

Universality Emerging in a Universality: Derivation of the Ericson Transition in Stochastic Quantum Scattering and Experimental Validation

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At lower energies, the resonances in scattering experiments are often isolated. In quantum chaotic many-body, disordered, or generically stochastic systems, the resonances overlap at larger energies. Eventually, the Ericson regime is reached in which the cross section behaves like a random function. The scattering-matrix elements then follow a universal Gaussian distribution. For more than 60 years, the emergence of this robust additional universal behavior on top of the universal system stochasticity has awaited a concise analytical treatment. We derive the transition to the Ericson regime in the universal Heidelberg approach and prove the universal Gaussian distribution by a proper asymptotic expansion. We also obtain explicit formulas for the moments and for the cross-section distribution. We compare with microwave experiments and numerical simulations.

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Introduction—Scattering experiments are ubiquitous in quantum and classical physics. Numerous examples for, in a broad sense, chaotic or, more generally, stochastic dynamics are found in nuclear [1–12], atomic and molecular physics [13–20], quantum graphs [21–26], microwave cavities and networks [27–37], and classical wave systems [38–40]. The theory of electronic transport [41–48] is closely related. The field of stochastic quantum scattering originated in nuclear physics and was pioneered by Ericson in the 1960s [49–51]. He showed that the cross section versus energy behaves like a random function when the resonances strongly overlap. The Ericson regime was first identified in nuclei [49–51], see a review in [9], and subsequently in numerous systems, e.g., [12], in electronic transport [52,53], in atoms [18,19,54–58], in ultracold atomic-molecular systems [59], in Bose-Einstein condensates [60], and in quantum graphs [61]. In the Ericson

regime, a universal Gaussian distribution of the scattering-matrix elements emerges. Up to now, it was merely phenomenologically understood, here we provide the full explanation.

Scattering process—The scattering process is encoded in the complex and unitary scattering matrix S [62–64]. If M channels are connected to the interaction zone described by a Hermitian Hamilton operator, the elements of the S matrix are $S_{ab} = \delta_{ab} - 2\pi i W_a^\dagger G W_b$, where

$$G = \left(E \mathbb{1}_N - H + i\pi \sum_{c=1}^M W_c W_c^\dagger \right)^{-1} \quad (1)$$

is the $N \times N$ matrix resolvent at energy E in a matrix notation. The limit $N \rightarrow \infty$ has to be taken, H is the Hamilton matrix and the N -component vectors W_c , $c = 1, \dots, M$ describe the coupling between bound states in the interaction zone and the channels. One may assume their orthogonality. Apart from kinematic factors, the cross sections are given by $\sigma_{ab} = |S_{ab}|^2$.

Heidelberg approach—In a generic chaotic, complex or stochastic setting, H can be chosen as a random matrix, drawn from the Gaussian Orthogonal, Unitary or Symplectic Ensemble, respectively, labeled GOE ($\beta = 1$), GUE ($\beta = 2$) or GSE ($\beta = 4$), where $\beta = 1, 4$ describe time-reversal invariant systems. Here, we focus on the time-reversal noninvariant unitary ($\beta = 2$) case of Hermitian $N \times N$ matrices H drawn from a Gaussian probability density with variance ν^2/N . The technically much more complicated cases $\beta = 1, 4$ will be treated in a forthcoming publication elsewhere.

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Ericson regime and Weisskopf estimate—The Ericson transition is governed by the ratio of average resonance width Γ and average mean level spacing D , i.e., by the dimensionless parameter

$$\Xi = \frac{1}{2\pi} \sum_{c=1}^M T_c = \frac{\Gamma}{D}, \quad (2)$$

where the second equation is known as the Weisskopf estimate [65] and T_c are the transmission coefficients. Importantly, the number M of channels must be large, see Appendix A for details.

We have two objectives: First, we prove analytically that the real and imaginary parts of S_{ab} for $a \neq b$ are Gaussian distributed in the Ericson regime. Up to now the reasoning was mainly heuristic [63]. Second, we provide asymptotic corrections to the Gaussian form, thereby revealing the complete transition to the Ericson regime.

Supersymmetry and starting point—In Refs. [64,66,67] a variant of the supersymmetry method was put forward [68–70] which made it possible to explicitly calculate the distributions $P_s(x_s)$ of the real part, $x_1 = \text{Re}(S_{ab})$, and the imaginary part, $x_2 = \text{Im}(S_{ab})$, of S_{ab} and the cross section σ_{ab} in terms of low-dimensional integrals. We employ those results to study the Ericson transition by means of asymptotic expansions in powers of $1/\Xi$. Previously, only the correlator of the S matrix for $\beta = 1$ was investigated in this way [71].

Derivation of the universal Gaussian—The characteristic function generates the moments $\bar{x}_s^n$ of the distribution $P_s(x_s)$, i.e.,

$$R_s(k) = \int_{-\infty}^{+\infty} dx_s P_s(x_s) e^{-ikx_s} = \sum_{n=0}^{\infty} (-i)^n \frac{k^n}{n!} \bar{x}_s^n. \quad (3)$$

The exact analytical result for the characteristic function is given by [68,69]

$$R_s(k) = 1 - \int_1^{\infty} d\lambda_1 \int_{-1}^1 d\lambda_2 \frac{k^2}{4(\lambda_1 - \lambda_2)^2} \mathcal{F}_U(\lambda_1, \lambda_2) \times (t_a^1 t_b^1 + t_a^2 t_b^2) J_0 \left(k \sqrt{t_a^1 t_b^1} \right), \quad s = 1, 2 \quad (4)$$

where $t_c^j = \sqrt{[\lambda_j^2 - 1]/(g_c^+ + \lambda_j)}$, $g_c^+ = 2/T_c - 1$ and J_0 denotes the Bessel function of order zero. The channel factor reads

$$\mathcal{F}_U(\lambda_1, \lambda_2) = \prod_{c=1}^M \frac{g_c^+ + \lambda_2}{g_c^+ + \lambda_1}. \quad (5)$$

The characteristic functions for $s = 1, 2$ and thus their distributions are equal for $\beta = 2$ [68,69].

We expand the moments asymptotically in powers of $1/\Xi$.

To this end, we expand $R_s(k)$ in Eq. (4), i.e., $J_0 \left(k \sqrt{t_a^1 t_b^1} \right)$ in powers of k . As J_0 is even in k , all odd moments vanish. All even moments necessarily exist due to the unitarity of the S matrix. The zeroth order moment equals 1 and is the Efetov-Wegner term [66,67,72] in the supersymmetry calculation. The integrand in Eq. (4) is singular at $\lambda_1 = \lambda_2 = 1$. This is typical for supersymmetry calculations, see Refs. [64,66–71,73–76], and entails that the computation of the remaining low-dimensional integrals is, in general, very challenging or even unfeasible. Yet, we succeeded in removing the singularity, with an exact change of variables $\lambda_1 = 1 + q'r'$, $\lambda_2 = 1 - q'(1 - r')$. As the integrand combined with the Jacobian contributes a factor of q'^2 this cancels the $(\lambda_1 - \lambda_2)^{-2} = q'^{-2}$ contribution at $q' = 0$ and the integrand is finite. We split the integration domain into two parts, $R_s(k) = R_s^{(d)}(k) + R_s^{(c)}(k)$, a disconnected part, $R_s^{(d)}(k)$, with independent integration variables, and a connected part, $R_s^{(c)}(k)$, where

$$R_s^{(d)}(k) = 1 - \frac{k^2}{4} \int_0^2 dq' \int_0^1 dr' f(q', r'; k), \quad (6)$$

$$R_s^{(c)}(k) = -\frac{k^2}{4} \int_2^{\infty} dq' \int_{(q'-2)/q'}^1 dr' f(q', r'; k). \quad (7)$$

The integrand is given by

$$f(q', r'; k) = \mathcal{F}_U(1 + q'r', 1 - q'(1 - r')) \quad (8)$$

$$\times \left[\frac{r'(q'r' + 2)}{(g_a^+ + q'r' + 1)(g_b^+ + q'r' + 1)} + \frac{(1 - r')[2 - q'(1 - r')]}{[g_a^+ + 1 - q'(1 - r')][g_b^+ + 1 - q'(1 - r')]} \right] \times J_0 \left(k \sqrt{\frac{q'r'(q'r' + 2)}{(g_a^+ + q'r' + 1)(g_b^+ + q'r' + 1)}} \right). \quad (9)$$

In a related but different context, the channel factor was approximated in Ref. [71] for infinitely many channels with $T_c \simeq 1/M$ in the form

$$\mathcal{F}_U(1 + q'r', 1 - q'(1 - r')) \simeq \exp(-\pi\Xi q'). \quad (10)$$

We use this limit to calculate the moments. A more general channel factor expansion is given in Appendix B. We obtain the asymptotes for the moments by employing Watson's lemma [77–80] to express the integral over q' in terms of the derivatives of $f(q', r'; k)/\mathcal{F}_U(1 + q'r', 1 - q'(1 - r'))$ with respect to q' at $q' = 0$ where only Eq. (6) contributes. There, the exponentially decaying channel factor becomes maximal. We find

$$\overline{x_s^{2n}} = \frac{\Gamma(2n+1)}{2^{2n}\Gamma^2(n)} \sum_{m=0}^{\infty} \left(\frac{1}{\pi\Xi}\right)^{m+1} \frac{\partial^m}{\partial q'^m} \int_0^1 dr' (q'r')^{n-1} \left[\frac{r'(q'r' + 2)}{(g_a^+ + q'r' + 1)(g_b^+ + q'r' + 1)} + \frac{(1-r')[2-q'(1-r')]}{[g_a^+ + 1 - q'(1-r')][g_b^+ + 1 - q'(1-r')]} \right] \left(\frac{q'r' + 2}{(g_a^+ + q'r' + 1)(g_b^+ + q'r' + 1)} \right)^{n-1} \Big|_{q'=0}. \quad (11)$$

After swapping the order of differentiation and integration, we carry out the derivatives over q' . The remaining integrand is a polynomial in r' . Evaluating their leading order term yields

$$\overline{x_s^{2n}} = \frac{(2n)!}{n!} \left(\frac{1}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right)^n \quad (12)$$

to order Ξ^{-n} . The explicit calculation is given in Appendix C. Inserting the leading order asymptote into Eq. (3) leads to $R_s(k) \simeq R_s^{(l)}(k)$, with

$$R_s^{(l)}(k) = \exp\left(-\frac{k^2}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}\right). \quad (13)$$

By taking the inverse Fourier transform of the characteristic function, we arrive at the Gaussian approximation of the distribution $P_s(x_s)$ for either the real part x_1 or imaginary part x_2 of the off-diagonal S -matrix elements. Even though our derivation is not related to the Central Limit Theorem, the asymptotics necessarily require a similar rescaling of x_s , namely $\xi_s = \sqrt{\Xi}x_s$. We find

$$P_s^{(l)}(\xi_s) \simeq \sqrt{\frac{(g_a^+ + 1)(g_b^+ + 1)}{2}} \exp\left(-\frac{\pi(g_a^+ + 1)(g_b^+ + 1)\xi_s^2}{2}\right) \quad (14)$$

for $s = 1, 2$. This proves the universal Gaussian, heuristically established by Ericson. The contribution from the connected part is globally decaying with Ξ and is thus negligible.

Transition and higher order corrections—To subleading orders in $1/\Xi$, the moments are

$$\overline{x_s^{2n}} \simeq \frac{(2n)!}{n!} \left(\frac{1}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right)^n + \frac{(2n)!}{\Gamma(n-1)} \frac{g_a^+ g_b^+ - g_a^+ - g_b^+ - 3}{[2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi]^{n+1}} \quad (15)$$

to order $\Xi^{-(n+1)}$. By resummation, the corresponding asymptote of the characteristic function becomes $R_s(k) \simeq R_s^{(l)}(k) + R_s^{(sl)}(k)$ with

$$R_s^{(sl)}(k) \simeq \exp\left(-\frac{k^2}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}\right) \times \frac{(g_a^+ g_b^+ - g_a^+ - g_b^+ - 3)k^4}{8[(g_a^+ + 1)(g_b^+ + 1)\pi\Xi]^3}. \quad (16)$$

The associated correction $P_s^{(sl)}(\xi_s)$ to the rescaled distribution, $P_s(\xi_s) \simeq P_s^{(l)}(\xi_s) + P_s^{(sl)}(\xi_s)$ is thus

$$P_s^{(sl)}(\xi_s) \simeq \sqrt{\frac{(g_a^+ + 1)(g_b^+ + 1)}{2}} \exp\left(-\frac{\pi(g_a^+ + 1)(g_b^+ + 1)\xi_s^2}{2}\right) \left(3 - 6\pi(g_a^+ + 1)(g_b^+ + 1)\xi_s^2 + [\pi(g_a^+ + 1)(g_b^+ + 1)]^2 \xi_s^4 \right) \frac{g_a^+ g_b^+ - g_a^+ - g_b^+ - 3}{8(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}, \quad (17)$$

which goes with $1/\Xi$. For $\Xi \gg 1$, the distribution $P_s(\xi_s) \simeq P_s^{(l)}(\xi_s) + P_s^{(sl)}(\xi_s)$ converges to a Gaussian. For details of the derivation, see Appendix D. The asymptotic approximation as well as the symmetry $a \leftrightarrow b$ of the scattering channels comply with the symmetry $P_s(\xi_s) = P_s(-\xi_s)$. The correction to the Gaussian maximum at $\xi_s = 0$ is

$$P_s^{(sl)}(0) = \frac{3(g_a^+ g_b^+ - g_a^+ - g_b^+ - 3)}{8\pi\Xi\sqrt{2(g_a^+ + 1)(g_b^+ + 1)}} \quad (18)$$

from Eq. (17). Thus, for $\Xi \approx 1$, i.e., for weakly overlapping resonances, the distribution is not Gaussian, rather it is shifted by the quantity $P_s^{(sl)}(0)$ that depends on the transmission coefficients T_a and T_b . This shift is positive if $1 - T_a - T_b > 0$ and negative otherwise, cf. Eq. (18).

Cross-section distribution—In many experiments only the cross sections are accessible. We may calculate the distribution of $\sigma_{ab} = \text{Re}^2 S_{ab} + \text{Im}^2 S_{ab}$ via a two-dimensional Fourier transform from a bivariate characteristic function [70]. Because of the radial symmetry of the latter, we can write the cross-section distribution as Hankel transform of the univariate characteristic function,

$$p(\sigma_{ab}) = \frac{1}{2} \int_0^\infty dk k R_s(k) J_0(\sqrt{\sigma_{ab}} k). \quad (19)$$

Using $R_s(k) \simeq R_s^{(l)}(k) + R_s^{(sl)}(k)$ in Eqs. (13) and (16) and rescaling $\tilde{\sigma}_{ab} = \sigma_{ab}/\langle\sigma_{ab}\rangle$, we arrive at $p(\tilde{\sigma}_{ab}) \simeq p^{(l)}(\tilde{\sigma}_{ab}) + p^{(sl)}(\tilde{\sigma}_{ab})$, with

$$\begin{aligned} p^{(l)}(\tilde{\sigma}_{ab}) &= \exp(-\tilde{\sigma}_{ab}), \\ p^{(sl)}(\tilde{\sigma}_{ab}) &= \frac{\exp(-\tilde{\sigma}_{ab})((4 - \tilde{\sigma}_{ab})\tilde{\sigma}_{ab} - 2)}{2\pi\Xi(g_a^+ + 1)(g_b^+ + 1)} \\ &\quad \times (3 + g_a^+ + g_b^+ - g_a^+ g_b^+). \end{aligned} \quad (20)$$

The rescaling amounts to normalizing the cross section in the distribution to its expectation value in the Ericson regime, given by $\langle\sigma_{ab}\rangle = 2/[\pi\Xi(g_a^+ + 1)(g_b^+ + 1)]$ to order

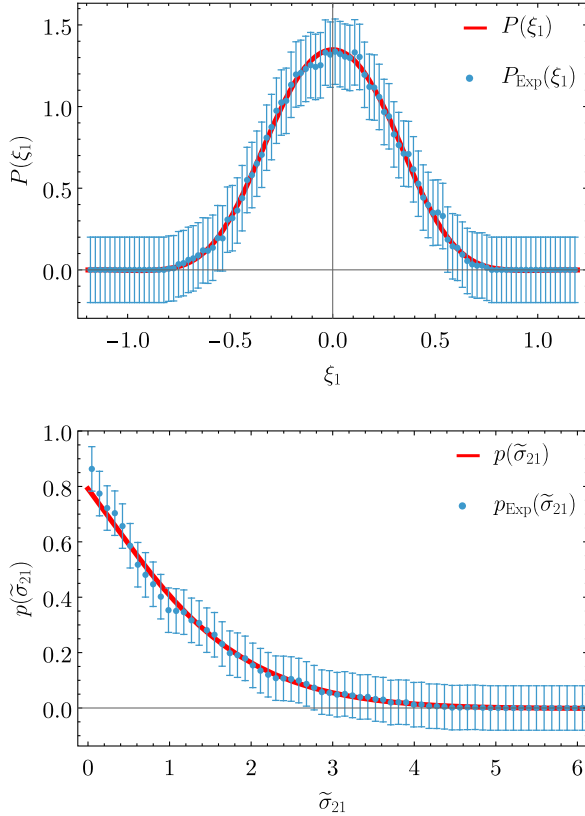


FIG. 1. Experimental and analytical results for the distributions of rescaled S -matrix elements (top) and of cross sections (bottom) for $\Xi = 1.424$ and $M = 52$. Blue error bars indicate the empirical standard deviation around each point of measurement.

Ξ^{-1} . There is an additional contribution to the pure exponential decay in the Ericson regime. At $\tilde{\sigma}_{ab} = 0$, we have

$$p(0) = 1 - \frac{3 + g_a^+ + g_b^+ - g_a^+ g_b^+}{\pi\Xi(g_a^+ + 1)(g_b^+ + 1)}. \quad (21)$$

The second term gives the shift relative to the maximum of the exponential decay. This quantitatively explains the observed deviations from the exponential decay in the onset of the Ericson regime [70]. The distribution of cross sections is closely related to that of the wave-function intensities I , $\mathcal{P}_M(I)$, in an open wave-chaotic cavity with violated time-reversal invariance [81]. By means of a similar analysis as presented above, asymptotes for $\mathcal{P}_M(I)$ are derived in Supplemental Material [82]. The leading-order term yields an exponential decay of $\mathcal{P}_M(I)$ in the limit $\Xi \gg 1$.

Validation with experimental data and numerical simulations—We compare our results to data from measurements with microwave networks [21]. These were designed such that they simulate open quantum graphs [22,23] with violated time-reversal invariance [34] exhibiting quantum chaotic scattering [61] for $\beta = 2$. The Heidelberg approach applies and numerical studies

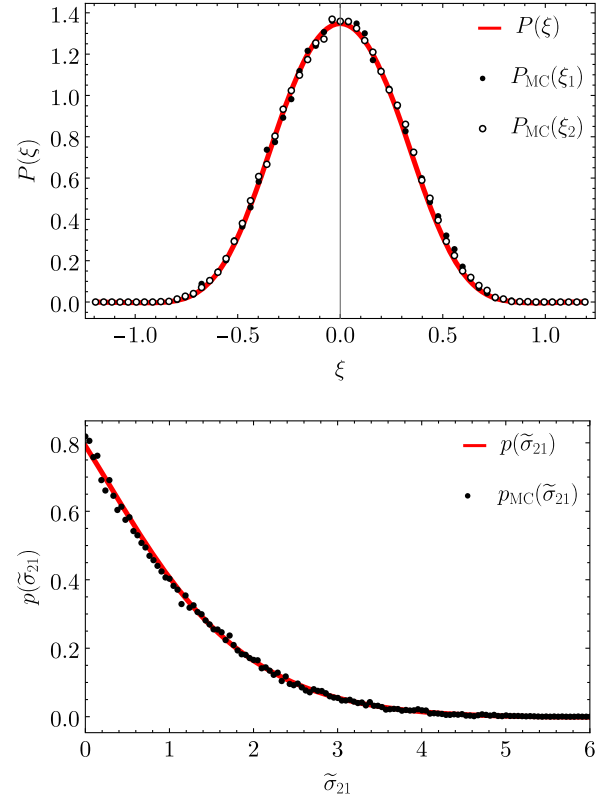


FIG. 2. Monte Carlo simulations and analytical results for the distributions of rescaled S -matrix elements (top) and of cross sections (bottom) for $\Xi = 1.424$ and $M = 52$.

revealed that the typical values of the transmission coefficients T_c and of M in such microwave experiments suffice to ensure applicability of the Weisskopf estimate [83]. We have two strong scattering channels $a = 2$, $b = 1$ with $T_1 = T_2 = 0.967$ and a parameter $\tau_{\text{abs}} = 7.013$ to account for absorption [32]. To model it, 50 fictitious channels of equal transmission were used. Using the Weisskopf estimate $\Xi = (T_1 + T_2 + \tau_{\text{abs}})/(2\pi) \approx 1.424$ we find that the measurements are taken in the onset of the Ericson regime, the resonances are weakly overlapping. We focus on the S -matrix element S_{21} . As seen in Fig. 1, the analytical result is in very good agreement with the experimental data. The measured distribution is equal for the real and imaginary parts. The peak of the distribution is non-Gaussian, as to be expected in the onset of the Ericson regime.

We also compare the analytical result with Monte Carlo simulations, which were done by sampling 30000 random matrices with dimension $N = 200$ from the GUE and numerically calculating the corresponding S -matrix elements. As displayed in Fig. 2, the agreement is very good. To visualize the transition to the Gaussian distribution $P_s^{(l)}(\xi_s)$ in the Ericson regime, we depict in Fig. 3 the subleading terms $P_s^{(sl)}(\xi_1)$ and $p^{(sl)}(\tilde{\sigma}_{ab})$ for the distributions in comparison with the experimental data. The sizeable dip at zero in Fig. 3 demonstrates the significant deviations from the Gaussian for weakly overlapping

resonances. The distribution is also broadened in the tails. The cross-section distribution shows a maximal deviation from the exponential decay at zero, which is clearly visible in Fig. 3. To numerically test the subleading corrections, we use the cumulative distribution function $F(\xi_s) = F^{(l)}(\xi_s) + F^{(sl)}(\xi_s)$ in Fig. 4. Since $P_s(\xi_s)$ is symmetric around $\xi_s = 0$, $F(\xi_s)$ and $F^{(sl)}(\xi_s)$ are antisymmetric in ξ_s , see Fig. 4.

An important comment is in order. One might question whether our asymptotic results are applicable for the relatively small value of $\Xi = 1.424$. Numerical simulations of the full problem compared to the analytically obtained subleading term are displayed in Fig. 4. The deviation is very small, strongly indicating that, even for this small value of Ξ , the higher order terms in $1/\Xi$ are of little relevance. Put differently, the transition to the Ericson regime is so fast that in typical situations the subleading term provides the relevant corrections. To further illustrate this important point, we numerically test our results for a set of parameters deep in the Ericson regime. We choose $T_c = 1$ for $M = 60$ channels, which corresponds to $\Xi \approx 9.55$. By doing so, the corrections in $1/\Xi$ become small compared to the leading order term and we indeed find Gaussian and exponential behavior, respectively, as depicted in Fig. 5.

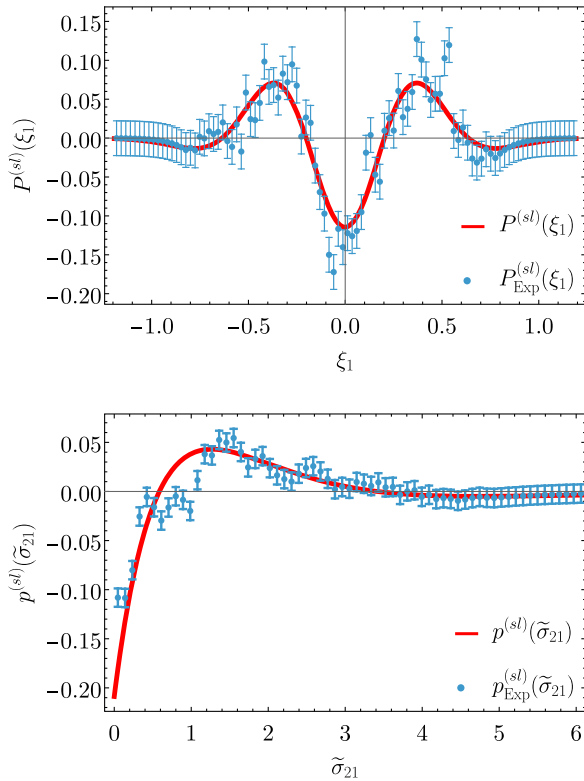


FIG. 3. Experimental data and the subleading terms for the distributions of rescaled S -matrix elements (top) and of cross sections (bottom) for $\Xi = 1.424$ and $M = 52$.

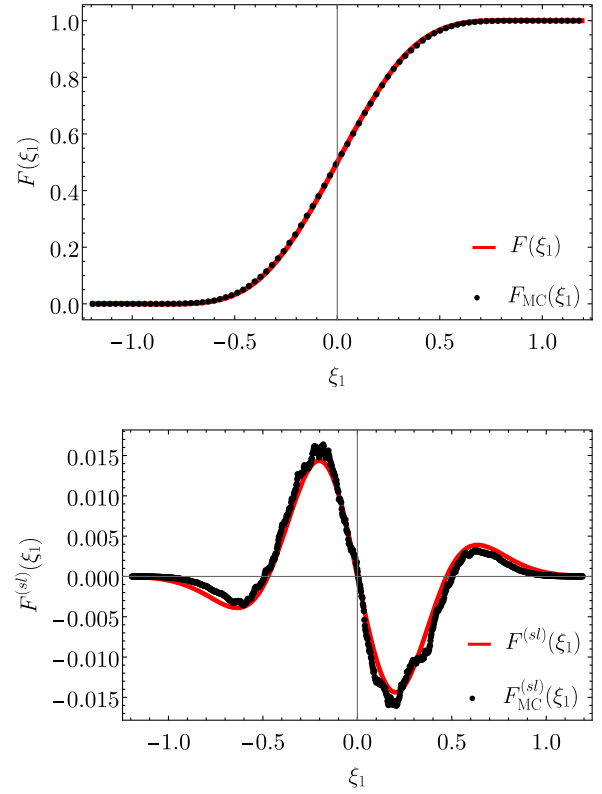


FIG. 4. Full cumulative distribution $F(\xi_1)$ of the rescaled S -matrix elements (top) and subleading term $F^{(sl)}(\xi_1)$ (bottom) for $\Xi = 1.424$ and $M = 52$.

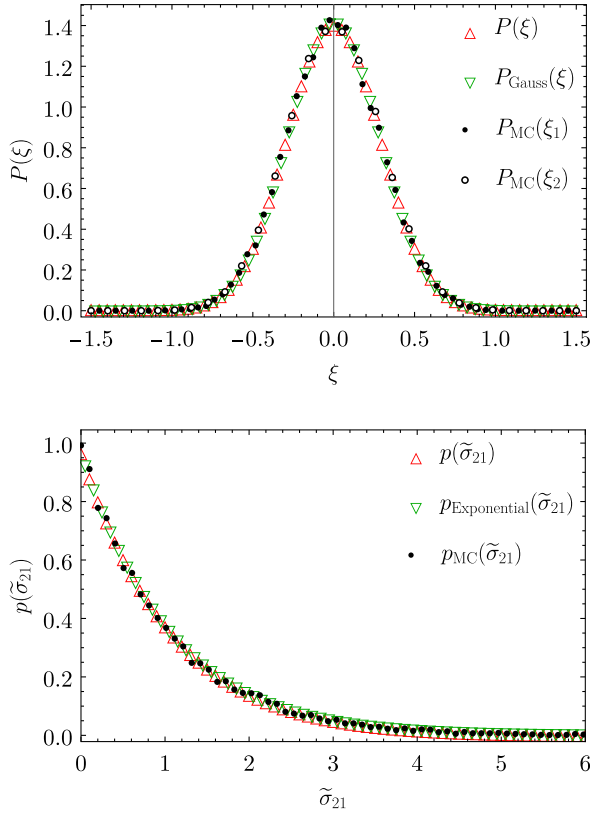


FIG. 5. Analytical results and Monte Carlo simulations for the distributions $P(\xi_1)$, $P(\xi_2)$ of rescaled real and imaginary parts of S -matrix elements (top) and for the distribution $p(\tilde{\sigma}_{21})$ of the rescaled cross-sections (bottom) for $\Xi = 9.55$ and $M = 60$.

Conclusion—We succeeded in fully solving a long-standing problem in quantum chaotic or stochastic scattering. We quantitatively captured the Ericson transition of time-reversal noninvariant systems ($\beta = 2$) within the Heidelberg approach, and derived the universal Gaussian distribution for the Ericson regime that emerges in the universal stochasticity of the scattering problem. The Heidelberg approach as such already incorporates universality as the random matrices modeling the Hamiltonian H in the interaction zone represent in a broad sense a generically chaotic or complex system. On top of this universality emerges another universality, i.e., the Gaussians, in the Ericson transition. Furthermore, we provided the full information on the transition to this Ericson limit by an asymptotic analysis. To this end, we calculated all moments for the probability distribution of the off-diagonal S -matrix elements and cross sections. Removing a cumbersome singularity was crucial. We carried out detailed validations with experimental data and numerical Monte Carlo simulations. The agreement with our analytical results is very good. We also found that the Ericson transition is very fast in the sense that the subleading term in the asymptotic expansion will typically suffice to understand deviations from the universal

Gaussian in experimental data. The methods developed here carry over to the case of orthogonally and symplectically invariant Hamiltonians, i.e., to the time-reversal invariant cases $\beta = 1$ and $\beta = 4$. As the calculations become technically much more involved, this material is beyond the scope of this contribution and will be presented elsewhere.

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Data availability—The data is available from the corresponding authors upon reasonable request.

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End Matter

Appendix A: Moldauer-Simonius relation and Weisskopf estimate—The Ericson regime is characterized by two prominent features. Resonances are strongly overlapping, that is, the ratio Γ/D of the average resonance width Γ and average mean level spacing D is large. Furthermore, the number M of open channels with associated transmission coefficients T_c is large. A relation between the transmission coefficients and Γ/D is given by the Moldauer-Simonius relation [83–88]

$$\frac{\Gamma}{D} = -\frac{1}{2\pi} \sum_{c=1}^M \ln(1 - T_c), \quad (\text{A1})$$

which was derived rigorously for $\beta = 2$ in [87]. For $T_c \ll 1, M \gg 1$, Eq. (A1) yields for the dimensionless parameter

$$\Xi = \frac{1}{2\pi} \sum_{c=1}^M T_c \quad (\text{A2})$$

the Weisskopf estimate

$$\frac{\Gamma}{D} = \Xi. \quad (\text{A3})$$

It provides a good approximation also for a large, but finite number of open channels and for larger values of T_c [83].

Appendix B: Channel factor expansion—We write the channel factor as exponential and identify the parametric dependencies in Ξ and T_c ,

$$\begin{aligned} F_U(\lambda_1, \lambda_2) &= \prod_{c=1}^M \frac{g_c^+ + \lambda_2}{g_c^+ + \lambda_1} = \exp \ln \left(\prod_{c=1}^M \frac{g_c^+ + \lambda_2}{g_c^+ + \lambda_1} \right) = \exp \left[\sum_{c=1}^M \ln \left(\frac{1 + \frac{T_c}{2}(\lambda_2 - 1)}{1 + \frac{T_c}{2}(\lambda_1 - 1)} \right) \right] \\ &= \exp \left(\sum_{c=1}^M \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \left[\left(\frac{T_c}{2}(\lambda_2 - 1) \right)^k - \left(\frac{T_c}{2}(\lambda_1 - 1) \right)^k \right] \right). \end{aligned} \quad (\text{B1})$$

In the last equation, we employed the Taylor series of the logarithm. Upon substituting $\lambda_1 = 1 + q'r', \lambda_2 = 1 - q'(1 - r')$ with $q' \in [0, 2]$ and $r' \in [0, 1]$, we arrive at

$$\begin{aligned} F_U(1 + q'r', 1 - q'(1 - r')) &= \exp \left(\sum_{c=1}^M \sum_{k=1}^{\infty} \left(\frac{T_c}{2} \right)^k \frac{(-1)^{k+1}}{k} \left((q'(r' - 1))^k - (q'r')^k \right) \right) \\ &= \exp \left(-\sum_{c=1}^M \frac{T_c}{2} q' \right) \exp \left(\sum_{c=1}^M \sum_{k=2}^{\infty} \left(\frac{T_c}{2} \right)^k \frac{(-1)^{k+1}}{k} \left((q'(r' - 1))^k - (q'r')^k \right) \right) \\ &= \exp(-\pi \Xi q') \exp \left(\sum_{c=1}^M \sum_{k=2}^{\infty} \left(\frac{T_c}{2} \right)^k \frac{(-1)^{k+1}}{k} \left((q'(r' - 1))^k - (q'r')^k \right) \right). \end{aligned} \quad (\text{B2})$$

From the exact expression above we can infer various regimes. The strongest case of the Ericson regime is given for $T_c = 1$ and large M , resulting in the Gaussian and exponential form of the aforementioned distributions. Moreover, we consider the case of many channels with small transmission coefficients. Now, when $T_c \simeq 1/M$, we get in the large M limit

$$\begin{aligned}
 & \lim_{M \rightarrow \infty} \exp(-\pi \Xi q') \exp \left(\sum_{c=1}^M \sum_{k=2}^{\infty} \left(\frac{T_c}{2} \right)^k \frac{(-1)^{k+1}}{k} \left([q'(r' - 1)]^k - (q'r')^k \right) \right) \\
 & \simeq \lim_{M \rightarrow \infty} \exp(-\pi \Xi q') \exp \left(\sum_{c=1}^M \sum_{k=2}^{\infty} \left(\frac{1}{2M} \right)^k \frac{(-1)^{k+1}}{k} \left([q'(r' - 1)]^k - (q'r')^k \right) \right) \\
 & = \exp(-\pi \Xi q'),
 \end{aligned} \tag{B3}$$

since the sum over the transmission coefficients scales linearly with M . Thereby we recover the result of [71]. The transmission coefficients are not necessarily equal for all channels. We conclude that only the term $\exp(-\pi \Xi q')$ significantly contributes in the large M limit of many small channels. However, if one has a few strong channels $T_c \simeq 1$ and $\Xi \simeq 1$, terms including powers T_c^j , $j \geq 2$ contribute to the asymptotic expansion in higher order corrections due to the occurrence of $(q'T_c/2)^j$. Therefore, the leading and subleading order terms of the asymptotic expansion in $1/\Xi$ are generally essential for an arbitrary number of channels, thereby corroborating the relevance of the analytical expansion presented in this Letter.

Appendix C: Explicit derivation of the Gaussian—Here, we explicitly calculate the integrals and derivatives arising in the leading order term of the $2n$ th moment. Splitting the integral from Eq. (11) into two parts and only calculating the leading order term ($m = n - 1$), we have

$$\begin{aligned}
 \int_0^1 dr' \frac{\partial^{n-1}}{\partial q'^{n-1}} q'^{n-1} \left(\frac{r'(q'r' + 2)}{(g_a^+ + q'r' + 1)(g_b^+ + q'r' + 1)} \right) \Big|_{q'=0} &= \int_0^1 dr' 2^n (n-1)! \left(\frac{r'}{(g_a^+ + 1)(g_b^+ + 1)} \right)^n \\
 &= 2^n (n-1)! \left(\frac{1}{(g_a^+ + 1)(g_b^+ + 1)} \right)^n \frac{1}{n+1}.
 \end{aligned} \tag{C1}$$

The second integral reads

$$\begin{aligned}
 & \int_0^1 dr' \frac{\partial^{n-1}}{\partial q'^{n-1}} q'^{n-1} \left[\left(\frac{r'(q'r' + 2)}{(g_a^+ + q'r' + 1)(g_b^+ + q'r' + 1)} \right)^{n-1} \left(\frac{(1-r')(2-q'(1-r'))}{[g_a^+ + 1 - q'(1-r')][g_b^+ + 1 - q'(1-r')]} \right) \right] \Big|_{q'=0} \\
 &= \int_0^1 dr' (n-1)! \left(\frac{2r'}{(g_a^+ + 1)(g_b^+ + 1)} \right)^{n-1} \frac{2(1-r')}{(g_a^+ + 1)(g_b^+ + 1)} \\
 &= 2^n (n-1)! \left(\frac{1}{(g_a^+ + 1)(g_b^+ + 1)} \right)^n \left(\frac{1}{n} - \frac{1}{n+1} \right) = 2^n (n-1)! \left(\frac{1}{(g_a^+ + 1)(g_b^+ + 1)} \right)^n \frac{1}{n(n+1)}.
 \end{aligned} \tag{C2}$$

All in all, the leading order asymptotic expression for the moments is

$$\begin{aligned}
 \overline{x_s^{2n}} &= \frac{\Gamma(2n+1)}{2^{2n}\Gamma^2(n)} \left(\frac{2}{2\pi\Xi} \right)^n \left(2^n (n-1)! \left(\frac{1}{(g_a^+ + 1)(g_b^+ + 1)} \right)^n \frac{1}{n+1} + 2^n (n-1)! \left(\frac{1}{(g_a^+ + 1)(g_b^+ + 1)} \right)^n \frac{1}{n(n+1)} \right) \\
 &= \frac{\Gamma(2n+1)}{\Gamma(n)n} \left(\frac{1}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right)^n = \frac{(2n)!}{n!} \left(\frac{1}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right)^n.
 \end{aligned} \tag{C3}$$

By using the moment generating property of the characteristic function, we arrive at

$$\begin{aligned}
 R_s^{(l)}(k) &= \sum_{n=0}^{\infty} (-1)^n k^{2n} \frac{1}{(2n)!} \frac{(2n)!}{n!} \left(\frac{1}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right)^n \\
 &= \sum_{n=0}^{\infty} (-1)^n k^{2n} \frac{1}{n!} \left(\frac{1}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right)^n = \exp \left(-\frac{k^2}{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi} \right).
 \end{aligned} \tag{C4}$$

Performing the inverse Fourier transform yields the Gaussian asymptote

$$\mathcal{F}^{-1}[R_s^{(l)}(k)](x_s) = \sqrt{\frac{\Xi(g_a^+ + 1)(g_b^+ + 1)}{2}} \exp\left(-\frac{\pi\Xi(g_a^+ + 1)(g_b^+ + 1)x_s^2}{2}\right). \quad (\text{C5})$$

Higher order corrections are similarly calculated.

Appendix D: Unscaled distributions $P_s(x_s)$ and $p(\sigma_{ab})$ —The leading order terms of the unscaled distribution $P_s(x_s) \simeq P_s^{(l)}(x_s) + P_s^{(sl)}(x_s)$ are found to be

$$\begin{aligned} P_s^{(l)}(x_s) &= \mathcal{F}^{-1}[R_s^{(l)}(k)](x_s) = \sqrt{\frac{\Xi(g_a^+ + 1)(g_b^+ + 1)}{2}} \exp\left(-\frac{\pi\Xi(g_a^+ + 1)(g_b^+ + 1)x_s^2}{2}\right), \\ P_s^{(sl)}(x_s) &\simeq \mathcal{F}^{-1}[R_s^{(sl)}(k)](x_s) = \sqrt{\frac{\Xi(g_a^+ + 1)(g_b^+ + 1)}{2}} \exp\left(-\frac{\pi\Xi(g_a^+ + 1)(g_b^+ + 1)x_s^2}{2}\right) [3 - 6\pi\Xi(g_a^+ + 1)(g_b^+ + 1)x_s^2 \\ &\quad + [\pi\Xi(g_a^+ + 1)(g_b^+ + 1)]^2 x_s^4] \times \frac{g_a^+ g_b^+ - g_a^+ - g_b^+ - 3}{8(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}. \end{aligned} \quad (\text{D1})$$

The unscaled asymptotes consists of a Gaussian multiplied with a series in noninteger powers of Ξ , calling for an appropriate rescaling as done in the main text. At $x_s = 0$, we have

$$P_s^{(sl)}(0) = \frac{3(g_a^+ g_b^+ - g_a^+ - g_b^+ - 3)}{8\sqrt{2(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}}. \quad (\text{D2})$$

We see that the first correction to the Gaussian maximum is of order $1/\sqrt{\Xi}$. The correction becomes negligible in the limit of large Ξ , which yields Gaussian behavior of the distribution. The leading order terms for the unscaled cross-section distribution $p(\sigma_{ab}) \simeq p^{(l)}(\sigma_{ab}) + p^{(sl)}(\sigma_{ab})$ read

$$\begin{aligned} p^{(l)}(\sigma_{ab}) &= \frac{1}{2} \exp\left(-\frac{\sigma_{ab}(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}{2}\right) (g_a^+ + 1)(g_b^+ + 1)\pi\Xi, \\ p^{(sl)}(\sigma_{ab}) &= \frac{1}{2} \exp\left(-\frac{\sigma_{ab}(g_a^+ + 1)(g_b^+ + 1)\pi\Xi}{2}\right) \\ &\quad \times (g_a^+ g_b^+ - g_a^+ - g_b^+ - 3) \left(1 - (g_a^+ + 1)(g_b^+ + 1)\pi\Xi\sigma_{ab} + \frac{1}{8}[(g_a^+ + 1)(g_b^+ + 1)\pi\Xi\sigma_{ab}]^2\right). \end{aligned} \quad (\text{D3})$$

Here, we have an exponential decay multiplied with a series in integer powers of Ξ , making a different rescaling compared to $P_s(x_s)$ necessary.