

Implication of the Existence of $J^{PC} = 0^{--} \bar{D}_s DK$ Bound State on the Nature of $D_{s0}^*(2317)$, and a New Configuration of Exotic State

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The discovery of numerous new hadrons over the past two decades has provided unprecedented opportunities to understand the nonperturbative QCD and hadron structure. The hadronic molecule picture plays an important role in explaining these new hadrons and enriching the configurations of exotic hadronic states. In this Letter, using the model-independent DK potential extracted from the relevant experimental data, a $J^{PC} = 0^{--} \bar{D}_s DK$ three-body hadronic molecule is predicted with a mass of 4310_{-24}^{+14} MeV. This state shows decoupling to conventional $c\bar{c}$ charmonia or the $\bar{D}_s D_{s0}^*(2317)$ two-body molecular state. It can be regarded as a compelling three-body hadronic molecular candidate. We further demonstrate that the $B^+ \rightarrow D^{*\pm} D^\mp K^+$ decays could be promising channels for searching for the predicted state in future high-luminosity LHCb runs.

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Introduction—Quantum chromodynamics (QCD), the fundamental theory of the strong interaction, exhibits strong coupling at low energies, resulting in color confinement. In this regime, the relevant degrees of freedom are hadrons rather than quarks and gluons, making quark-level studies of low-energy strong interactions particularly challenging. Consequently, hadron spectroscopy has become indispensable for investigating nonperturbative strong interactions, especially in light of the numerous new heavy hadrons discovered. A remarkable feature of the spectrum of these heavy hadrons is that most are near the threshold of a pair of hadrons. The recent studies from the unquenched quark model [1–4], effective field theories [5–10], and lattice QCD [11–15] indicate that the heavy hadrons have strong couplings to a pair of hadrons, where the hadron-hadron interactions represent a fundamental manifestation

of nonperturbative QCD dynamics. Importantly, such nonperturbative effects are not limited to spectroscopy but also emerge prominently in both the final-state interactions of heavy hadron decays [16,17] and their production [18–20]. Therefore, the hadron-hadron interactions are crucial for understanding the properties of heavy hadrons.

A salient example is the DK interaction, which has garnered renewed interest following the discovery of the $D_{s0}^*(2317)$ [21]. If $D_{s0}^*(2317)$ is interpreted as a $c\bar{s}$ charmed strange meson with $J^P = 0^+$, its mass is around 160 MeV lower than the theoretical predictions from the Godfrey-Isgur model [22]. The mass puzzle of $D_{s0}^*(2317)$ is solved if the DK molecular component is embodied [2–4,23–25]. Lattice QCD calculations of the DK scattering parameters [11,14,26] reveal that the molecular component constitutes over 70% of the $D_{s0}^*(2317)$ wave function [2,4,6,9,10]. However, the limited experimental data on $D_{s0}^*(2317)$ has hindered independent verification of this picture through alternative observables. Recent theoretical studies have proposed investigating the DK interaction and $D_{s0}^*(2317)$ properties within three-body hadron systems [27], inspiring many studies on analogous three-body hadronic molecules [27–34]. These three-body hadronic molecules represent novel configurations of hadronic matter that offer unique insights into nonperturbative QCD dynamics. These studies

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not only deepen our understanding of hadron structure but may also establish new paradigms in hadron spectroscopy.

Originally, the Valencia group employed the fixed center approximation to the Faddeev equations and predicted the existence of two exotic hadrons: three-body NDK [35] and $DK\bar{K}$ [36] molecules, where the DK interaction is determined by reproducing the $D_{s0}^*(2317)$ mass. However, verifying the DK interaction via the above three-body hadron systems is less optimal since the DN and $K\bar{K}$ potentials are strongly attractive [35,36]. This motivated our investigation of the DDK system, where the weak DD interaction allows the DK component to dominate the molecular binding [30,37,38]. Although theoretically promising, the DDK molecule presents experimental challenges: its production in e^+e^- collisions is highly suppressed [39], although it is possible in B_c decays [27]. However, this pathway suffers from the intrinsically low B_c production cross section. Subsequently, a $D\bar{D}K$ molecule was also predicted [40], but its yield in the inclusive process of e^+e^- collisions is lower by 3 orders of magnitude than that of the two-body DK molecule [41]. In addition, other components can mix with the three-body hadronic molecule [42]. The current landscape thus lacks an unambiguous three-body molecular candidate that simultaneously satisfies two crucial criteria: minimal contamination from other hadronic configurations and experimentally accessible production rates. The discovery of such a state would provide decisive evidence for three-body hadronic molecules and offer unique insights into the fundamental interactions between hadrons.

In this Letter, we predict a $J^{PC} = 0^{--}\bar{D}_s DK$ three-body molecule with a binding energy of about a few tens of MeV. Such a quantum number is forbidden for conventional $c\bar{c}$ charmonia, ensuring negligible mixing with ordinary quarkonium states [43]. Notably, while a $J^{PC} = 0^{--}D_{s0}^*\bar{D}_s$ two-body configuration is unlikely to form a bound state [44,45], the three-body $\bar{D}_s DK$ system exhibits binding, further distinguishing it from possible two-body molecular interpretations. Remarkably, this exotic state can be produced through B decays. The observation in the decays $B^+ \rightarrow D^{*\pm} D^\mp K^+$ by the LHCb Collaboration [46] is promising, considering its decay behavior and production rates. In the following, we will first demonstrate the existence of the $J^{PC} = 0^{--}\bar{D}_s DK$ bound state with its interaction inputs, denoted by $X(4310)$. Subsequently, we systematically address its decay properties and production mechanisms in B decays.

*Existence of a $J^{PC} = 0^{--}\bar{D}_s DK$ bound state $X(4310)$ —*The three-body wave function for the $\bar{D}_s DK$ system of good C parity reads

$$\Psi^C = \frac{1}{\sqrt{2}} \left(\Psi_{\bar{D}_s DK} + C \Psi'_{D_s \bar{D} \bar{K}} \right), \quad (1)$$

where $C = \pm 1$ represents the C -parity eigenvalue, and Ψ (Ψ') denotes the wave function of the $\bar{D}_s DK$ ($D_s \bar{D} \bar{K}$)

system. The wave function Ψ^C can be solved by the Schrödinger equation with the Hamiltonian $H = T + T' + V + V' + V^C$, where T (T') and V (V') are the kinetic energy term and the hadron-hadron potentials of Ψ (Ψ'), respectively.

In this Letter, we employ the contact-range effective field theory to construct the DK , $\bar{D}_s K$, and $D\bar{D}_s$ potentials [47,48]. The contact-range potential in momentum space can be parametrized by a Gaussian-shaped potential in coordinate space

$$V(r) = C_a \frac{e^{(r/R_c)^2}}{\pi^{3/2} R_c^3}, \quad (2)$$

where R_c is a cutoff radius of the order of a typical hadronic size. From systematic studies of exotic state masses and applications of SU(3)-flavor symmetry relations [11,49–52], we establish the following approximate ratio for the contact potentials for a cutoff $\Lambda = 1$ GeV: $C_a^{DK} : C_a^{\bar{D}_s K} : C_a^{D\bar{D}_s} = 1 : 0.5 : 0.1$. Given the dominant role of the DK interaction, we focus particularly on determining this potential by precisely reproducing the $D_{s0}^*(2317)$ mass.

We assume the $D_{s0}^*(2317)$ as a mixture of a $DK - D_s \eta$ molecular state and a $c\bar{s}$ bare state rather than a pure DK molecule. The lattice QCD simulations or reanalyses of lattice QCD results [2,4,6,9–11,14,26] found that the molecular component accounts for more than 70% of the $D_{s0}^*(2317)$ wave function. Therefore, we assume that the molecular and bare components account for 70% and 30% of the physical $D_{s0}^*(2317)$, respectively. In the mixture picture, the extracted DK potential forms a bound state with a binding energy of 14 MeV, which is less attractive than assuming the $D_{s0}^*(2317)$ as a pure DK molecule. To estimate the uncertainty of the extracted DK potential, we vary the molecular compositeness of the $D_{s0}^*(2317)$ from 50% to 100%.

The potential V^C , dependent on the C parity, could be a three-body interaction correlating the wave functions Ψ and Ψ' . Here, since the DK potential can form a bound state D_{s0}^* , we use an effective two-body $\bar{D}_s D_{s0}^*$ potential utilizing the one-boson-exchange (OBE) model [45,53]. To account for the composite nature of the interacting hadrons, the dipole form factor is usually introduced in the OBE model [54]:

$$F(q, \Lambda, m_E) = \left(\frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2} \right). \quad (3)$$

m_E represents the mass of the exchanged particle, and Λ is a cutoff parameter, which can be further parametrized as $\Lambda = \alpha \Lambda_{\text{QCD}} + m_E$ [55]. Following the studies of the heavy hadron decays [55,56] and the spectrum of hadronic molecular states [27], we vary the parameter α from 1 to 2. The subsequent final-state interactions in the strong decays of the $X(4310)$ and the production of the $X(4310)$ in B decays

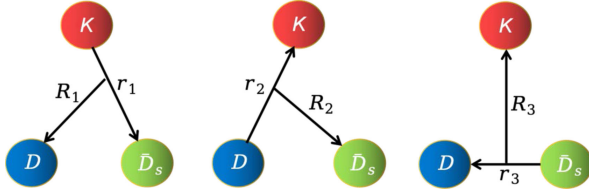


FIG. 1. Three permutations of the Jacobi coordinates for the $\bar{D}_s DK$ system.

are described by the OBE model as well. The C -parity-dependent potential of $\bar{D}_s D_{s0}^*$ in the OBE model arises from the η -meson exchange, which is attractive for the $C = +$ configuration and repulsive for the $C = -$ configuration. The η exchange potential for the $\bar{D}_s D_{s0}^*$ system is written as

$$V^{C=\pm} = \mp \frac{2}{3} \frac{k^2}{f_\pi^2} q_0^2 \left(\frac{e^{-mr} - e^{-\Lambda r}}{4\pi r} - \frac{\Lambda^2 - m^2}{8\pi\Lambda} e^{-\Lambda r} \right), \quad (4)$$

where $q_0 = m_{D_{s0}^*} - m_{D_s}$, $k = 0.56$, and $f_\pi = 130$ MeV [53].

With the pairwise two-body interactions, the Schrödinger equation of Ψ^C can be simplified as

$$\langle \Psi | (T + V_{DK} + V_{\bar{D}_s K} + V_{\bar{D}_s D} + V^C - E) | \Psi \rangle = 0, \quad (5)$$

which can be solved by the Gaussian expansion method [57], with the three Jacobi channels as shown in Fig. 1.

The results are shown in Table I, the $0^{--} \bar{D}_s DK$ system is predicted to be a bound state with a binding energy of 21_{-14}^{+24} MeV. Our results indicate that when the $DK - D_s \eta$ molecular component of the $D_{s0}^*(2317)$ ranges from 50% to 100%, the $\bar{D}_s DK$ system always remains bound. The weight of each Jacobi channel Φ_i is calculated by $P_i = \langle \Psi | \hat{P}_i^G | \Psi \rangle$, with $|\Psi\rangle = \sum_i |\Phi_i\rangle$ and the generalized projection operator $\hat{P}_i^G = \sum_j |\Phi_i\rangle S_{ij}^{-1} \langle \Phi_j|$ suited for a nonorthogonal basis [58], where S_{ij}^{-1} is the element of

TABLE I. Binding energy (in units of MeV), weights of Jacobi channels, root mean square radii (in units of fm), and expectation values of the Hamiltonian (in units of MeV) of the $0^{--} \bar{D}_s DK$ molecule.

Scenarios	B.E. (0^{--})	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha = 1$	22_{-14}^{+23}	$11_{-1}^{+1}\%$	$78_{-2}^{+1}\%$	$11_{-1}^{+0}\%$
$\alpha = 2$	20_{-13}^{+22}	$10_{-1}^{+1}\%$	$80_{-2}^{+1}\%$	$10_{-1}^{+0}\%$
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T \rangle$
$\alpha = 1$	$1.6_{+0.9}^{-0.4}$	$1.2_{+0.5}^{-0.3}$	$1.4_{+0.8}^{-0.3}$	177_{-74}^{+81}
$\alpha = 2$	$1.7_{+1.4}^{-0.4}$	$1.2_{+0.7}^{-0.3}$	$1.6_{+1.3}^{-0.5}$	169_{-78}^{+82}
Scenarios	$\langle V_{D_s \bar{K}} \rangle$	$\langle V_{DK} \rangle$	$\langle V_{D_s \bar{D}} \rangle$	$\langle V_{D_s \bar{D}_{s0}^*}^{C=-} \rangle$
$\alpha = 1$	-40_{+20}^{-25}	-147_{+62}^{-73}	-14_{+6}^{-7}	2_{-1}^{+0}
$\alpha = 2$	-37_{+22}^{-25}	-143_{+64}^{-73}	-13_{+7}^{-6}	3_{-1}^{+2}

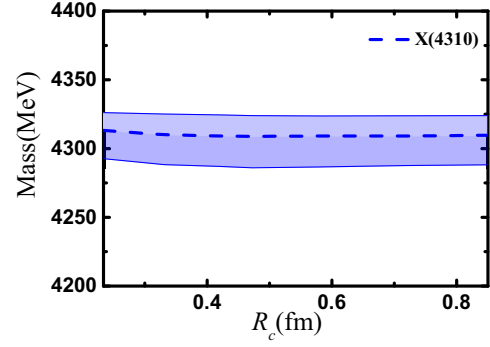


FIG. 2. Mass of $X(4310)$ as a function of the cutoff R_c .

the inverse of the overlap S matrix of Jacobi channels with $S_{ij} = \langle \Phi_i | \Phi_j \rangle$ ($i, j = 1-3$). Noting that the calculated weights P_i should not be viewed as percentage contributions of individual components [59], but rather as the relative importance of the Jacobi channels in forming the three-body bound state. Interestingly, the weights of Jacobi channels $i = 1-3$ in Fig. 1 are stable, which are about 10%, 80%, and 10% of $\bar{D}_s K - D$, $DK - \bar{D}_s$, and $\bar{D}_s D - K$, respectively, varying only 1%–2%. This indicates the 0^{--} three-body bound state is mainly contributed by the $(DK) - \bar{D}_s$ channel, weakly dependent on the molecular component of the $D_{s0}^*(2317)$. To test the impact of the cutoff on our results, the X mass as a function of the cutoff R_c is shown in Fig. 2, which indicates that its mass is weakly dependent on the cutoff. The root mean square radii and expectation values of the Hamiltonian of the $0^{--} \bar{D}_s DK$ bound state are also presented in Table I, thereby providing a more intuitive demonstration of the spatial extent of the predicted molecular state and the relative contributions of individual interaction terms in the Hamiltonian to its formation.

The $J^{PC} = 0^{-+} \bar{D}_s DK$ three-body system is also predicted to be bound with a binding energy of 27_{-17}^{+24} MeV, which is slightly more bound than its C -parity counterpart $X(4310)$. The predicted $J^{PC} = 0^{-+} \bar{D}_s DK$ molecule can couple to two other configurations: an excited $0^{-+} c\bar{c}$ state [22] and a $0^{-+} \bar{D}_s D_{s0}^*(2317)$ two-body molecule [44]. The intricate nonperturbative dynamics involving all three configurations require a more systematic study on the $J^{PC} = 0^{-+} \bar{D}_s DK$ molecule. In contrast, the $X(4310)$ molecule exhibits no coupling to either conventional $c\bar{c}$ states nor the $\bar{D}_s D_{s0}^*(2317)$ two-body configuration. It can be regarded as a compelling three-body hadronic molecular candidate. Therefore, we mainly focus on the $X(4310)$ in this Letter.

Decays and productions of $X(4310)$ —To facilitate potential observation of the predicted state $X(4310)$, we investigate its decay properties and production mechanisms through triangle diagrams, focusing particularly on B decays [55,60]. The Jacobi coordinate analysis (Fig. 1 and Table I) indicates that the $X(4310)$ predominantly decays via its subsystem $\bar{D}_s D_{s0}^*$, which subsequently

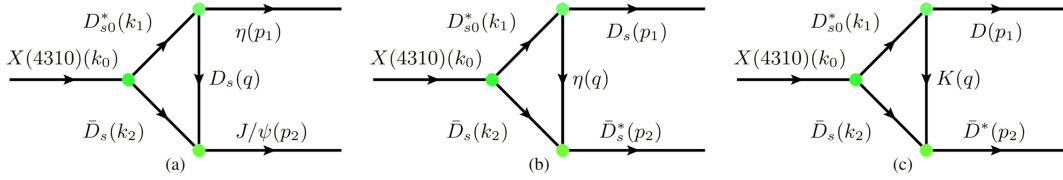


FIG. 3. Triangle diagrams of the $X(4310)$ via the subsystem $D_{s0}^* \bar{D}_s$ decaying into $J/\psi\eta$ (a), $D_s \bar{D}_s^*$ (b), and $D \bar{D}^*$ (c).

undergoes inelastic scattering into final states including $J/\psi\eta$, $\bar{D}^* D$, and $\bar{D}_s^* D_s$ through meson-exchange processes (Fig. 3). With the Lagrangian for each vertex in Fig. 3, we obtain the corresponding amplitudes as

$$i\mathcal{M}_a = g_{XD_{s0}^* \bar{D}_s} g_{D_{s0}^* D_s \eta} g_{\psi \bar{D}_s D_s} \int \frac{d^4 q}{(2\pi)^4} (k_2^\mu - q^\mu) \frac{1}{k_1^2 - m_{D_{s0}^*}^2} \frac{1}{k_2^2 - m_{\bar{D}_s}^2} \frac{1}{q^2 - m_{D_s}^2} \epsilon_\mu(p_2) F^2(q, \Lambda, m_E), \quad (6)$$

$$i\mathcal{M}_b = g_{XD_{s0}^* \bar{D}_s} g_{D_{s0}^* D_s \eta} g_{\bar{D}_s D_s^* \eta} \int \frac{d^4 q}{(2\pi)^4} q^\mu \frac{1}{k_1^2 - m_{D_{s0}^*}^2} \frac{1}{k_2^2 - m_{\bar{D}_s}^2} \frac{1}{q^2 - m_{\eta}^2} \epsilon_\mu(p_2) F^2(q, \Lambda, m_E), \quad (7)$$

$$i\mathcal{M}_c = g_{XD_{s0}^* \bar{D}_s} g_{D_{s0}^* D K} g_{\bar{D}_s D^* K} \int \frac{d^4 q}{(2\pi)^4} q^\mu \frac{1}{k_1^2 - m_{D_{s0}^*}^2} \frac{1}{k_2^2 - m_{\bar{D}_s}^2} \frac{1}{q^2 - m_K^2} \epsilon_\mu(p_2) F^2(q, \Lambda, m_E), \quad (8)$$

where g with subscripts denotes the couplings of the interaction vertices in the triangle diagrams.

It is well known that many charmonium and charmoniumlike states are produced in B decays, proceeding via the decay $b \rightarrow c \bar{c} s$ at the quark level [27,61], similar to the production mechanism of the $X(4310)$ in B decays, as shown in Fig. 4(a). However, the decay $B \rightarrow X(4310)K$ poses significant theoretical challenges due to its complex nonperturbative dynamics, making first-principles calculations at the quark level particularly difficult. To account for the significant nonperturbative effects in such heavy hadron weak decays, the final state interaction approach has proven to be an essential theoretical tool to understand the hadronization process and final-state rescattering effects that dominate these decays [55,62–67].

In this Letter, we assume that the B meson first decays into a $\bar{D}^*$ meson and a D_{s0}^* hadronic molecule, and then the $\bar{D}^*$ meson scatters into a $\bar{D}_s$ and a K meson. Finally, the $\bar{D}_s DK$ molecule is dynamically generated by the subsystem $\bar{D}_s D_{s0}^*$ as shown in Fig. 4(b). In Refs. [48,68], the production of D_{s0}^* as a DK hadronic molecule in B decays is investigated, which laid the foundation for the present study. Similarly, the amplitude of the Feynman diagram of Fig. 4(b) is written as

$$\begin{aligned} \mathcal{M} = & g_{\bar{D}^* \bar{D}_s K} g_{D_{s0}^* \bar{D}_s X} \mathcal{A}(B \rightarrow D_{s0}^* \bar{D}^*)^\mu p_1^\nu \\ & \times \frac{-g_{\mu\nu} + \frac{k_{1\mu} k_{1\nu}}{k_1^2}}{(k_1^2 - m_{D_{s0}^*}^2)(k_2^2 - m_{\bar{D}_s}^2)(q^2 - m_{\bar{D}_s}^2)} F^2(q, \Lambda, m_E). \end{aligned} \quad (9)$$

The relevant effective Lagrangians and the values of the couplings in Eqs. (6)–(9) are listed in the Supplemental Material [69]. The partial decay widths of Figs. 3 and 4(b)

can be finally obtained as

$$\Gamma = \frac{1}{2J+1} \frac{1}{8\pi M^2} |\vec{p}| \overline{|\mathcal{M}|^2}, \quad (10)$$

where J is the total angular momentum of the initial state M , the overline indicates the sum over the polarization vectors of final states, and $|\vec{p}|$ is the momentum of either final state in the rest frame of the M meson.

In this study, in addition to the uncertainties of parameter α in Eq. (3), the dominant uncertainties originate from the couplings of the three vertices of the triangle diagrams. As a result, we obtain the uncertainties of the decay widths originating from the uncertainties of these parameters via a Monte Carlo sampling in their 1σ intervals.

In Fig. 5, we show the partial decay widths of X with $M_X = 4310$ as a function of α , where the uncertainties of the couplings of vertices induce the bands. We can see that the partial widths of the decays $X \rightarrow J/\psi\eta$, $X \rightarrow \bar{D}_s D_s^*$, and $X \rightarrow \bar{D}^* D$ are of the order of 10^1 , 10^2 , and 10^3 keV, indicating that the partial decay width of $X \rightarrow \bar{D}^* D$ is larger than those of $X \rightarrow \bar{D}_s D_s^*$ and $X \rightarrow J/\psi\eta$ by 1 and 2 orders

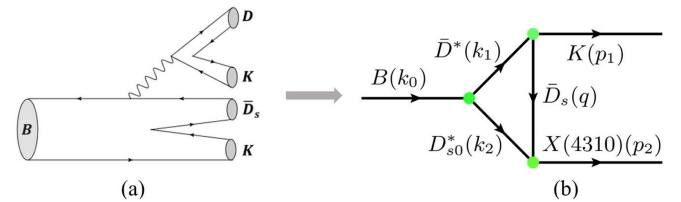


FIG. 4. Topology diagram at the quark level (a) and triangle diagram at the hadron level (b) accounting for the weak decays of $B \rightarrow X(4310)K$.

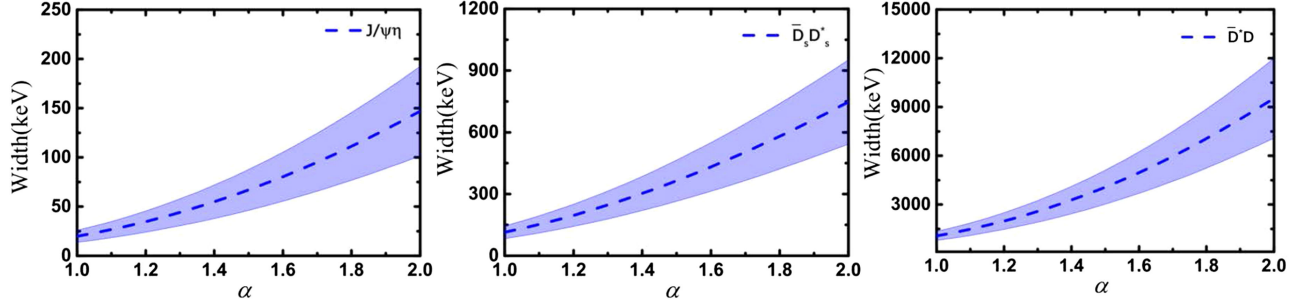


FIG. 5. Partial decay widths of $X(4310) \rightarrow J/\psi\eta$, $X(4310) \rightarrow \bar{D}_s D_s^*$, and $X(4310) \rightarrow \bar{D}^* D$ as functions of α . The dashed line and band correspond to their central value and uncertainties.

of magnitude, respectively. Taking into the lower and center mass of $X(4310)$, we find that the ratio of $\Gamma(X \rightarrow \bar{D}^* D)/\Gamma(X \rightarrow \bar{D}_s D_s^*)$ lies in the range of 9.9–13.6, 9.3–12.6, and 8.7–11.8 for the lower, center, and upper mass of X , and the corresponding ratio of $\Gamma(X \rightarrow \bar{D}_s D_s^*)/\Gamma(X \rightarrow J/\psi\eta)$ lies in the range of 5.2–6.1, 5.1–5.7, and 4.9–5.4. Regardless of the mass of X , the ratio of $\Gamma(X \rightarrow \bar{D}^* D):\Gamma(X \rightarrow \bar{D}_s D_s^*):\Gamma(X \rightarrow J/\psi\eta) \approx 50:5:1$. Therefore, we conclude that $X(4310)$ dominantly decays into $\bar{D}^* D$.

In Fig. 6, we show the branching fraction of the decay $B \rightarrow KX$ as a function of α , where $m_X = 4310$ MeV. One can see that the branching fraction of the decay $B \rightarrow KX(4310)$ is approximately 10^{-6} . Since the production rate of $X(4310)$ in B decays remains of the order of 10^{-6} for both its lower and upper mass limits, the production rate of $X(4310)$ in B decays is of the order of 10^{-6} . The results of strong decays and production rates of $X(4310)$ for the lower and upper limits are given in Supplemental Material [69]. Compared with the yield of $D_{s0}^*(2317)$ in B decays [48], the production rate of the two-body molecule DK in B decays is larger than that of the three-body molecule $\bar{D}_s DK$ in B decay by 3 orders of magnitude, consistent with the ratio of the yield of the two-body molecule DK to that of the three-body molecule $\bar{D}DK$ in e^+e^- collisions [41], indicating that the ratio of the production rates of the DK molecule to those of the

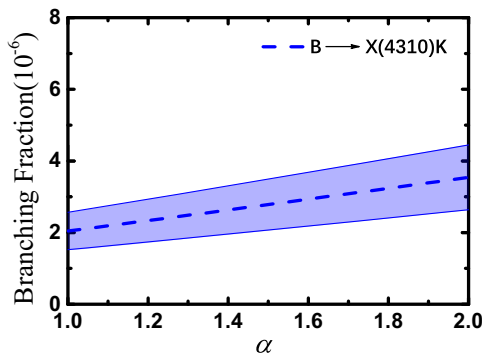


FIG. 6. Branching fraction of the decays $B \rightarrow KX(4310)$ as a function of α .

$D\bar{D}_{(s)}K$ molecule is only dependent on the long-range interaction but independent on the short-range interaction in the production process.

Our results indicate that the $X(4310)$ dominantly decays into $\bar{D}^* D$. In the isospin limit, the branching fraction of the $X(4310)$ decays into $\bar{D}^{*0} D^0$, $\bar{D}^{*+} D^0$, $D^{*+} D^-$, and $D^{*0} D^+$ is around 0.25. As a result, we estimate the branching fractions of the decays $B^+ \rightarrow [X(4310) \rightarrow D^{*+} D^+ K^+]$ to be 5×10^{-7} . Referring to the branching fractions $\mathcal{B}(B^+ \rightarrow D^{*+} D^+ K^+) = 6 \times 10^{-4}$ [86,87], we estimate the ratios of $\mathcal{B}[B^+ \rightarrow (X(4310) \rightarrow D^{*+} D^+) K^+]/\mathcal{B}(B^+ \rightarrow D^{*+} D^+ K^+) \sim 10^{-3}$. The event number of the decays $B^+ \rightarrow D^{*+} D^+ K^+$ of the LHCb experiment corresponding to an integrated luminosity of 9 fb^{-1} is around 2×10^3 [46]. One can expect that the event number of the decays $B^+ \rightarrow [X(4310) \rightarrow D^{*+} D^+] K^+$ would reach at least 10 and 10^2 corresponding to the integrated luminosity of 50 fb^{-1} and 350 fb^{-1} .

Summary and outlook—Over the past two decades, the discovery of numerous exotic hadronic states—many considered strong candidates for hadronic molecules—has revolutionized our understanding of hadron-hadron interactions, opening new avenues to explore three-body molecular configurations. In this Letter, with the DK interaction determined by the $D_{s0}^*(2317)$ assumed as a mixture state, we predicted a rather special three-body hadronic molecule $J^{PC} = 0^{--} \bar{D}_s DK$ with a mass of about 4310 MeV as shown in Fig. 7. A careful study of its decay

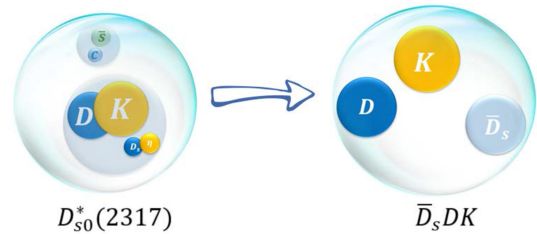


FIG. 7. With the DK potential determined by reproducing the mass of the $D_{s0}^*(2317)$ as a mixture of a $DK - D_s\eta$ molecular state and a $c\bar{s}$ bare state, a three-body $\bar{D}_s DK$ molecule is predicted.

behavior and production mechanism shows that it dominantly decays into $\bar{D}^*D$, and its production rate in B decays is of the order of 10^{-6} . Notably, we found that the ratio of the production rates of the DK molecule to those of the $D\bar{D}_{(s)}K$ molecules is only dependent on the long-range interaction of the production processes. This crucial observation demonstrates that nonperturbative strong interaction effects fundamentally govern the three-body molecular formation.

Moreover, the event number of the decays $B^+ \rightarrow [X(4310) \rightarrow D^{*\pm}D^\mp]K^+$ at the LHC corresponding to the integrated luminosity of 50 fb^{-1} and 350 fb^{-1} is estimated to be at least 10 and 10^2 , respectively. It should be noted that the LHCb Collaboration observed the signal of $J^{PC} = 0^{--}$ charmonium and charmoniumlike states in the decays $B^+ \rightarrow D^{*\pm}D^\mp K^+$ [46]. Therefore, the observation of the $J^{PC} = 0^{--} \bar{D}_s DK$ molecule by the LHCb Collaboration in the future is promising. On the other hand, the $0^{--} \bar{D}_s DK$ three-body bound state is easily distinguished from charmonium states and two-body molecules. The discovery of the $0^{--} \bar{D}_s DK$ three-body hadronic molecule will extend the configuration of the hadron spectrum and help determine the relevant hadron-hadron interaction and even pinpoint the molecular nature of $D_{s0}^*(2317)$. We strongly recommend experimental searches for the $0^{--} \bar{D}_s DK$ molecule in the decay channels $B^+ \rightarrow D^{*\pm}D^\mp K^+$.

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