

Experimental Distributed Quantum Sensing in a Noisy Environment

J. Bate¹, A. Hamann², M. Canteri¹, A. Winkler¹, Z. X. Koong¹, V. Krutyanskiy¹, W. Dür², and B. P. Lanyon^{1,*}

¹*Universität Innsbruck, Institut für Experimentalphysik, Technikerstraße 25, 6020 Innsbruck, Austria*

²*Universität Innsbruck, Institut für Theoretische Physik, Technikerstr. 21a, 6020 Innsbruck, Austria*



(Received 10 January 2025; revised 9 May 2025; accepted 15 October 2025; published 24 November 2025)

The precision advantages offered by harnessing the quantum states of sensors can be readily compromised by noise. However, when the noise has a different spatial function than the signal of interest, recent theoretical work shows how the advantage can be maintained and even significantly improved. In this Letter, we experimentally demonstrate the associated sensing protocol, using trapped-ion sensors. An entangled state of multidimensional sensors is created that isolates and optimally detects a signal, while being insensitive to otherwise overwhelming noise fields with different spatial profiles over the sensor locations. The quantum protocol is found to outperform a perfect implementation of the best comparable strategy without sensor entanglement. While our demonstration is carried out for magnetic and electromagnetic fields over a few microns, the technique is readily applicable over arbitrary distances and for arbitrary fields, thus present a promising application for emerging quantum sensor networks.

DOI: 10.1103/3hgx-wcdn

Measuring physical quantities with ever higher precision lies at the heart of the natural sciences and is the key to many technological applications. Quantum sensors can achieve Heisenberg-limited scaling, where the standard deviation of the parameter estimate scales as $1/N$, while classical sensors are limited by the standard quantum limit, with standard deviation scaling as $1/\sqrt{N}$, where N is the number of sensors used [1–7]. In scenarios in which a local field strength or frequency should be measured, there are many demonstrations of quantum enhanced precision, e.g., for electric and magnetic fields [8–10], gravitational waves [11], and momentum changes [12–14]. In many cases, a spatially distributed signal is of interest, such as a difference in field strength or frequency. In such “distributed sensing” scenarios, quantum sensor networks could also offer Heisenberg-limited scaling [15–20], as well as unconditional privacy [21–23], and the first experimental investigations have been done [24–29]. However, in general, noise jeopardizes accessing the scaling advantage of both local and distributed quantum sensing as the targeted precision is increased [30–34].

Quantum error correction offers a powerful future approach to overcome noise in quantum sensing [30,31,35–38]. However, when noise and signal have a different spatial dependence, a simpler approach has been

proposed [39] based on decoherence-free subspaces (DFSs) [40,41] that is equally as powerful [42] and far less resource intensive. Here, the choice of initial entangled state established between distributed sensors allows optimal estimations of any desired targeted signal, while being resilient against fluctuations in noise fields. Such DFSs lead to a quantum advantage even in the presence of strong noise, and have been theoretically constructed for general signals and noise sources [39,42–44]. Those results show how to generalize techniques that exploit, e.g., two-sensor states to avoid decoherence due to spatially constant noise [8,27,45].

In this Letter, we experimentally demonstrate the SWD (Sekatski-Wölk-Dür) protocol [39]. As an example, we consider the task of sensing the strength of a field that varies quadratically in space, while being insensitive to fluctuations in the strength of fields with both constant and gradient profiles. The sensors are three atomic $^{40}\text{Ca}^+$ ions, 4.9 μm apart. The performance of our implementation of the optimal entangled sensor state is compared with the best comparable scheme without entanglement.

We consider the scenario in which three sensors are positioned equidistantly along one dimension. The total scalar field strength at position x is given by $B(x) = B^c + B^l x + B^q x^2/2$ [Fig. 1(a)], where B^j are the coefficients of three linearly independent field components, corresponding to the first three orders of a Taylor expansion. Each of the L_i eigenstates (or “levels”) of sensor i ($|s_k^i\rangle$) experience an energy shift proportional to the total field strength at the sensor position ($B(x_i)$). Specifically, the Hamiltonian of sensor i is given by $H(x_i) = \hbar\kappa \sum_{k=1}^{L_i} B(x_i) s_k^i |s_k^i\rangle \langle s_k^i|$, where κ is a constant

*Contact author: ben.lanyon@uibk.ac.at

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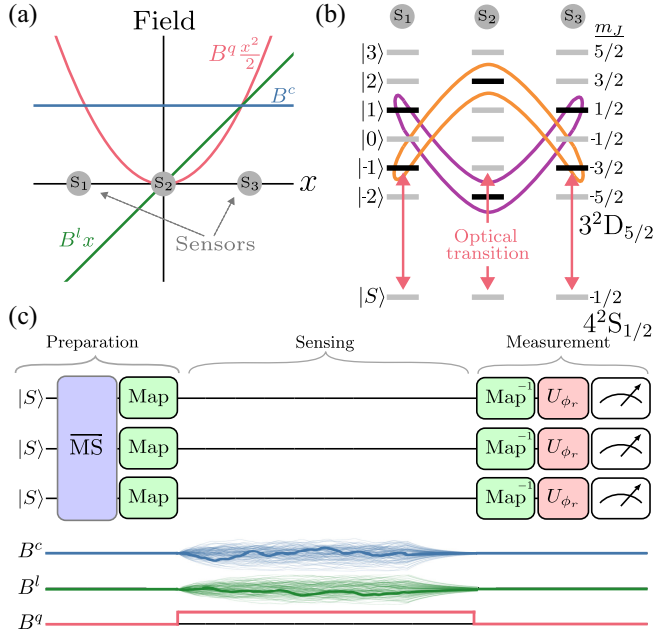


FIG. 1. Experimental demonstration with trapped-ion sensors. (a) Three sensors in a line are exposed to spatially constant (B^c), linear ($B^l x$), and quadratic ($B^q x^2/2$) fields. (b) Encoding of sensor state $|\psi_{(1,-2,1)}^{\text{SWD}}\rangle = (|1, -2, 1\rangle + |-1, 2, -1\rangle)/\sqrt{2}$ into superpositions of the $3^2D_{5/2}$ manifolds of three $^{40}\text{Ca}^+$ ions. (c) Protocol. $\overline{\text{MS}}$ is equivalent to an entangling Mølmer-Sørensen gate on the optical transition. “Map” moves electron population to the $3^2D_{5/2}$ manifold. After state preparation, fluctuating noise (B^c and B^l) and signal (B^q) fields are turned on. U_{ϕ_r} is described in the text.

determining the sensor-field interaction strength, and each s_k^i is a real number proportional to the sensitivity of the k th eigenstate. The strength of each field component at the three sensor positions $x = (-1, 0, 1)$ is described by the component vectors $\mathbf{f}^c = B^c(1, 1, 1)$, $\mathbf{f}^l = B^l(-1, 0, 1)$, and $\mathbf{f}^q = B^q(1, 0, 1)$, where the i th element is the field strength of the corresponding field component at sensor i .

The SWD protocol [39] requires preparing sensors into entangled states of the form $|\psi_s^{\text{SWD}}\rangle = (|s_1, s_2, s_3\rangle + |-s_1, -s_2, -s_3\rangle)/\sqrt{2}$, where $\mathbf{s} = (s_1, s_2, s_3)$. Each s_i is constrained such that $|s_i\rangle$ is an eigenstate of the sensor Hamiltonian $H(x_i)$. Exposure to any three field components, described by component vectors $\{\mathbf{f}^1, \mathbf{f}^2, \mathbf{f}^3\}$, for a time t causes the state to develop a relative phase $e^{-2ikt \sum_{j=1}^3 s_j f_j^i}$. The SWD protocol describes how to choose $\mathbf{s}$ such that the phase evolves due to any one of the field components ($\mathbf{f}^j$) at the maximal rate ($\max_s |\mathbf{s} \cdot \mathbf{f}^j|$) [46], while the other field components generate no phase ($\mathbf{s} \cdot \mathbf{f}^{j' \neq j} = 0$). For example, up to a scaling factor in $\mathbf{s}$, the optimal state to sense B^q while being insensitive to B^c and B^l is $|\psi_{(1,-2,1)}^{\text{SWD}}\rangle$. That state exists in a DFS with respect to constant and gradient fields, meaning that it is insensitive

to fluctuations in the field strengths B^c and B^l . $|\psi_{(1,-2,1)}^{\text{SWD}}\rangle$ is a GHZ-type entangled state [60] of two-level sensors, but with differing sensitivities to the surrounding fields.

We encode sensor states into the metastable $3^2D_{5/2}$ manifold of each of three $^{40}\text{Ca}^+$ ions, following $|s_i\rangle = |3^2D_{5/2, m_j=(s_i-1/2)}\rangle$ [Fig. 1(b)]. The sensor states couple to magnetic fields via the linear Zeeman effect: the frequency shift of sensor state $|s_i\rangle$ is given by $\kappa B(x_i)(s_i + 1/2)/(2\pi)$, where $\kappa = g_{5/2}\mu_B/\hbar = (2\pi)0.0168 \text{ Hz pT}^{-1}$. A spatially constant principle magnetic field of $4.14686(1) \times 10^{-4} \text{ T}$ is applied at all times, yielding a level splitting of $2\pi \times 6.96486(2) \text{ MHz}$. Each experimental shot of the SWD protocol (and its classical counterpart) consists of three steps: state preparation, sensing and measurement [Fig. 1(c)]. State preparation begins with Doppler cooling, sideband cooling, and optical pumping that prepares each ion into the state $|S\rangle = |4^2S_{1/2}, m_j = -1/2\rangle$ and the ground state of the axial center of mass motional mode. Next, a Mølmer-Sørensen logic gate (MS gate) [61] followed by a $\pi/2$ pulse, both driven by a 729 nm laser on the $|S\rangle \leftrightarrow |-1\rangle$ transition, generates the state $|G\rangle = (|S, S, S\rangle + |-1, -1, -1\rangle)/\sqrt{2}$. A final sequence of laser pulses map $|G\rangle$ to $|\psi_{(1,-2,1)}^{\text{SWD}}\rangle$ [46].

During sensing, the ions are exposed to fluctuating magnetic fields generated by two pairs of current-carrying coils mounted outside the vacuum chamber. Each pair is driven by a separate noisy voltage generator that produces currents fluctuating at acoustic frequencies. One pair—referred to as the “constant coils” (CC)—is operated in Helmholtz-like configuration: the currents flow in the same direction and generate a spatially constant field B^c varying in the range $\pm 1.02(5) \mu\text{T}$ at the positions of the ions. The other coil pair—referred to as the “gradient coils” (GC)—is in anti-Helmholtz-like configuration with counter-propagating currents. The GC coil is used to generate a spatial-gradient field B^l that varies in the range $\pm 0.905(3) \text{ mTm}^{-1}$: corresponding to field differences in the range $\pm 8.88(2) \text{ nT}$ between the outer ions. The GC pair also creates a spatially constant field that does not affect interpretations of the results. The scalar fields of the SWD protocol correspond to the projections of the vector fields, generated by the coil pairs, onto the principle magnetic field axis. The fluctuating fields are switched off during state preparation and measurement, which corrupt those steps. To avoid this, one could use logic gates that incorporate dynamical decoupling [62].

A laser imprints an effective spatially quadratic field across the ion sensors, shifting the frequency of the $|-2\rangle$ state of all ions in proportion to the laser intensity via the AC-Stark effect. The laser’s effect on the Hilbert space, spanned by the bold states in Fig. 1(b), is equivalent to a spatially quadratic field across the ions: shifting the relevant energy gap of the middle sensor but not those of the outer sensors. The following calibrated quadratic

field strengths are applied to the sensors: $B_{\text{cal}}^q = [0.0, 2.1, 4.7, 7.6, 9.5, 11.9, 15.2]$ pT μm^{-2} [46], corresponding to laser-induced frequency shifts of $|-2\rangle$ by up to 12.3 Hz (an effective magnetic field change of the central ion by 365 pT with respect to the outer ions).

The measurement step begins with 729 nm laser pulses that map the sensor's states from the $3^2D_{5/2}$ manifold to qubit superpositions of $|S\rangle$ and $|-1\rangle$ for each ion. Next, a $\pi/2$ pulse realizes the global rotation $U_{\phi_r} = e^{-i(\pi/4)(\sigma_x \cos \phi_r + \sigma_y \sin \phi_r)}$, where ϕ_r is the angle of rotation, and $\sigma_j = \sum_{l=1}^3 \sigma_j^l$, with σ_j^l being the Pauli j operator acting on ion l . Finally, the state of each ion-qubit is measured via the standard electron shelving method on all ions simultaneously. One experimental shot is now complete. Each experimental shot is repeated n times, and from those n shots an estimate P_{est} for the parity $P = \langle U_{\phi_r}^\dagger \sigma_z^1 \sigma_z^2 \sigma_z^3 U_{\phi_r} \rangle$ is calculated. For the initial state $|\psi_{(1,-2,1)}^{\text{SWD}}\rangle$, the parity has the form

$$P(B^q, \phi_r) = A \cos(\omega B^q + 3\phi_r + \phi_0), \quad (1)$$

where $A = 1$, $\omega = 2\kappa td^2$, $d = 4.9 \mu\text{m}$ is the distance between neighboring ions, and ϕ_0 is a phase offset discussed later. An estimate for the quadratic field strength B_{est}^q is given by $B_{\text{est}}^q = \hat{B}^q(P_{\text{est}})$, where the estimator function $\hat{B}^q$ is the inverse function of P , corresponding to the maximum likelihood estimator [46]. The combination of the initial state $|\psi_s^{\text{SWD}}\rangle$, the measurement P , and the estimator $\hat{B}^q$ constitute our implementation of the SWD protocol. To quantify the accuracy of the estimates we use the normalized root mean squared error defined as $\overline{\text{RMSE}} = \sqrt{(n/m) \sum_{i=1}^M (B_{\text{cal}}^q - B_{\text{est},i}^q)^2}$, where m is the number of independent estimates, each containing n shots. The $\overline{\text{RMSE}}$ combines both systematic and statistical errors, and the factor $\sqrt{n}$ cancels the expected shot-noise scaling. Using the initial state and measurement of the SWD protocol it is possible to achieve the minimum $\overline{\text{RMSE}}$ over all states and measurements in the $3^2D_{5/2}$ manifold that are insensitive to the noise fields, as set by the Heisenberg limit [46]. Achieving that minimum value requires the optimal estimator, which, in the limit of infinite shots, is the maximum likelihood estimator.

Our separable protocol is achieved by removing the MS gate, which results in the preparation of the separable state $|\psi_s^{\text{sep}}\rangle = \otimes_i (|s_i\rangle + |-s_i\rangle)/\sqrt{2}$, with $s = (1, -2, 1)$. When exposed to overwhelming fluctuations in B^c and B^l during sensing—which eliminate all off-diagonal coherence terms that are sensitive to them—the separable state $|\psi_{(1,-2,1)}^{\text{sep}}\rangle$ is partially projected onto the DFS spanned by $\{|1, -2, 1\rangle, |-1, 2, -1\rangle\}$. The expected parity has the same form as Eq. (1) with a reduced amplitude of $A = 0.25$. The initial state and measurement of the separable protocol can

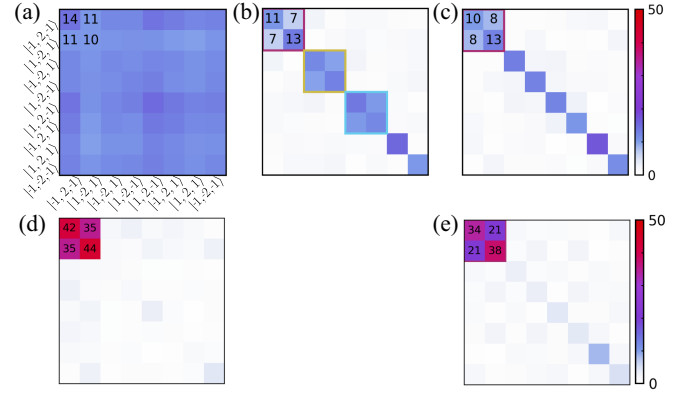


FIG. 2. Experimentally-reconstructed density matrices of three-sensor states. Numbers in cells are matrix elements multiplied by 100. (a) Initial separable state ρ^{sep} . (b) ρ^{sep} after 80 ms of noise from CC coils. Red, yellow and blue boxes show DFSs with respect to spatially-constant noise. (c) ρ^{sep} after 80 ms of noise from both the CC and CG coils. Red box shows DFS with respect to both spatially constant and gradient noise. (d) ρ^{SWD} . (e) ρ^{SWD} after 80 ms noise from both CC and CG coils.

achieve the minimum $\overline{\text{RMSE}}$ over all noise-protected separable states restricted to occupying two levels of the $3^2D_{5/2}$ manifold and projective measurements that do not utilize entanglement [46]. The separable protocol corresponds to the established technique of correlation spectroscopy [27,28,63]. For characterization purposes, state tomography is used to reconstruct the generated states [46]. The reconstructed density matrices ρ^{SWD} and ρ^{sep} are ideally given by $|\psi_{(1,-2,1)}^{\text{SWD}}\rangle\langle\psi_{(1,-2,1)}^{\text{SWD}}|$ and $|\psi_{(1,-2,1)}^{\text{sep}}\rangle\langle\psi_{(1,-2,1)}^{\text{sep}}|$, respectively.

We begin by investigating the spatial properties of the fluctuating fields produced by the two coil pairs across the ion string. Tomographic reconstruction of ρ^{sep} is seen to populate all coherence terms between eigenstates [Fig. 2(a)], making this a useful state to probe and verify the action of the noise fields. Next, we subject the state ρ^{sep} to 80 ms of noise generated by the CC coils. The reconstructed state shows that the initial state has been partially projected into an incoherent mixture of three two-dimensional subspaces [Fig. 2(b)]; coherences that lie outside those subspaces have essentially been eliminated, while those within them remain. The three remaining subspaces are the ones that are expected to be decoherence-free with respect to spatially constant noise [46], verifying that the action of the CC coils is to produce spatially constant noise across the ion string. Next, we subject the state ρ^{sep} to 80 ms of noise generated by both the CC and GC coils. The reconstructed state shows that the initial state has now been partially projected into a single subspace, spanned by $\{|1, -2, 1\rangle, |-1, 2, -1\rangle\}$ [Fig. 2(c)]. That subspace is the one that is expected to be decoherence-free with respect to both spatially constant and gradient noise. The fields generated from both coil pairs over 80 ms

therefore cause overwhelming noise with both spatially constant and gradient functions over the ion string.

We perform tomographic reconstruction of the SWD state before [Fig. 2(d)] and after exposure to the 80 ms-long noise channel from both coil pairs [Fig. 2(e)]. The fidelities of the initial and final state with a GHZ state [46] are 0.780(14) and 0.564(12), respectively. Since both values are larger than 0.5, the presence of genuine GHZ-type multipartite entanglement is proven [64]. The potential of the generated states to serve as sensors of a spatially quadratic field is determined by the amplitude A_ρ of the $|1, -2, 1\rangle\langle -1, 2, -1|$ coherence term. Specifically, the amplitude of the parity signal in Eq. (1) is ideally given by $A = 2A_\rho$. After exposure to our noise fields, the observed separable state achieves only $A = 0.16(2)$ [Fig. 2(c)], while the observed entangled state [Fig. 2(e)] achieves $A = 0.42(4)$. The ideal values for the separable and entangled states are $A = 0.25$ and $A = 1$, respectively. Imperfections in the prepared initial states are predominantly due to laser phase noise [46]. Additional imperfections in states reconstructed after 80 ms are predominantly due to spontaneous decay of the $3^2D_{5/2}$ manifold, which has a lifetime of 1.045 s [46].

Next we perform the full sensing protocol, aiming to sense calibrated quadratic field strengths (B_{cal}^q) generated by the laser, while the sensors are exposed to the noise fields generated by both coils for 80 ms. For each applied quadratic field strength, measurements of parity are made for 60 values of the analysis phase ϕ_r that are equally spaced between $\{0, 1.6\pi\}$ rad. The protocol is carried out both when preparing ρ^{SWD} and ρ^{sep} . The obtained parity data exhibit the expected sinusoidal oscillations, with phases shifted by different applied quadratic field strengths [exemplified in Fig. 3(a)]. Each parity oscillation is separately fit to Eq. (1). The phase offset parameter ϕ_0 , caused by a background quadratic field [46], is determined from the fit to the case $B_{\text{cal}}^q = 0$ [Fig. 3(a), blue solid line]. The total phase of each oscillation is given by $\Phi(B^q) = \omega B^q + 3\phi_r + \phi_0$. A is calculated from the average fitted amplitude, yielding $A = 0.45(2)$ and $A = 0.146(9)$ for the SWD and separable protocols, respectively. Those fitted amplitudes are statistically consistent with the ones predicted from tomography.

We turn our attention to extracting estimates of the applied quadratic field strengths B_{est}^q from the data, and then calculating the $\overline{\text{RMSE}}$ to quantify the accuracy of those estimates. Parity estimates are selected around the maximum slope of the parity function, which minimizes the $\overline{\text{RMSE}}$ [46]. Specifically, parity estimates are chosen in the range $\text{mod}_\pi(\Phi(B_{\text{cal}}^q)) \in (\pi/2) \pm 0.73$ [Fig. 3(a), bold markers]. Quadratic field strength estimates are computed for each selected parity estimate. The estimates are obtained using the inverse parity function P^{-1} [inverse of Eq. (1)], using the relevant average fitted amplitude and the relevant fitted phase offset parameter (ϕ_0). Histograms

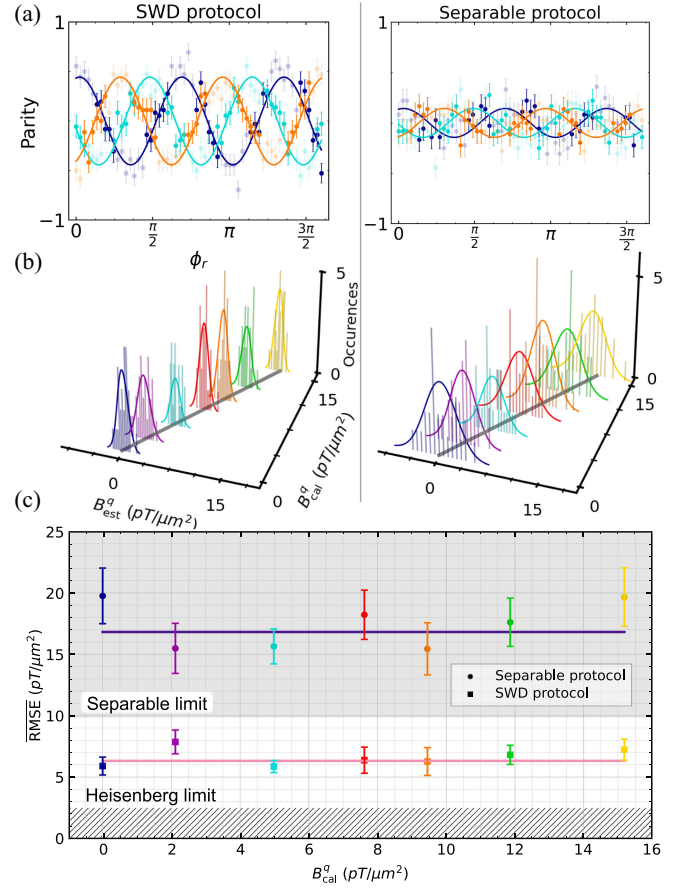


FIG. 3. Sensing results. Data coloring reflects applied signal strength B_{cal}^q . (a) Example parity estimates. Colored shapes show data. Lines show fits of $P(B^q, \phi_r)$ to data. Bold markers are used for estimating B_{est}^q in (b). (b) Histograms showing that quadratic field strength estimates B_{est}^q are centered around the calibrated values B_{cal}^q . Gray lines show $B_{\text{est}}^q = B_{\text{cal}}^q$. Colored curves show Gaussian fits. (c) Markers show $\overline{\text{RMSE}}$ from data in each histogram in (b). Solid lines show corresponding weighted averages. Gray area is achievable by two-level separable estimation protocols. Striped area is beyond the Heisenberg limit and inaccessible. Error bars are one standard deviation.

comparing the estimated and calibrated quadratic field strengths show that the SWD state yields consistently lower variance than the separable state [Fig. 3(b)]. The mean values of B_{est}^q obtained from the two protocols are statistically consistent with each other, but not with those of B_{cal}^q [Fig. 3(b), gray line]. We attribute this discrepancy to a 6% change in the intensity of the Stark shift laser between calibration and sensing stages. This systematic error is masked by the larger statistical fluctuations in the $\overline{\text{RMSE}}$ values discussed later.

Figure 3(c) compares the measured $\overline{\text{RMSE}}$ values—obtained from the histograms [Fig. 3(b)]—with those for ideal implementations. First, one sees that the experimental SWD protocol significantly outperforms the experimental separable protocol. The average $\overline{\text{RMSE}}$ measured for the

SWD protocol is a factor of 2.6(1) smaller than the average measured for the separable protocol, corresponding to a 4.1(2) dB improvement. Second, the performance of the measured SWD protocol even surpasses an ideal implementation of the separable protocol. The average $\overline{\text{RMSE}}$ measured for the SWD protocol is a factor 1.49(6) lower than an ideal implementation of the separable protocol, corresponding to a 1.7(2) dB improvement. We conclude that the implemented SWD protocol outperforms the best that could possibly be achieved with a separable protocol that utilizes two levels of each sensor. The deviation between the measured and ideal performances for both protocols is statistically consistent with the imperfect amplitudes (A).

Finally, we present new theoretical results. Consider the task of sensing the highest order of an M order Taylor expansion, whilst being insensitive to fluctuations in all lower orders, using the minimum number of sensors, $N_{\min}$. That task requires a minimum of $N_{\min} = M + 1$ sensors all at different locations. We find that the SWD protocol achieves a lower $\overline{\text{RMSE}}$ compared to the best two-level separable protocol by an amount that grows exponentially in M [46]. Moreover, we prove that this exponential advantage extends to arbitrary spatial field distributions [46].

We experimentally demonstrated a protocol that exploits entanglement to optimally sense a spatially distributed signal, while being immune to noise with different spatial dependencies. Exploiting that protocol in future requires positioning sensors at more locations and the ability to establish multipartite entanglement between them. Over distances of hundreds of microns these capabilities are currently under development in various experimental platforms (e.g., [65–68]), and over distances of meters to tens of kilometers there has been significant success establishing entanglement between many of those platforms using traveling photons [69–78], including an ion in the trap used in the present Letter and another 230 m away [79]. Converting photons to telecom wavelengths [80] and using quantum repeaters [81] could establish entanglement between ions separated by hundreds of kilometers or more, allowing for the SWD protocol to detect large-scale spatial field variations. Challenges include significantly increasing the rates at which remote ion-qubits can be entangled beyond the state of the art [72] and developing robust ion-qubit memories [82]. Over global scales, we speculate that the protocol could be used to investigate gravitational waves emitted by specific sources, while hiding from others.

Acknowledgments—We thank Denis Vasilyev for valuable discussions. This work was funded in part by; the Austrian Science Fund (FWF) [Grants DOI: 10.55776/P34055, DOI: 10.55776/P36009, DOI: 10.55776/P36010, and DOI: 10.55776/COE1]; the European Union under the

DIGITAL-2021-QCI-01 Digital European Program under Project No. 101091642 and project name “QCI-CAT,” and the European Union’s Horizon Europe research and innovation programme under Grant Agreement No. 101102140 and project name “QIA-Phase 1”; the Österreichische Nationalstiftung für Forschung, Technologie und Entwicklung (AQUnet project). We acknowledge funding for B. P. L. by the CIFAR Quantum Information Science Program of Canada.

J. B. took data. J. B., A. H., and B. P. L. analyzed and interpreted data. J. B. and A. H. performed theoretical modeling. J. B., M. C., A. W., Z. X. K., and V. K. contributed to the experimental setup. A. H. derived theoretical results, with supervision from W. D. The manuscript was written by J. B., A. H., and B. P. L., with all authors providing detailed comments. The project was conceived and supervised by B. P. L. with support from W. D.

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Data availability—The data that support the findings of this article are openly available [83].

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