

Phase-Dependent Squeezing in Dual-Comb Interferometry

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Manipulating the quantum noise of continuous-wave lasers through squeezing has reshaped optical interferometry. However, progress in optical frequency comb interferometry with pulsed squeezed sources has been limited, despite the role of frequency combs in ultraprecise optical metrology. Here, we introduce a new time-domain approach to characterizing squeezed femtosecond light pulses using dual-comb interferometry. Time-domain interferograms are generated via multiheterodyne beating between the modes of a Kerr soliton-squeezed frequency comb and a coherent state comb. The interferogram noise reveals phase-dependent squeezing and antisqueezing, dipping as much as 3.8 ± 0.2 dB below the shot noise level at alternating zero crossings. We model this nonstationary quantum noise as a periodic optical displacement of the squeezed comb by the coherent comb. These results support a route toward quantum-enhanced dual-comb timing applications and high-speed quantum state tomography with dual-comb interferometers.

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Overcoming the intrinsic quantum noise of light for precision interferometry has been a long-standing challenge in laser physics and optical metrology. While techniques for squeezing the quantum noise distribution between the amplitude and phase quadratures of a continuous-wave (CW) laser field were introduced four decades ago [1–5], interferometric gravitational wave detection below the standard quantum limit was not demonstrated until recently [6]. In parallel to these developments, optical frequency combs based on pulsed mode-locked laser sources have emerged over the past 25 years as powerful frequency metrology tools for ultraprecise optical clocks and atomic or molecular spectroscopy [7,8]. Although squeezed pulsed laser sources have been studied [9–12], the application of pulsed squeezed states in comb interferometry and femtosecond timekeeping remains an open area of investigation.

A highly effective approach to comb interferometry involves multifrequency heterodyne comparison between pairs of combs [13]. This technique, also known as

dual-comb interferometry (DCI), down-converts the optical comb spectrum to accessible and traceable radio frequency (rf) signals. DCI has found widespread application to distance ranging [14], free space optical time transfer [15,16], holography [17], microimaging [18,19], fiber-based optical networking [20], and spectroscopy [21]. However, only in recent years has DCI reached the quantum limit associated with coherent state combs [16,22]. Theoretical proposals [23–25] and some initial experiments [26–30] have shown that squeezed combs can provide quantum metrological advantages. However, the observation of phase-dependent quantum noise arising from squeezing in time-domain DCI measurements has yet to be reported.

A DCI experiment consists of two mutually coherent comb sources with slightly different repetition rates ($f_{\text{rep},1}$ and $f_{\text{rep},2} = f_{\text{rep},1} + \Delta f_{\text{rep}}$) that interfere on a photodiode to yield an interferogram (IGM) signal that is periodic at Δf_{rep} . For spectroscopy, the IGM is Fourier-transformed to recover an rf comb whose amplitude is proportional to the geometric mean of the individual comb spectra [21]. In time transfer and ranging, the IGM centerburst is cross-correlated and phase-demodulated to yield timing information [31]. Using a second comb as a tracking oscillator with $\Delta f_{\text{rep}} = 0$, dual-comb timing measurements achieve quantum-noise-limited operation by constantly measuring timing jitter at a zero crossing of the interferogram (IGM) carrier fringe [16,22].

Nonstationary quantum noise in dual-comb metrology can be analyzed using a formalism that incorporates the

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role of temporal mode matching [32]. This treatment separates DCI into a quadraturelike measurement regime for small optical path differences near the centerburst region of the IGM and an intensitylike measurement regime for large optical path differences. Early experiments recognized that the quantum noise of pulsed lasers is inherently nonstationary and that temporal gating of optical pulses can reveal squeezed states with improved contrast [11]. Related techniques allow for improved classical noise limits for comb measurements including microwave generation [33], CW-comb heterodyne comparison [34] and dual-comb spectroscopy [35]. Recently, we utilized amplitude squeezing to gain a ~ 3 dB metrological advantage in dual-comb spectroscopy by reducing quantum noise in the intensitylike measurement regime [28]. This quantum noise reduction allows higher signal-to-noise ratios at large optical path differences where the molecular free induction decay signal is present [36].

In this Letter, we introduce a squeezed DCI measurement scheme and demonstrate its potential to impact dual-comb time transfer, ranging, and networking by revealing sub-shot-noise-limited statistics on the carrier fringes of a dual-comb interferogram for small optical path differences. This new form of time-domain pulsed squeezing measurement can also be applied to characterize laser quantum states generated by optical nonlinearities [37]. And while nonlinear electro-optical sampling can probe ultrafast quantum states [26,38], our results show that linear optical sampling with dual combs extracts similar information.

To demonstrate phase-dependent squeezing in the centerburst of a dual-comb signal, we use a method that resembles a quadrature measurement [see Figs. 1(a) and 1(b)]. We generate a squeezed comb using the $\chi^{(3)}$ Kerr nonlinearity in a polarization-maintaining highly nonlinear fiber (PM-HNLF). Because the Kerr effect is number preserving, the squeezing angle of an undisplaced Kerr state yields the same amplitude noise as the original coherent state before Kerr interaction [39]. The squeezed quadrature of the Kerr state is revealed through a small optical displacement followed by intensity detection of the resulting interferogram [see Fig. 1(c)] [40]. For shot-noise-limited laser sources, squeezing can be measured with a single photodiode [28]. Here, we displace a squeezed pulse using a field derived from an entirely independent laser system [see Fig. 1(d)].

To accomplish the optical displacement operation, we inject two cross-polarized combs into a PM-HNLF (see Fig. 2) at a power ratio of 10:1. Both combs have repetition rates near 1 GHz, are centered at 1563 ± 1 nm, and are temporally compressed in fiber to ~ 260 fs. This geometry minimizes noncommon technical noise with the stronger comb experiencing significant Kerr squeezing, while the weaker comb remains approximately in a coherent state. Both combs are phase locked to a narrow linewidth 1564 nm CW laser with feedback to cavity piezoelectric

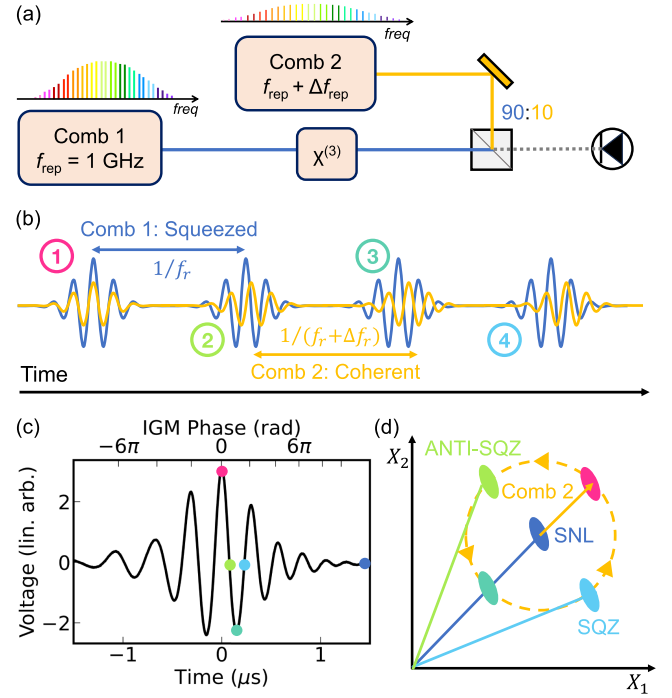


FIG. 1. Conceptual schematics. (a) Simplified experimental block diagram and frequency-domain picture of multiheterodyne between a $\chi^{(3)}$ -squeezed comb and a coherent state comb. (b) Time-domain pulse pair picture of DCI with subsequent pulses overlapping with linearly varying phases labeled by ①–④. (c) Interferogram (IGM) generated from the interfering pulse pairs, including phase markers at 0 radians (red), $\pi/2$ radians (green), π radians (teal), $3\pi/2$ radians (light blue), and large path difference (dark blue). These phases correspond to the cases 1–4 of (b). (d) Phase space representation indicating antisqueezed state (ANTI-SQZ; green), squeezed state (SQZ; light blue), IGM maximum (red), IGM minimum (teal), and the shot-noise-limited Kerr state at large path differences (SNL; dark blue). X_1 and X_2 refer to the bosonic quadrature operators. Note that the displacement phase is given by the differential optical phase between the pulse pairs, which maps to the interferogram phase.

actuators with less than 100 kHz bandwidth. The repetition rate difference of the comb lasers is set to ~ 700 Hz. After PM-HNLF, the combs are recombined with a power ratio of 100:1.

A single high quantum efficiency InGaAs photodiode is used to detect the dual-comb IGMs, which are centered at 3 MHz and span 1 MHz to 5 MHz in the rf domain. The IGMs are low-pass filtered at 225 MHz and then split into two signals. The first signal is amplified using a 20 dB gain rf amplifier and recorded using a 500 MS/s analog-to-digital converter (ADC). The second signal is high-pass filtered at 25 MHz to remove the IGM itself, and the high frequency noise is then amplified using a low-noise rf amplifier and recorded on a second ADC channel. We refer to the second signal as the “noise-gram” (NGM). The low IGM center frequency allows us to oversample the IGM fringe and fully capture the minimum values of the

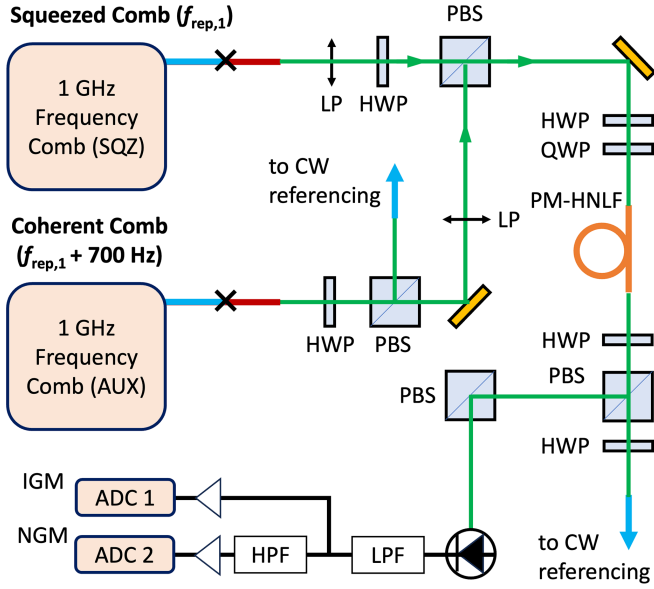


FIG. 2. Experimental schematic. Two 1 GHz combs are optically isolated and compressed to the transform limit in polarization-maintaining normal dispersion (PM-ND) fiber. Polarization optics combine the combs with a variable splitting ratio and send cross-polarized combs onto orthogonal axes of a PM-HNLF. The two combs are recombined on a half-wave plate (HWP) and polarization beam splitter (PBS). The IGM is recorded on a photodiode and digitized using a high-speed analog-to-digital converter (ADC). Both combs share the common phase reference from a narrow linewidth CW laser. LP: linear polarizer; QWP: quarter-wave plate; LPF: low-pass rf filter; HPF: high-pass rf filter; blue/red/orange lines indicate PM-1550/PM-ND/PM-HNLF fiber; green lines indicate free space optical path; Black x's indicate fiber splice; black lines indicate rf signal.

quantum noise. Because the carrier-envelope offset (CEO) frequencies of our combs are not stabilized, we postselect only the IGMs that have similar CEO phases.

A continuous stream of 730 IGMs and NGMs is acquired on two ADC channels. We perform a Hilbert transform on the individual IGMs and then cross-correlate each IGM against the first IGM to align them based on their envelopes. The delays calculated from the IGM cross-correlation are also applied to the NGMs. We then downselect the IGMs and NGMs by the CEO phase calculated using Hilbert transformation. This selection process yields 50 IGMs within a $\Delta\phi_{\text{ceo}} = \phi_{\text{ceo},1} - \phi_{\text{ceo},2}$ range of < 50 mrad. After CEO binning, we take the standard deviation along the IGM number axis, i.e., the standard deviation of points from every IGM at the properly aligned time [see Fig. 3(a)]. The high frequency NGM is known to be dominated by shot noise at large path differences [28], and this noise is well above the electrical noise floor, which allows us to derive the measurement shot-noise limit (SNL) from these data. Zooming in on the IGM centerburst region, the NGM exhibits nonstationarity that is phase coherent with the IGM.

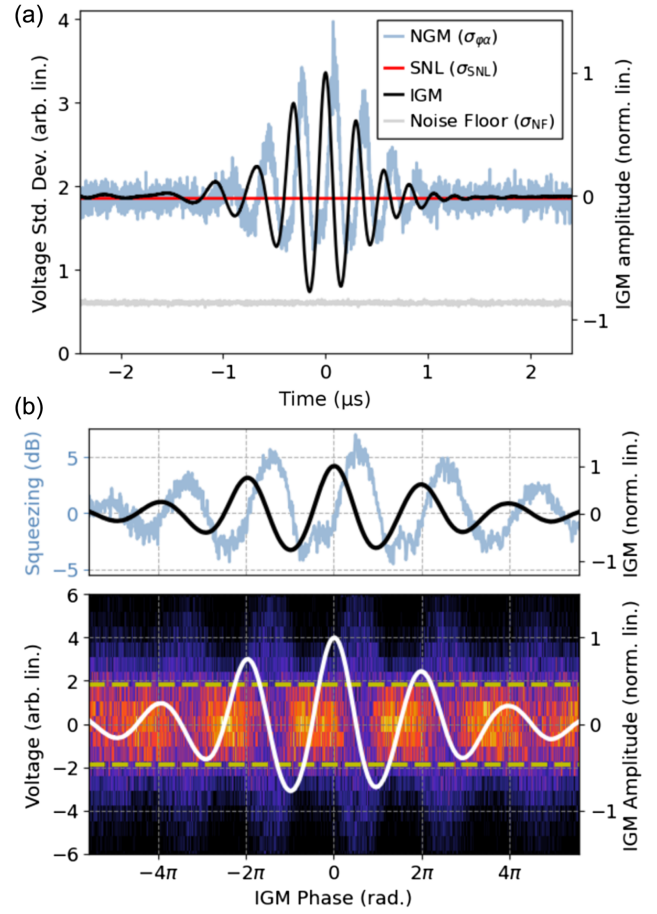


FIG. 3. Time-dependent quantum noise fluctuations during a dual-comb IGM. (a) Quantum noise reduction and amplification are visible during the centerburst. Blue, NGM; black, normalized IGM; red, shot-noise limit (SNL); gray electrical noise floor. The NGM, SNL, and noise floor are plotted as voltage standard deviation, while the IGM is plotted as a normalized voltage. (b) Enlarged image of the centerburst showing the squeezing factor V_{SQZ} plotted in dB relative to the SNL versus unwrapped IGM phase. Normalized IGM is shown in black. Below the squeezing is a false color histogram of NGM voltage counts. Brighter colors indicate larger counts. Dotted yellow line, SNL; white line, normalized IGM. The detected optical power for Comb 1 (Comb 2) is 14.6 mW (170 μ W).

We quantify the squeezing factor (V_{SQZ}) as

$$V_{\text{SQZ}} = \frac{\sigma_{\varphi\alpha}^2 - \sigma_{\text{NF}}^2}{\sigma_{\text{SNL}}^2 - \sigma_{\text{NF}}^2}, \quad (1)$$

where $\sigma_{\varphi\alpha}^2$ is the IGM variance at a given IGM phase φ_α , σ_{SNL}^2 is the SNL variance at large path differences, and σ_{NF}^2 is the variance of the electrical noise floor. The squeezing results shown in Fig. 3(b) are plotted versus the demodulated, unwrapped, and averaged IGM phase axis. The maximum measured squeezing level is 3.8 ± 0.2 dB, which is close to the level measured using single-comb

self-displacement [28]. The squeezing level is limited by system optical loss ($\sim 25\%$) and parasitic nonlinearities in the optical fiber [41]. We observe a maximum antisqueezing of ~ 6 dB. In Fig. 3(b), we also plot the full histogram of NGM values for each IGM phase, clearly showing the changing statistics as the IGM phase evolves. Of particular note is the appearance of squeezing and antisqueezing at alternating zero crossings of the IGM. This form of nonstationarity should only be present if the displaced state is squeezed. A small amount of nonstationarity is to be expected during an IGM as the power on the detector changes slightly, but for our comb power ratio (14.6 mW from Comb 1 and 170 μ W from Comb 2) this effect is small. For an unbalanced measurement, the varying SNL is expected to be directly correlated with the IGM level rather than shifted to the zero crossings.

The observed statistics can be explained using a simple model of the displacement operation that incorporates the pulse width normalized to the duty cycle of the lasers, the measured squeezing level from the single comb experiment, and the evolving mode mismatch across the IGM (see Supplemental Material, Note I [42]). Specifically, the model predicts that the NGM voltage variance, $V = \sigma_{\varphi\alpha}^2$, takes the following form:

$$V = |\eta|^2 |\beta + \gamma\alpha|^2 (2\sinh^2(r) - \sinh(2r) \cos[2(\phi - \varphi)]) + |\eta|(|\beta|^2 + |\alpha|^2 + 2\text{Re}(\beta^*\gamma\alpha)), \quad (2)$$

where η is detection efficiency (including losses), α is the weak coherent state amplitude (displacing state), β is the strong squeezed state amplitude, r is the squeezing parameter, γ is the mode overlap of the pulses, ϕ is the squeezing angle, and φ is the phase angle of the mean field state. The mean field phase can be written in terms of the phase of the α state (Comb 2), which also acts as the IGM phase (φ_α) for a frame that corotates with the β state (Comb 1):

$$\varphi \approx \frac{|\gamma\alpha| \sin(\varphi_\alpha)}{|\beta|}. \quad (3)$$

Equation (2) predicts a “bifurcation point” in the NGMs as the ratio $R = |\gamma\alpha|/|\beta|$ is varied, which is viewed geometrically in Fig. 4(a). For small R , the intensity noise-gram has two extrema per 2π of displacement phase [see Fig. 4(b)]. This statement is confirmed by Taylor expanding the φ -dependent term in Eq. (2) for small R :

$$\cos[2\phi - 2R \sin(\varphi_\alpha)] \approx \cos(2\phi) + 2R \sin(2\phi) \sin(\varphi_\alpha). \quad (4)$$

As R increases, two more extrema will appear (four total extrema per IGM cycle), resulting in frequency doubling of the NGM [see Figs. 4(c) and 4(d)].

To confirm the model prediction, we conducted a series of measurements using a dual-detector setup where we

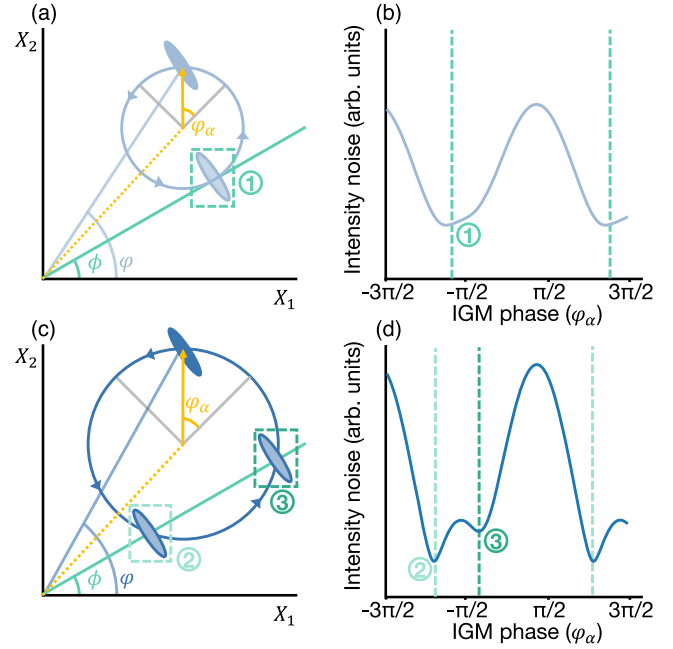


FIG. 4. Geometric picture of power-dependent noise properties in optical displacement operation. (a) Phase space (Wigner representation) schematic demonstrating the time-varying noise within the dual-comb IGM. The light blue phase space path has intensity noise with one local maximum and one local minimum. The teal line indicates the squeezing angle and sets the phase angle for local minima of the amplitude variance. (b) Intensity noise of phase space path from (a) with teal vertical dotted lines indicating the local minima points. (c) The darker blue phase space path (after the bifurcation point) has intensity noise with two local minima/maxima. (d) Intensity noise of phase space path from (c).

performed a photocurrent subtraction/addition in postprocessing over a range of power ratios. The setup follows Fig. 2 except an additional 50:50 beam splitter is placed before the photodetector. Two identical high quantum efficiency photodetection chains record the signals at each of the beam splitter outputs. As before, the signals are separated into IGM/NGM filtered components. A four-channel ADC is used to record the two IGMs and two NGMs. The detection chains use matched cable lengths and components to minimize relative delays, which is confirmed by cross-correlating the IGM recorded on each channel. After alignment, summing the two NGM channels yields the NGM measured from a single-channel measurement, and differencing the two NGM channels yields the fringe-varying SNL.

The dual-detector measurements are shown in Figs. 5(a)–5(d). We clearly see the frequency doubling associated with the bifurcation. We notice that the squeezing is reduced after the bifurcation point although the antisqueezing dramatically increases to > 9 dB (see Supplemental Material, Note II [42]). The highly nonclassical behavior of the noise is nearly completely

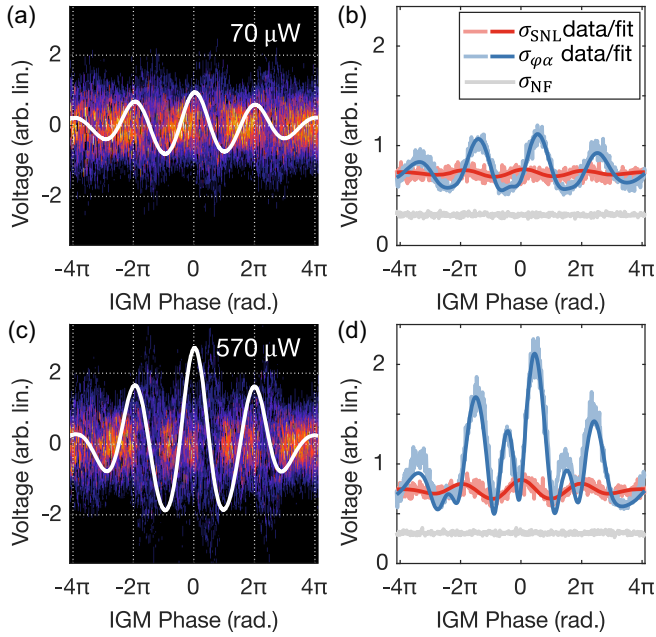


FIG. 5. Dual-detector measurements of quantum noise reduction in dual-comb centerbursts. (a) False color histogram of noise-gram (sum) voltage counts for coherent power of $70\ \mu\text{W}$ and squeezed power of $15\ \text{mW}$. The white line is the average interferogram, and brighter colors indicate higher voltage counts. (b) Corresponding voltage standard deviations for power ratio from (a). A sinusoidal nonstationary noise-gram pattern in the sum voltages (blue) is observed with one local maximum per 2π . The time-dependent SNL is calculated from the difference voltage (red). Fits to the data are shown as dark lines. The noise floor of the measurement is shown in gray. (c) Same as (a) for coherent power of $570\ \mu\text{W}$ and squeezed power of $15\ \text{mW}$. (d) Corresponding voltage standard deviations for power ratio from (c). A nonstationary pattern is observed in the sum voltages (blue) with two local maxima per 2π . A larger variation of the SNL is also measured.

captured by our simple model given by Eq. (2). The model deviates from the data as the mode mismatch between the pulses increases, possibly due to the effects of temporal chirp and spectral mismatch. However, this effect may also arise from the distribution of CEO phases of our down-selected IGMs.

This demonstration of phase-dependent quantum noise in a dual-comb IGM is a critical step toward the development of quantum-enhanced dual-comb timing systems. With time-programmable combs [16,22], we can control the phase difference between the squeezed comb and coherent state comb such that the coherent state always displaces the squeezed comb to reveal sub-SNL behavior on the IGM fringe [57]. When a coherent state interferes with a squeezed comb, less optical power is required to achieve the same SNR on the time-programmable IGM fringe signal. This property enables low-power optical timing systems without the need for amplification, which could introduce additional noise and system complexity.

For most timing and ranging applications, a balanced homodyne measurement setup is required [27]. Quantum advantage in future balanced homodyne measurements can be achieved by using a weak squeezed comb as a sensing probe and a strong coherent state as a local oscillator (LO) [58]. Balanced homodyne dual-comb measurements also constitute a new method for continuous-variable quantum state tomography [59] (see Supplemental Material, Note III [42]).

In summary, we introduced DCI as a valuable tool for quantum metrology, using the technique to characterize the phase dependence of pulsed squeezing with two independent combs. With this advance, we establish the ability to arbitrarily scan the relative phase between a quantum comb and a coherent state comb. Replacing differential optical path changes generated by piezoelectric transduction or electro-optic modulation with dual-comb scanning allows for high-speed time-programmable quantum measurement with long, arbitrary phase delays [22]. Quantum DCI should lead to improvements for comb-based timing, ranging, and networking. DCI also provides a promising method for characterizing exotic quantum states generated via extreme nonlinear optical interactions [60].

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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