

Novel Ringdown Tests of General Relativity with Black Hole Graybody Factors

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We present GreyRing, a new model for the postmerger signal in black-hole binary coalescences based on the graybody factor of the remnant. The model accurately reproduces the full frequency-domain ringdown signal for a large set of comparable-mass, aligned-spin numerical relativity waveforms, including the $(\ell, m) = (2, 2)$, $(3, 3)$, and $(4, 4)$ modes. It achieves mismatches of order $\mathcal{O}(10^{-6})$ for the dominant $(2, 2)$ mode, demonstrating that the graybody-factor parametrization provides a very accurate phenomenological description of the postmerger signal. Building on this model, we introduce a novel consistency test of strong gravity based on the graybody factor: the remnant mass and spin inferred from GreyRing can be compared with those obtained through standard black-hole spectroscopy. This agnostic test relies exclusively on the postmerger signal and does not require the inclusion of overtones or the choice of very early ringdown starting times, combining the advantages of inspiral-merger-ringdown consistency tests and traditional black-hole spectroscopy. We apply the test to GW250114 and find that the remnant mass and spin inferred from GreyRing are consistent with those measured from the full signal. Remarkably, the inferred parameters can be measured with a precision comparable to, or slightly better than, that achieved with standard black-hole spectroscopy. Our graybody-factor waveform model allows for new precision tests of strong gravity using the ringdown signal.

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Introduction—The final stage of a binary black-hole (BH) merger, the so-called “ringdown,” provides one of the cleanest probes of strong-field gravity. During this phase the newly formed BH relaxes toward equilibrium through damped oscillations, and the gravitational waveform is commonly described as a superposition of quasinormal modes (QNMs) of the remnant spacetime [1]. In general relativity (GR), the remnant is a rotating (Kerr) BH and the entire QNM spectrum is uniquely determined by its mass M and dimensionless spin $\chi = J/M^2$ [2–7] (we use geometrical units, $G = c = 1$). This property underlies the program of BH spectroscopy, in which multiple QNMs extracted from the data are checked for consistency with the Kerr prediction [8–10]. Such measurements enable

stringent tests of GR [11–15], of the nature of compact remnants [16–19], and of near-horizon environments [20,21].

In practice, however, accurate ringdown measurements remain challenging. While BH perturbation theory predicts an infinite tower of QNMs, only a few are expected to be observable in realistic mergers. Even in the loudest event observed to date, GW250114, only two QNMs have been confidently measured, allowing for a ringdown test of GR at the level of $\approx 30\%$ for the frequency of the prograde quadrupolar first overtone [22,23]. The starting time of the ringdown is inherently uncertain and the number of modes that should be included in the analysis must be carefully guided by theoretical models. Only certain QNM sets are excited and detectable at any given time postmerger [24], and ringdown tests that include too many free parameters can overfit the data. This has fueled an ongoing debate on the robustness of ringdown measurements (see, e.g., Refs. [25–28], and [1] for a review).

Because only a small subset of QNMs can be measured, an independent determination of M and χ is highly desirable. Current consistency tests estimate M and χ either from the inspiral-merger signal or from the full waveform [15,29,30], and compare these values with those inferred from the ringdown. In this approach, systematics in the modeling of the preringdown signal may propagate into

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the inspiral-merger or full inspiral-merger-ringdown estimate of the remnant properties [31,32] (see Refs. [33–36] for proposed mitigation strategies based on numerical relativity surrogates and postpeak analyses). For instance, unmodeled eccentricity or precession effects in the inspiral-merger analysis could bias the inferred values of M and χ , potentially mimicking a deviation from GR [31,37]. These considerations motivate the search for complementary descriptions of the postmerger signal that do not rely exclusively on a QNM decomposition.

In this context, recent attention has focused on the BH graybody factor, which characterizes the scattering of perturbations by the spacetime curvature surrounding the BH. The graybody factor depends only on the underlying geometry, so it is a natural “no-hair” quantity that can be used to infer M and χ from the observed ringdown waveform. Moreover, graybody factors are stable under small perturbations of the effective potential [38–40], in contrast with the well-known spectral instability of QNMs [20,41–44]. Recent work suggested that the graybody factor modulates the ringdown amplitude in the frequency domain [45]. Although originally investigated within BH perturbation theory for a plunging test particle [38,45,46], this description was later shown to capture remarkably well the spectral amplitude of comparable-mass mergers [47,48].

In this Letter we present GreyRing, a new model for the postmerger ringdown signal based on the graybody factor of the remnant. Building on recent theoretical insights [49], the model accurately reproduces *both* the amplitude and the phase of the frequency-domain postmerger signal, extending previous models that were limited to the amplitude [38,45,46,48]. We show that GreyRing reproduces a large representative set of comparable-mass, aligned-spin numerical relativity waveforms with typical mismatches of order $\mathcal{O}(10^{-6})$ for the dominant quadrupole mode, thus enabling new ringdown tests of gravity with current and future detectors.

GreyRing—The gravitational-wave signal in the time domain can be written as

$$h_+ - ih_\times = \frac{M_T}{d_L} \sum_{\ell m} h_{\ell m}(t) {}_{-2}Y_{\ell m}(\theta, \phi), \quad (1)$$

where h_+ and $h_\times$ denote the plus and cross polarizations of the gravitational wave, M_T is the total mass of the system, d_L is the luminosity distance to the source, and ${}_{-2}Y_{\ell m}$ are spin-weighted spherical harmonics with angular multipole numbers (ℓ, m) , evaluated at the polar angle θ (inclination) and azimuthal angle ϕ specifying the direction of the observer in the source frame. The frequency-domain counterpart of each multipole is $\tilde{h}_{\ell m}(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} h_{\ell m}(t)$. Recently, Ref. [49] showed that the graybody factor is directly encoded in the component of the Green’s function governing the linear ringdown response of a BH in the frequency domain,

providing an analytical foundation for previous phenomenological proposals describing the spectral amplitude of the signal [38,45,46,48]. Based on this theoretical insight, we propose to model each multipole as

$$\tilde{h}_{\ell m}(\omega) = \frac{MA_{\ell m}}{(M\omega)^{p_{\ell m}}} R_{\ell m}(M\omega, \chi) e^{i\frac{c_{\ell m}}{M\omega}} e^{i(\phi_c + \omega t_c)}, \quad (2)$$

where M and χ are the remnant BH’s mass and spin. Here, $R_{\ell m}$ is the complex reflection amplitude, defined in terms of the graybody factor as the ratio between the outgoing and ingoing wave amplitudes in the remnant geometry (see Supplemental Material [50] and [61–68]); ϕ_c and t_c are the phase and time of reference, respectively; and $A_{\ell m}$, $p_{\ell m}$, and $c_{\ell m}$ are source-dependent fitting parameters. In particular, the model for the amplitude,

$$|\tilde{h}_{\ell m}(\omega)| = M \frac{A_{\ell m}}{(M\omega)^{p_{\ell m}}} |R_{\ell m}(M\omega, \chi)|, \quad (3)$$

was previously validated against numerical relativity simulations of binary BH mergers, and $(A_{\ell m}, p_{\ell m})$ were fitted as functions of the progenitor parameters [48]. Here, for the first time, we complete the waveform template by including a model for the phase

$$\arg(\tilde{h}_{\ell m}(\omega)) = \arg(R_{\ell m}(M\omega, \chi)) + \omega t_c + \phi_c + \frac{c_{\ell m}}{M\omega}. \quad (4)$$

The term $\sim c_{\ell m}/(M\omega)$ is the simplest among a family of parametrizations discussed in Supplemental Material [50]. It can be viewed as the leading term in a high-frequency expansion of the residual source-induced dephasing, after the trivial time and phase shifts have been factored out.

Overall, each (ℓ, m) multipole is modeled by seven real parameters: (M, χ, ϕ_c, t_c) , and three source-dependent parameters $(A_{\ell m}, p_{\ell m}, c_{\ell m})$.

Validation against numerical simulations—We validate the GreyRing model against 311 comparable-mass, quasircular, nonprecessing binary BH simulations from the Simulating eXtreme Spacetimes (SXS) Collaboration catalog [69,70], with mass ratios $q \in [1, 4]$. We perform two complementary tests.

First, we assess whether the model can recover the properties of the remnant geometry. To this end, we employ the mismatch

$$\mathcal{M} = 1 - \frac{\langle h_{\ell m}^{\text{NR}} | h_{\ell m}^{\text{model}} \rangle}{\sqrt{\langle h_{\ell m}^{\text{NR}} | h_{\ell m}^{\text{NR}} \rangle \langle h_{\ell m}^{\text{model}} | h_{\ell m}^{\text{model}} \rangle}}, \quad (5)$$

where $h_{\ell m}^{\text{NR}}$ is the numerical relativity waveform, $h_{\ell m}^{\text{model}}$ is given by Eq. (2), and the inner product is defined without detector-noise weighting, i.e.,

$$\langle h_1 | h_2 \rangle = \int_{\omega_i}^{\omega_f} d\omega \tilde{h}_1(\omega) \tilde{h}_2^*(\omega), \quad (6)$$

as our goal is to assess the model's intrinsic accuracy. We minimize $\mathcal{M}$ by varying the seven model parameters. The best-fit values $(M_{\text{fit}}, \chi_{\text{fit}})$ are then compared with the remnant quantities reported in the SXS metadata. Across the catalog, GreyRing recovers the remnant mass and spin with typical relative errors of order 10^{-3} for the dominant (2,2) mode, both in $|M_{\text{fit}} - M_{\text{NR}}|/M_{\text{NR}}$ and in $|\chi_{\text{fit}} - \chi_{\text{NR}}|$, consistent with the accuracy obtained in standard QNM-based remnant-parameter recovery [71,72]. Details of the analysis are presented in Supplemental Material [50].

Second, fixing M and χ to their numerical relativity values, we quantify the intrinsic waveform accuracy of the model through the mismatch between numerical and modeled multipoles. In Fig. 1 we illustrate the performance of the model on the numerical relativity simulation SXS:BBH:3617. The model is fitted over the frequency interval $\omega \in [\omega_i, \omega_f]$, chosen following the same stability criterion introduced in Ref. [48] and here applied consistently to the extended GreyRing model. In the upper panel we show that the model of Eq. (3) accurately reproduces the numerical amplitude $|\tilde{h}_{\ell m}(\omega)|$. In the lower panel we present a similar comparison for the phase. We detrend the numerical phase by subtracting its best-fit linear component, related to the term $\omega t_c + \phi_c$ in Eq. (4). A minimal model in which we similarly detrend the phase of $R_{\ell m}(M\omega, \chi)$ (shown in blue) already exhibits very good agreement. Including the sub-leading term $c_{\ell m}/(M\omega)$ yields excellent agreement between the numerical phase and the model, shown as a dashed orange line, across the entire frequency range.

We do not extremize over t_c and ϕ_c , as these degrees of freedom are already included within the phase model and determined by the fitting procedure. For the case shown in Fig. 1, we find $\mathcal{M} \simeq 2.0 \times 10^{-6}$.

We test the model against 311 SXS simulations of comparable-mass, quasicircular, nonprecessing binary BHs with mass ratios in the range [1,4] [69,70]. In Fig. 2 we show the mismatch distribution across the catalog for the $\ell = m = 2, 3, 4$ multipoles. For $(\ell, m) = (2, 2)$, GreyRing achieves mismatches in the range $\mathcal{M} \in [3 \times 10^{-8}, 5 \times 10^{-5}]$, and typically $\mathcal{M} = \mathcal{O}(10^{-6})$. The mismatch for $(\ell, m) = (3, 3)$ and $(\ell, m) = (4, 4)$ is at the level of $\mathcal{O}(10^{-5})$ and $\mathcal{O}(10^{-3})$, respectively. While the reduced accuracy may partly reflect the numerical precision of the SXS catalog, we also expect that incorporating effects currently neglected in our model—such as spherical-spheroidal mode mixing [73] and nonlinear mode coupling [74–76]—will further improve the agreement for higher-order multipoles. Nevertheless, when the modeled multipoles are combined to reconstruct the full angular dependence of the waveform, the mismatch remains typically $\mathcal{O}(10^{-5})$ because the signal is dominated by the (2,2)

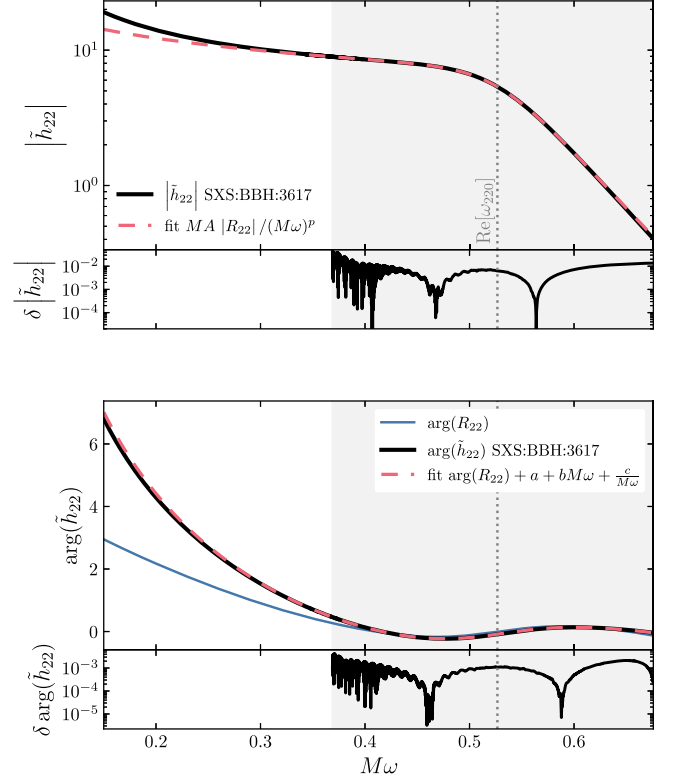


FIG. 1. Fit of the GreyRing model to the postmerger quadrupolar signal of a comparable-mass binary BH simulation (SXS:BBH:3617). Top panel: amplitude of the Fourier-domain waveform (black), compared with the model in Eq. (3) (dashed orange). Bottom panel: detrended phase of the numerical signal (black) and of the graybody factor R_{22} (blue), together with the fitted model including the $c_{\ell m}/(M\omega)$ correction (dashed orange); see Eq. (4). The dashed orange curves are also extrapolated outside the fitting window (gray shaded region) toward lower frequencies, where they still provide a reasonable description of the numerical signal, especially for the phase. As a reference, the dotted vertical line marks the real part of the fundamental QNM. In the bottom inset of each panel, we plot the residual $\delta X = |X_{\text{SXS}} - X_{\text{fit}}|$ for the corresponding quantity X .

mode. A detailed discussion is provided in Supplemental Material [50].

The mismatches for the present model are at least one order of magnitude lower than for the amplitude-only model [48]. Remarkably, typical GreyRing model mismatches are comparable to (or better than) state-of-the-art time-domain models calibrated using numerical relativity simulations [24], such as the TEOBPM template [77–80].

Tests of gravity with GreyRing—The high accuracy of our model enables a novel agnostic test of GR: we can use GreyRing to infer M and χ from the frequency-domain signal, and compare the resulting posteriors with those obtained with a standard QNM-based ringdown template. This is conceptually similar to inspiral-merger-ringdown consistency tests, which infer the remnant properties from the progenitor binary using fits calibrated to numerical

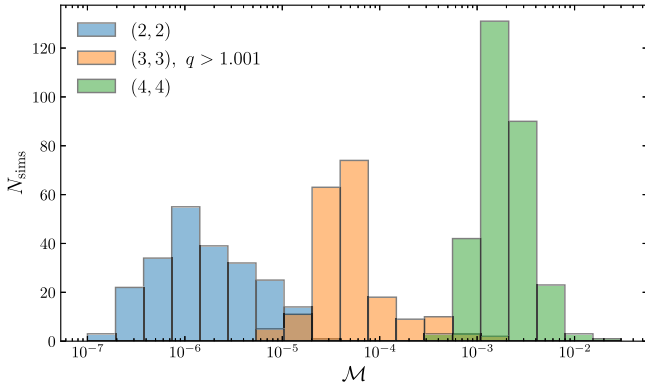


FIG. 2. Mismatch distribution of the GreyRing model for the $(\ell, m) = (2, 2), (3, 3)$, and $(4, 4)$ multipoles over 311 comparable-mass, spin-aligned binary BH simulations from the SXS catalog [69,70]. For the $(3, 3)$ mode, we restrict to the subset of simulations with mass ratio $q > 1.001$, because the mode amplitude vanishes in the equal-mass limit.

relativity simulations in GR [14]. Unlike those tests, our approach relies exclusively on the postmerger signal, and it does not require connecting the inspiral to the ringdown via numerical relativity modeling. Moreover, a consistency test can be performed by extracting (M, χ) using only the $\ell = m = 2$ multipole in GreyRing and only the fundamental $\ell = m = 2$ QNM in standard BH spectroscopy, without the need for higher modes [10,81] or overtones [25,26,28,33,71,72,82–87]. This is a significant advantage because (i) $\ell > 2$ modes are generically harder to measure; (ii) for highly symmetric binaries—including GW250114 [23,88], which has enabled the most stringent ringdown tests to date [15]—odd- m multipoles are strongly suppressed; and (iii) multimode QNM fits are subject to ambiguities in the starting time and in the number of basis elements retained in the template. These become especially relevant when overtones are included, since the nonorthogonality of the fitted basis can lead to overfitting [26,72,89].

As a first step, we have applied this test by performing Bayesian parameter estimation on synthetic data (see Supplemental Material [50] for details). We assume nonprecessing binaries, so that $\pm m$ multipoles are related by $\tilde{h}_{\ell m}(\omega) = (-1)^\ell \tilde{h}_{\ell -m}^*(-\omega)$. We consider the GW250114-like numerical simulation SXS:BBH:3617 and perform injection-recovery tests with two independent models. We first generate injections using the QNM model calibrated to the simulation and perform a standard QNM analysis with PYCBC INFERENCE [90]. We inject QNMs with physical parameters consistent with GW250114 in the $\ell = m = 2$ multipole and recover the posterior distributions of (M, χ) .

Then, we perform an independent parameter estimation using GreyRing, in which we inject a signal constructed from the same simulation and recover it using only the frequency-domain $\ell = m = 2$ multipole. As expected for consistency, the posteriors on (M, χ) recovered in this case

are compatible with those from BH spectroscopy. Furthermore, the uncertainties of the two methods are similar.

Application to GW250114—We apply the GreyRing consistency test to real data, focusing on GW250114. For this event (the loudest detected so far) the real and imaginary parts of the $\ell = m = 2$ fundamental QNM frequency were measured at the level of 6% and 10%, the first $\ell = m = 2$ overtone frequency was measured with an accuracy of approximately 30%, and there is some evidence for other modes in the ringdown [23,88].

Our main results are shown in Fig. 3, where we apply GreyRing to the Fourier transform of the GW250114 time series. The resulting posteriors (blue contours) are compared with the standard LIGO-Virgo-KAGRA ringdown analysis using either the fundamental $\ell = m = 2$ mode alone (red) or a model including also the first overtone (green) [23,88,91]. The ringdown analysis starts at $t_{\text{start}} = 13M$ or $t_{\text{start}} = 8M$ after the peak of the strain in the two cases, respectively, since including the first overtone better captures the early postmerger phase [25,71]. The GreyRing posteriors are compatible with (and slightly narrower than) those from the standard ringdown analysis [23,88], yielding $(1+z)M = 66.9^{+6.1}_{-6.4} M_\odot$ and $\chi = 0.69^{+0.10}_{-0.16}$ (90% credible intervals) for the final detector-frame mass and spin,

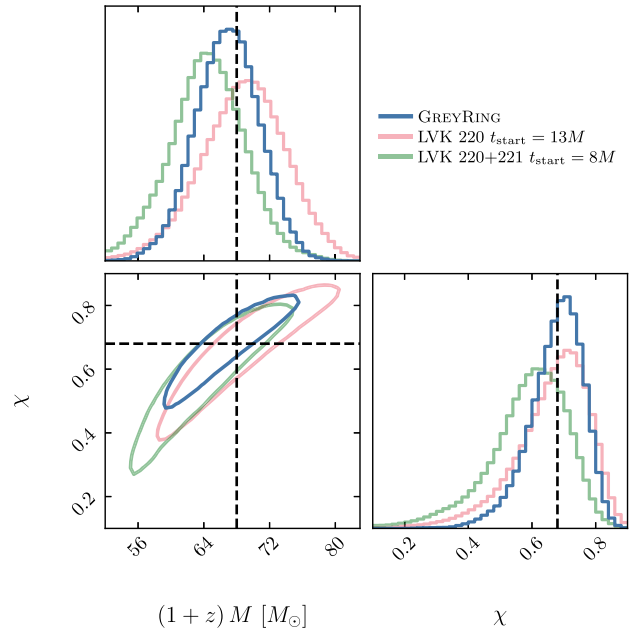


FIG. 3. 90% credible region of the joint posterior distribution for the redshifted remnant mass and spin of GW250114. The blue contour corresponds to our graybody-factor model applied in the frequency range $f \in [100, 512]$ Hz. The red and green contours show the results of a standard QNM analysis performed with pyRing [92] using the 220 mode and the 220 + 221 modes, respectively [23,88,91]. Black dashed lines mark the maximum-likelihood estimate from the LIGO-Virgo-KAGRA analysis with the NRSur7dq4 inspiral-merger-ringdown waveform model [93].

respectively. The lower cutoff frequency in Fig. 3 is chosen using the stability criterion of Ref. [48], applied here to real-data posteriors: we select the range where the inferred remnant parameters are stable under changes in $f_{\min}$ and not yet affected by the loss of high-frequency signal power. The Supplemental Material [50] presents the full corner plots, an independent validation of the standard ringdown analysis, and variations of both t_{start} and the frequency range, showing that the GreyRing posteriors on M and χ remain compatible and stable as $f_{\min}$ is varied, unlike the stronger starting-time dependence of standard QNM spectroscopy.

Discussion—Our graybody-factor model opens up a new class of tests of gravity in the ringdown regime. Although we have focused here on the dominant $\ell = m = 2$ mode, analogous tests can be performed with subdominant harmonics, in parallel with standard QNM-based studies.

Compared to ordinary BH spectroscopy, GreyRing offers several key advantages: (i) the graybody factor encodes the full linear response of the remnant in the frequency domain and does not require a QNM decomposition. This effectively resums *all* overtones and captures features of the response that are not included in a QNM expansion [49,94–99], such as late-time tails [49]. (ii) Unlike QNMs, the graybody factor is a complex, frequency-dependent function. The consistency tests proposed here probe the *entire* functional form of $R_{\ell m}(M\omega, \chi)$ rather than a discrete set of frequencies and damping times, thereby exploiting more of the signal’s information to reconstruct a no-hair property of the remnant. (iii) Compared to standard QNM measurements, the graybody factor captures a broad frequency range, including relatively high frequencies. Therefore, it is expected to be more sensitive to short-length-scale modifications in the strong-field regime, such as higher-order curvature corrections to GR [100] or horizon-scale physics [40,46].

The GreyRing model can be extended to perform additional consistency tests of gravity that are complementary to (and independent of) traditional inspiral-merger-ringdown tests. For example, a less conservative implementation of the agnostic test proposed in this Letter would rely on numerical relativity simulations to map the progenitor masses and spins to the model parameters $(A_{\ell m}, p_{\ell m}, c_{\ell m})$ along the lines of Ref. [48], thus enabling consistency checks with the information extracted from the inspiral signal.

The increasing sensitivity of current detectors, together with the prospects of next-generation ground- and space-based observatories such as the Einstein Telescope [101–103], Cosmic Explorer [104,105], and LISA [106], strongly motivates the development of accurate and robust frameworks to parametrize and constrain deviations from the Kerr metric in the ringdown regime. Systematic effects will be dominant at large signal to noise, particularly relevant in the context of ringdown, where modeling

uncertainties in the starting time and in the number of modes can significantly bias the inference [107,108]. In this context, approaches that minimize such systematics are especially valuable [31,109].

GreyRing achieves mismatches of order $\mathcal{O}(10^{-6}-10^{-5})$ for signals including modes up to $\ell = m = 4$, typically outperforming state-of-the-art time-domain models [24]. Using the conservative criterion that two waveforms are indistinguishable for parameter-estimation purposes when $\mathcal{M} \lesssim (2 \text{ SNR}^2)^{-1}$ [110,111], where SNR is the signal-to-noise ratio, this level of accuracy is already sufficient for ringdown signals with $\text{SNR} \sim 100-1000$, as those expected for next-generation detectors [112] and (in optimistic scenarios) for LISA [106,113,114].

Several extensions of our work are possible and desirable. The model should be validated and generalized to precessing and eccentric binaries, possibly by developing fitting formulas for the model parameters that extend the analysis of Ref. [48]. Another important direction is the inclusion of spheroidal-spherical mode mixing [73] and nonlinearities [74–76], which is relevant for rapidly spinning remnants and in the high-frequency regime. It will be interesting to attach the model to inspiral-merger waveforms in the frequency domain, such as those of the IMRPhenom family [115,116] or reduced-order frequency-domain implementations of effective-one-body models [117–119], and to quantify the resulting overall faithfulness. Finally, it is important to extend GreyRing to beyond-GR theories, complementing ordinary BH spectroscopy models beyond GR. Some of this work is currently underway and will be presented elsewhere.

Inference simulations were carried out with PYCBC INFERENCE [90]. Parameter estimation was performed in Bilby [120]. The manuscript content has been derived using publicly available software: MATPLOTLIB, CORNER, JSON, and NUMPY [121–124].

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Data availability—The data supporting the findings of this study are publicly available in Ref. [125]. The code used in this work is publicly available.

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