

Hidden Simplicity in AdS Spinning Mellin Amplitudes via Scaffolding

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We uncover surprising hidden simplicity in Mellin amplitudes for tree-level anti-de Sitter (AdS) holographic correlators for AdS “gluon” (spin-1) operators. We define Mellin amplitudes with n spinning operators via the so-called scaffolding of $2n$ -scalar ones with specific projection operators for each spin state, which are rational functions of Mellin variables of $2n$ scalars generalizing flat-space scaffolding amplitudes. We classify possible three-point structures with spin 1, which take the same form as massive three-point amplitudes in flat space, and match them with special solutions such as those extracted from 6-scalar ones in $\text{AdS}_5 \times S^3$. Focusing on AdS_5 gluons, we directly bootstrap spinning amplitudes in scaffolding form up to $n = 6$ gluons (which amounts to $2n = 12$ scalars) using factorizations, multilinearity, and flat-space limit. The results take a remarkably simple form in analogy with flat-space amplitudes, which can be constructed from familiar 3- and 4-vertices as well as propagators of massive spin-1 particles. Surprisingly, we find that vertices with any descendant levels are proportional to the primary ones with nice combinatorial coefficients, which makes manifest the correct flat-space limit in the simplest possible way.

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Introduction—Recent years have witnessed remarkable progress in computing and revealing new structures of holographic correlators, or “scattering amplitudes,” in anti-de Sitter (AdS) space (cf. tree [1–9] and loop [10–16] level). A natural language for holographic correlators is the Mellin representation [17–19], where amplitudes factorize at physical poles with the residues encoding the operators exchanged in the operator product expansion [20], while the flat-space limit is captured by poles at infinity. Following pioneering works [21–25] in AdS super-Yang-Mills (sYM) theories (see Refs. [26–29] for loop level), a new method has been proposed in [30,31] for constructing higher-point supergluon amplitudes purely from lower-point ones. Despite the above progress, most results and understandings of Mellin amplitudes have been restricted to cases with at most one spinning operator. Beyond that,

computations involving more spinning operators have been carried out by keeping explicit tensor indices [32–34]. The precise definition of Mellin amplitudes with arbitrary number of spinning operators or legs has only been carefully studied very recently in [35], where one introduces additional Mellin variables η to represent the “polarization” degrees of freedom.

On the other hand, a new way for representing spinning particles, such as gluons and gravitons, as pairs of “scaffolding” scalars for flat-space amplitudes has proved to be very fruitful [36,37]. It allows one to extract all-loop n -gluon amplitudes in Yang-Mills theory from $2n$ -scalar amplitudes obtained from a simple kinematical shift of the so-called stringy $\text{Tr}\phi^3$ amplitudes expressed using (combinatorial) “curve integrals” on surfaces [38–43]. Already at tree level, the n -gluon scaffolding amplitude becomes a canonical rational function of kinematic variables of $2n$ scalars, where important notions based on gluon polarizations, such as gauge invariance, multilinearity, and factorizations, all have natural combinatorial origins from curves on this simplest surface, the $2n$ -gon [37].

In this Letter, we combine these ideas and propose a new way for representing AdS correlators with arbitrary numbers of spinning operators in terms of “scaffolding scalars”: the Mellin amplitudes with n spinning operators can be obtained by acting the $2n$ -scalar one with appropriate projection operators. As a warm-up exercise, we classify

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three-point correlators with spin 1, which is equivalent to massive amplitudes in flat space, and we match our results with those from scaffolding 6-scalar amplitudes of lowest Kaluza-Klein (KK) modes in $\mathcal{N} = 1$ sYM on $\text{AdS}_5 \times S^3$ (see Sec. A of Supplemental Material [44] for a review). Focusing on tree-level amplitudes of the Yang-Mills-scalar (YMS) sector in AdS_5 background, we then directly bootstrap n -gluon amplitudes using factorizations, multilinearity, and flat-space limit (which are nicely the scaffolding Yang-Mills amplitudes [37]). As we will see, it is straightforward to construct these amplitudes explicitly through $n = 6$, which amounts to the most interesting parts of “supergluon” amplitudes up to $2n = 12$ [30,31]. Moreover, we reveal hidden simplicity of these new results in scaffolding variables; quite nicely we find that the n -gluon amplitudes can be built from Feynman rules with 3- and 4-vertices (including all descendant levels), as well as Proca propagators, which are formally identical to massive spin-1 particles in flat space. In other words, through $n = 6$ we discover a set of “emergent” Feynman rules where 3- and 4-vertices with any descendant levels are proportional to flat-space ones (in scaffolding variables). Moreover, these descendant coefficients enjoy a nice combinatorial interpretation, and for any Feynman diagram, they conspire to give the same overall coefficient (following from a combinatorial identity), which makes flat-space limit completely manifest.

Spinning correlators from scaffolded and projected Mellin amplitudes—In this section, we show how to extract n -point AdS spinning correlators in Mellin representation from $2n$ -scalar ones, in a way very similar to the flat-space case proposed in [37]. Recall that it is natural to represent n gluons (same works for gravitons) using n pairs of scalars, e.g., in $2n$ -scalar amplitudes of Yang-Mills-scalar theory, by taking the scaffolding residue $X_{1,3} = X_{3,5} = \dots = X_{1,2n-1} = 0$, and identifying the gluon kinematic data (momenta k_i and polarizations e_i) with linear combinations of scalar momenta p_{2i-1}, p_{2i} via

$$e_i^\mu \equiv (1 - \alpha)p_{2i}^\mu - \alpha p_{2i-1}^\mu, \quad k_i^\mu \equiv p_{2i}^\mu + p_{2i-1}^\mu. \quad (1)$$

In this way, we fuse n pairs of colored scalars into n on-shell gluons, as shown in Fig. 1.

Our goal is to extend this formalism to $2n$ -point Mellin amplitudes of scalar correlators [17]

$$G_{12\dots 2n} = \int [d\delta] \mathcal{M}_{2n}(\delta_{ij}) \prod_{i < j} \frac{\Gamma(\delta_{ij})}{(-2P_i \cdot P_j)^{\delta_{ij}}}. \quad (2)$$

While the scaffolded spinning amplitude is independent of details of the $2n$ scalars, we employ the scalar operators with conformal dimension $\Delta = 2$ for simplicity. Here we use the embedding formalism following [20], where $P_i \cdot P_j = -\frac{1}{2}(x_i - x_j)^2$. Although the physical P_i live in $\mathbb{R}^{d+1,1}$, the Mellin representation (2) is defined for

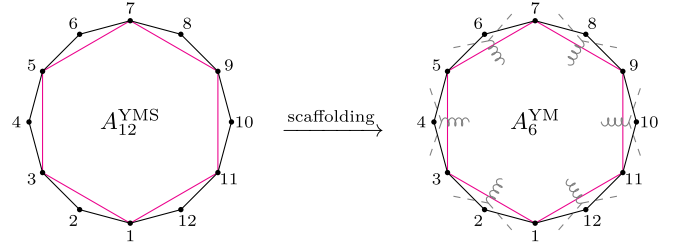


FIG. 1. Scaffolding from $2n = 12$ scalars to $n = 6$ gluons.

generic $P_i \cdot P_j$, with the resulting correlator restricted to $P_i \in \mathbb{R}^{d+1,1}$ only at the end [46]. The Mellin variables are constrained as $\delta_{ij} = p_i \cdot p_j$ for auxiliary momenta satisfying $\sum_i p_i = 0$ and $\delta_{ii} = p_i^2 = -\tau_i = -2$, with conformal twist $\tau_i := \Delta_i - J_i$ (J_i is the spin of an operator). Inspired by discussions in flat space [40], we also introduce the $[2n(2n-3)/2]$ planar variables

$$\mathbf{X}_{i,j} \equiv -1 - \frac{1}{2} \sum_{i \leq k, l < j} \delta_{kl} = -1 - \frac{1}{2} \left(\sum_{i \leq k < j} p_k \right)^2, \quad (3)$$

where we have $\mathbf{X}_{i,j} \equiv \mathbf{X}_{j,i}$, $\mathbf{X}_{i,i+1} = 0$, and $\mathbf{X}_{i,i} = -1$, with indices understood modulo $2n$. The inverse transform, motivated by the associahedron in [40,42], reads as follows:

$$\delta_{ij} = \mathbf{X}_{i,j} + \mathbf{X}_{i+1,j+1} - \mathbf{X}_{i,j+1} - \mathbf{X}_{i+1,j}. \quad (4)$$

With the planar variables in hand, we now proceed to extract the scaffolded n -spinning amplitudes from the $2n$ -scalar Mellin amplitudes by taking the residue at $\mathbf{X}_{2i-1,2i+1} = 0$ for $i = 1, \dots, n$, leaving $2n(n-2)$ independent variables. However, unlike flat-space YMS where scaffolding cleanly isolates the n -gluon amplitude, the AdS scaffolding residue generally mixes exchange channels from cubic couplings of the scaffolded scalar pairs to bulk fields of various spins: scalars or vectors (see, e.g., Fig. 2 for a mixture of spin-0,1 exchanges). To address this issue, we employ spin projectors to disentangle these contributions by selecting terms with specific powers of $\mathbf{e}_i$. Motivated by the flat-space multilinearity condition [37], we define $\mathcal{P}_i^{(\mathbf{e})}$ as the AdS analog of $\mathbf{e}_i \cdot \partial_{\mathbf{e}_i}$ acting on the i th external leg as follows:

$$\mathcal{P}_i^{(\mathbf{e})} = \sum_{j \neq 2i-1, 2i+1} \left(\mathbf{X}_{2i,j} - \frac{\mathbf{X}_{2i-1,j} + \mathbf{X}_{2i+1,j} + 1}{2} \right) \frac{\partial}{\partial \mathbf{X}_{2i,j}}, \quad (5)$$

where we have adopted the choice $\alpha = 1/2$ here and throughout the rest of this Letter. The operator $\mathcal{P}_i^{(\mathbf{e})}$ serves as the building block for the general spin- ℓ projector $\mathcal{P}^{(\ell)}$ for $\ell = 0, 1$, which acts as $\mathbf{e}_i \cdot \partial_{\mathbf{e}_i}$ to extract the term linear in “polarization” $\mathbf{e}_i$ (see Sec. B of Supplemental Material [44]),

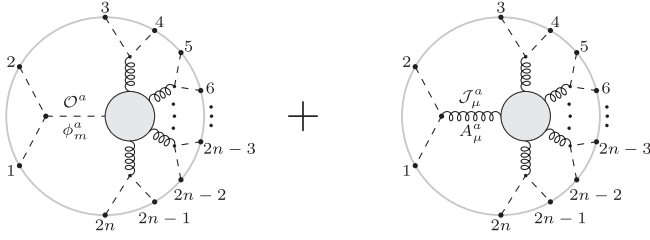


FIG. 2. Mixture of spin-0 (left) and spin-1 (right) contribution in scaffolding residue on $\mathbf{X}_{1,3}$.

implying $\mathcal{P}_i^{(1)} = \mathcal{P}_i^{(e)}$ and $\mathcal{P}_i^{(0)} = 1 - \mathcal{P}_i^{(e)}$. By this means, the spinning amplitude $\mathcal{A}_n^{(s_1, \dots, s_n)}$ for $s_i = 0, 1$ follows from the formula

$$\mathcal{A}_n^{(s_1, \dots, s_n)} \equiv \prod_{i=1}^n \mathcal{P}_i^{(s_i)} \text{Res}_{\mathbf{x}_{2i-1, 2i+1}=0} \mathcal{M}_{2n}. \quad (6)$$

Note that in general, one may also define spinning amplitudes analogous to (6), but with residues taken from descendant poles. Here we restrict to the case of primary spinning correlators based on available data [30,48].

Three-point amplitudes—As a warm-up, we determine possible structures of three-point spinning amplitudes with $s = 1$, which turn out to be in one-to-one correspondence with those for massive three-point amplitudes in flat space. We impose two conditions, the *multilinearity*, i.e.,

$$\prod_{i=1}^n \mathcal{P}_i^{(1)} \mathcal{A}_n^{(1, \dots, 1)} = \mathcal{A}_n^{(1, \dots, 1)}, \quad (7)$$

as well as *flat-space limit*, i.e., with rescaling $\mathbf{X} \rightarrow \beta \mathbf{X}$, AdS spinning amplitudes must reproduce flat-space amplitudes in the limit $\beta \rightarrow \infty$. For simplicity, we focus on the leading power-counting case, namely the flat-space limit corresponds to Yang-Mills for spins $(1, 1, 1)$ (we also elaborate on amplitudes with mixed spins in Sec. A of Supplemental Material [44]), and construct an ansatz of a degree-2 polynomial of $\mathbf{X}$ variables; imposing both conditions fixes the answer (up to an overall constant) as follows:

$$\mathcal{A}_3^{(1,1,1)} = A_3 + \sum_{i=1}^3 \left[\mathbf{X}_{2i, 2i+2} - \frac{1}{2} \mathbf{X}_{i, i+3} \right] - \frac{3}{4}, \quad (8)$$

where $A_3 = 2 \sum_{i=1}^3 \mathbf{X}_{i, i+3} (\mathbf{X}_{i+1, i+4} - \mathbf{X}_{2-2i, 4-2i})$ is the leading quadratic term matching the flat-space scaffolding Yang-Mills three-point amplitude [37], and the linear and constant terms are completely fixed by multilinearity.

In fact, with a simple change of variables, we can make these results formally identical to flat-space amplitudes with massive spinning particles. Analogous to the flat-space scaffolding gluon kinematic (1), we introduce formal

momenta $\mathbf{k}_i$ and polarizations $\mathbf{e}_i$ with $\alpha = 1/2$, together with their “contractions” defined as follows:

$$\begin{aligned} \mathbf{e}\mathbf{e}_{ij} &= \frac{1}{4} (\delta_{2i, 2j} + \delta_{2i-1, 2j-1} - \delta_{2i, 2j-1} - \delta_{2i-1, 2j}), \\ \mathbf{e}\mathbf{k}_{ij} &= \frac{1}{2} (\delta_{2i, 2j} - \delta_{2i-1, 2j-1} + \delta_{2i, 2j-1} - \delta_{2i-1, 2j}), \\ \mathbf{k}\mathbf{k}_{ij} &= \delta_{2i, 2j} + \delta_{2i-1, 2j-1} + \delta_{2i, 2j-1} + \delta_{2i-1, 2j}, \end{aligned} \quad (9)$$

where we still have “transversality” $\mathbf{e}\mathbf{k}_{ii} = 0$ and massive “on-shell” conditions $\mathbf{e}\mathbf{e}_{ii} = -\frac{3}{2}$ and $\mathbf{k}\mathbf{k}_{ii} = -2$. We also have the analog of “momentum conservation” $\sum_j \mathbf{e}\mathbf{k}_{ij} = 0$ and $\sum_j \mathbf{k}\mathbf{k}_{ij} = 0$. It is easy to count that the number of variables is still $2n(n-2)$. Equation (9) goes back to the familiar transformation between $e_i \cdot e_j$, $e_i \cdot k_j$, $k_i \cdot k_j$, and $\delta_{i,j}$ in the flat-space limit where those boundary values of $\mathbf{e}\mathbf{e}_{ii}$, $\mathbf{k}\mathbf{k}_{ii}$ all go to zero as δ_{ii} goes to zero.

Surprisingly, the scaffolding amplitudes become homogeneous functions of these formal polarizations $\mathbf{e}_i$. For $n = 3$, Eq. (8) becomes formally identical to the three-gluon amplitude, or more precisely, the amplitude for three massive spin-1 particles as follows:

$$\mathcal{A}_3^{(1,1,1)} = 2(\mathbf{e}\mathbf{e}_{12}\mathbf{e}\mathbf{k}_{31} + \mathbf{e}\mathbf{e}_{23}\mathbf{e}\mathbf{k}_{12} + \mathbf{e}\mathbf{e}_{31}\mathbf{e}\mathbf{k}_{23}). \quad (10)$$

Very nicely, this agrees with three-gluon amplitudes in $\text{AdS}_5 \times S^3$ (extracted from 6-scalar results in [25,30]). We emphasize that (10) is a general result for $s_i = 1$. As “massive amplitudes,” they do not respect gauge invariance under “ $\mathbf{e}_i \rightarrow \mathbf{e}_i + \beta \mathbf{k}_i$,” though such invariance is recovered in the flat-space limit [37].

Bootstrap N -gluon amplitudes and Feynman rules—Next we focus on the Yang-Mills-scalar sector of $\mathcal{N} = 1$ sYM in $\text{AdS}_5 \times S^3$, with the lowest KK modes as scalars: we show how to bootstrap n -gluon amplitudes and reveal hidden simplicity in our explicit results up to $n = 6$.

Bootstrap: We denote $\mathcal{A}_n^{\text{YMS}} = \mathcal{A}_n^{(1, \dots, 1)}$, and in addition to the two conditions, *multilinearity* and *flat-space limit*, we also impose *factorizations*; the exchange of spin-1 operators dictates that $\mathcal{A}_n^{\text{YMS}}$ factorizes on residues of primary and descendant poles as follows [49]:

$$\text{Res}_{\mathbf{x}_{1, 2j+1}=m} \mathcal{A}_n^{\text{YMS}} \sim f(\mathbf{X}, m) \partial_{\mathbf{x}_{i, I}} \mathcal{A}_L^{\text{YMS}(m)} \partial_{\mathbf{x}_{j, I'}} \mathcal{A}_R^{\text{YMS}(m)}, \quad (11)$$

where $m = 0, 1, 2, \dots$ denotes the descendant level whose truncation was studied in, e.g., Ref. [31]; I, I' denote the exchange indices, while i and j denote the indices carried by the amplitudes on the left and right, respectively. The precise formula is discussed in Sec. B of Supplemental Material [44].

We use four-point amplitude as an illustrative example, which has the following structure:

$$\mathcal{A}_4^{\text{YMS}} = \frac{A_{15}}{\mathbf{X}_{1,5}} + \frac{B_{15}}{\mathbf{X}_{1,5} - 1} + \frac{A_{37}}{\mathbf{X}_{3,7}} + \frac{B_{37}}{\mathbf{X}_{3,7} - 1} + R_4. \quad (12)$$

The A 's and B 's are completely determined by factorization where we glue two $\mathcal{A}_3^{(1,1,1)}$ together. To determine R_4 , the flat-space limit fixes the highest-power quadratic terms in $\mathbf{X}$ while multilinearity fixes the lower-power terms. The result is given compactly in (17). The same procedure extends straightforwardly to all n , which can be done without any difficulty at least up to $n = 6$. Nevertheless, they correspond to the most complicated and interesting part of “supergluon” amplitudes with $2n \leq 12$ scalars, which had been worked out completely only up to $2n = 8$ in the literature [30].

Emergent Feynman rules: Recall that we introduced a change of variables in (9), which renders the three-point expressions particularly simple. These variables not only trivialize the projector ($\mathcal{P}_i^{(e)} \sim \mathbf{e}_i \cdot \partial / \partial \mathbf{e}_i$), but they also make “spinning” factorizations, flat-space limit, etc. manifest. We show that, much like their flat-space counterparts, AdS gluon amplitudes exhibit hidden simplicity and novel structures.

We find that “a remarkable set of ‘Feynman rules’ emerges” from our explicit results: not only have we found no “contact” vertices beyond four points (all those contact terms in $\mathbf{X}$ variables are gone), but the “effective” 3- and 4-vertices all take identical form as the flat-space ones. We denote these “contracted” vertices as follows:

$$V(123) \equiv \mathbf{ee}_{12}(\mathbf{ek}_{31} - \mathbf{ek}_{32}) + \text{cyclic},$$

$$V(1234) \equiv -2\mathbf{ee}_{13}\mathbf{ee}_{24} + \mathbf{ee}_{12}\mathbf{ee}_{34} + \mathbf{ee}_{14}\mathbf{ee}_{23}, \quad (13)$$

and a highly nontrivial observation through $n = 6$ (which we conjecture to hold for any n) is that 3- and 4-vertices with any *descendant levels*, $\{m_1, \dots, m_k\}$ for $k = 3, 4$, take the same form and the only dependence on $\{m_i\}$ is the overall coefficient defined by $V_{m_1, m_2, m_3}(123) \equiv \alpha_{m_1, m_2, m_3} V(123)$ and $V_{m_1, \dots, m_4}(1234) \equiv \alpha_{m_1, \dots, m_4} V(1234)$. We find, for primary ones, $\alpha_{000} = 2$, $\alpha_{0000} = 6$, and for descendant cases needed up to $n = 6$, $\alpha_{001} = \alpha_{012} = -1$, $\alpha_{011} = 2$, $\alpha_{111} = -4$, and $\alpha_{0001} = -6$, $\alpha_{0011} = 12$, $\alpha_{0002} = 2$; we conjecture that the coefficients $\alpha_{m_1, m_2, \dots, m_k}$ for $k = 3, 4$ read:

$$\sum_{n_1=0}^{m_1} \cdots \sum_{n_k=0}^{m_k} \Gamma\left(\sum_{a=1}^k n_a + k\right) \prod_{a=1}^k \frac{(-m_a)_{n_a}}{n_a!(n_a + 1)!}, \quad (14)$$

with the Pochhammer symbol $(a)_b \equiv \Gamma(a + b)/\Gamma(a)$. Remarkably, these coefficients exactly match Witten-diagram computations in [50], and we present a nice recursion relation for $\alpha_{m_1, m_2, \dots, m_k}$ in Sec. C of Supplemental Material [44].

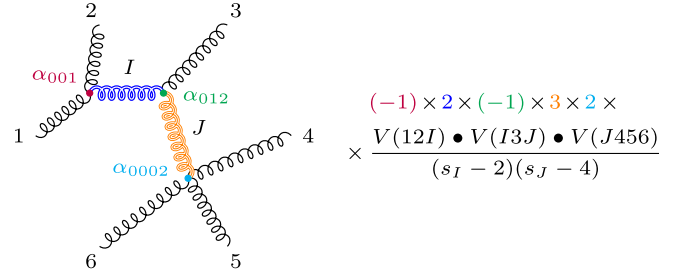


FIG. 3. An example of 6-pt Feynman diagram ($3 \times 3 \times 4$ channel) where the two propagators have descendant levels $m_I = 1$ and $m_J = 2$, respectively.

When gluing two vertices $V_L(\dots, I)$ and $V_R(I, \dots)$ on an internal leg I , it is understood as if $\mathbf{ee}$, $\mathbf{ek}$, and $\mathbf{kk}$ come from contractions of “vectors” $\mathbf{e}^\mu$ and $\mathbf{k}^\mu$, multiplied by a “Proca” propagator (at level $m_I = 0, 1, \dots$), i.e.,

$$V_L \bullet V_R \equiv \frac{\partial V_L}{\partial \mathbf{e}_I^\mu} \left(\eta^{\mu\nu} + \frac{\mathbf{k}_I^\mu \mathbf{k}_I^\nu}{2 + 2m_I} \right) \frac{\partial V_R}{\partial \mathbf{e}_I^\nu}, \quad (15)$$

where $\mathbf{k}_I^\mu \equiv -\sum_{i \in L} \mathbf{k}_i^\mu = \sum_{i \in R} \mathbf{k}_i^\mu$ denotes the “momentum” of I , and the derivative with respect to “polarization” $\mathbf{e}_I^\mu$ is via those with respect to $\mathbf{ee}_{I,i}$, $\mathbf{ek}_{I,i}$. For example, when two 3-vertices are glued, $V(12I) \bullet V(I34)$, the contraction gives

$$\begin{aligned} & 4[\mathbf{ee}_{12}(\mathbf{ek}_{31}\mathbf{ek}_{42} - \mathbf{ek}_{32}\mathbf{ek}_{41}) + (12 \leftrightarrow 34) \\ & + \mathbf{ek}_{12}(\mathbf{ee}_{24}\mathbf{ek}_{34} - \mathbf{ee}_{23}\mathbf{ek}_{43}) + (13 \leftrightarrow 24)] \\ & - 2\mathbf{ee}_{12}\mathbf{ee}_{34}(2\mathbf{kk}_{23} + \mathbf{kk}_{12} - 2). \end{aligned}$$

At least through $n = 6$, the n -gluon amplitudes exactly follow from these Feynman rules including all descendants, which take the form (see Fig. 3)

$$\begin{aligned} \mathcal{A}_n^{\text{YMS}} &= \sum_g \sum_{\{m\}} \frac{N_{g, \{m\}}}{\prod_{I \in E(g)} D_I^{(m_I)}} \\ &= \sum_g \sum_{\{m\}} (V \bullet V \bullet \dots \bullet V) \frac{\prod_{v \in V(g)} \alpha_{\{m_v\}}}{\prod_{I \in E(g)} D_I^{(m_I)}}, \quad (16) \end{aligned}$$

where we sum over all planar tree graphs with 3- and 4-vertices [51], g , as well as possible descendant levels for all propagators I , $\{m_I\}$; the poles are normalized according to the convention [50]: $D_I^{(m_I)} \equiv [(s_I - 2m_I)/(m_I + 1)]$ with $s_I \equiv 2\mathbf{X}_{2i-1, 2j-1}$ for $I = \{i, i+1, \dots, j-1\}$ [52], and the numerator $N_{g, \{m\}}$ obtained from gluing V 's together; the dependence of $N_{g, \{m\}}$ on descendant levels is only through Proca propagators in the gluing and the product of α 's for vertices v [53]. For example, the four-point amplitude is a sum of two cubic graphs and a quartic one as follows:

$$\mathcal{A}_4^{\text{YMS}} = V(12I) \cdot V(I34) \left(\frac{2^2}{s_{12}} + \frac{(-1)^2 \times 2}{s_{12} - 2} \right) + (i \rightarrow i+1) + 6 \times V(1234), \quad (17)$$

and the five-point amplitude consists of $5 + 5$ graphs [54] as follows:

$$\begin{aligned} \mathcal{A}_5^{\text{YMS}} = & V(12I) \cdot V(I3J) \cdot V(J45) \left(\frac{2^3}{s_{12}s_{45}} + \frac{(-1)^2 \times 2^2}{(s_{12} - 2)s_{45}} + \frac{(-1)^2 \times 2^2}{s_{12}(s_{45} - 2)} + \frac{(-1)^2 \times 2^3}{(s_{12} - 2)(s_{45} - 2)} \right) + \text{cyclic} \\ & + V(12I) \cdot V(I345) \left(\frac{6 \times 2}{s_{12}} + \frac{(-6) \times (-1) \times 2}{s_{12} - 2} \right) + \text{cyclic}. \end{aligned} \quad (18)$$

For $n = 6$, there are 38 graphs from seven cyclic seeds as we enumerate in Sec. C of Supplemental Material [44] (see Fig. 3 for an example). Very nicely, this makes the flat-space limit completely manifest, where (13) reduces to flat-space vertices and the sum of scalar graphs (with descendant levels) in (16) reduces to the flat-space graph times a coefficient made of products of α 's. For any graph of $n = 4, 5, 6$, the products of α 's indeed conspire to give $3!, 4!$, and $5!$, respectively; as we show in Sec. C of Supplemental Material [44], the magic behind this is the beautiful recursion relation, $\alpha_{m_1 \dots m_n} = \sum_{m_I=0}^{\infty} \alpha_{m_1 \dots m_I m_I} (m_I + 1) \alpha_{m_I m_{I+1} \dots m_n}$ (and the fact that it is crossing symmetric), which guarantees that any Feynman diagram picks the *same* overall coefficient in the flat-space limit, which is $\alpha_{0, \dots, 0} = (n-1)!$.

Outlook—In this Letter, we have initiated a study on Mellin amplitudes for AdS spinning correlators by scaffolding the scalar ones similar to that for flat-space amplitudes. We set up the formalism and study general structures of three-point spinning amplitudes, which we then match with AdS amplitudes of “gluons” from scaffolding. Focusing on Yang-Mills-scalar theory, we then bootstrap n -gluon amplitudes based on flat-space limit, multilinearity, and factorizations. Remarkably we find an emergent set of Feynman rules of massive spin-1 particles in scaffolding variables, where vertices with different descendant levels all take identical form up to nice coefficients with combinatorial interpretations.

Our preliminary investigations open up numerous directions to explore. It would be highly desirable to derive Feynman rules for all descendant levels directly, and find more structures underlying n -gluon amplitudes such as color-kinematics duality [55,56] and possible “universal expansions” [57]. Practically it would also be advantageous to have direct recursion relations generalizing the flat-space ones [58,59]. Could there be a “curve-integral” formulation for these scaffolding gluon amplitudes [38,39], and even for string amplitudes in AdS [60–68]? Moving beyond gluons, it would be interesting to discover whether there also exist projectors for extracting higher-spin states and, e.g., bootstrap $n \geq 3$ amplitudes in supergravity (which amounts to the most interesting part with $2n \geq 6$ scalars) and unveil their underlying structures such as double

copy [8]. It would also be highly desirable to study general Feynman rules including arbitrary KK modes, for AdS spinning amplitudes in and beyond super-Yang-Mills and supergravity theories, where we expect hidden symmetries [60–63,68–74].

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Data availability—The data that support the findings of this article are openly available [75].

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