

Erratum: Thermal Area Law in Long-Range Interacting Systems [Phys. Rev. Lett. 134, 020402 (2025)]

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In our Letter, we have identified a minor error in the proof of Theorem 2. Here we briefly report how to correct it.

Notation and overview—Fix a finite set of sites $\Lambda = [N]$. Let $L \subset \Lambda$ and $Z_1, \dots, Z_n \subset \Lambda$ be subsets with $|Z_s| \leq k$ ($1 \leq s \leq n$). Write $\{L, Z_1, \dots, Z_n\}$: nonsep to mean that every Z_s overlaps at least one member of $L, Z_1, \dots, Z_{s-1}, Z_{s+1}, \dots, Z_n$. Throughout the paper, the following locality bound was assumed:

$$\max_{i \in \Lambda} \sum_{Z \ni i} \|h_Z\| \leq 2gu =: \tilde{g}. \quad (1)$$

In the proof of Theorem 2 in the Letter, we needed to treat the following summation in the cluster expansion, which we here denote by S_n : $S_n := \sum_{\{L, Z_1, \dots, Z_n\}: \text{nonsep}} \prod_{s=1}^n \|h_{Z_s}\|$. From the first line to the second line in the Supplemental Material inequality (S.45), we realized that the inequality does not hold. We show here how to fix it using the ideas from the authors' other papers ([1] Proposition 4 therein).

Layer decomposition—We define L as the zeroth layer. We define $w_1 := \{Z_1^{(1)}, \dots, Z_{q_1}^{(1)}\}$ as the first layer such that they intersect with L , i.e., $\{Z_s^{(1)} : L \cap Z_s^{(1)} \neq \emptyset\}$. The second layer is defined by $w_2 := \{Z_1^{(2)}, \dots, Z_{q_2}^{(2)}\}$ such that they intersect with only w_1 not L . Inductively, we define $w_s := \{Z_1^{(s)}, \dots, Z_{q_s}^{(s)}\}$ such that each $Z_j^{(s)}$ intersects with at least one set in w_{s-1} and to none in earlier layers. Set $q_0 := |L|/k$; then $\sum_{s \geq 1} q_s = n$.

Here, we iteratively estimate the contribution from each of the layers. We first consider the case where the number of layers is r and $\{q_1, q_2, \dots, q_r\}$ are fixed. For the deepest r th layer, the summation is upper-bounded using (1):

$$\prod_{j=1}^{q_r} \sum_{Z_j^{(r)} : Z_j^{(r)} \cap V_{w_{r-1}} \neq \emptyset} \|h_{Z_j^{(r)}}\| \leq (q_{r-1} k \tilde{g})^{q_r}, \quad (2)$$

where $V_{w_{r-1}} := \bigcup_{j=1}^{q_{r-1}} Z_j^{(r-1)}$ and $|V_{w_{r-1}}| \leq k q_{r-1}$. Repeating upwards gives the upper bound for the summation as follows:

$$\prod_{s=1}^r (q_{s-1} k \tilde{g})^{q_s}, \quad q_0 = |L|/k. \quad (3)$$

We now have all the ingredients to estimate the total nonseparable summation. For a fixed $\{q_1, q_2, \dots, q_r\}$, the possible number of assignment of $\{Z_s\}_{s=1}^n$ is at most $n!/(q_1! q_2! \cdots q_r!)$. Hence, combining the multinomial count with (3),

$$\begin{aligned} S_n &\leq n! (k \tilde{g})^n \sum_{r=1}^n \sum_{\substack{q_1 + \dots + q_r = n \\ q_s \geq 1}} \prod_{s=1}^r \frac{q_{s-1}^{q_s}}{q_s!} \\ &\leq n! (k \tilde{g})^n e^{|L|/k + 2n} \sum_{r=1}^n \binom{n-1}{r-1} \\ &\leq n! (2e^2 k \tilde{g})^n e^{|L|/k}, \end{aligned} \quad (4)$$

where, from the second line to the third line, we use $q_{s-1}^{q_s}/q_s! \leq (q_s + q_{s-1})^{q_s} e^{q_s}/q_s^{q_s} \leq e^{q_{s-1} + q_s}$.

Conclusion—This gives the condition for the threshold temperature β_c as $\beta_c = W(1/(2e^2))/(\tilde{g}k)$ instead of $W(1/e)/(\tilde{g}k)$. However, we stress that the essence has not been changed.

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- [1] T. Kuwahara and K. Saito, Gaussian concentration bound and ensemble equivalence in generic quantum many-body systems including long-range interactions, [Ann. Phys. \(Amsterdam\)](#) **421**, 168278 (2020).