

# Comment on “Hydrodynamic Equations for Space-Inhomogeneous Aggregating Fluids with First-Principle Kinetic Coefficients”

We show that the Maxwellian ansatz for the velocity distributions for clusters adopted in [Phys. Rev. Lett. **133**, 217201 (2024)] is in general not valid. In Ref. [1] the authors develop a hydrodynamic description of aggregation kinetics of monomers into clusters. They extend the Smoluchowski equations to the case where clusters of different sizes have different mean velocities. The basic ansatz allowing for the hydrodynamic description is an assumption [Eq. (5) in [1]] of a Maxwellian (Gaussian) distribution of velocities for each cluster size:  $f_k(\vec{V}_k) = n_k(2\pi\theta_k)^{-3/2} \exp[(\vec{V}_k - \vec{u}_k)^2/2\theta_k]$ , where  $\vec{u}_k$  is the mean velocity of clusters consisting of  $k$  monomers and  $n_k$  is the number of clusters. Below I show that this ansatz is generally not valid. To see this let us consider for simplicity a one-dimensional setup where at some moment of time there are only monomers ( $k=1$ , mean velocity  $A$ ) and dimers ( $k=2$ , mean velocity  $B$ ) with Gaussian distributions of velocities  $f_1(V_1; t) = N_1(2\pi\theta_1)^{-1/2} \exp[-(V_1 - A)^2/(2\theta_1)]$ ,  $f_2(V_2; t) = N_2(2\pi\theta_2)^{-1/2} \exp[-(V_2 - B)^2/(2\theta_2)]$  [Fig. 1(a)]. Furthermore, for simplicity we assume that aggregation rates are velocity independent.

Let us first consider dimers at a later time  $t_1 > t$ . There are altogether  $M_2$  dimers, which can be divided in two groups. (i) There are  $M_2^{(11)}$  dimers that appeared due to merging of two monomers. A velocity of such a dimer is  $\tilde{V}_2 = (V'_1 + V''_1)/2$ , where  $V'_1$  and  $V''_1$  are two random numbers from the distribution  $f_1(V_1; t)$ . Thus, the mean value is  $\langle \tilde{V}_2 \rangle = A$ , and the variance is  $\theta_1/2$ . The distribution density of these dimers is  $f_2(\tilde{V}_2; t_1) = M_2^{(11)}(\pi\theta_1)^{-1/2} \exp[-(\tilde{V}_2 - A)^2/\theta_1]$ . (ii) There are  $M_2^{(2)}$  dimers that existed already at time  $t$  and have not experienced any merging event up to time  $t_1$ . Clearly  $M_2^{(2)} < N_2$ . The distribution of their velocities  $\hat{V}_2$  is the renormalized distribution  $f_2(V_2; t)$ :  $f_2(\hat{V}_2; t_1) = M_2^{(2)}(2\pi\theta_2)^{-1/2} \exp[-(\hat{V}_2 - B)^2/(2\theta_2)]$ . Thus, the total distribution density of the dimers is [Fig. 1(b)]

$$f_2(V_2; t_1) = \frac{M_2^{(11)}}{\sqrt{\pi\theta_1}} \exp[-(V_2 - A)^2/\theta_1] + \frac{M_2^{(2)}}{\sqrt{2\pi\theta_2}} \exp[-(V_2 - B)^2/(2\theta_2)],$$

and it is evidently non-Gaussian.

The same conclusion holds for trimers. At some time  $t_1 > t$  there are altogether  $M_3 = M_3^{(111)} + M_3^{(12)}$  trimers, which can be divided in two groups. (i) There are  $M_3^{(111)}$  trimers that consist of three initial monomers. A velocity of such a trimer is  $\tilde{V}_3 = (V_1^a + V_1^b + V_1^c)/3$ , where  $V_1^{a,b,c}$  are three random numbers from the distribution  $f_1(V_1; t)$ . The mean value is  $\langle \tilde{V}_3 \rangle = A$ , and the variance is  $\theta_1/3$ . The distribution density of these trimers is  $f_3(\tilde{V}_3; t_1) = M_3^{(111)}(2\pi\theta_1/3)^{-1/2} \exp[-(\tilde{V}_3 - A)^2/(2\theta_1/3)]$ . (ii) There are  $M_3^{(12)}$  trimers that consist of merged initial monomer and initial dimer. Momentum conservation yields  $\hat{V}_3 = (V_1 + 2V_2)/3$ , where  $V_1, V_2$  are random variables with distributions  $f_1(V_1; t)$  and  $f_2(V_2; t)$ , respectively. The mean velocity for such trimers is  $\langle \hat{V}_3 \rangle = (A + 2B)/3$ , and the variance is  $(\theta_1 + 4\theta_2)/9$ . The density of these trimers is  $f_3(\hat{V}_3; t) = M_3^{(12)}[2\pi(\theta_1 + 4\theta_2)/9]^{-1/2} \exp\{-[\hat{V}_3 - (A + 2B)/3]^2/[2(\theta_1 + 4\theta_2)/9]\}$ . Thus, the total distribution density of the trimers is [Fig. 1(c)]

$$f_3(V_3; t_1) = \frac{M_3^{(111)}}{\sqrt{2\pi\theta_1/3}} \exp\left[-\frac{(V_3 - A)^2}{2\theta_1/3}\right] + \frac{M_3^{(12)}}{\sqrt{2\pi(\theta_1 + 4\theta_2)/9}} \exp\left\{-\frac{[V_3 - (A + 2B)/3]^2}{2(\theta_1 + 4\theta_2)/9}\right\},$$

and it is evidently non-Gaussian.

I have demonstrated that even if the Gaussian ansatz is valid at some time, in the course of aggregation it becomes strongly violated. The only situation where the Gaussian ansatz remains approximately valid is the case where all the mean velocities  $\vec{u}_k$  of all clusters are equal (in the example above  $A = B$ ). But in this case the problem is trivially solved by transforming to the moving reference frame where all the mean velocities vanish and a Smoluchowski-type equation without advection [e.g., Eq. (1) of Ref. [1]] holds. A situation with nearly equal mean velocities may appear at the long-time stage of the evolution, where all the clusters have large masses  $k \gg 1$ . At this long-time stage, the central limit theorem arguments suggest validity of the Gaussian ansatz, but, on the other hand, the law-of-large-numbers arguments suggest that the mean velocities of all large clusters are nearly equal [cf. Figs. 1(a) and 1(b) of [1], where clusters of different sizes are shown to have nearly the same mean velocity]. Thus, the Gaussian ansatz adopted in [1] is not valid at the nontrivial initial evolution

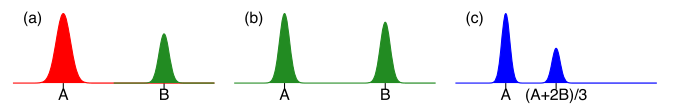


FIG. 1. (a) Initial distributions  $f_1(V_1; t)$  (red) and  $f_2(V_2; t)$  (green). (b) Distribution density of velocities of dimers at some  $t_1 > t$ . (c) Distribution density of velocities of trimers at  $t_1 > t$ . Scales of vertical axes are arbitrary.

stage, and at the long-time stage, the evolution is described by a Smoluchowski-type equation without advection.

Arkady Pikovsky 

Department of Physics and Astronomy  
University of Potsdam  
Karl-Liebknecht-Str. 24/25  
14476, Potsdam-Golm, Germany



Received 9 December 2024; accepted 10 October 2025;  
published 13 November 2025

DOI: [10.1103/2djl-jzw4](https://doi.org/10.1103/2djl-jzw4)

- [1] A. I. Osinsky and N. V. Brilliantov, Hydrodynamic equations for space-inhomogeneous aggregating fluids with first-principle kinetic coefficients, *Phys. Rev. Lett.* **133**, 217201 (2024).