

Gravitational Waves from a Dilaton-Induced, First-Order QCD Phase Transition

Aleksandr Chatrchyan^{1,2,*}, M. C. David Marsh^{1,†}, and Charalampos Nikolis^{1,‡}

¹*The Oskar Klein Centre for Cosmoparticle Physics, Department of Physics, Stockholm University, AlbaNova, 10691 Stockholm, Sweden*

²*Nordita, KTH Royal Institute of Technology and Stockholm University, Hannes Alfvéns väg 12, SE-106 91 Stockholm, Sweden*



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We show that a QCD dilaton field, whose vacuum expectation value sets the strong coupling, can render the quantum chromodynamic (QCD) confinement transition first order. The QCD dilaton is cosmologically attracted to a false vacuum at weak coupling in the early Universe. Quantum tunneling toward the true vacuum triggers prompt chiral symmetry breaking and confinement of QCD, leading to detonating bubbles of the hadronic phase. We find that plasma sound waves produced by this dilaton-induced, first-order QCD phase transition generate a stochastic gravitational wave signal strikingly similar to the recently detected gravitational wave background from pulsar timing arrays. We briefly comment on how this theory can be probed through collider experiments and cosmology.

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Despite decades of enormous progress in observational and theoretical cosmology, the first microsecond of the Universe remains poorly known. Direct evidence for events taking place prior to big bang nucleosynthesis (BBN) is scarce, which limits the ability to use cosmology as a “poor man’s accelerator” [1,2]. The gateway to higher energies is provided by the quantum chromodynamic (QCD) phase transition (PT), in which the hot big bang plasma transitioned from the high-temperature quark-gluon phase into the low-temperature confined, hadronic phase. In the standard model (SM) of particle physics, the QCD PT is a smooth crossover occurring at temperatures around $T_{\text{QCD}} \sim 160$ MeV [3]. Crossovers are comparatively calm cosmic events, leaving few observational imprints. Thus, in the SM, the prospects for directly probing the QCD PT are dim.

The QCD confinement PT is accompanied by chiral symmetry breaking and the growth of the strong coupling constant g_3 . In the standard model, g_3 is perturbatively small at high energies and runs to strong coupling at around the QCD scale, which through dimensional transmutation is given by

$$\Lambda_{\text{QCD}} = \mu \exp \left[-\frac{8\pi^2}{\beta_0 g_3^2(\mu)} \right], \quad (1)$$

*Contact author: aleksandr.chatrchyan@su.se

†Contact author: david.marsh@fysik.su.se

‡Contact author: charalampos.nikolis@fysik.su.se

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at one-loop order, where μ is an energy scale, $\beta_0 = 11 - \frac{2}{3}N_f$ for N_f flavors. The ultraviolet value of the coupling constant $g_3^{\text{UV}} = g_3(\mu_{\text{UV}})$ is, like all standard model parameters, assumed to be constant: once fixed, it is unchanging over cosmic time.

By contrast, quantum gravity is expected to have no free parameters or true constants. All parameters of low-energy effective theories, like the standard model, are expected to be secretly field dependent. We refer to the scalar field $\phi(x)$ that makes the strong coupling constant dynamical as the “QCD dilaton” (or just dilaton, for short).

In this Letter, we show that the dynamics of ϕ can effectively turn the QCD phase transition first order, leading to detonating bubbles of the confined phase, separated from the quark-gluon plasma by a dilaton domain wall. The expanding bubbles quickly reach relativistic speeds and produce a stochastic gravitational wave background with an amplitude and spectrum that agree well with recent observations from pulsar timing arrays (PTAs) [4–7], thus begging the question: Have we already observed the first signals from a dilaton-induced, first-order QCD phase transition?

The mechanism introduced in this Letter is based on multiple-field dynamics and quantum vacuum transitions and should be contrasted to the much-studied scenario of thermal PTs. It is well established that physics beyond the standard model (BSM) can turn the QCD crossover into a thermal first-order PT. If the resulting PT is sufficiently strong, such models can produce gravitational waves in the frequency band probed by PTAs [8–10]. However, recent work has shown that the relevant parameter space is hard to realize in explicit models, either due to weak transition strengths [11–13], nonrelativistic wall velocities [14], or

failure to percolate [15]. Conversely, we will now show that the dilaton-induced QCD PT naturally leads to relativistic bubble walls and offers a compelling, and rather minimal, explanation of the PTA observations.

QCD and its dilaton—The Lagrangian of QCD and its dilaton ϕ is given by

$$\mathcal{L} = -\frac{1}{2}\tilde{f}(\phi)\text{tr}(G_{\mu\nu}G^{\mu\nu}) + \sum_i \bar{q}_i(i\not{D} - m_i)q_i + \frac{1}{2}(\partial\phi)^2 - V_0(\phi), \quad (2)$$

where $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, q_i denotes quark species i , and $\not{D} = \gamma^\mu(\partial_\mu - iA_\mu)$ for the non-Abelian gauge potential A_μ . The real-valued gauge kinetic function $\tilde{f}(\phi)$ sets the QCD coupling constant as $\tilde{f}(\langle\phi\rangle) = 1/(g_3^{\text{UV}})^2$ at some high-energy scale.

We assume that the zero-temperature dilaton potential $V_0(\phi)$ has two minima: the true vacuum (TV) at ϕ_{TV} and the false vacuum (FV) at ϕ_{FV} , with $\Delta V = V_0(\phi_{\text{FV}}) - V_0(\phi_{\text{TV}}) > 0$, and, importantly, $\Delta\tilde{f} = \tilde{f}(\phi_{\text{FV}}) - \tilde{f}(\phi_{\text{TV}}) > 0$. This assumption means that QCD is more weakly coupled in the false vacuum than in the true vacuum. We do not model the microphysical origins of $V_0(\phi)$ and $\tilde{f}(\phi)$, but note that the requisite structure can be readily realized with simple ingredients. Scalar potentials with localized bumps (or dips) can, e.g., be generated through the Coleman-Weinberg potential by integrating out bosons (or fermions) whose masses are minimized at finite values of ϕ . In potential string theory realizations of our scenario, ϕ would be a modulus field whose potential can be quite complex, receiving both perturbative and nonperturbative contributions [16–18]. Stabilized string compactifications with positive vacuum energy automatically involve multiple local minima (some asymptotic in field space) [19,20], and long-standing general arguments suggest that our Universe is located in regions of field space where the scalar potential is rather complex, with multiple important contributions [21,22], generating a rich landscape of local minima. Similarly, the gauge kinetic function may descend from the holomorphic gauge kinetic function of supergravity, which receives both one-loop and nonperturbative corrections [23,24]. Below, we provide a concrete toy model that realizes our scenario and draws on ingredients commonly found in string compactifications; however, many generalizations are possible. Recent work on related theories include [25–34].

In the present Universe, $\phi = \phi_{\text{TV}}$, and the properties of the dilaton are experimentally constrained. Collider searches at the Large Hadron Collider (LHC) have been used to constrain dilaton theories with linear gauge kinetic functions, $f \sim \phi/\Lambda$, leading to limits on Λ in the TeV range for dilaton masses $2 \leq m_\phi \leq 5$ TeV [35]. In this Letter, we assume $m_{\phi_{\text{TV}}} \sim m_{\phi_{\text{FV}}} \sim 10$ TeV, for simplicity. Moreover,

the QCD coupling constant is set by experiments to $\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$ (cf. [36] and references therein), which implies

$$\tilde{f}_{\text{TV}} \equiv \tilde{f}(\phi_{\text{TV}}) = \frac{\beta_0}{8\pi^2} \ln\left(\frac{\Lambda_{\text{UV}}}{m_Z}\right) + \frac{1}{4\pi\alpha_s(m_Z)}.$$

For example, a UV scale of $\Lambda_{\text{UV}} \sim 10^{19}$ GeV, implies $\tilde{f}_{\text{TV}} \simeq 4.2$ for $\beta_0 = 7$ (as appropriate for six quark flavors).

The dilaton-dependent QCD scale—The weaker gauge coupling in the false vacuum implies an exponentially lower strong coupling scale Λ_{QCD} than in standard QCD,

$$\Lambda_{\text{QCD}}^{\text{FV}} = \Lambda_{\text{QCD}}^{\text{TV}} \exp\left(-\frac{8\pi^2\Delta\tilde{f}}{\beta_0}\right). \quad (3)$$

For $\Delta\tilde{f} \sim \mathcal{O}(1)$, QCD with $\phi = \phi_{\text{FV}}$ remains weakly coupled well below the standard model crossover temperature $T_{\text{QCD}}^{\text{TV}}$, suggesting that chiral symmetry breaking and confinement would only happen at $T_{\text{QCD}}^{\text{FV}} \ll T_{\text{QCD}}^{\text{TV}}$.

Below the confinement temperature (for any ϕ), we model the low-energy QCD dynamics with a two-quark linear σ model featuring an explicit chiral symmetry breaking [37],

$$V_\chi(\phi, \chi) = \frac{\lambda}{4}(\chi^4 - 2f_\pi^2\chi^2) - m_\pi^2 f_\pi \chi, \quad (4)$$

with χ the chiral condensate, where $f_\pi(\phi)$ and $m_\pi(\phi)$ are the pion decay constant and the pion mass, respectively. The quantities f_π, m_π inherit their ϕ dependence from their dependence on the confinement scale and obey the scaling relations (cf. [28] for a similar discussion)

$$\frac{f_\pi(\phi_{\text{FV}})}{f_\pi(\phi_{\text{TV}})} = \frac{m_\pi(\phi_{\text{FV}})}{m_\pi(\phi_{\text{TV}})} = \exp\left[-\frac{8\pi^2\Delta\tilde{f}}{\beta_0}\right]. \quad (5)$$

Note that, in (4), we have omitted a χ -independent (but ϕ -dependent) contribution $\sim f_\pi^4$, as this contribution can be absorbed into $V_0(\phi)$. Taking $\lambda = 20$ reproduces the condensate mass of $m_\chi = 600$ MeV [38].

More elaborate effective theories modeling the confinement transition in addition to the chiral symmetry breaking can be constructed [39,40]; however, the simpler linear σ model suffices to capture the relevant physics of our scenario, which we now discuss.

A dilaton-induced, first-order QCD phase transition—Early in cosmic history, the dilaton was in thermal equilibrium with the quark-gluon plasma and its dynamics was governed by the temperature-dependent effective potential

$$V_{\text{eff}}(\phi) = V_0(\phi) + V_{\text{T}}(\phi) + V_{\text{P}}(\phi), \quad (6)$$

where

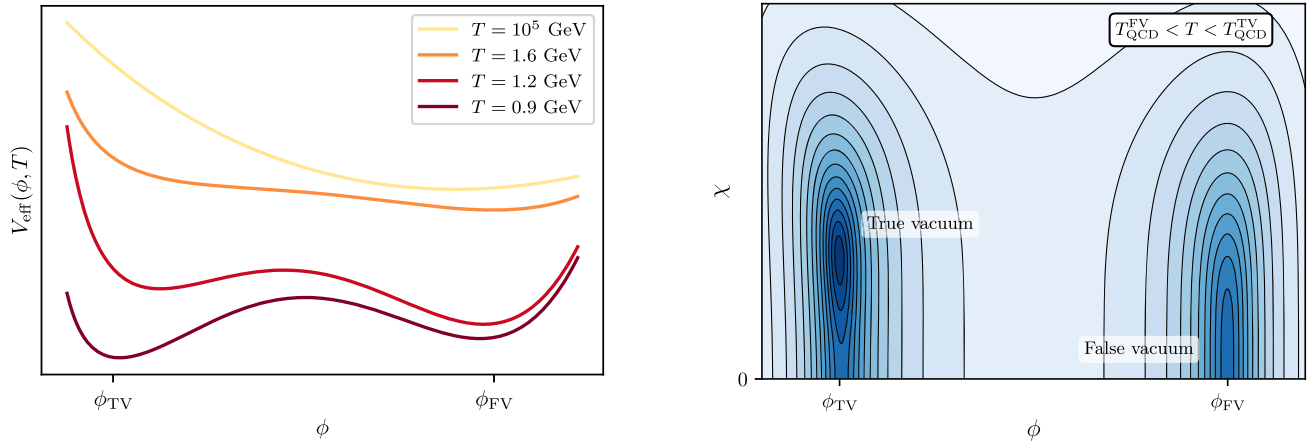


FIG. 1. Left: the dilaton potential $V_{\text{eff}}(\phi, T)$ at different temperatures. The values of the potential are rescaled for visual clarity. For $T \sim 100 \text{ TeV} \gg m_\phi$ the minimum is located in between the tree-level vacua. As temperature drops [$T \sim \mathcal{O}(1) \text{ GeV} \ll m_\phi$], the minimum moves toward the ϕ_{FV} while a local minimum emerges near ϕ_{TV} . When temperature corrections become unimportant, ϕ_{TV} becomes the global minimum. Right: schematic contour plot for the two-field potential $V_{\text{tot}}(\chi, \phi)$ for $T_{\text{QCD}}^{\text{FV}} < T < T_{\text{QCD}}^{\text{TV}}$.

$$V_{\text{T}}(\phi) = \frac{T^4}{2\pi^2} J_{\text{B}} \left(\frac{m_\phi^2(\phi)}{T^2} \right)$$

denotes the one-loop thermal effective potential due to dilaton self-interactions [41]. Here, J_{B} is the thermal bosonic function and $m_\phi(\phi)$ is the field-dependent mass. The term $V_{\text{P}}(\phi)$ represents the thermal QCD pressure corrections ($\mathcal{P}_{\text{QCD}} = -V_{\text{P}}$) in a fixed dilaton background for $N_c = 3$ and N_f flavors [42],

$$V_{\text{P}}(\phi) = -\frac{8\pi^2}{45} T^4 \left(\left(1 + \frac{21N_f}{32} \right) - \frac{15\alpha_s(\phi, T)}{4\pi} \left(1 + \frac{5N_f}{12} \right) + \mathcal{O}(\alpha_s^{3/2}) \right), \quad (7)$$

where $\alpha_s(\phi) = 1/[4\pi\mathfrak{f}(\phi)]$. The contribution of the pressure term due to the coupling of the dilaton to the standard model in the context of PTs was discussed recently in [43] and in the context of dark matter in [44,45,77].

The effective potential of Eq. (6) is shown schematically in the left panel of Fig. 1 for four different temperatures. At sufficiently high temperatures ($T \gg m_\phi$), V_{eff} is dominated by the stabilizing, leading-order contribution from V_{T} , $\sim m^2(\phi)T^2$, and the loop-corrected part of V_{P} , which push the dilaton toward weak coupling (corresponding to larger field values). The potential has a single minimum in the range $\phi_{\text{TV}} < \phi < \phi_{\text{FV}}$, slightly offset toward ϕ_{FV} .

At intermediate temperatures, the thermal contributions from self-interactions become Boltzmann suppressed and can be ignored, and V_{eff} develops a second minimum at smaller field values. Notably, due to V_{P} , the large-field minimum approaching ϕ_{FV} is the global minimum of V_{eff} at these temperatures, and the dilaton is attracted to the false vacuum. This is a key property of our scenario and a

general feature of theories with field-dependent couplings: the theory is pushed to weaker couplings at high temperatures. This effect traps the dilaton at ϕ_{FV} , providing natural initial conditions for the weak-to-strong PT.

At sufficiently low temperatures, V_{eff} becomes dominated by the zero-temperature potential $V_0(\phi)$, and ϕ_{TV} becomes the new global minimum of the potential. However, the dilaton remains classically trapped at ϕ_{FV} . The right panel of Fig. (1) shows a schematic representation of the full two-field potential, $V_{\text{tot}}(\phi, \chi) = V_{\text{eff}}(\phi) + V_\chi(\phi, \chi)$, for $T_{\text{QCD}}^{\text{FV}} < T < T_{\text{QCD}}^{\text{TV}}$. For $\langle \phi \rangle = \phi_{\text{TV}}$, chiral symmetry is broken by $\langle \chi \rangle \neq 0$ in the vacuum. For $\langle \phi \rangle = \phi_{\text{FV}}$, QCD is in the unconfined phase, and, consistently, $\langle \chi \rangle = 0$ and chiral symmetry is unbroken.

With ϕ in the false vacuum, the quark-gluon plasma may cool below $T_{\text{QCD}}^{\text{TV}}$ without triggering the crossover transition to the hadron gas (which for $\phi = \phi_{\text{FV}}$ would happen around $T_{\text{QCD}}^{\text{FV}}$). However, the dilaton vacuum is nonperturbatively unstable and eventually decays to ϕ_{TV} through quantum tunneling, proceeding through the nucleation of true-vacuum bubbles. The nucleation rate per unit volume is given by $\Gamma/V = Ae^{-S_4}$ [78,79], with S_4 the time-independent $O(4)$ symmetric bounce action and A a prefactor. The nucleation temperature T_n is bounded from above by the requirement that the global minimum of V_{eff} is at ϕ_{TV} . This gives $T_n \ll m_\phi$ and, more stringently, $T_n < T_c$, where T_c denotes the “critical temperature” at which the two minima have the same energy: $V_0(\phi_{\text{TV}}) + V_{\text{P}}(\phi_{\text{TV}}, T_c) = V_0(\phi_{\text{FV}}) + V_{\text{P}}(\phi_{\text{FV}}, T_c)$. Lattice simulations have found that the QCD pressure at strong coupling is lower than the corresponding ideal gas pressure [46,80] (see Ref. [81] for a review). Parametrically, we expect $T_c^3 T_{\text{QCD}} \sim \Delta V$ when $\mathcal{O}(300) \text{ MeV} \lesssim T_c \lesssim 2 \text{ GeV}$ and $T_c^4 \sim \Delta V$ for higher critical temperatures; cf. Supplemental Material [47] for a more

detailed discussion. Moreover, the nucleation temperature is bounded from below by the BBN temperature $T_n \gtrsim T_{\text{BBN}} \approx 4 \text{ MeV}$ [82–85]. For $\Delta f \sim \mathcal{O}(1)$, $T_{\text{BBN}} \gg T_{\text{QCD}}^{\text{FV}}$, so the false-vacuum quark-gluon plasma does not undergo a QCD crossover transition prior to the dilaton tunneling. In the main text of this Letter, we explore T_n in the range

$$T_{\text{BBN}} < T_n < T_{\text{QCD}}^{\text{TV}}. \quad (8)$$

The complementary range $T_{\text{QCD}}^{\text{TV}} < T_n < T_c$ naturally predicts ultrarelativistic “runaway” bubbles that, upon collision, generate gravitational wave signals at higher frequencies. Such models can be probed by future gravitational wave detectors, but do not affect the QCD crossover transition; cf. Supplemental Material for a more detailed discussion [47]. Fixing the nucleation temperature corresponds to fixing the decay rate through the approximate equation $\Gamma/\mathcal{V} \simeq H^4(T_n)$, expressing that, at T_n , one bubble is nucleated per Hubble volume and Hubble time.

Outside the bubbles, the plasma remains in the pre-confined quark-gluon phase (as $T_n > T_{\text{QCD}}^{\text{FV}}$), but inside the bubbles, the temperature is lower than the local QCD confinement scale (i.e., $T_{\text{QCD}}^{\text{TV}}$), leading to prompt hadronization and chiral symmetry breaking. In the two-quark linear σ model, the approximate timescale of confinement can be found from $t_{\text{roll}} \sim \mathcal{O}(1/m_\chi) \ll H^{-1}$, with $m_\chi = \sqrt{2\lambda f_\pi}$, so chiral symmetry breaking inside the true-dilaton-vacuum bubbles is approximately instantaneous. Thus, in this scenario, the QCD PT is no longer a smooth crossover, but characterized by expanding bubbles of the hadronic phase inside the quark-gluon plasma; this scenario effectively turns the QCD phase transition first order. We stress, however, that the mechanism is intrinsically quantum and multiple field and lies outside the much-studied framework of thermal QCD PTs.

We require that the transition percolates (at plasma temperature $T_\star$) which, for a PT with constant Γ , requires $\alpha \lesssim 20$ [48], where $\alpha = \Delta V/\rho_{\text{rad}}(T_n)$. For such transitions, the inverse duration of the PT β is set by α and satisfies $8 \gtrsim \beta/H_\star > 3$ [48]. As the energy density ΔV is channeled into the standard model, the plasma is reheated to the temperature $T_{\text{rh}} = (1 + \alpha)^{1/4} T_n$, here assuming $T_\star \approx T_n$. We restrict our discussion to $T_{\text{rh}} \lesssim T_{\text{QCD}}^{\text{TV}}$, so that the hadrons do not deconfine (and then undergo a “second” QCD transition, this time a crossover).

The subsequent cosmology and observational predictions of this scenario depend on the speed at which the bubbles expand. As we show in Supplemental Material [47], the bubble walls reach ultrarelativistic terminal velocities before the bubbles collide. This is in notable contrast to BSM or holographic models of thermal first-order QCD PT, in which the latent heat is typically too small to overpower the pressure from the quark-gluon plasma, thereby making ultrarelativistic velocities unattainable.

Gravitational wave signal—As the relativistic dilaton bubbles grow large, collide, and percolate, they produce a stochastic gravitational wave (GW) background, potentially observable today [11,86]. This is particularly interesting given the recent observation of the Hellings-Down correlation in PTA datasets [4–7], providing evidence for a stochastic GW background in the nanohertz range. Such a background could potentially be explained astrophysically through supermassive black hole binary mergers [87], but this explanation has difficulties with accommodating the observed amplitude and spectrum [88,89]. By contrast, several studies have found that GWs generated by a first-order PT provide a better fit to the data [49,90–103] (see Ref. [9] for an alternative gravitational wave explanation generated by turbulent sources).

Since the bubbles are expected to reach a terminal velocity before they collide, the main contribution to the GW signal in this scenario is from the dynamics of sound waves generated in the plasma [50,51,104–106] (the contribution to the signal coming from bubble wall collisions, which is relevant for runaway PTs, is discussed in Supplemental Material [47]).

Estimates of the amplitude and spectrum of the GW background can be parametrized through rather simple formulas, depending on $T_\star \approx T_n$, α , β , and v_w (cf. [107] and, for a recent review, [108]). The peak frequency is given by

$$f_{\text{sw}} = \frac{17.53 \text{ nHz}}{v_w} \frac{\beta}{H_\star} \frac{T_n}{100 \text{ MeV}} \left(\frac{g_\star}{61.75} \right)^{1/6}. \quad (9)$$

Thus, the dilaton-induced, first-order QCD PT proposed in this Letter results in peak frequencies in the $\mathcal{O}(10\text{--}100 \text{ nHz})$ range.

The corresponding GW energy spectra $h^2 \Omega_{\text{GW}}$ give the fractional energy density per logarithmic frequency interval, rescaled by the squared Hubble parameter. The GW spectrum from sound waves is then

$$\begin{aligned} h^2 \Omega_{\text{sw}}(f) = & 3.11 \times 10^{-6} \left(\frac{H_\star}{\beta} \right) \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \\ & \times \left(\frac{61.75}{g_\star(T_\star)} \right)^{1/3} v_w \left(\frac{f}{f_{\text{sw}}} \right)^3 \\ & \times \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}, \end{aligned} \quad (10)$$

where the efficiency factor κ_v has been inferred from simulations of ultrarelativistic walls [109], $\kappa_v = \alpha/(0.73 + \alpha + 0.083\sqrt{\alpha})$. We note that Eq. (10) is motivated by simulations of detonations with $\alpha \leq 0.67$ [110,111]; for larger values of α , it provides an often-used extrapolation.

In Fig. 2, we show the GW energy spectrum generated by the dilaton-induced phase transition for several benchmark values of the parameters. The spectra are overlaid with the

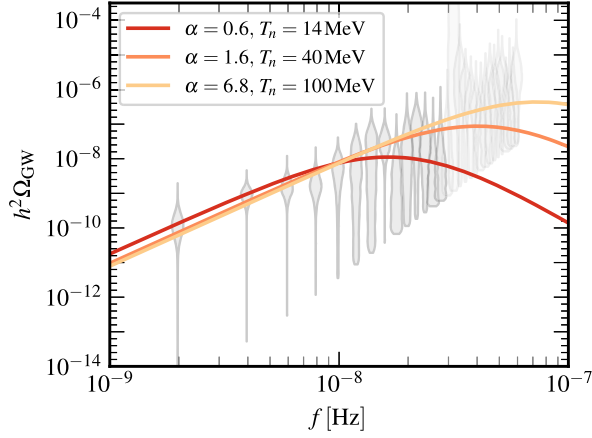


FIG. 2. Gravitational wave spectra from a dilaton-induced QCD PT overlaid with NANOGrav 15-yr free-spectrum constraints (two shades of gray violins). The red curve corresponds to the best-fit spectrum to the 14 lowest-frequency bins (darker gray). For more information, see the main text.

posterior distributions from the NANOGrav 15-yr free-spectrum analysis [4], shown as violins in two shades of gray (see below). Remarkably, the predictions of our model are broadly compatible with the observed spectrum. In Supplemental Material [47], we follow [49] and determine the parameter space favored by the NANOGrav data by performing a Markov chain Monte Carlo analysis using the PTArade code [52,53]. Only the 14 lowest-frequency bins are used to constrain the spectrum, since the evidence for the GW background is dominated by the low-frequency modes [49]. These bins cover the frequency range $2 \leq f \leq 28$ nHz and are shaded in darker gray in Fig. 2. For clarity, we also plot the higher-frequency range up to 59 nHz, shown in lighter gray, that is used for fitting the pulsar-intrinsic red noise. The predicted spectrum provides an excellent fit to the data, with 68% posterior credible intervals given by $11.7 \leq T_n \leq 22.7$ MeV and $0.47 \leq \alpha \leq 0.84$. The best-fit spectrum is shown in red in Fig. 2, together with two illustrative examples of spectra for other parameter values. We note that the slight dip in the inferred spectrum above 20 nHz drives the preference for a low nucleation temperature. Higher values of T_n push the peak of the spectrum to higher frequencies, while increasing α lowers the peak frequency and increases the overall strength of the signal.

An explicit model—The dilaton-induced QCD PT can be realized in a range of models. Here, we consider one simple, explicit example for concreteness. We defer the construction of an ultraviolet completion of this model to future work. We start from the Lagrangian

$$\mathcal{L} \supset \Lambda_1^2 \frac{\partial_\mu \phi \partial^\mu \phi}{2\phi^2} - \frac{\phi}{2\Lambda} \text{tr} G_{\mu\nu} G^{\mu\nu}, \quad (11)$$

for the energy scales Λ_1 and Λ . This general form of the Lagrangian, with noncanonical kinetic terms and a linear

dependence of the gauge kinetic function on ϕ , is often found in the four-dimensional supergravity description of string compactifications, where it is common for moduli fields to have logarithmic Kähler potentials and linear holomorphic gauge kinetic functions at tree level.

In terms of the canonically normalized dilaton field $\phi = \Lambda_1 \ln(\varphi/\Lambda)$, the Lagrangian is given by

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{e^{\phi/\Lambda_1}}{2} \text{tr} G_{\mu\nu} G^{\mu\nu}, \quad (12)$$

i.e., with $\tilde{f} = \exp(\phi/\Lambda_1)$.

An exponential hierarchy between $\Lambda_{\text{QCD}}^{\text{TV}}$ and $\Lambda_{\text{QCD}}^{\text{FV}}$ is generated if $8\pi^2 \Delta \tilde{f}/\beta_0 \gtrsim \mathcal{O}(1)$, which gives

$$\frac{8\pi^2}{\beta_0} \Delta \phi f'_{\text{TV}} = \frac{8\pi^2 \tilde{f}_{\text{TV}}}{\beta_0} \frac{\Delta \phi}{\Lambda_1} \gtrsim \mathcal{O}(1),$$

where we have Taylor expanded $\tilde{f}$ to leading order. For $\tilde{f}_{\text{TV}} = 4.2$ this gives a weak lower bound on the separation of the vacua: $\Delta \phi/\Lambda_1 \gtrsim 3 \times 10^{-2}$. We assume a toy model effective potential for ϕ given by

$$V_0(\phi) = \frac{m^2}{2\Delta\phi^2} (\phi - \phi_{\text{TV}})^2 (\phi_{\text{FV}} - \phi)^2 + \frac{\epsilon(\phi - \phi_{\text{FV}})}{\Delta\phi} + V_{\text{np}}(\phi), \quad (13)$$

for which we treat ϕ_{TV} , ϕ_{FV} , m^2 , and ϵ as free parameters. Note that $\Delta\phi = \phi_{\text{FV}} - \phi_{\text{TV}}$, and $m_{\text{TV/FV}}^2 = m^2$. The term V_{np} includes contributions that become important for small field values, including the contribution $\sim f_\pi^4(\phi)$ from the linear σ model, the gluon condensate $\sim -(\beta_0/32)\Lambda_{\text{QCD}}^4(\phi)$ [112], and further nonperturbative contributions of the form $\sim c e^{-8\pi^2 \tilde{f}(\phi)/\beta_0}$. The potential V_{np} is necessary to ensure that the minimum for the two-field potential $V_{\text{tot}}(\phi, \chi)$ is located at $(\phi_{\text{TV}}, f_\pi^{\text{SM}})$. Finally, we denote as $\Delta V \equiv \epsilon + V_{\text{np}}(\phi_{\text{FV}}) - V_{\text{np}}(\phi_{\text{TV}})$.

To realize the hierarchy of Eq. (8) with a nucleation temperature of $T_n = 100$ MeV one may, for example, choose $m \sim 10$ TeV, $\Delta\phi \simeq 2 \times 10^{-3}$ GeV, $\Delta\phi/\Lambda_1 \sim \mathcal{O}(1)$, and $\Delta V \simeq 0.7 \rho_{\text{rad}}(T = 100 \text{ MeV})$. In this case, the barrier height in ϕ direction is large, and the thin-wall approximation can be used to estimate S_4 and the decay rate. Thus, our scenario can be realized with a rather minimal set of ingredients.

Discussion—We have shown that quantum tunneling of the QCD dilaton can turn the QCD phase transition first order and generate a gravitational wave signal intriguingly similar to that observed by pulsar timing arrays. Notably, in our scenario, the PT is induced by multifield dynamics rather than generated from thermal fluctuations, and its predictions differ quantitatively and qualitatively from the

much-studied framework of thermal first-order QCD transitions.

Our theory can be probed by collider experiments. Dilatons produced through gluon fusion can be searched for through dijet resonances. Data from Run 2 of the LHC has already been used to constrain the theory up to $m_\phi \sim 5$ TeV [35]. Data from LHC Run 3, the planned high-luminosity LHC, and future collider experiments at higher energy will probe some of our theory's most interesting parameter space.

A first-order QCD PT may source inhomogeneities in the plasma at the onset of BBN, which can affect the abundances of heavy elements [113,114] (see also Ref. [115]). We expect to explore the impact of our dilaton-induced QCD PT on BBN in future work.

An interesting extension of our theory includes the axion, which can be regarded as the pseudoscalar partner of the dilaton. The mass of the axion increases exponentially from the false vacuum to the true, and an ambient “seed” abundance of axions can be trapped in relic pockets of the false-vacuum state [32] (see also Refs. [11,116]). If Δf is sufficiently large, such “axion relic pockets” can be stable and may inherit a substantial fraction of the energy density ΔV , leading to an overclosure of the Universe unless $\alpha \ll 1$. For moderate Δf , the axion relic pockets are transient “droplets” and provide an novel source of small-scale isocurvature.

A further extension of this theory includes an axionlike particle (ALP) coupled to a dark sector confining gauge theory in addition to QCD (i.e., a “QCD ALP” [117]). Such theories often suffer from a problem where stable cosmic domain walls come to dominate the energy density, in conflict with observations. Drawing on the findings of [118,119], we note that this problem may be solved if the combined ALP and dilaton tunneling “eats” the domain walls, which may be achieved in theories where $f(\phi)$ is nonmonotonic. We expect to make this mechanism more explicit in future work.

While the explicit model presented in this Letter illustrates how it may be realized with rather simple ingredients, it would be interesting to embed this toy model in a more complete theoretical framework.

Finally, the mechanism proposed in this Letter may find applications outside cosmology. Emergent gauge theories with tunable dilaton parameters have been discovered in several condensed matter systems [120–123] and may potentially host nontrivial PTs similar to our scenario. More generally, the essence of our proposal is that the order-parameter space of certain theories can naturally be augmented, leading to rich multiple-field dynamics that can transform the nature of predicted PTs.

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Data availability—Some of the data that support the findings of this article are openly available [54]. Numerical evaluation of equations given in the article and evaluation of the bounce action are available within the article.

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