

Anyonization of Bosons in One Dimension: An Effective Swap Model

Botao Wang^{1,2,*} Amit Vashisht^{1,2,†} Yanliang Guo^{3,4,‡} Sudipta Dhar³ Manuele Landini³
 Hanns-Christoph Nägerl³ and Nathan Goldman^{1,2,5,§}

¹*Center for Nonlinear Phenomena and Complex Systems, Université Libre de Bruxelles,
 CP 231, Campus Plaine, 1050 Brussels, Belgium*

²*International Solvay Institutes, 1050 Brussels, Belgium*

³*Institut für Experimentalphysik und Zentrum für Quantenphysik, Universität Innsbruck,
 Technikerstraße 25, Innsbruck 6020, Austria*

⁴*Key Laboratory of Quantum State Construction and Manipulation (Ministry of Education), School of Physics,
 Renmin University of China, Beijing 100872, China*

⁵*Laboratoire Kastler Brossel, Collège de France, CNRS, ENS-Université PSL,
 Sorbonne Université, 11 Place Marcelin Berthelot, 75005 Paris, France*



(Received 12 May 2025; revised 31 October 2025; accepted 13 November 2025; published 17 December 2025)

Anyons emerge as elementary excitations in low-dimensional quantum systems and exhibit behavior distinct from bosons or fermions. In one dimension, anyons can arise from unconventional scattering processes or density-dependent hopping on a lattice. Here, we introduce a novel framework for realizing anyonic correlations using the internal degrees of freedom of a spinor quantum gas. We propose a “swap” model, which assigns a complex phase factor to the swapping processes between two different species, referred to as “host particles” and “impurities.” The anyonic characteristics are demonstrated through the one-body correlator of the impurity, using a spin-charge separation analysis. The impurity plays the role of both creating and directly probing the anyonic correlations in a quantum many-body system, which are not accessible in the conventional anyon-Hubbard model. For a single impurity, our framework can be effectively implemented by applying tilt potentials in a strongly interacting quantum gas [Dhar *et al.*, Observing anyonization of bosons in a quantum gas, *Nature (London)* **642**, 53 (2025)]. We further explore the dynamical properties of anyonic correlations and extend our analysis to the case of multiple impurities. Our Letter provides new avenues for engineering many-body anyonic behavior in quantum simulation platforms.

DOI: 10.1103/2np8-mp39

Introduction—Exotic behavior can emerge in lower-dimensional systems. A prime example is given by the emergence of anyons obeying fractional statistics, interpolating between bosons and fermions [1–5]. Arising from a mathematical concept, anyons are often found to exist as elementary excitations of topologically ordered states of matter [6–13]. The idea of utilizing non-Abelian anyons for topological quantum computing has spurred efforts to observe their fractional statistics across a variety of physical platforms, such as solid-state materials [14,15], superconducting quantum circuits [16–19], Rydberg atom

arrays [20], and trapped-ion processors [21]. Since anyons are typically defined through the exchange of quasiparticles in two dimensions (2D) with topologically distinct windings, fractional statistics seems to be absent in one dimension (1D).

The generalization of fractional statistics to arbitrary dimensions was provided by Haldane [22]. He demonstrated that the spinon excitations in a 1D spin chain, described by the Haldane-Shastry model (HSM) [23,24], satisfy fractional exclusion statistics [22]. Subsequent investigations predicted the presence of 1D (Abelian) anyonic excitations in other models, including the Kuramoto-Yokoyama model [25–28] and the Calogero-Sutherland model [29–31]. It was recognized that these excitations exhibit a common feature: a statistical phase can be associated to their unidirectional crossing, leading to a fractional shift in the linear momenta in 1D that is analogous to the fractional relative angular momentum in 2D [5,32]. By leveraging the notion of statistical interactions [22], one can embed fractional statistics into 1D scattering processes, offering an interesting perspective

*Contact author: botao.wang@ulb.be

†Contact author: amit.vashisht@ulb.be

‡Contact author: yanliang.guo@uibk.ac.at

§Contact author: nathan.goldman@lkb.ens.fr

Published by the American Physical Society under the terms of the *Creative Commons Attribution 4.0 International* license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

to reinterpret interacting bosons [33–39]. Experimentally, realizing and probing anyons in such 1D models has been much less explored. Recently, the chiral BF theory [37,40,41], originally constructed as a candidate model for 1D anyons, has been realized in a Raman-coupled Bose-Einstein condensate [41,42].

Another route to realize 1D Abelian anyons is offered by engineering density-dependent Peierls phases on a lattice, following the bosonic formulation of the anyon-Hubbard model (AHM) [43–47]. This approach has been shown to give rise to intriguing phenomena, such as statistically driven equilibrium phases [43,48–52] and nonequilibrium anyonic dynamics [53–59]. Despite extensive theoretical efforts [43–64], experimental realization of 1D anyons was only recently observed in an optical lattice using two ultracold atoms, where the density-dependent hopping was engineered using Floquet driving [65]. However, a direct measurement of the asymmetric momentum distribution, the hallmark signature of anyonic correlations [66–73], is not possible in that setting: anyonic correlations need to be manually reconstructed upon extracting the momentum distribution of the bare bosons.

More recently, anyonic correlations were observed by injecting an impurity into a strongly interacting Bose gas [74], wherein the statistical phase was encoded in the momentum of the impurity, hence allowing for a direct measurement of the asymmetric momentum distribution [75–77]. Here, we present a simple yet insightful model to capture the essence of anyonization in a strongly interacting 1D spinor quantum gas, providing a general framework to engineer anyonic correlations in 1D. We propose a new “swap” model, which associates the exchange between the two species of the spinor quantum gas with a tunable complex phase. Using a spin-charge separation analysis, we demonstrate anyonic signatures in the impurity’s one-body correlation function. For a single impurity, this scheme can be implemented by generating a spin wave, for instance, by applying a weak constant force to the impurity. This simple picture offers an effective description of the recent experimental observations of Ref. [74]. We then extend our approach to study the real-time evolution of anyonic correlations as well as scenarios with multiple impurities. Altogether, this Letter lays the groundwork for engineering and probing anyonic many-body behavior in 1D quantum simulators.

Anyon-Hubbard model—We start by briefly reviewing the properties of anyons in a 1D lattice, as described by the paradigmatic AHM,

$$\hat{H}_{\text{AHM}} = -J \sum_{\ell} (\hat{a}_{\ell}^{\dagger} \hat{a}_{\ell+1} + \text{H.c.}) + \frac{U}{2} \sum_{\ell} \hat{n}_{\ell} (\hat{n}_{\ell} - 1), \quad (1)$$

where J denotes the tunneling amplitude, U quantifies the on-site interaction strength between anyons, and $\hat{n}_{\ell} = \hat{a}_{\ell}^{\dagger} \hat{a}_{\ell}$ is the number operator at site ℓ . The anyonic

operators $\hat{a}_{\ell}$ obey the generalized commutation relations,

$$\hat{a}_j \hat{a}_k^{\dagger} - e^{i\theta \text{sgn}(j-k)} \hat{a}_k^{\dagger} \hat{a}_j = \delta_{jk} \quad (2a)$$

$$\hat{a}_j \hat{a}_k - e^{-i\theta \text{sgn}(j-k)} \hat{a}_k \hat{a}_j = 0, \quad (2b)$$

where θ is the statistical angle. The sign function $\text{sgn}(j-k)$ equals $+1$ for $j > k$, -1 for $j < k$ and 0 for $j = k$. By applying a fractional version of the Jordan-Wigner (JW) transformation, $\hat{a}_{\ell} = \hat{b}_{\ell} e^{i\theta \hat{N}_{\ell}}$, where $\hat{N}_{\ell} = \sum_{m < \ell} \hat{n}_m$ and $\hat{b}_{\ell}$ are bosonic operators, the one-body correlation operator reads $\hat{a}_{\ell}^{\dagger} \hat{a}_{\ell'} = \hat{b}_{\ell}^{\dagger} e^{i\theta(\hat{N}_{\ell'} - \hat{N}_{\ell})} \hat{b}_{\ell'}$. Writing the ground state in the Fock state basis $|\Psi\rangle = \sum_i c_i |\zeta_i\rangle$, with c_i being complex coefficients, one can express the anyonic correlation function as

$$\langle \hat{a}_{\ell}^{\dagger} \hat{a}_{\ell'} \rangle = \begin{cases} \sum_{i,j} c_i^* c_j \langle \zeta_i | \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell'} | \zeta_j \rangle e^{i\theta(N_{\ell'}^j - N_{\ell}^j)}, & \ell' \geq \ell, \\ \sum_{i,j} c_i^* c_j \langle \zeta_i | \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell'} | \zeta_j \rangle e^{i\theta(N_{\ell'}^j - N_{\ell}^j + 1)}, & \ell' < \ell, \end{cases} \quad (3)$$

where N_{ℓ}^j denotes the number of particles after applying the operator $\hat{N}_{\ell}$ on the Fock state $|\zeta_j\rangle$, and $\langle \dots \rangle$ denotes an expectation value over the ground state of the system. As shown in Eq. (3), the string operators entering the JW transformation lead to a phase factor depending on the density difference in the one-body correlation [see Fig. 1(a)]. By solving the time-independent Schrödinger equation associated with the AHM in the bosonic representation, and performing the Fourier transformation of the one-body correlation function (3), one obtains the θ -dependent asymmetric quasimomentum distribution of the anyons [71,78]; see the dashed lines in Fig. 1(c). This asymmetric behavior stands as a hallmark signature of anyons in 1D systems.

Swap model—Previous 1D anyon models often include the fractional statistics either through nontrivial collisions [5,32,36–39] or utilizing density-dependent hopping processes [43–46]. Here, instead, we propose a two-component model that encodes the anyonic correlations using an internal (“spin”) degree of freedom. Considering a strongly interacting spinor quantum gas in a 1D lattice, the proposed swap model reads,

$$\begin{aligned} \hat{H}_{\text{swap}} = & -J \sum_{\ell} (\hat{b}_{\uparrow\ell}^{\dagger} \hat{b}_{\uparrow\ell+1} + \hat{b}_{\downarrow\ell}^{\dagger} \hat{b}_{\downarrow\ell+1}) \\ & - J_{\text{ex}} e^{i\theta} \sum_{\ell=1}^{L-1} \hat{b}_{\uparrow\ell}^{\dagger} \hat{b}_{\downarrow\ell+1}^{\dagger} \hat{b}_{\downarrow\ell} \hat{b}_{\uparrow\ell+1} + \text{H.c.}, \end{aligned} \quad (4)$$

where $\hat{b}_{\sigma\ell}^{\dagger}$ ($\hat{b}_{\sigma\ell}$) are the creation (annihilation) operators of hard-core bosons with spin $\sigma = \{\uparrow, \downarrow\}$ at site ℓ . We use spin indices $\uparrow$ and $\downarrow$ to denote the host and the impurity

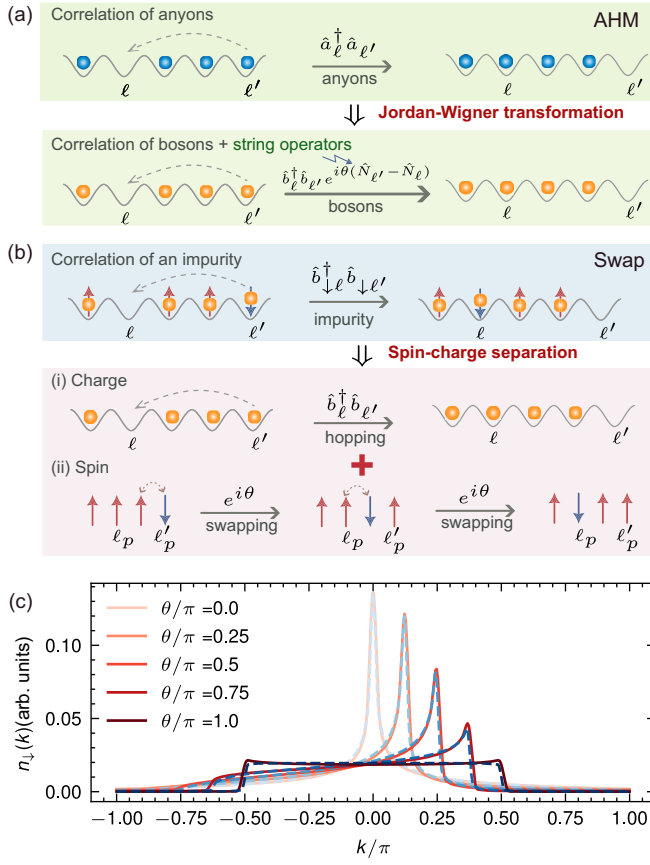


FIG. 1. (a) Sketch of one-body correlations $\langle \hat{a}_{\ell}^{\dagger} \hat{a}_{\ell'} \rangle$ of anyons in the AHM (with respect to a given Fock state configuration). By means of a Jordan-Wigner transformation, the anyonic one-body correlator can be expressed as that of bosons accompanied by a phase factor, i.e., $\langle \hat{a}_{\ell}^{\dagger} \hat{a}_{\ell'} \rangle = \langle \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell'} e^{i\theta(\hat{N}_{\ell'} - \hat{N}_{\ell})} \hat{b}_{\ell'} \rangle$. (b) Sketch of one-body correlation $\langle \hat{b}_{\downarrow \ell}^{\dagger} \hat{b}_{\downarrow \ell'} \rangle$ of a single impurity in our swap model. As there are two host particles between site ℓ and ℓ' , the long-range hopping indicated here involves two swaps. Since each swap is associated with $e^{i\theta}$, the swapping processes contribute a phase factor $e^{i\theta(\ell'_p - \ell_p)} = e^{i2\theta}$ that recovers the contribution from the string operators in (a). (c) Quasimomentum distribution of the impurity with different phase θ . Solid lines: Using our swap Hamiltonian (4) with $N_{\uparrow} = 50, N_{\downarrow} = 1, L = 100, J_{\text{ex}} = 0.1$. Dashed lines: results of AHM with $N = 50, L = 100$ and hard-core interaction. The peak has been rescaled according to a normalization factor determined at $\theta = 0$.

particles, respectively; J represents the strength of nearest-neighbor hopping and J_{ex} is the strength of the swapping between different spins on neighboring sites. Here we assume no double occupancy, and thus our model applies in the regime of strong on-site interaction. This is reminiscent of the tJ -model studied in quantum magnetism [79]. One major difference is the complex phase factor $e^{i\theta}$ assigned to the swapping terms. In the following, we will show that θ plays the role of the fractional statistical angle of anyons, as captured by the one-body correlation function. We use L as the number of lattice sites and $N_{\uparrow}(N_{\downarrow})$ as the number of

host particles (impurities). We set $J = 1$ as the unit of energy. Similar to the experiment reported in Ref. [74], we first focus on the case of a single impurity.

We compute the one-body correlation of the impurity based on a spin-charge separation analysis [80,81]. In the Fock state basis $\{|\psi_i\rangle\}$, the one-body correlation function of the impurity reads,

$$\langle \hat{b}_{\downarrow \ell}^{\dagger} \hat{b}_{\downarrow \ell'} \rangle = \sum_{i,j} \tilde{c}_i^* \tilde{c}_j \langle \psi_i | \hat{b}_{\downarrow \ell}^{\dagger} \hat{b}_{\downarrow \ell'} | \psi_j \rangle, \quad (5)$$

with $\tilde{c}_i$ being complex coefficients. Each Fock state can be written as the product of its charge sector $|\psi_i^c\rangle$ and the spin sector $|\psi_i^s\rangle$, i.e. $|\psi_i\rangle = |\psi_i^c\rangle \otimes |\psi_i^s\rangle$. Meanwhile, we can decompose the one-body correlation operator of the impurity $\hat{b}_{\downarrow \ell}^{\dagger} \hat{b}_{\downarrow \ell'}$ as a hopping of the charge and a swapping of the spins. Taking the Fock state configuration sketched in Fig. 1(b) as a concrete example, the one-body correlator consists of the hopping of the charge from site ℓ' to ℓ , combined with a series of swapping processes that start from the spin labeled by $\ell'_p = N_{\ell'}^j + 1 = 4$ and end with the spin $\ell_p = N_{\ell}^j + 1 = 2$. Here, we used ℓ and ℓ_p to label the lattice sites and the spins, respectively, and N_{ℓ}^j denotes the number of particles to the left of site ℓ in a given Fock state $|\psi_j^c\rangle$. It is apparent that the number of swaps is determined by the number of particles between site ℓ and ℓ' , i.e., $\ell'_p - \ell_p = N_{\ell'}^j - N_{\ell}^j$.

Based on this observation, one can write the correlator in the given Fock states as

$$\begin{aligned} \langle \psi_i | \hat{b}_{\downarrow \ell}^{\dagger} \hat{b}_{\downarrow \ell'} | \psi_j \rangle &= \sum_{\ell_p, \ell'_p} \langle \psi_i^c | \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell'} | \psi_j^c \rangle \delta_{N_{\ell}^j}^{\ell_p-1} \delta_{N_{\ell'}^j}^{\ell'_p-1} \\ &\times \langle \psi_i^s | \hat{\mathcal{E}}_{\ell_p, \ell'_p \mp 1} \cdots \hat{\mathcal{E}}_{\ell_p \pm 1, \ell'_p} | \psi_j^s \rangle. \end{aligned} \quad (6)$$

Here, the Kronecker delta, $\delta_{N_{\ell}^j}^{\ell_p-1} = 1$ if $\ell_p - 1 = N_{\ell}^j$ and 0 otherwise, ensures that the ℓ' th site is occupied by the ℓ'_p th spin, and after hopping, the ℓ th site is occupied by the ℓ_p th spin [81]. We use $\hat{\mathcal{E}}_{\ell_p \pm 1, \ell_p}$ to denote the swapping between the impurity labeled by ℓ_p and the host particle labeled by $\ell_p \pm 1$. Assigning a phase factor $e^{\pm i\theta}$ in the swapping between the impurity and the host particle to its left (right), we obtain the following one-body correlation function of the impurity:

$$\langle \hat{b}_{\downarrow \ell}^{\dagger} \hat{b}_{\downarrow \ell'} \rangle = \begin{cases} \sum_{i,j} \tilde{c}_i^* \tilde{c}_j \langle \psi_i^c | \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell'} | \psi_j^c \rangle e^{i\theta(N_{\ell'}^j - N_{\ell}^j)}, & \ell' \geq \ell, \\ \sum_{i,j} \tilde{c}_i^* \tilde{c}_j \langle \psi_i^c | \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell'} | \psi_j^c \rangle e^{i\theta(N_{\ell'}^j - N_{\ell}^j + 1)}, & \ell' < \ell. \end{cases} \quad (7)$$

Hence, we obtain that the anyonic correlations, equivalent to Eq. (3), are formally established in the one-body

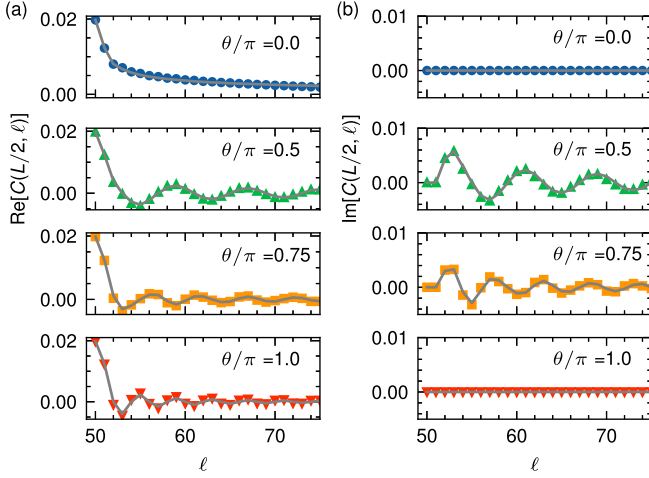


FIG. 2. The real (a) and the imaginary (b) part of the one-body correlation function of the single impurity in our swap model for different θ . We have used $N_\uparrow = 50, N_\downarrow = 1, L = 100$ and $J_{\text{ex}} = 0.1$. Gray solid lines: results of the AHM with $N = 50, L = 100$ and hard-core interaction.

correlation of the impurity. The spin swapping processes in our model play the role of the operators $\hat{N}_\ell$ in the JW transformation of the AHM.

This result is illustrated in Fig. 2: the swap model yields identical correlations as the AHM (up to a normalization factor [82]). For $\theta = 0$, the bosonic correlation shows a power law decay; in the case $\theta = \pi$, we recover the k_F oscillation (with $k_F = \pi\rho_\uparrow$ being the Fermi momentum and $\rho_\uparrow$ the density of the host particles) due to the fermionic statistics [81]. For values of θ between 0 and π , we observe oscillations of period of $2\pi/\theta\rho_\uparrow$ in both the real and the imaginary part of the one-body correlation, which is a manifestation of the anyonic features therein.

The momentum distribution, derived from the Fourier transform of anyonic correlation functions, serves as a useful observable of anyonic characteristics. In our Letter, the impurity's quasimomentum profile mirrors the anyonic behavior seen in the AHM; see Fig. 1(c). Crucially, this opens up a new experimental pathway: using an impurity allows one to directly probe anyonic momentum distributions, which are not directly accessible in standard bosonic implementations of AHM [43–46, 78].

Effective realization—We now demonstrate that the ground state $|\Psi_{\text{swap}}(\theta)\rangle$ can be prepared by exciting a spin wave in the swap model at $\theta = 0$. This offers a concrete strategy to explore anyonic correlations in two-component Bose gases operating in the strongly correlated regime. Considering a closed spinful chain under the constraint of no double occupancy, spin waves are known to be the low energy excitations [87, 88]. They are eigenstates of the spin cyclic permutation operator $\hat{P}$ defined as $\hat{P}|\sigma_1, \sigma_2, \dots, \sigma_N\rangle = |\sigma_N, \sigma_1, \dots, \sigma_{N-1}\rangle$ with $\sigma = \{\uparrow, \downarrow\}$ and $N = N_\uparrow + N_\downarrow$. In the case of a single impurity, the

operation of $\hat{P}$ is equivalent to the application of spin swaps. For instance, the spin configuration $|\downarrow\uparrow\uparrow\uparrow\rangle = \hat{P}|\uparrow\uparrow\uparrow\downarrow\rangle$ can be equivalently obtained from $\hat{\mathcal{E}}_{1,2}\hat{\mathcal{E}}_{2,3}\hat{\mathcal{E}}_{3,4}|\uparrow\uparrow\uparrow\downarrow\rangle$. Thus, for an open chain described by our swap model with $\theta = 0$, where the spin swapping processes are allowed, the spin-wave excitations are expected to exist in the thermodynamic limit. Relating the phase factor $e^{i\theta}$ in our swap model to the eigenvalues e^{iq} of the operator $\hat{P}$, the ground state $|\Psi_{\text{swap}}(\theta)\rangle$ can be associated to the spin-wave state with momentum $q = \theta$ [82]. This intuitive picture not only leads to an alternative way of deriving the anyonic correlation [82], but also indicates that the ground state $|\Psi_{\text{swap}}(\theta)\rangle$ can be effectively realized via a slow injection of momentum to the impurity such that the spin-wave states are adiabatically prepared.

To support this intuitive picture, we start from the ground state of our swap model with $\theta = 0$ and then dynamically evolve the system by applying a weak constant force. The time evolution is governed by the following Hamiltonian:

$$\hat{H}_F = \hat{H}_{\text{swap}}(\theta = 0) - \sum_\ell F a \ell \hat{n}_{\downarrow\ell}, \quad (8)$$

where a constant force F is applied on the impurity only and we set the lattice constant $a = 1$ in the following. To characterize how close the time-evolved state $|\Psi_F(t)\rangle$ is to the ground state $|\Psi_{\text{swap}}(\theta)\rangle$, we define the fidelity as

$$\mathcal{F}(\theta, t) = |\langle\Psi_{\text{swap}}(\theta)|\Psi_F(t)\rangle|^2. \quad (9)$$

In Fig. 3(a), the fidelity $\mathcal{F}(\theta, t)$ is plotted as a function of θ and t . It is observed that the fidelity approaches 1 in proximity to the diagonal, indicating that the ground state $|\Psi_{\text{swap}}(\theta)\rangle$ can be effectively approximated by applying a weak force over a specific duration. Taking $\theta/\pi = \{0.2, 0.4, 0.6\}$ as examples, Fig. 3(b) illustrates the maximum fidelity values as a function of the constant force F . It demonstrates that the fidelity converges towards unity as F decreases, which corresponds to a more adiabatic preparation process.

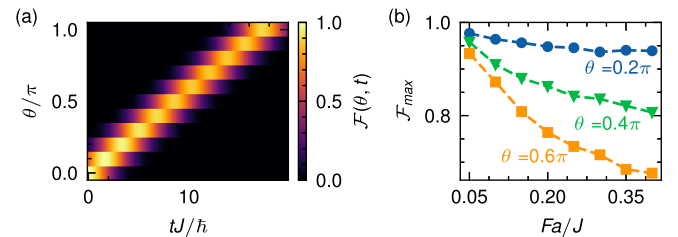


FIG. 3. (a) The fidelity $\mathcal{F}(\theta, t)$ as a function of θ and evolution time t for $Fa/J = 0.1$. (b) The maximum of $\mathcal{F}(\theta, t)$ as a function of force F for different θ . Here we use $N_\downarrow = 1, N_\uparrow = 10, L = 20, J_{\text{ex}} = 0.1$.

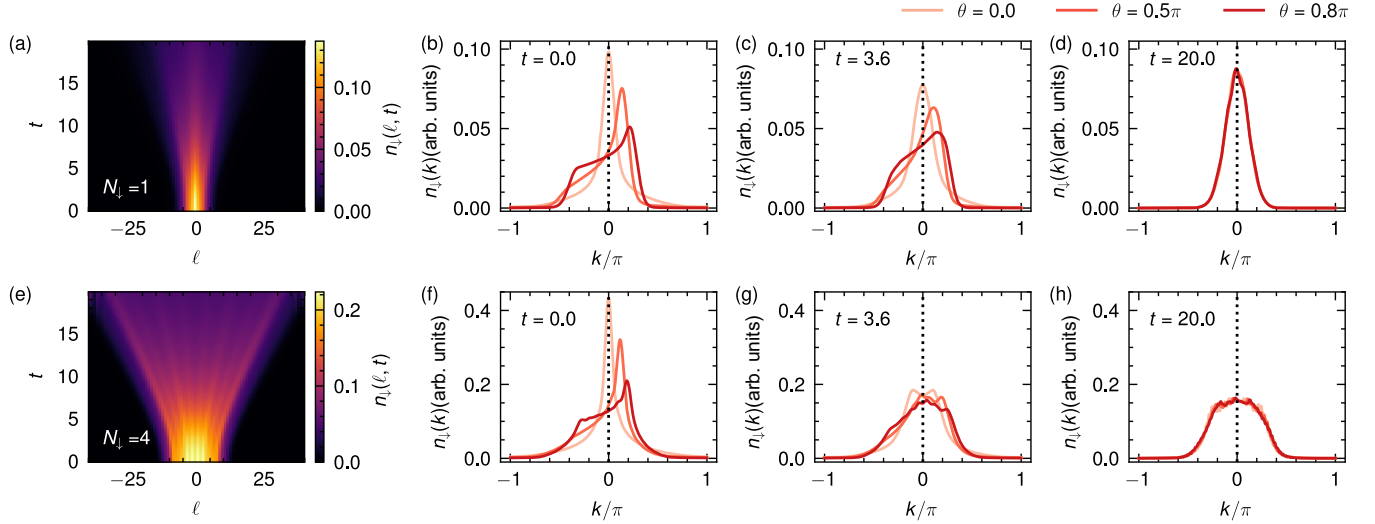


FIG. 4. (a) Spatial-temporal density distribution of the single impurity after abruptly releasing the trap for $\theta = \pi/2$. (b)–(d) The quasimomentum distribution of the impurity at different values of expansion time t (in unit of $\hbar/J$). We consider an open chain with $L = 80, N_\downarrow = 1, N_\uparrow = 6, J_{\text{ex}} = 0.1$, and $V_{\text{tr}} = 0.02$. (e)–(h) Same as (a)–(d) but considering $N_\downarrow = 4$ impurities.

The dynamical evolution governed by the Hamiltonian (8) can be readily implemented in experiments. Due to its tJ -type nature, $\hat{H}_{\text{swap}}(\theta = 0)$ is a good approximation for the spinful Bose-Hubbard model in the strongly interacting regime [82]. In the continuum limit, it can be used to describe the recent experimental observation of anyonization in bosonic systems, where a Bose-Einstein condensate of Cs atoms was loaded into a 2D optical lattice, realizing an array of 1D Bose gases in the Tonks-Girardeau regime [74]. A spin impurity, introduced via a radio-frequency pulse, performed adiabatic motion under a weak force. By varying the evolution time, the system exhibited anyonic features in the impurity's momentum distribution [74]. The lattice nature of our models [Eqs. (4) and (8)] can also find other applications, such as tilted optical lattices and a closed chain under rotation. For instance, based on the driving protocol introduced in Ref. [89], our swap model could also be approximately realized using Floquet engineering [82,90,91].

Quench dynamics—It is an interesting perspective to investigate the dynamical properties of the anyonic correlations in our swap model. We start with the ground state of the swap model at a given value of θ with an overall harmonic trapping potential of strength V_{tr} , which exhibits anyonic quasimomentum distribution for different values of θ ; see Fig. 4(b). At $t = 0$, we abruptly remove the trap and monitor the quasimomentum distribution of the impurity. Despite distinct initial anyonic momentum distributions for different θ , all curves converge to the same asymptotic distribution after long timescales of expansion; see Fig. 4(d). Such an asymptotic distribution of the impurity reflects its spatial density distribution before removing the external trap [92]. Such a rapidity measurement potentially offers a useful tool for probing the *in situ* distribution of a spinor quantum gas.

Multi-impurities—So far we have focused our discussion on the case of a single impurity. The anyonic correlations can also be qualitatively engineered in the case of multiple impurities [82]. As shown in Fig. 4(f), a very similar asymmetric quasimomentum distribution can be observed in the system with more impurities (e.g. $N_\downarrow = 4$ and $N_\uparrow = 6$). A long-time free expansion leads to a fermion-type quasimomentum distribution, which is reminiscent of the phenomenon of dynamical fermionization of anyons [93].

Conclusion and outlook—We have introduced a minimal 1D swap model that features anyonic correlations, which can be revealed by a simple spin-charge separation analysis. In this framework, spin-swapping processes play a similar role in modifying the one-body correlations as the more conventional Jordan-Wigner string of the anyon-Hubbard model. Moreover, the ground state of the swap model can be prepared to good approximation via a straightforward protocol, offering a realistic route to engineer anyonic behavior in quantum engineered systems. Generalizing our scheme to non-Abelian anyons [94]—or to other exotic particles such as traid anyons [95–97], parafermions [98,99], and the R-matrix paraparticles [100] encoded in the impurity dynamics—remains an exciting open question.

Acknowledgments—We thank Xi-Wen Guan, Felix A. Palm, Bing Yang, and Yunbo Zhang for helpful discussions. Work in Brussels is supported by the ERC (LATIS project), the EOS project CHEQS, the FRS-FNRS Belgium, and the Fondation ULB. The Innsbruck team acknowledges funding by a Wittgenstein Prize grant with the Austrian Science Fund (FWF), Project No. Z336-N36, by the European Research Council (ERC) with Projects

No. 789017 and No. 101201611, by an FFG infrastructure grant with Project No. FO999896041, and by the FWF's COE 1 and quantA. Y.G. is supported by the FWF with Project No. 10.55776/COE1. Computational resources have been provided by the Consortium des Équipements de Calcul Intensif (CÉCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Grant No. 2.5020.11 and by the Walloon Region. Numerical simulations are based on matrix product states implemented using ITensors [101,102].

B. W. and A. V. contributed equally to this work.

Data availability—The data that support the findings of this article are openly available [103].

- [1] J. M. Leinaas and J. Myrheim, On the theory of identical particles, *Il Nuovo Cimento B* (1971–1996) **37**, 1 (1977).
- [2] G. A. Goldin, R. Menikoff, and D. H. Sharp, Representations of a local current algebra in nonsimply connected space and the Aharonov–Bohm effect, *J. Math. Phys. (N.Y.)* **22**, 1664 (1981).
- [3] F. Wilczek, Quantum mechanics of fractional-spin particles, *Phys. Rev. Lett.* **49**, 957 (1982).
- [4] A. Khare, *Fractional Statistics and Quantum Theory*, 2nd ed. (World Scientific, Singapore, 2005).
- [5] M. Greiter and F. Wilczek, Fractional statistics, *Annu. Rev. Condens. Matter Phys.* **15**, 131 (2024).
- [6] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Two-dimensional magnetotransport in the extreme quantum limit, *Phys. Rev. Lett.* **48**, 1559 (1982).
- [7] R. B. Laughlin, Anomalous quantum Hall effect: An incompressible quantum fluid with fractionally charged excitations, *Phys. Rev. Lett.* **50**, 1395 (1983).
- [8] B. I. Halperin, Statistics of quasiparticles and the hierarchy of fractional quantized Hall states, *Phys. Rev. Lett.* **52**, 1583 (1984).
- [9] D. Arovas, J. R. Schrieffer, and F. Wilczek, Fractional statistics and the quantum Hall effect, *Phys. Rev. Lett.* **53**, 722 (1984).
- [10] D. E. Feldman and B. I. Halperin, Fractional charge and fractional statistics in the quantum Hall effects, *Rep. Prog. Phys.* **84**, 076501 (2021).
- [11] A. Kitaev, Anyons in an exactly solved model and beyond, *Ann. Phys. (Amsterdam)* **321**, 2 (2006).
- [12] H. Yao and S. A. Kivelson, Exact chiral spin liquid with non-Abelian anyons, *Phys. Rev. Lett.* **99**, 247203 (2007).
- [13] B. Bauer, L. Cincio, B. Keller, M. Dolfi, G. Vidal, S. Trebst, and A. Ludwig, Chiral spin liquid and emergent anyons in a Kagome lattice Mott insulator, *Nat. Commun.* **5**, 5137 (2014).
- [14] H. Bartolomei, M. Kumar, R. Bisognin, A. Marguerite, J.-M. Berroir, E. Bocquillon, B. Plaçais, A. Cavanna, Q. Dong, U. Gennser, Y. Jin, and G. Fève, Fractional statistics in anyon collisions, *Science* **368**, 173 (2020).
- [15] J. Nakamura, S. Liang, G. C. Gardner, and M. J. Manfra, Direct observation of anyonic braiding statistics, *Nat. Phys.* **16**, 931 (2020).
- [16] Y. P. Zhong, D. Xu, P. Wang, C. Song, Q. J. Guo, W. X. Liu, K. Xu, B. X. Xia, C.-Y. Lu, S. Han, J.-W. Pan, and H. Wang, Emulating anyonic fractional statistical behavior in a superconducting quantum circuit, *Phys. Rev. Lett.* **117**, 110501 (2016).
- [17] K. J. Satzinger, Y.-J. Liu, A. Smith, C. Knapp, M. Newman, C. Jones, Z. Chen, C. Quintana, X. Mi, A. Dunsworth *et al.*, Realizing topologically ordered states on a quantum processor, *Science* **374**, 1237 (2021).
- [18] Google Quantum AI and Collaborators, Non-Abelian braiding of graph vertices in a superconducting processor, *Nature (London)* **618**, 264 (2023).
- [19] S. Xu, Z.-Z. Sun, K. Wang, H. Li, Z. Zhu, H. Dong, J. Deng, X. Zhang, J. Chen, Y. Wu *et al.*, Non-Abelian braiding of Fibonacci anyons with a superconducting processor, *Nat. Phys.* **20**, 1469 (2024).
- [20] G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletić, and M. D. Lukin, Probing topological spin liquids on a programmable quantum simulator, *Science* **374**, 1242 (2021).
- [21] M. Iqbal, N. Tantivasadakarn, R. Verresen, S. L. Campbell, J. M. Dreiling, C. Figgatt, J. P. Gaebler, J. Johansen, M. Mills, S. A. Moses *et al.*, Non-Abelian topological order and anyons on a trapped-ion processor, *Nature (London)* **626**, 505 (2024).
- [22] F. D. M. Haldane, Fractional statistics in arbitrary dimensions: A generalization of the Pauli principle, *Phys. Rev. Lett.* **67**, 937 (1991).
- [23] F. D. M. Haldane, Exact Jastrow-Gutzwiller resonating-valence-bond ground state of the spin- $\frac{1}{2}$ antiferromagnetic Heisenberg chain with $1/r^2$ exchange, *Phys. Rev. Lett.* **60**, 635 (1988).
- [24] B. S. Shastry, Exact solution of an $s = 1/2$ Heisenberg antiferromagnetic chain with long-ranged interactions, *Phys. Rev. Lett.* **60**, 639 (1988).
- [25] Y. Kuramoto and H. Yokoyama, Exactly soluble supersymmetric t - J -type model with long-range exchange and transfer, *Phys. Rev. Lett.* **67**, 1338 (1991).
- [26] Y. Kuramoto and Y. Kato, Spin-charge separation at finite temperature in the supersymmetric t - J model with long-range interactions, *J. Phys. Soc. Jpn.* **64**, 4518 (1995).
- [27] Y. Kato, Hole dynamics of the one-dimensional supersymmetric $t - J$ model with a long-range interaction: An exact result, *Phys. Rev. Lett.* **81**, 5402 (1998).
- [28] M. Arikawa, Y. Saiga, and Y. Kuramoto, Electron addition spectrum in the supersymmetric $t - J$ model with inverse-square interaction, *Phys. Rev. Lett.* **86**, 3096 (2001).
- [29] Z. N. C. Ha, Exact dynamical correlation functions of Calogero-Sutherland model and one-dimensional fractional statistics, *Phys. Rev. Lett.* **73**, 1574 (1994).
- [30] Z. Ha, Fractional statistics in one dimension: View from an exactly solvable model, *Nucl. Phys. B* **435**, 604 (1995).
- [31] M. V. N. Murthy and R. Shankar, Thermodynamics of a one-dimensional ideal gas with fractional exclusion statistics, *Phys. Rev. Lett.* **73**, 3331 (1994).
- [32] M. Greiter, Statistical phases and momentum spacings for one-dimensional anyons, *Phys. Rev. B* **79**, 064409 (2009).

- [33] S. B. Isakov, Statistical mechanics for a class of quantum statistics, *Phys. Rev. Lett.* **73**, 2150 (1994).
- [34] Y.-S. Wu, Statistical distribution for generalized ideal gas of fractional-statistics particles, *Phys. Rev. Lett.* **73**, 922 (1994).
- [35] Y.-S. Wu and Y. Yu, Bosonization of one-dimensional exclusions and characterization of Luttinger liquids, *Phys. Rev. Lett.* **75**, 890 (1995).
- [36] M. T. Batchelor, X.-W. Guan, and N. Oelkers, One-dimensional interacting anyon gas: Low-energy properties and Haldane exclusion statistics, *Phys. Rev. Lett.* **96**, 210402 (2006).
- [37] A. Kundu, Exact solution of double δ function Bose gas through an interacting anyon gas, *Phys. Rev. Lett.* **83**, 1275 (1999).
- [38] T. Hansson, J. Leinaas, and J. Myrheim, Dimensional reduction in anyon systems, *Nucl. Phys.* **B384**, 559 (1992).
- [39] T. Posske, B. Trauzettel, and M. Thorwart, Second quantization of Leinaas-Myrheim anyons in one dimension and their relation to the Lieb-Liniger model, *Phys. Rev. B* **96**, 195422 (2017).
- [40] U. Aglietti, L. Griguolo, R. Jackiw, S.-Y. Pi, and D. Seminara, Anyons and chiral solitons on a line, *Phys. Rev. Lett.* **77**, 4406 (1996).
- [41] C. S. Chisholm, A. Frölian, E. Neri, R. Ramos, L. Tarruell, and A. Celi, Encoding a one-dimensional topological gauge theory in a raman-coupled Bose-Einstein condensate, *Phys. Rev. Res.* **4**, 043088 (2022).
- [42] A. Frölian, C. S. Chisholm, E. Neri, C. R. Cabrera, R. Ramos, A. Celi, and L. Tarruell, Realizing a 1D topological gauge theory in an optically dressed BEC, *Nature (London)* **608**, 293 (2022).
- [43] T. Keilmann, S. Lanzmich, I. McCulloch, and M. Roncaglia, Statistically induced phase transitions and anyons in 1D optical lattices, *Nat. Commun.* **2**, 361 (2011).
- [44] S. Greschner and L. Santos, Anyon Hubbard model in one-dimensional optical lattices, *Phys. Rev. Lett.* **115**, 053002 (2015).
- [45] L. Cardarelli, S. Greschner, and L. Santos, Engineering interactions and anyon statistics by multicolor lattice-depth modulations, *Phys. Rev. A* **94**, 023615 (2016).
- [46] C. Sträter, Shashi C. L. Srivastava, and A. Eckardt, Floquet realization and signatures of one-dimensional anyons in an optical lattice, *Phys. Rev. Lett.* **117**, 205303 (2016).
- [47] L. Yuan, M. Xiao, S. Xu, and S. Fan, Creating anyons from photons using a nonlinear resonator lattice subject to dynamic modulation, *Phys. Rev. A* **96**, 043864 (2017).
- [48] J. Arcila-Forero, R. Franco, and J. Silva-Valencia, Critical points of the anyon-Hubbard model, *Phys. Rev. A* **94**, 013611 (2016).
- [49] J. Arcila-Forero, R. Franco, and J. Silva-Valencia, Three-body-interaction effects on the ground state of one-dimensional anyons, *Phys. Rev. A* **97**, 023631 (2018).
- [50] F. Lange, S. Ejima, and H. Fehske, Anyonic Haldane insulator in one dimension, *Phys. Rev. Lett.* **118**, 120401 (2017).
- [51] M. Bonkhoff, K. Jägering, S. Hu, A. Pelster, S. Eggert, and I. Schneider, Anyonic phase transitions in the 1D extended Hubbard model with fractional statistics, *Phys. Rev. Lett.* **135**, 036601 (2025).
- [52] W. Zhang, L. Qian, H. Sun, and X. Zhang, Anyonic bound states in the continuum, *Commun. Phys.* **6**, 139 (2023).
- [53] Y. Hao and S. Chen, Dynamical properties of hard-core anyons in one-dimensional optical lattices, *Phys. Rev. A* **86**, 043631 (2012).
- [54] S. Longhi and G. Della Valle, Anyonic Bloch oscillations, *Phys. Rev. B* **85**, 165144 (2012).
- [55] L. Wang, L. Wang, and Y. Zhang, Quantum walks of two interacting anyons in one-dimensional optical lattices, *Phys. Rev. A* **90**, 063618 (2014).
- [56] T. M. Wright, M. Rigol, M. J. Davis, and K. V. Kheruntsyan, Nonequilibrium dynamics of one-dimensional hard-core anyons following a quench: Complete relaxation of one-body observables, *Phys. Rev. Lett.* **113**, 050601 (2014).
- [57] F. Liu, J. R. Garrison, D.-L. Deng, Z.-X. Gong, and A. V. Gorshkov, Asymmetric particle transport and light-cone dynamics induced by anyonic statistics, *Phys. Rev. Lett.* **121**, 250404 (2018).
- [58] S. Greschner, L. Cardarelli, and L. Santos, Probing the exchange statistics of one-dimensional anyon models, *Phys. Rev. A* **97**, 053605 (2018).
- [59] Q.-W. Wang, Exact dynamical correlations of hard-core anyons in one-dimensional lattices, *Phys. Rev. B* **105**, 205143 (2022).
- [60] C. Yannouleas and U. Landman, Anyon optics with time-of-flight two-particle interference of double-well-trapped interacting ultracold atoms, *Phys. Rev. A* **100**, 013605 (2019).
- [61] M. D. Girardeau, Anyon-fermion mapping and applications to ultracold gases in tight waveguides, *Phys. Rev. Lett.* **97**, 100402 (2006).
- [62] H. Wang, Y. Chen, and X. Cui, Boson-anyon-fermion mapping and anyon construction in one dimension, *Phys. Rev. Res.* **7**, L022075 (2025).
- [63] M. Bonkhoff, K. Jägering, S. Eggert, A. Pelster, M. Thorwart, and T. Posske, Bosonic continuum theory of one-dimensional lattice anyons, *Phys. Rev. Lett.* **126**, 163201 (2021).
- [64] F. Theel, M. Bonkhoff, P. Schmelcher, T. Posske, and N. L. Harshman, Chirally protected state manipulation by tuning one-dimensional statistics, *Phys. Rev. Lett.* **135**, 063401 (2025).
- [65] J. Kwan, P. Segura, Y. Li, S. Kim, A. V. Gorshkov, A. Eckardt, B. Bakkali-Hassani, and M. Greiner, Realization of one-dimensional anyons with arbitrary statistical phase, *Science* **386**, 1055 (2024).
- [66] P. Calabrese and M. Mintchev, Correlation functions of one-dimensional anyonic fluids, *Phys. Rev. B* **75**, 233104 (2007).
- [67] O. I. Păţu, V. E. Korepin, and D. V. Averin, Correlation functions of one-dimensional Lieb-Liniger anyons, *J. Phys. A* **40**, 14963 (2007).
- [68] R. Santachiara, F. Stauffer, and D. C. Cabra, Entanglement properties and momentum distributions of hard-core anyons on a ring, *J. Stat. Mech.* (2007) L05003.
- [69] R. Santachiara and P. Calabrese, One-particle density matrix and momentum distribution function of one-dimensional anyon gases, *J. Stat. Mech.* (2008) P06005.

- [70] Y. Hao, Y. Zhang, and S. Chen, Ground-state properties of one-dimensional anyon gases, *Phys. Rev. A* **78**, 023631 (2008).
- [71] Y. Hao, Y. Zhang, and S. Chen, Ground-state properties of hard-core anyons in one-dimensional optical lattices, *Phys. Rev. A* **79**, 043633 (2009).
- [72] M. Mintchev and P. Sorba, Luttinger liquid in a non-equilibrium steady state, *J. Phys. A* **46**, 095006 (2013).
- [73] S. Scopa, L. Piroli, and P. Calabrese, One-particle density matrix of a trapped Lieb–Liniger anyonic gas, *J. Stat. Mech.* (2020) P093103.
- [74] S. Dhar, B. Wang, M. Horvath, A. Vashisht, Y. Zeng, M. B. Zvonarev, N. Goldman, Y. Guo, M. Landini, and H.-C. Nägerl, Observing anyonization of bosons in a quantum gas, *Nature (London)* **642**, 53 (2025).
- [75] L. Yang and H. Pu, One-body density matrix and momentum distribution of strongly interacting one-dimensional spinor quantum gases, *Phys. Rev. A* **95**, 051602(R) (2017).
- [76] O. Gamayun, O. Lychkovskiy, and M. B. Zvonarev, Zero temperature momentum distribution of an impurity in a polaron state of one-dimensional Fermi and Tonks-Girardeau gases, *SciPost Phys.* **8**, 053 (2020).
- [77] O. Gamayun, E. Quinn, K. Bidzhiev, and M. B. Zvonarev, Emergence of anyonic correlations from spin and charge dynamics in one dimension, *Phys. Rev. A* **109**, 012209 (2024).
- [78] G. Tang, S. Eggert, and A. Pelster, Ground-state properties of anyons in a one-dimensional lattice, *New J. Phys.* **17**, 123016 (2015).
- [79] H.-P. Eckle, *Models of Quantum Matter: A First Course on Integrability and the Bethe Ansatz* (Oxford University Press, New York, 2019).
- [80] L. Yang, L. Guan, and H. Pu, Strongly interacting quantum gases in one-dimensional traps, *Phys. Rev. A* **91**, 043634 (2015).
- [81] S. Basak and H. Pu, Generalized effective spin-chain formalism for strongly interacting spinor gases in optical lattices, *Phys. Rev. A* **108**, 063315 (2023).
- [82] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/2np8-mp39> for the discussions of the variations in the swap model, spin waves, time evolution, Floquet engineering, and multiple impurities, which includes Refs. [83–86].
- [83] L. Vanderstraeten, J. Haegeman, and F. Verstraete, Tangent-space methods for uniform matrix product states, *SciPost Phys. Lect. Notes* **7** (2019).
- [84] F. Meinert, M. Knap, E. Kirilov, K. Jag-Lauber, M. B. Zvonarev, E. Demler, and H.-C. Nägerl, Bloch oscillations in the absence of a lattice, *Science* **356**, 945 (2017).
- [85] L. Amico, D. Anderson, M. Boshier, J.-P. Brantut, L.-C. Kwek, A. Minguzzi, and W. von Klitzing, Colloquium: Atomtronic circuits: From many-body physics to quantum technologies, *Rev. Mod. Phys.* **94**, 041001 (2022).
- [86] L. Su, A. Douglas, M. Szurek, R. Groth, S. F. Ozturk, A. Krahn, A. H. Hébert, G. A. Phelps, S. Ebadi, S. Dickerson *et al.*, Dipolar quantum solids emerging in a Hubbard quantum simulator, *Nature (London)* **622**, 724 (2023).
- [87] I. Ivantsov, H. B. Xavier, A. Ferraz, and E. Kochetov, Stable and metastable kinetic ferromagnetism on a ring, *Phys. Rev. B* **101**, 195107 (2020).
- [88] K. K. Kesharpu and E. A. Kochetov, Spin-flip-induced superfluidity in a ring of spinful hard-core bosons, *Phys. Rev. A* **106**, 033323 (2022).
- [89] A. Defossez, L. Vanderstraeten, L. Peralta Gavensky, and N. Goldman, Dynamic realization of Majorana zero modes in a particle-conserving ladder, *Phys. Rev. Res.* **7**, 023183 (2025).
- [90] N. Goldman and J. Dalibard, Periodically driven quantum systems: Effective Hamiltonians and engineered gauge fields, *Phys. Rev. X* **4**, 031027 (2014).
- [91] A. Eckardt, Colloquium: Atomic quantum gases in periodically driven optical lattices, *Rev. Mod. Phys.* **89**, 011004 (2017).
- [92] S. S. Alam, T. Skaras, L. Yang, and H. Pu, Dynamical fermionization in one-dimensional spinor quantum gases, *Phys. Rev. Lett.* **127**, 023002 (2021).
- [93] A. del Campo, Fermionization and bosonization of expanding one-dimensional anyonic fluids, *Phys. Rev. A* **78**, 045602 (2008).
- [94] A. Y. Kitaev, Unpaired majorana fermions in quantum wires, *Phys. Usp.* **44**, 131 (2001).
- [95] N. Harshman and A. Knapp, Anyons from three-body hard-core interactions in one dimension, *Ann. Phys. (Amsterdam)* **412**, 168003 (2020).
- [96] N. L. Harshman and A. C. Knapp, Topological exchange statistics in one dimension, *Phys. Rev. A* **105**, 052214 (2022).
- [97] S. Nagies, B. Wang, A. C. Knapp, A. Eckardt, and N. L. Harshman, Beyond braid statistics: Constructing a lattice model for anyons with exchange statistics intrinsic to one dimension, *SciPost Phys.* **16**, 086 (2024).
- [98] J. Alicea and P. Fendley, Topological phases with parafermions: Theory and blueprints, *Annu. Rev. Condens. Matter Phys.* **7**, 119 (2016).
- [99] I. E. Nielsen, J. Schulenburg, R. Egger, and M. Burrello, Dynamics of parafermionic states in transport measurements, *SciPost Phys.* **15**, 189 (2023).
- [100] Z. Wang and K. R. A. Hazzard, Particle exchange statistics beyond fermions and bosons, *Nature (London)* **637**, 314 (2025).
- [101] M. Fishman, S. R. White, and E. M. Stoudenmire, The ITensors Software Library for tensor network calculations, *SciPost Phys. Codebases* **4** (2022).
- [102] M. Fishman, S. R. White, and E. M. Stoudenmire, Codebase release 0.3 for ITensors, *SciPost Phys. Codebases* **4** (2022).
- [103] B. Wang, A. Vashisht, Y. Guo, S. Dhar, M. Landini, H.-C. Nägerl, and N. Goldman, 2025, [10.6084/m9.fig-share.29042813.v3](https://arxiv.org/abs/2504.2813v3).