frac{b_T Q}{v}\right) \right] = 0, \quad (14)$$

where

$$J_q^{\text{EEC}}\left(b_T, \mu, \frac{b_T Q}{v}\right) = \sum_i \int_0^1 dx x \mathcal{K}_{qi}\left(x, b_T, \mu, \frac{b_T Q}{v}\right) \quad (15)$$

is the EEC jet function [49,56,58]. Using the newly determined expression for J^{QQC} and the three loop results for J^{EEC} from Ref. [58] we verified explicitly that both Eqs. (13) and (14) hold to three loops, providing a strong analytic cross check on the three loop determination of J^{QQC} in this Letter. The corresponding RGEs for the ingredients of the factorization theorem are given in Refs. [56,58] and we report them in Ref. [61] (which, in addition, uses Refs. [62–68]) for completeness. Their solution is expressed in terms of the anomalous dimensions and boundary conditions leading to the resummed formula for the QQC in the back-to-back limit

$$\begin{aligned} \frac{d\sigma}{dz} = & \sum_q \frac{\sigma_0^q}{2} \int_0^\infty d(b_T Q)^2 J_0(b_T Q \sqrt{z}) H_{q\bar{q}}(Q, \mu_H) \\ & \times \mathcal{J}_q^{\text{QQC}}\left(b_T, \mu_J, \frac{Q b_T}{v_n}\right) \\ & \times \mathcal{J}_{\bar{q}}^{\text{QQC}}\left(b_T, \mu_J, Q b_T v_{\bar{n}}\right) \left(\frac{v_n}{v_{\bar{n}}}\right)^{\frac{1}{2}\gamma_s^q(b_T, \mu_J)} \\ & \times \exp\left[4 \int_{\mu_J}^{\mu_H} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}}^q[\alpha_s(\mu')] \ln \frac{\mu'}{Q} - \gamma_H^q[\alpha_s(\mu')]\right]. \end{aligned} \quad (16)$$

Here μ_J, μ_H, v_n , and $v_{\bar{n}}$ are the scales at which the boundary conditions of the RGEs are evaluated, to be taken of the order of the canonical scales. The determination of the QQC jet function to N^3LO allows for the evaluation of Eq. (16) to N^4LL accuracy.

Analytic and numerical validation—At LO, $[\mathcal{O}(\alpha_s)]$, the QQC is IRC safe, we can therefore obtain the back-to-back limit from the expansion of its full angle expression as [8,69]

$$\frac{\text{QQC}^{\text{LO}}(z)}{\frac{\alpha_s}{4\pi}} \stackrel{z \rightarrow 1}{=} C_F (8\mathcal{L}_1(\bar{z}) + 12\mathcal{L}_0(\bar{z}) + (8 + 8\zeta_2)\delta(\bar{z})), \quad (17)$$

which matches the $\mathcal{O}(\alpha_s)$ expansion of our formula, including for the contact terms proportional to $\delta(\bar{z})$.

Furthermore, a powerful consistency check can be performed by considering $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory. In $\mathcal{N} = 4$ SYM, the operator defining the QQC is identical to that of the energy-energy correlator (EEC), as both relate to correlation functions of operators $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \mathcal{O}(x_3) \mathcal{O}(x_4) \rangle$ within the stress-tensor supermultiplet [7,10,12,20]. Consequently, we have

$$\text{QQC}_{\mathcal{N}=4}(z) = \text{EEC}_{\mathcal{N}=4}(z). \quad (18)$$

To verify this relation, we apply the principle of maximal transcendentality [70] to the QQC and substitute $C_F \rightarrow C_A, n_f \rightarrow C_A$ to obtain its $\mathcal{N} = 4$ form from QCD and compare the known results for the EEC in the back-to-back

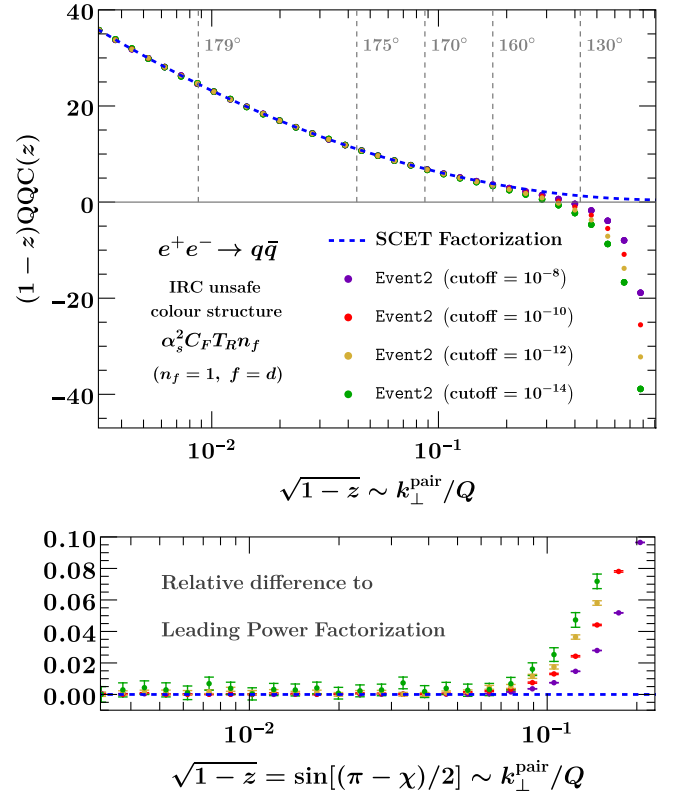


FIG. 3. Numerical validation of the leading power factorization in Eq. (10) with Event2 [71]. In the back-to-back limit the numerical result is cutoff independent and matches the analytic prediction of the SCET factorization theorem. Away from the limit, the numerical results deviate from the leading power prediction and show a stronger cutoff dependence due to the IRC unsafety of the QQC at subleading powers in $1-z$.

limit in $\mathcal{N} = 4$ SYM [16,58,68]. We carried out this procedure to three loops, finding perfect agreement between the two results. This yields an independent prediction for the leading transcendental terms in the back-to-back limit, providing a valuable cross check against our results obtained via our factorization theorem.

To ensure our factorization framework accurately describes the contributions responsible for the IRC unsafety of the QQC, we performed a direct comparison with a fixed-order result focusing specifically on the problematic channel involving gluon splitting into a $q\bar{q}$ pair in the back-to-back limit ($z \rightarrow 1$). We employed the parton-level generator Event2 [71] to obtain the NLO QCD prediction for this specific $g \rightarrow q\bar{q}$ contribution to the QQC as $z \rightarrow 1$. Crucially, the Event2 calculation yields a finite result and independent of the IRC cutoff in the back-to-back limit, numerically confirming the anticipated IRC safety of the QQC in this configuration. Most importantly, the fixed-order result exactly matches the prediction obtained from our factorization theorem evaluated in the same limit. This agreement, illustrated in Fig. 3, provides a highly nontrivial cross check of our theoretical framework, demonstrating

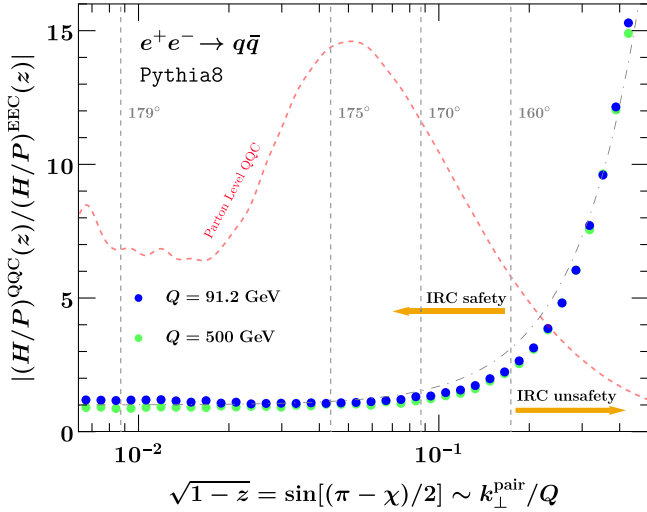


FIG. 4. Ratio of hadronization effects in QQC and EEC in the back-to-back limit. While for a generic angle the hadronization effects in the QQC are much larger than the ones for an IRC safe observable like the EEC, as we approach the back-to-back region we see a strong suppression of such effects in accordance with LP safety. The red dashed line depicts the parton level QQC to help visualize the IRC safe range of the observable. Finally, the gray dot-dashed line corresponds to $y \sim (1-z) \sim (k_{\perp}/Q)^2$ indicating that the hadronization model indicates an hadronization corrections quadratic in the transverse momentum of the radiation.

that our perturbative description based on the factorization theorem accurately captures the intricate cancellation of divergences and correctly predicts the perturbative result.

As a further check beyond this perturbative order, we have verified that a parton-level prediction obtained with the PanScales NNLL parton shower [72,73] for the QQC is independent of the infrared cutoff of the algorithm in the back-to-back regime (cf. Ref. [61]). This is a further nontrivial confirmation of the LP safety of this observable.

Hadronization effects—A further test of our results is the sensitivity of the back-to-back limit of QQC to hadronization effects, simulated with PYTHIA8 [74]. In Fig 4, we see that the hadronization effects for the QQC in the bulk are very large compared to the ones of an IRC safe observable like the EEC. However, they dramatically decrease as we approach the back-to-back limit, in line with the IRC safety of the QQC in this regime. In particular, we see that at center-of-mass energies corresponding to the Z peak, there is a ten-degree-wide angular region in which hadronization effects are comparable to the ones of the EEC. Although this might seem a narrow range, this window can be measured very precisely on charged tracks and thus has significant potential for phenomenological studies. As an example, a recent reanalysis of ALEPH data [75] reports $\mathcal{O}(70)$ bins in this window for the tracks-based EEC. We leave a quantitative study of hadronization for future work.

Conclusions—In this Letter, we have shown that the charge-charge correlation, a fundamental probe of gauge

theory dynamics known to be IRC unsafe, becomes perturbatively calculable in the back-to-back kinematic limit $z \rightarrow 1$. The IRC safety of the QQC in this asymptotic regime is a consequence of the suppression of the source of IRC divergences by powers of the angular variable $(1-z)$, a mechanism that we dub leading-power safety. At leading power, we have derived a SCET factorization theorem which does not rely on nonperturbative fragmentation functions. This factorization features a novel theoretical ingredient, the QQC jet function, which we determined to $\mathcal{O}(\alpha_s^3)$. While nothing was known beyond $\mathcal{O}(\alpha_s)$ in QCD for the QQC, our theoretical framework has allowed us to carry out the resummation for this observable to an unprecedented N⁴LL accuracy. Our framework has been rigorously validated against analytic constraints from $\mathcal{N} = 4$ SYM and direct numerical calculations. This Letter transforms the QQC from a incalculable, IRC unsafe observable into a high-precision phenomenological tool. It opens a new avenue for testing the intricate charge-flow dynamics of QCD and provides a powerful new observable for experimental analyses at e^+e^- colliders. The generalization of our results to ep and pp collisions is straightforward following [76–78]. Analogous considerations apply to correlators of other conserved global charges, that deserve a similar level of theoretical scrutiny in the future. By operating at the brink of IRC safety, we have unveiled a new, calculable window into the structure of quantum field theory.

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Data availability—The data that support the findings of this article are openly available at [57].

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