Bag Parameters for Heavy Meson Lifetimes from a Lattice QCD Calculation at the $SU(3)_F$-symmetric Point

Matthew Black,[1,2,*] Robert V. Harlander,[3] Jonas T. Kohnen,[3] Fabian Lange,[4,5] Antonio Rago,[6] Andrea Shindler,[3,7,8] and Oliver Witzel[9]

[1]*Higgs Centre for Theoretical Physics, School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom*

[2]*Theoretische Physik 1, Center for Particle Physics Siegen, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany*

[3]*Institute for Theoretical Particle Physics and Cosmology, RWTH Aachen University, 52056 Aachen, Germany*

[4]*Physik-Institut, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland*

[5]*PSI Center for Neutron and Muon Sciences, 5232 Villigen PSI, Switzerland*

[6]*IMADA and Quantum Theory Center, University of Southern Denmark, Odense, Denmark*

[7]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

[8]*Department of Physics, University of California, Berkeley, California 94720, USA*

[9]*Theoretische Physik 1, Center for Particle Physics Siegen, Naturwissenschaftlich-Technische Fakultät, and Cluster of Excellence "Color meets Flavor," Universität Siegen, 57068 Siegen, Germany*

(Received 16 April 2026; revised 15 July 2026; accepted 23 September 2026; published 5 November 2026)

We calculate the dimension-six $\Delta Q = 0$ four-quark matrix elements describing heavy-meson lifetime ratios using the gradient flow with its short flow-time expansion (SFTX) as a renormalization procedure. On six RBC/UKQCD $2+1$-flavor domain-wall fermion ensembles, we determine flowed bag parameters for physical charm and strange quarks and match to the $\overline{\text{MS}}$ scheme with perturbative SFTX coefficients through next-to-next-to-leading order (NNLO). A multiscale matching procedure using renormalization-group running improves the extrapolation to zero flow time. For the operators relevant to $\tau(D_s)/\tau(D^0)$ at the $\text{SU}(3)_\text{F}$ symmetric point, we obtain $B_1^{\overline{\text{MS}}}(3\text{ GeV}) = 1.0524(97)$, $B_2^{\overline{\text{MS}}}(3\text{ GeV}) = 0.9621(70)$, $\epsilon_1^{\overline{\text{MS}}}(3\text{ GeV}) = -0.2275(76)$, and $\epsilon_2^{\overline{\text{MS}}}(3\text{ GeV}) = -0.0005(8)$ using a specific choice of evanescent operators. This is the first lattice-QCD determination of $\Delta Q = 0$ four-quark operators with a full error budget. It opens the path toward higher-precision predictions of heavy-meson lifetimes and similar quantities exhibiting operator mixing under renormalization.

DOI: 10.1103/2vg9-34y9

Introduction—The physics of heavy hadrons provides insight into several crucial aspects of the standard model (SM). While the binding of the quarks is governed by strong interactions, mixing and decay of the hadrons is induced by weak interactions, with strong sensitivity to the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements. The comparison of precise measurements and theoretical predictions of the heavy-hadron properties, thus, provides crucial input for the indirect search of physics beyond the SM. A summary of the theoretical and experimental status can be found in Refs. [1–3].

Theoretical predictions of heavy-hadron properties need to account for both perturbative (short-distance) and nonperturbative (long-distance) effects. A suitable framework for this is provided by effective field theories, where the perturbative effects are captured by the Wilson coefficients, while the determination of the associated matrix elements requires nonperturbative methods.

To date, sum-rule techniques have been used to estimate these nonperturbative effects for lifetimes and lifetime ratios in the framework of heavy-quark effective theory (HQET) [4–9]. Lattice field theory (LFT) calculations have so far delivered phenomenologically relevant results for short-distance contributions to heavy-meson mixing [10–18], but, despite similarities in the calculation and early work [19–21], LFT is yet to provide the nonperturbative input for heavy-meson lifetimes with full error budgets. An alternative approach using HQET has been proposed in Ref. [22].

*Contact author: matthew.black@ed.ac.uk

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

In this Letter, we consider D_s mesons, which constitute an ideal laboratory for establishing and validating our method to nonperturbatively renormalize the matrix elements on the lattice. The gradient flow is used for the operator renormalization and enables us to take the continuum limit. Subsequently, we convert the results to the $\overline{\text{MS}}$ scheme using perturbatively evaluated matching coefficients. In this proof-of-principle calculation, we focus on lifetime ratios in the limit of SU(3)-flavor symmetry between u, d, and s quarks, which provide clean probes of the leading spectator effects in the heavy-quark expansion (HQE). In these ratios, contributions due to so-called "eye diagrams" and mixing with lower-dimensional operators cancel up to $\text{SU}(3)_\text{F}$-breaking effects. The effect of eye diagrams was found to be $O(0.2\%)$ in HQET using the sum-rules approach, and the strange quark-mass corrections were found to be $O(0.1\%)$ [7,8]. The error budget quoted in this work applies to the bag parameters computed in this $\text{SU}(3)_\text{F}$-symmetric setup, while a direct lattice calculation of the corresponding $\text{SU}(3)_\text{F}$-breaking corrections is left to future work.

Nevertheless, our approach is applicable also to the calculation of absolute lifetimes in principle, even if some of the steps may require additional efforts in order to reach similar precision. The analogous calculation for the simpler case of heavy-meson mixing and further details on the lifetime calculation are presented in the companion paper [23], while preliminary results have been presented in Refs. [24–26].

Operators and bag parameters—Within the heavy-quark expansion (HQE) [27,28], the lifetime of a heavy hadron H is governed by the imaginary part of the forward $H \to H$ matrix elements of local $\Delta Q = 0$ operators. The leading spectator effects incorporating the meson's light valence quark arise at order $1/m_Q^3$ and are given by the following dimension-six four-quark operators (see, e.g., Refs. [4,29]):

$$\begin{aligned}
\mathcal{O}_1 &= [\bar{Q}\gamma_\mu(1-\gamma_5)q][(\bar{q}\gamma^\mu(1-\gamma_5)Q], \\
\mathcal{O}_2 &= [\bar{Q}(1-\gamma_5)q][\bar{q}(1+\gamma_5)Q], \\
T_1 &= [\bar{Q}\gamma_\mu(1-\gamma_5)T^A q][\bar{q}\gamma^\mu(1-\gamma_5)T^A Q], \\
T_2 &= [\bar{Q}(1-\gamma_5)T^A q][\bar{q}(1+\gamma_5)T^A Q],
\end{aligned} \tag{1}$$

where the T^A are the generators of SU(3) color. A leading order (LO) Feynman diagram contributing to the forward matrix elements of these operators is shown in Fig. 1(a). These matrix elements are parametrized as

$$\begin{aligned}
\langle H|\mathcal{O}_1|H\rangle &= f_H^2 M_H^2 B_1^H, \\
\langle H|\mathcal{O}_2|H\rangle &= \frac{M_H^2}{(m_Q+m_q)^2} f_H^2 M_H^2 B_2^H, \\
\langle H|T_1|H\rangle &= f_H^2 M_H^2 \epsilon_1^H, \\
\langle H|T_2|H\rangle &= \frac{M_H^2}{(m_Q+m_q)^2} f_H^2 M_H^2 \epsilon_2^H,
\end{aligned} \tag{2}$$

with M_H the mass of the heavy meson H. The bag parameters B_1^H, B_2^H, ϵ_1^H, and ϵ_2^H as well as the decay constant f_H encode contributions due to strong interactions whose determinations require nonperturbative methods. In this Letter, we focus on a bound charm-strange state, i.e., $H = D_s$. We expect our method to apply also to other D or B mesons; we leave this for future work.

Aside from diagrams like Fig. 1(a), the operators in Eq. (1) also lead to diagrams where the light valence quark of the meson is not directly connected to the composite operator, sometimes called "eye diagrams"; see Fig. 1(b). Their consistent treatment requires an extension of the operator basis in Eq. (1). In a first step toward the full calculation of heavy-meson lifetimes, we determine the matrix elements relevant to $SU(3)_\text{F}$-*symmetric ratios* of lifetimes like $\tau(D_s)/\tau(D^0)$, where these eye diagrams cancel.

Lattice calculation—Figure 2 illustrates the calculation of a three-point function on the lattice. Mesons are created at Euclidean times t_0 and $t_0 + \Delta T$. The four-quark interactions described by the operators defined in Eq. (1) are contracted at $t \in [t_0, t_0 + \Delta T]$. By placing the sources at equal time separations, propagators can be reused to calculate three-point functions with different separation ΔT concurrently (cf. Ref. [32]).

Adjusting the spin-color algebra factors in the ratios seen in Ref. [32] to the $\Delta Q = 0$ case, the bag parameters can be obtained from the following ratio:

$$R_X(t_0, t, \Delta T) = \frac{C_X^{\text{3pt}}(t_0, t, t_0+\Delta T)}{C_X(t_0, t) C_X(t_0+\Delta T, t)} \to B_X, \tag{3}$$

where the arrow indicates the limits $t_0 \ll t \ll t_0 + \Delta T$ and $\Delta T \to \infty$. The two- and three-point functions are given by

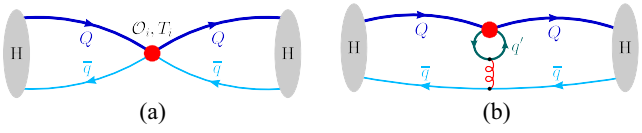


FIG. 1. (a) LO diagram for the four-quark operator contribution to heavy-meson decay in HQE; (b) eye diagram (not considered in this Letter). The diagrams were produced with the help of FeynGame [30,31].

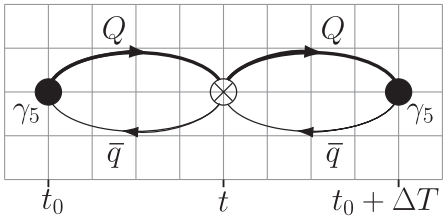


FIG. 2. Quark-line diagram for the lattice calculation of $\Delta Q = 0$ operators.

$$C_X^{3\mathrm{pt}}(t_1,t_2,t_3) = \langle 0|j_P(t_1)X(t_2)j_P^\dagger(t_3)|0\rangle,$$
$$C_X(t_1,t_2) = \langle 0|j_X(t_2)j_P^\dagger(t_1)|0\rangle, \quad (4)$$

where $t_1 < t_2 < t_3$ are generic positions, and $j_P = \bar{q}\gamma_5 Q$ and $j_X = \bar{q}\Gamma_X Q$ are zero-momentum currents. Note that we are using X both as an operator and as an index in Eq. (4). In order to have appropriate overlap with the bag parameters, $\Gamma_{\mathcal{O}_1} = \Gamma_{T_1} = \gamma_0\gamma_5$, and $\Gamma_{\mathcal{O}_2} = \Gamma_{T_2} = \gamma_5$.

Gradient flow—Bare correlators as well as the ratio of Eq. (3) are ultraviolet (UV) divergent. We obtain the $\overline{\mathrm{MS}}$-renormalized hadronic matrix elements by closely following the method of Refs. [33–39], which is based on the gradient flow (GF) [40–43] and the SFTX [33,44,45]. To be specific, we replace the operators X and the currents j_X in Eq. (4) by their flowed versions $\tilde{X}(\tau)$ and $\tilde{j}_X(\tau)$, i.e., the fields in these quantities are evolved according to

$$\partial_\tau \mathcal{A}_\mu(\tau) = D_\nu(\tau)G_{\nu\mu}(\tau), \qquad \mathcal{A}_\mu(\tau=0) = A_\mu,$$
$$\partial_\tau \chi(\tau) = D^2(\tau)\chi(\tau), \qquad \chi(\tau=0) = \psi, \quad (5)$$

with τ indicating the flow time. $\chi(\tau)$ represents the quark fields, $G_{\mu\nu}$ is the field-strength tensor, and D_μ is the covariant derivative, each with the regular gluon field A_μ replaced by its flowed version $\mathcal{A}_\mu(\tau)$. On the lattice, we use the Wilson kernel to evolve the gauge fields and apply the lattice Laplace operator to the propagators [45]. For finite $\tau > 0$, the GF regularizes the UV divergences, and the continuum limit combining results from the different ensembles can be taken. Power-divergent operator mixing on the lattice is absent in this approach and shifted to the SFTX. Note that any wave function renormalizations for the flowed and unflowed quark fields cancel in the ratio of Eq. (3).

At this point, the continuum bag parameters depend on the flow time τ; we may consider them as renormalized in the GF scheme denoting them by $\tilde{B}_X(\tau)$. They can be converted to the $\overline{\mathrm{MS}}$ scheme at a scale μ as follows. Using the SFTX [33,44,45], the GF-renormalized bag parameters are multiplied by the matching matrices,

$$\begin{pmatrix} B_i(\tau,\mu) \\ \epsilon_i(\tau,\mu) \end{pmatrix} = \zeta_{B,i}^{-1}(\tau,\mu) \begin{pmatrix} \tilde{B}_i(\tau) \\ \tilde{\epsilon}_i(\tau) \end{pmatrix}, \quad (6)$$

for $i \in \{1,2\}$. The matching matrices are given by

$$\zeta_{B,1} = \zeta_1\zeta_A^{-2}, \qquad \zeta_{B,2} = \zeta_2\zeta_P^{-2}, \quad (7)$$

where ζ_P and ζ_A are the SFTX matching coefficients for the pseudoscalar and the axial-vector current, and the ζ_i are the matching matrices of the four-quark operators $\{\mathcal{O}_i, T_i\}$ in Eq. (1). They can be calculated perturbatively and are known through next-to-next-to-leading order (NNLO) in QCD [23,46,47]. The quantities on the lhs of Eq. (6) are related to the $\overline{\mathrm{MS}}$ bag parameters by taking the $\tau \to 0$ extrapolation, i.e.,

$$B_X^{\overline{\mathrm{MS}}}(\mu) = \lim_{\tau\to 0} B_X(\tau,\mu). \quad (8)$$

This limit is nontrivial and involves identifying a suitable region from which such an extrapolation can be performed as we discuss later.

Numerical analysis—Using the six RBC/UKQCD 2 + 1-flavor Shamir domain-wall fermion (DWF) and Iwasaki gauge field ensembles [32,48–50] listed in Table I, we create multiple $\mathbb{Z}_2$ wall sources [51,52] per gauge-field configuration. For strange quarks, we apply Gaussian smearing [53] to the source before inverting a Shamir DWF [54–56], whereas for charm quarks we directly invert a heavy, stout-smeared Möbius DWF [57–59]. In both cases, the mass is tuned close to its physical value.

Evolving the gauge field and quark propagators along the GF requires solving the ordinary differential equations of Eq. (5). We implement them using a third-order Runge-Kutta algorithm in the `Grid` [60] and `Hadrons` [61,62] lattice-QCD software libraries. The GF evolution runs with step size $\delta\tau/a^2 = 0.01$, with measurements of the matrix elements carried out every 10 steps (increasing to 40 steps toward larger τ).

For each value of the flow time, we perform combined correlated fits to Eq. (3) for a range of values of ΔT where excited states are not statistically resolved and consistent plateaus can be identified; the minimum ΔT considered on each ensemble is listed in Table I. Details of the numerical analysis are discussed in Ref. [23].

Since the gradient flow is only effective if the "smearing radius" $\sqrt{8\tau}$ is sufficiently large in relation to the lattice spacing, we only include data in our analysis with $\tau > \tau_{\mathrm{min}}$, where $8\tau_{\mathrm{min}} = 2a^2_{\mathrm{max}}$. Using the coarsest lattice spacing of our dataset, this translates to $\tau_{\mathrm{min}} = 0.08\ \mathrm{GeV}^{-2}$. Using $\mathcal{O}(a)$-improved actions, we take the continuum limit using a linear ansatz in a^2 and also parametrize sea-pion mass effects using

$$\tilde{B}_i^{\mathrm{latt}}(a^2, M_\pi^{\mathrm{latt}}; \tau) = \tilde{B}_i(\tau) + Ca^2 + Da^2\left[(M_\pi^{\mathrm{latt}})^2 - (M_\pi^{\mathrm{phys}})^2\right], \quad (9)$$

where $\tilde{B}_i(\tau)$ denotes the continuum-extrapolated value at physical pion mass [2], while C and D are fit parameters describing the leading discretization and sea-pion effects. Estimates of systematic uncertainties are discussed in Ref. [23].

Zero-flow-time limit—In the final step of our procedure, the flowed bag parameters are multiplied by the inverse matching matrices defined in Eqs. (6) and (7), and the result is extrapolated to $\tau = 0$, resulting in the bag parameters in the $\overline{\mathrm{MS}}$ scheme.

To derive the bag parameters in the $\overline{\mathrm{MS}}$ scheme at a certain value of $\mu = \mu_0$, we can either insert the (inverse) matching matrices at this value, $\zeta^{-1}(\tau,\mu_0)$, into Eq. (6), or

evaluate them at a different scale μ, and subsequently evolve them to μ_0 according to regular RG evolution [63],

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \zeta(\tau, \mu) = \zeta(\tau, \mu) \gamma[\alpha_s(\mu)], \tag{10}$$

where γ is the regular (unflowed) anomalous dimension of R_X [64–67]. We denote the resulting matching matrix as $\zeta(\tau, \mu \to \mu_0)$. Different initial values of μ typically change the τ dependence of $\zeta^{-1}(\tau, \mu \to \mu_0)\tilde{B}(\tau)$ and, therefore, lead to small variations in the $\tau \to 0$ limit, which we will use to estimate part of the systematic uncertainty. The value of $\alpha_s(\mu)$ is obtained from our reference value $\alpha_s(3\ \mathrm{GeV}) = 0.2535$ by RG evolution to the corresponding perturbative order with the help of RUNDEC [68,69].

Furthermore, we can improve the perturbative convergence of our matching procedure by resumming the logarithmic τ dependence in the matching coefficients via the flow-time RG equation [63],

$$\tau \frac{\mathrm{d}}{\mathrm{d}\tau} \tilde{\mathcal{O}}(\tau) = \tilde{\gamma}\, \tilde{\mathcal{O}}(\tau), \quad \text{with} \quad \tilde{\gamma} = \left(\tau \frac{\mathrm{d}}{\mathrm{d}\tau} \zeta\right) \zeta^{-1}, \tag{11}$$

where the flowed anomalous dimensions $\tilde{\gamma}$ are provided in Ref. [23]. This allows us to run our data for the bag parameters between different flow times; in particular, we can evolve all our data from τ to a specific flow time τ_0 where higher-order corrections to the matching matrix ζ are particularly small. Specifically, we choose $\tau_0 = \mathrm{e}^{-\gamma_\mathrm{E}}/(2\mu_0^2)$ [70], where $\gamma_\mathrm{E} = 0.5772\ldots$ is the Euler-Mascheroni constant.

Our final results for $B_i^{\overline{\mathrm{MS}}}(\mu_0)$ and $\epsilon_i^{\overline{\mathrm{MS}}}(\mu_0)$ are obtained by performing a single global fit to each $B(\mu \to \mu_0, \tau \to \tau_0)$ for $\mu \in \{2.5, 3.0, 3.5, 4.0, 4.5\}$ GeV and $\mu_0 = 3$ GeV, with the functional form

$$f(\tau, \mu) = c_0 + \tau\left(c_1^{(\mu)} + c_2^{(\mu)} \ln(\mu^2 \tau)\right), \tag{12}$$

where c_0 is a common parameter to all fit functions, and $c_{1,2}^{(\mu)}$ are independent for each μ used. By constraining c_0 to be global to all fit functions, we ensure a solution for which the perturbative RG evolution is consistent across the chosen range of μ. We evaluate fits within all possible intervals from τ_min up to at most $\tau_\mathrm{max} = 0.4\ \mathrm{GeV}^{-2}$ and identify the spread of all fit results for c_0 with p value > 0.05 as the symmetrized uncertainty interval for the bag parameters at scale μ_0. This interval accounts for both

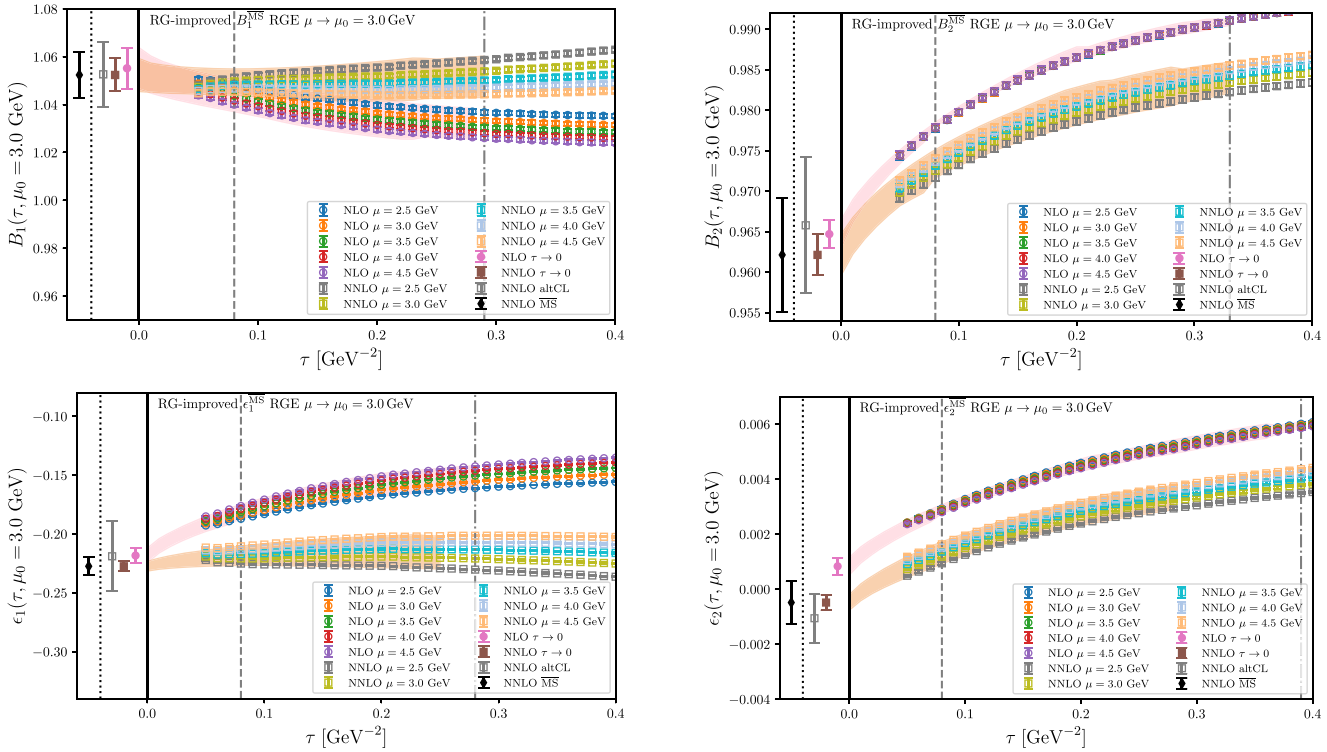


FIG. 3. Zero-flow-time extrapolation of the $\Delta Q = 0$ bag parameters. B_1 (top left), B_2 (top right), ϵ_1 (bottom left), ϵ_2 (bottom right). The matching to $\overline{\mathrm{MS}}$ is performed at both NLO and NNLO using $\mu \in \{2.5, 3.0, 3.5, 4.0, 4.5\}$ GeV before running back to $\mu_0 = 3$ GeV and uses the flow-time RG equation to set flow-time logarithms to zero and improve the extrapolations. The pink and orange shaded areas indicate the spread of all fits with p value > 0.05 at NLO and NNLO, resulting in the pink and brown data points in the left panel, respectively. The gray data points estimate systematic effects due to discarding our coarsest ensembles; the black data points show our final result with all errors added in quadrature. The dashed lines indicate $\tau_\mathrm{min} = 0.08\ \mathrm{GeV}^{-2}$, whereas dash-dotted lines mark the maximum flow time included in our fit results.

statistical and systematic uncertainties of the $\tau \to 0$ extrapolation.

Results—Figure 3 shows the SFTX and $\tau \to 0$ limits for the bag parameters of the $\Delta Q = 0$ operators for the lifetime ratio $\tau(D_s)/\tau(D^0)$. The colored bands show the spread of all fit variations considered. The results at $\tau = 0$ at next-to-leading order (NLO) and NNLO are shown in the left panel as pink and brown data points, respectively. The gray data points are based on an alternative analysis to estimate systematic effects by discarding our coarsest ensembles (C1, C2) in the continuum extrapolation. This removes $1/3$ of our total dataset and an entire lattice spacing, leading to larger uncertainties since the continuum extrapolations are less constrained. Finally, the black data points show our final results incorporating in addition systematic uncertainties due to truncating the perturbative series as well as estimates for discretization and quark mass tuning uncertainties. We find

$$\begin{aligned} B_1^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) &= 1.0524(97), \\ B_2^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) &= 0.9621(70), \\ \epsilon_1^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) &= -0.2275(76), \\ \epsilon_2^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) &= -0.0005(8) \end{aligned} \tag{13}$$

for the bag parameters of lifetime operators with a full error budget, which is provided in Table II and described in further detail in Ref. [23]. While no recent lattice calculation exists for these quantities, they were computed using HQET sum rules with light spectator quarks in Ref. [9] and then matched to QCD for both B and D mesons. Accounting for different schemes of evanescent operators used (for details, see Ref. [23]), we find agreement with Ref. [9] for $B_2^{\overline{\mathrm{MS}}}$ and $\epsilon_2^{\overline{\mathrm{MS}}}$ within 1.5σ; however, our results for $B_1^{\overline{\mathrm{MS}}}$ and $\epsilon_1^{\overline{\mathrm{MS}}}$ differ significantly. An improved HQET-QCD matching in the sum-rules approach that includes subleading $O(1/m_Q)$ corrections may resolve this discrepancy. A confirmation of the increase of $\epsilon_1^{\overline{\mathrm{MS}}}$ by approximately a factor of 2 with respect to Ref. [9] would be of particular phenomenological interest due to the relatively large Wilson coefficient associated with this parameter.

The $\Delta Q = 0$ operators also contribute to inclusive semileptonic B-meson decays [71] and are leading uncertainties at high q^2. High-precision determinations will, therefore, have implications to the broader community.

Conclusion—We have presented the first lattice-QCD determination of the $\mathrm{SU}(3)_\mathrm{F}$-symmetric dimension-six $\Delta Q = 0$ four-quark bag parameters that govern the leading spectator effects for the lifetime ratios of heavy mesons with full error budget away from the static limit. Ratios of heavy-meson lifetimes play an important role in validating the HQE but also enter many phenomenological determinations such as decay-rate ratios of $b \to s\ell\ell$ currents. A direct treatment of eye contractions, which contribute to absolute lifetimes and $\mathrm{SU}(3)_\mathrm{F}$-breaking lifetime ratios, is the natural next step of the program.

Our result is based on GF combined with the SFTX to obtain renormalized bag parameters in the $\overline{\mathrm{MS}}$ scheme. UV divergences are regularized at positive flow time such that the continuum limit can be taken without encountering power-divergent mixing, and results can be matched to the $\overline{\mathrm{MS}}$ scheme using perturbatively computed SFTX coefficients (here through NNLO). A robust $\tau \to 0$ extrapolation —restricted to a window where both discretization effects and perturbative instabilities are under control—completes the connection to standard continuum phenomenology. The determination of lifetime-ratio operators for a charm-strange system demonstrates that the GF + SFTX is a viable renormalization procedure for lattice calculations, which can be further applied to complex renormalization structures including power-divergent operator mixing. Further possible applications may include other four-quark operators exhibiting operator mixing under renormalization such as, for example, nucleon electric dipole moments.

Note added—While this paper was under review, Ref. [72] presented the NNLO Wilson coefficients for heavy-meson lifetime ratios and combined them with the bag parameters determined here. It finds that our lattice results significantly change the charm lifetime-ratio predictions compared to the HQET sum-rule results [9], reduce the associated uncertainties, and improve the agreement with experiment.

Acknowledgments—We thank the RBC/UKQCD Collaboration for generating and making their gauge field ensembles publicly available. Special thanks are given to Felix Erben, Ryan Hill, and J. Tobias Tsang for assistance in setting up the simulation code, and we thank Anna Hasenfratz and Martin Lang for constructive discussions. Fruitful input and encouragement from the members of the CRC TRR 257 is highly appreciated. Special thanks in this respect go to Alexander Lenz, Uli Nierste, and Thomas Mannel. We are also grateful to the authors of Ref. [72], especially Francesco Moretti, for comparing the anomalous dimensions of the general basis of evanescent operators. Similarly, we thank the authors of Ref. [67], especially Pol Morell, for help comparing to our anomalous dimensions. These computations used resources provided by the OMNI cluster at the University of Siegen, the HAWK cluster at the High-Performance Computing Center Stuttgart, the DeiC Large Memory HPC Type3/Hippo System managed by the eScience Center at the University of Southern Denmark, and LUMI-G at the CSC data center Finland (DeiC National HPC g.a. DEIC-SDU-L5-13 and DEIC-SDU-N5-2024053). This research was supported by Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through Grant No. 396021762—TRR 257

"Particle Physics Phenomenology after the Higgs Discovery," through Grant No. 513989149, Germany's Excellence Strategy—EXC 3107—Project No. 533766364, UK STFC Grant No. ST/X000494/1, the Swiss National Science Foundation (SNSF) under Contract No. TMSGI2_211209, the National Science Foundation under Grant No. PHY-2209185, and the DOE Topical Collaboration "Nuclear Theory for New Physics" Award No. DE-SC0023663.

Data availability—The data that support the findings of this article are openly available [73].

[1] S. Banerjee *et al.* (Heavy Flavor Averaging Group (HFLAV)), Averages of b-hadron, c-hadron, and τ-lepton properties as of 2023, Phys. Rev. D **113**, 012008 (2026).
[2] S. Navas *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **110**, 030001 (2024).
[3] J. Albrecht, F. Bernlochner, A. Lenz, and A. Rusov, Lifetimes of b-hadrons and mixing of neutral B-mesons: Theoretical and experimental status, Eur. Phys. J. Special Topics **233**, 359 (2024).
[4] M. Kirk, A. Lenz, and T. Rauh, Dimension-six matrix elements for meson mixing and lifetimes from sum rules, J. High Energy Phys. 12 (2017) 068; 06 (2020) 162(E).
[5] M. J. Kirk, Charming New Physics in Beautiful Processes?, Ph.D. thesis, Durham University, IPPP, 2018, 10.1007/978-3-030-19197-9.
[6] D. King, A. Lenz, M. L. Piscopo, T. Rauh, A. V. Rusov, and C. Vlahos, Revisiting inclusive decay widths of charmed mesons, J. High Energy Phys. 08 (2022) 241.
[7] D. King, A. Lenz, and T. Rauh, SU(3) breaking effects in B and D meson lifetimes, J. High Energy Phys. 06 (2022) 134.
[8] D. J. King, Three loop corrections to HQET Sum Rules for the Bag Parameter, Ph.D. thesis, Durham University, 2022, https://etheses.durham.ac.uk/id/eprint/14487/.
[9] M. Black, M. Lang, A. Lenz, and Z. Wüthrich, HQET sum rules for matrix elements of dimension-six four-quark operators for meson lifetimes within and beyond the Standard Model, J. High Energy Phys. 04 (2025) 081.
[10] N. Carrasco *et al.*, $D^0 - \bar{D}^0$ mixing in the standard model and beyond from $N_f = 2$ twisted mass QCD, Phys. Rev. D **90**, 014502 (2014).
[11] N. Carrasco, P. Dimopoulos, R. Frezzotti, V. Lubicz, G. C. Rossi, S. Simula, and C. Tarantino (ETM Collaboration), $\Delta S = 2$ and $\Delta C = 2$ bag parameters in the standard model and beyond from $N_f = 2 + 1 + 1$ twisted-mass lattice QCD, Phys. Rev. D **92**, 034516 (2015).
[12] A. Bazavov *et al.*, Short-distance matrix elements for D^0-meson mixing for $N_f = 2 + 1$ lattice QCD, Phys. Rev. D **97**, 034513 (2018).
[13] N. Carrasco *et al.* (ETM Collaboration), B-physics from $N_f = 2$ tmQCD: The Standard Model and beyond, J. High Energy Phys. 03 (2014) 016.
[14] Y. Aoki, T. Ishikawa, T. Izubuchi, C. Lehner, and A. Soni, Neutral B meson mixings and B meson decay constants with static heavy and domain-wall light quarks, Phys. Rev. D **91**, 114505 (2015).
[15] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu, and M. Wingate (HPQCD Collaboration), Neutral B meson mixing in unquenched lattice QCD, Phys. Rev. D **80**, 014503 (2009).
[16] A. Bazavov *et al.* (Fermilab Lattice Collaboration and MILC Collaboration), $B^0_{(s)}$-mixing matrix elements from lattice QCD for the Standard Model and beyond, Phys. Rev. D **93**, 113016 (2016).
[17] R. J. Dowdall, C. T. H. Davies, R. R. Horgan, G. P. Lepage, C. J. Monahan, J. Shigemitsu, and M. Wingate, Neutral B-meson mixing from full lattice QCD at the physical point, Phys. Rev. D **100**, 094508 (2019).
[18] P. Boyle, F. Erben, A. Jüttner, T. Kaneko, M. Marshall, A. Portelli, O. Witzel, L. Del Debbio, and J. T. Tsang, BSM $B - \bar{B}$ mixing on JLQCD and RBC/UKQCD $N_f = 2 + 1$ DWF ensembles, Proc. Sci. LATTICE2021 (**2022**) 224 [arXiv:2111.11287].
[19] M. Di Pierro and C. T. Sachrajda (UKQCD Collaboration), A Lattice study of spectator effects in inclusive decays of B mesons, Nucl. Phys. **B534**, 373 (1998).
[20] M. Di Pierro, C. T. Sachrajda, and C. Michael (UKQCD Collaboration), An Exploratory lattice study of spectator effects in inclusive decays of the Λ_b baryon, Phys. Lett. B **468**, 143 (1999); **525**, 360(E) (2002).
[21] D. Becirevic, Theoretical progress in describing the B meson lifetimes, Proc. Sci. HEP2001 (**2001**) 098 [arXiv:hep-ph/0110124].
[22] J. Lin, W. Detmold, and S. Meinel, Lattice study of spectator effects in b-hadron decays, Proc. Sci. LATTICE2022 (2023) 417 [arXiv:2212.09275].
[23] M. Black, R. V. Harlander, J. T. Kohnen, F. Lange, A. Rago, A. Shindler, and O. Witzel, companion paper, Heavy-meson bag parameters using gradient flow, Phys. Rev. D **114**, 094503 (2026).
[24] M. Black, R. Harlander, F. Lange, A. Rago, A. Shindler, and O. Witzel, Using gradient flow to renormalise matrix elements for meson mixing and lifetimes, Proc. Sci. LATTICE2023 (**2024**) 263 [arXiv:2310.18059].
[25] M. Black, Improving Precision for Hadronic Observables in the Standard Model and Beyond, Ph.D. thesis, Universität Siegen, 2024, 10.25819/ubsi/10603
[26] M. Black, R. Harlander, F. Lange, A. Rago, A. Shindler, and O. Witzel, Gradient flow renormalisation for meson mixing and lifetimes, Proc. Sci. LATTICE2024 (**2025**) 243 [arXiv:2409.18891].
[27] N. N. Nikolaev, Gauge model for weak interaction and supercharged hadrons, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 447 (1973), https://inspirehep.net/literature/87709.
[28] M. Neubert, Heavy quark symmetry, Phys. Rep. **245**, 259 (1994).
[29] M. L. Piscopo, Higher order corrections to the lifetime of heavy hadrons, Ph.D. thesis, Siegen University, 2021, 10.25819/ubsi/10024.
[30] R. Harlander, S. Klein, and M. Lipp, FeynGame, Comput. Phys. Commun. **256**, 107465 (2020).
[31] L. Bündgen, R. V. Harlander, S. Y. Klein, and M. C. Schaaf, FeynGame 3.0, Comput. Phys. Commun. **314**, 109662 (2025).

[32] P. A. Boyle, L. Del Debbio, N. Garron, A. Jüttner, A. Soni, J. T. Tsang, and O. Witzel (RBC/UKQCD Collaboration), SU(3)-breaking ratios for $D_{(s)}$ and $B_{(s)}$ mesons, arXiv:1812.08791.

[33] H. Suzuki, Energy–momentum tensor from the Yang–Mills gradient flow, Prog. Theor. Exp. Phys. **2013**, 083B03 (2013); **2015**, 079201(E) (2015).

[34] H. Makino and H. Suzuki, Lattice energy–momentum tensor from the Yang–Mills gradient flow—inclusion of fermion fields, Prog. Theor. Exp. Phys. **2014**, 063B02 (2014); **2015**, 079202(E) (2015).

[35] Y. Taniguchi, S. Ejiri, K. Kanaya, M. Kitazawa, H. Suzuki, and T. Umeda (WHOT-QCD Collaboration), $N_f = 2 + 1$ QCD thermodynamics with gradient flow using two-loop matching coefficients, Phys. Rev. D **102**, 014510 (2020); **102**, 059903(E) (2020).

[36] H. Suzuki and H. Takaura, $t \to 0$ extrapolation function in the small flow time expansion method for the energy–momentum tensor, Prog. Theor. Exp. Phys. **2021**, 073B02 (2021).

[37] M. Black, R. V. Harlander, A. Hasenfratz, A. Rago, and O. Witzel, Renormalized quark masses using gradient flow, Phys. Rev. D **113**, 094504 (2026).

[38] A. Francis, P. Fritzsch, R. Karur, J. Kim, G. Pederiva, D. A. Pefkou, A. Rago, A. Shindler, A. Walker-Loud, and S. Zafeiropoulos, Moments of parton distributions functions of the pion from lattice QCD using gradient flow, Phys. Rev. D **113**, 074520 (2026).

[39] A. Francis *et al.*, Gradient flow for parton distribution functions: First application to the pion, Phys. Rev. Lett. **136**, 171903 (2026).

[40] R. Narayanan and H. Neuberger, Infinite N phase transitions in continuum Wilson loop operators, J. High Energy Phys. 03 (2006) 064.

[41] M. Lüscher, Trivializing maps, the Wilson flow and the HMC algorithm, Commun. Math. Phys. **293**, 899 (2010).

[42] M. Lüscher, Properties and uses of the Wilson flow in lattice QCD, J. High Energy Phys. 08 (2010) 071; 03 (2014) 92.

[43] M. Lüscher, Chiral symmetry and the Yang–Mills gradient flow, J. High Energy Phys. 04 (2013) 123.

[44] M. Lüscher and P. Weisz, Perturbative analysis of the gradient flow in non-abelian gauge theories, J. High Energy Phys. 02 (2011) 051.

[45] M. Lüscher, Future applications of the Yang-Mills gradient flow in lattice QCD, Proc. Sci. LATTICE2013 (**2014**) 016 [arXiv:1308.5598].

[46] R. V. Harlander and F. Lange, Effective electroweak Hamiltonian in the gradient-flow formalism, Phys. Rev. D **105**, L071504 (2022).

[47] J. Borgulat, R. V. Harlander, J. T. Kohnen, and F. Lange, Short-flow-time expansion of quark bilinears through next-to-next-to-leading order QCD, J. High Energy Phys. 05 (2024) 179.

[48] C. Allton *et al.* (RBC-UKQCD Collaboration), Physical results from $2 + 1$ flavor domain wall QCD and SU(2) chiral perturbation theory, Phys. Rev. D **78**, 114509 (2008).

[49] Y. Aoki *et al.* (RBC Collaboration and UKQCD Collaboration), Continuum limit physics from $2 + 1$ flavor domain wall QCD, Phys. Rev. D **83**, 074508 (2011).

[50] T. Blum *et al.* (RBC Collaboration and UKQCD Collaboration), Domain wall QCD with physical quark masses, Phys. Rev. D **93**, 074505 (2016).

[51] C. McNeile and C. Michael (UKQCD Collaboration), Decay width of light quark hybrid meson from the lattice, Phys. Rev. D **73**, 074506 (2006).

[52] P. A. Boyle, A. Jüttner, C. Kelly, and R. D. Kenway, Use of stochastic sources for the lattice determination of light quark physics, J. High Energy Phys. 08 (2008) 086.

[53] S. Güsken, U. Löw, K. H. Mütter, R. Sommer, A. Patel, and K. Schilling, Nonsinglet axial vector couplings of the baryon octet in lattice QCD, Phys. Lett. B **227**, 266 (1989).

[54] D. B. Kaplan, A method for simulating chiral fermions on the lattice, Phys. Lett. B **288**, 342 (1992).

[55] Y. Shamir, Chiral fermions from lattice boundaries, Nucl. Phys. **B406**, 90 (1993).

[56] V. Furman and Y. Shamir, Axial symmetries in lattice QCD with Kaplan fermions, Nucl. Phys. **B439**, 54 (1995).

[57] C. Morningstar and M. J. Peardon, Analytic smearing of SU(3) link variables in lattice QCD, Phys. Rev. D **69**, 054501 (2004).

[58] R. C. Brower, H. Neff, and K. Orginos, The Möbius domain wall fermion algorithm, Comput. Phys. Commun. **220**, 1 (2017).

[59] Y.-G. Cho, S. Hashimoto, A. Jüttner, T. Kaneko, M. Marinkovic, J.-I. Noaki, and J. T. Tsang, Improved lattice fermion action for heavy quarks, J. High Energy Phys. 05 (2015) 072.

[60] P. A. Boyle *et al.*, Grid: A next generation data parallel C++ QCD library, Proc. Sci. LATTICE2015 (**2016**) 023.

[61] A. Portelli *et al.*, Hadrons, 10.5281/zenodo.4293902.

[62] A. Portelli *et al.*, 10.5281/zenodo.4063666 (2022).

[63] R. V. Harlander, F. Lange, and T. Neumann, Hadronic vacuum polarization using gradient flow, J. High Energy Phys. 08 (2020) 109.

[64] A. J. Buras and P. H. Weisz, QCD nonleading corrections to weak decays in dimensional regularization and 't Hooft-Veltman schemes, Nucl. Phys. **B333**, 66 (1990).

[65] M. Ciuchini, E. Franco, V. Lubicz, G. Martinelli, I. Scimemi, and L. Silvestrini, Next-to-leading order QCD corrections to $\Delta F = 2$ effective Hamiltonians, Nucl. Phys. **B523**, 501 (1998).

[66] A. J. Buras, M. Misiak, and J. Urban, Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the standard model, Nucl. Phys. **B586**, 397 (2000); **B1002**, 116529(E) (2024).

[67] J. Aebischer, P. Morell, M. Pesut, and J. Virto, Two-loop anomalous dimensions in the LEFT: Dimension-six four-fermion operators in NDR, J. High Energy Phys. 05 (2026) 043.

[68] K. G. Chetyrkin, J. H. Kühn, and M. Steinhauser, RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses, Comput. Phys. Commun. **133**, 43 (2000).

[69] F. Herren and M. Steinhauser, Version 3 of RunDec and CRunDec, Comput. Phys. Commun. **224**, 333 (2018).

[70] R. V. Harlander, Y. Kluth, and F. Lange, The two-loop energy–momentum tensor within the gradient-flow formalism, Eur. Phys. J. C **78**, 944 (2018); **79**, 858(E) (2019).

[71] M. Fael, J. Jenkins, E. Lunghi, and Z. Polonsky, The Standard Model prediction for the rare decay $B \to X_s \nu\bar{\nu}$, J. High Energy Phys. 03 (2026) 217.

[72] F. Moretti, U. Nierste, P. Reeck, and M. Steinhauser, Next-to-next-to-leading QCD corrections to the B^+-B_d^0, D^+-D^0, and D_s^+-D^0 lifetime ratios, J. High Energy Phys. 09 (2026) 250.

[73] M. Black, R. Harlander, J. Kohnen, F. Lange, A. Rago, A. Shindler, and O. Witzel, Dataset: Heavy-meson bag parameters using gradient flow, 10.25819/67d6vsvpskcu5g9w (2026).

End Matter

End Matter: Bag Parameters for Heavy Meson Lifetimes—In this End Matter, we provide two tables with supporting information to this Letter. Table I lists the set of gauge-field ensembles used in this study, specifying the relevant ensemble parameters as well as statistics used and minimum source separation ΔT considered in the analysis. The final bag parameter results are presented in Table II with a full error budget.

TABLE I. RBC/UKQCD's $N_f = 2 + 1$ Shamir DWF ensembles with Iwasaki gauge action [32,48–50] characterized by inverse lattice spacing (a^{-1}), unitary pion mass (M_π), valence strange (am_s^{val}), and charm (am_c) quark masses. We use coarse (C1,C2), medium (M1, M2,M3), and one fine ensemble (F1S). The Jacobi smearing procedure was applied on the C and M ensembles with smearing width σ and N_σ iterations. ΔT_{min} indicates the minimum source separation ΔT considered to have sufficient resolution of the ground state.

	L/a	T/a	a^{-1}[GeV]	M_π[MeV]	am_s^{sea}	am_s^{val}	am_c	$N_{\mathrm{src}} \times \mathrm{N}_{\mathrm{conf}}$	σ	N_σ	ΔT_{min}
C1	24	64	1.7848(50)	339.8(1.2)	0.04	0.03224	0.64	32×101	4.5	400	20
C2	24	64	1.7848(50)	430.6(1.4)	0.04	0.03224	0.64	32×101	4.5	400	20
M1	32	64	2.3833(86)	303.6(1.4)	0.03	0.02477	0.45	32×79	6.5	400	22
M2	32	64	2.3833(86)	360.7(1.6)	0.03	0.02477	0.45	32×89	6.5	100	22
M3	32	64	2.3833(86)	410.8(1.7)	0.03	0.02477	0.45	32×68	6.5	100	22
F1S	48	96	2.785(11)	267.6(1.3)	0.02144	0.02167	0.37	24×98			28

TABLE II. Values and error budgets for the $\Delta Q = 0$ bag parameters in the $\overline{\mathrm{MS}}$ scheme for our choice of evanescent operators at $\mu_0 = 3$ GeV. Individual errors are added in quadrature to obtain the total uncertainty. The "GF" error refers to the combined statistical and systematic uncertainty obtained from our set of fits performing the zero-flow-time extrapolation; "CL" is an estimate of systematic effects in our continuum limit by performing our analysis without the coarse C1 and C2 ensembles and taking half the difference of its zero-flow-time result with our central result (shown by the gray data points in Fig. 3); "PT" estimates the effect of truncating the perturbative series to NNLO by taking half the difference between the NNLO results (brown squares) and the NLO results (pink circles); "OS" incorporates other systematic effects, specifically quark-mass tuning and residual chiral-symmetry breaking of the light quarks. For further details, see Ref. [23].

Bag parameter	$B_1^{\overline{\mathrm{MS}}}$	$B_2^{\overline{\mathrm{MS}}}$	$\epsilon_1^{\overline{\mathrm{MS}}}$	$\epsilon_2^{\overline{\mathrm{MS}}}$
Central value at $\mu_0 = 3$ GeV	1.0524	0.9621	−0.2275	−0.0005
	Error budget			
Stat. error and $\tau \to 0$ extrapolation ("GF")	0.0069	0.0025	0.0041	0.0003
Continuum limit ("CL")	0.0001	0.0018	0.0042	0.0000
Perturbative truncation ("PT")	0.0014	0.0013	0.0046	0.0007
Other systematics ("OS")	0.0067	0.0062	0.0014	0.0000
Total	0.0097	0.0070	0.0076	0.0008