

Late-Time Tails in Nonlinear Evolutions of Merging Black Holes

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We uncover late-time gravitational-wave tails in fully nonlinear $3 + 1$ dimensional numerical relativity simulations of merging black holes, using the highly accurate *SpEC* code. We achieve this result by exploiting the strong magnification of late-time tails due to binary eccentricity, recently observed in perturbative evolutions, and showcase here the tail presence in head-on configurations for several mass ratios close to unity. We validate the result through a large battery of numerical tests and detailed comparison with a perturbative evolution, which display striking agreement with full nonlinear ones in the ringdown regime, and very similar tail morphologies. Our results offer yet another confirmation of the highly predictive power of black hole perturbation theory in the presence of a source, even when applied to nonlinear solutions. The late-time tail signal is much more prominent than anticipated until recently, and possibly within reach of gravitational-wave detector measurements, unlocking observational investigations of an additional set of general relativistic predictions on the long-range gravitational dynamics.

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Introduction—In the last few decades, the merger dynamics and subsequent relaxation regime (quasinormal “ringing”) of compact binaries with comparable masses have been the subject of considerable modeling efforts, chiefly enabled by fully nonlinear numerical relativity (NR) evolutions [1–4]. Gravitational-wave (GW) observations of

coalescing black hole (BH) binaries [5–8], granting an unprecedented opportunity of accessing the highly nonlinear regime of dynamical spacetimes, were the main driver for these efforts. In contrast, the subsequent late-time dynamics received much less attention from the physics community, partly because its signatures were thought to be out of reach of foreseeable experimental efforts.

BH perturbation theory [9] predicts that the dominant late-time effect in the relaxation of compact objects is a power-law contribution (“tail”), both in spherical symmetry [10–22] and axisymmetry [23–38]. At future null infinity ($\mathcal{I}^+$), metric perturbations of a Schwarzschild BH decay with the leading-order power law $u^{-(\ell+2)}$, with u the retarded time and ℓ the waveform multipole. (An especially clear discussion of the spherically symmetric case can be

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found in [16,18], and Chapter 12 of [39].) Tails constitute a clean probe of the long-range structure of dynamical spacetimes and allow for distinct predictions on the relaxation of compact object within general relativity [25]. Even more interestingly, as shown in [40] and discussed below, source-driven tails at intermediate times bear direct imprints of the *strong-field binary dynamics*. So far, late-time tails have only been studied numerically in a perturbative framework [12,21,24,38,40–43], but never observed in nonlinear NR simulations of comparable-mass BH mergers. This was mostly due to the very high accuracy required to unveil them, together with solid control over boundary and initial data (ID) effects needed. Here, we present the first robust extraction of late-time tails from fully nonlinear NR simulations of comparable-mass BH binaries.

Characterizing tail effects in numerical simulations is challenging because excitations of quasinormal modes (QNMs) can take a long time to decay [38]. Hence, to identify tail contributions we must look for them in a regime where QNMs are short-lived, and the tail amplitude is large. The first is easy to achieve by targeting remnant BHs with small angular momentum (shorter relaxation time). The second condition is harder to obtain because we do not know *a priori* the dependence of the tail amplitude on the binary’s initial conditions. Recently, some of us [42] found through effective-one-body (EOB) [44–46] calculations in the small-mass-ratio limit that the binary eccentricity induces a strong enhancement of the tail amplitude. The mechanism behind this enhancement and whether its explanation necessitated a modification to Price’s law were initially unknown. In [40] some of us subsequently constructed an analytical model that, for a given bounded trajectory, accurately describes the source-driven tail behavior in generic orbital configurations. The model shows how the simple picture of a single “Price tail” needs to be replaced by a superposition of a large number of inverse power-law components giving rise to a long slowly decaying transient. A key prediction of the model is that the tail amplitude reaches its maximum for radial infalls; hence in this Letter we restrict to head-on collisions. Tails in perturbative radial infalls, with a similar source-driven setup that we employ, were computed in, e.g., [47] and for the first time at $\mathcal{I}^+$ in [48]. These analytical predictions, together with considerations presented in [49,50], equip us with the necessary understanding of ID and boundary effects to extract a tail signal in nonlinear mergers, and constitute the required semianalytical tool to verify the robustness of numerical evolutions.

Exploiting the very high accuracy of the `SpEC` code [51], we demonstrate the extraction of tail effects in fully nonlinear $3+1$ evolutions, displayed in Fig. 1. The figure shows the amplitude of the GW news quadrupolar mode extrapolated to $\mathcal{I}^+$ for several head-on binary simulations with mass ratios close to unity. Around $140M$ after the peak, the amplitude transitions from an exponentially damped quasinormal-driven regime to a slowly decaying

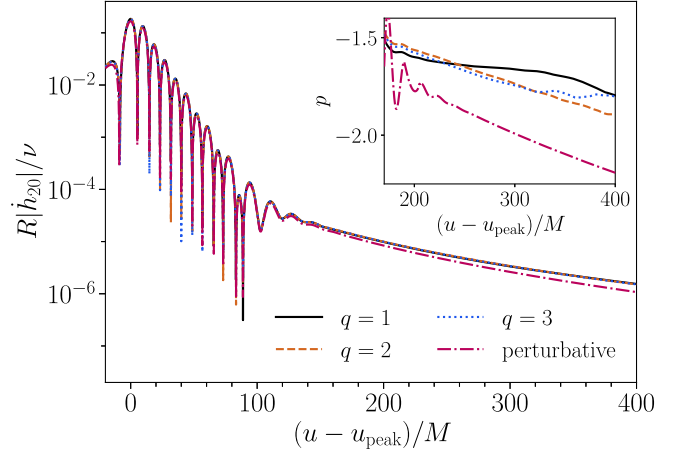


FIG. 1. Mass-rescaled quadrupolar news amplitude as a function of retarded time from the peak. The inset shows the behavior of the tail exponent, as defined in Eq. (1). Thick lines represent nonlinear evolution of head-on comparable-mass collisions, while the dot-dashed line represents a perturbative evolution of a radial infall with compatible initial data.

nonoscillatory behavior. The result aligns remarkably well with perturbative linear evolutions of an infalling test particle with compatible ID until the tail-dominated regime, further validating the numerical computation and pointing toward a suppression of nonlinear corrections [52]. The tail signal portions display very similar morphologies as well, with a consistent initial tail amplitude.

Below, we report on the numerical methods employed both in the nonlinear and perturbative cases. We discuss in more detail the results obtained, their interpretation, and nonlinear effects behind the small differences between linear and nonlinear evolutions at later times. We conclude summarizing open questions and future detectability avenues for tail signals. In Supplemental Material (SM) [53] we give further details of the linearized analytical computation, and present a series of tests to confirm the robustness of our numerical results.

Conventions—We use geometric units $c = G = 1$. The GW strain is decomposed in spin-weight -2 spherical harmonics modes, $h_{\ell m}(t)$. To avoid memory contributions or gauge effects that could spoil tail extraction, entering as a constant offset, we focus on the GW news function $\dot{h}_{\ell m}$. Modes beyond the quadrupolar are subdominant; hence we are going to focus on $\ell = 2$. Exploiting the cylindrical symmetry of the problem, we present all results in a frame [62] in which the two BHs collide along the z axis and hence the $m = 0$ is the only nonzero $\ell = 2$ waveform multipole. At asymptotically late times, it holds: $h_{\ell m} \propto u^{\bar{p}}$, $\dot{h}_{\ell m} \propto u^{\bar{p}-1}$, with $\bar{p} = -(\ell + 2)$ for Schwarzschild BHs. Hence it is convenient to define a “strain tail exponent” as

$$p(t) = 1 + \frac{d \ln |\dot{h}_{\ell m}|}{d \ln u} \quad (1)$$

so that $p(t) = \bar{p}$ at asymptotically late times. With $m_{1,2}$ we indicate the Christodoulou masses [63] of the individual black holes at the relaxation time of the simulation (i.e., when the high-frequency oscillations in the Christodoulou masses have settled down), $M = m_1 + m_2$ is the total Christodoulou mass of the system, $\nu = m_1 m_2 / M^2$ the symmetric mass ratio, and $q = m_1 / m_2 \geq 1$ the binary mass ratio. The time axis is constructed by setting $t = 0$ at the peak of $|\dot{h}_{20}(t)|$, and is quoted in units of M . The tortoise coordinate in perturbative evolutions is $r_* = r + 2M \log[(r/2M) - 1]$, with r the standard Schwarzschild coordinate. We indicate the distance between the binary and the observer with R .

Numerical methods—We generate nonlinear evolutions of head-on collisions with the SpEC code [51,64–67], whose methods are summarized in [68,69]. The ID [70–73] are constructed using the extended conformal thin sandwich equations [74] and the evolution is carried out with the generalized harmonic formulation [75,76]. Standard SpEC runs stop at a retarded time of $100M$ after merger, which is typically enough to only resolve the quasinormal ringdown. In order to capture the waveform behavior on longer timescales, here we carry out simulations with much longer postmerger components, stopping at retarded time $400M$ after merger.

We compare these NR waveforms with linear perturbative waveforms of radial infalls into a Schwarzschild BH, numerically computed using the RWZHyP code [77,78]. We solve for the Regge-Wheeler and Zerilli equations, governing the evolution in time of the gauge-invariant quantities $\Psi_{\ell m}^{(e/o)}$,

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}^{(e/o)}(r_*) \right] \Psi_{\ell m}^{(e/o)}(t, r_*) = S_{\ell m}^{(e/o)}(t, r). \quad (2)$$

These master variables encode the odd and even metric degrees of freedom at linear level [47]. In the linear evolutions, we set null ID $\Psi_{\ell m}^{(e/o)}(t = 0, r) = \partial_t \Psi_{\ell m}^{(e/o)}(t = 0, r) = 0$, as the perturbations are driven by the presence of the source $S_{\ell m}^{(e/o)}$. The latter is evaluated on the infalling test particle’s trajectory. We refer to [40] for additional details and the source explicit expression. For radial infalls with null ID, the odd sector is identically zero; hence in what follows we drop the superscript. Following an EOB-inspired approach, we compute the particle’s trajectory, solving the associated Hamiltonian equations of motion (see Ref. [42] for the explicit expressions), where dissipative effects linked to the GW emission are taken into account by including a radiation-reaction force [46,47]. Such a force was computed for generic orbits by means of a post-Newtonian-based, EOB-resummed analytical expansion [79,80] for the fluxes of energy and angular momentum observed at infinity, and shown to be consistent with the corresponding numerical quantities in [81]. Note that,

due to the short timescale of the dynamics in the configurations we consider, the inclusion of the radiation-reaction force has a small impact on the evolution.

At large distances, the master function $\Psi_{\ell m}$ is related to the linearized strain’s spin-weight -2 spherical harmonic modes $h_{\ell m}$ through [82]

$$h_{\ell m} = \frac{1}{r} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \Psi_{\ell m} + \mathcal{O}\left(\frac{1}{r^2}\right). \quad (3)$$

Moreover, the source term $S_{\ell m}(t, r)$ is linearly proportional to the mass ratio, that in the test-particle limit is equivalent to the symmetric mass ratio, so $\Psi_{\ell m} \propto \nu$. To compare perturbative waveforms with NR comparable-mass results, we will thus always rescale the perturbative results $\Psi_{\ell m}$ by ν . The NR waveforms are also rescaled with their symmetric mass ratio.

To compare full NR results against perturbative test-mass limit ones, we initialize the two systems with compatible ID. We only consider nonspinning black holes. The SXS simulation ID are given by setting the angular momentum to zero and imposing that the Arnowitt-Deser-Misner (ADM) energy E_{ADM} is equal to the total rest mass of the system M within relative accuracy of 10^{-4} , so that the two black holes are at rest at infinite separation, and the initial binding energy is close to zero. We generate equivalent test-mass data by imposing that the test particle is at rest at infinity, i.e., by setting its initial energy equal to its mass.

Outer boundary—The SpEC code imposes data on the outer boundary, located on a sphere with radius R_{out} , such that constraints are preserved [75], and the physical degrees of freedom are chosen assuming that there is no gravitational radiation entering the numerical domain [83,84]. In curved spacetimes, however, a certain amount of radiation is always scattered back. Backscattering, giving rise to wave propagation well within the light cone, is precisely the mechanism responsible for tails generation [40,85–88], and a small R_{out} would imply missing a significant fraction of such contribution. Hence, to capture the vast majority of the backscattered radiation, we place the outer boundary exceptionally far away from the binary location (see Table I).

Moreover, due to imperfect boundary conditions, when radiation reaches the boundary, a nonphysical numerical artifact is generated, contaminating the signal with numerical noise. As pointed out in Refs. [49,50], this contamination can alter the structure of tails. Therefore, to study the long-range and late-time tails contribution, it is important that our simulations remain causally disconnected from the boundary. To avoid such contamination, we chose R_{out} to be large enough such that the boundary is never in causal contact with the extraction spheres for the entire evolution (see Table I). This causal structure, our evolution domain, and the locations of finite-radius observers are visualized

TABLE I. List of simulations and relevant parameters: D_0 is the initial separation, R_{out} the radius of the outer boundary and $u_{\text{con}} = R_{\text{out}} - 2R_{\text{ext}}$ denotes the approximate retarded time at which the outermost extraction radius (located at $1200M$) would enter in causal contact with the outer boundary. The first column states the identifier in the SXS waveform catalog [69,89].

SXS identifier	q	D_0/M	R_{out}/M	u_{con}/M
SXS:BBH:3991 [90]	1	100	4000	1600
SXS:BBH:3997 [91]	2	100	4000	1600
SXS:BBH:3998 [92]	3	100	4000	1600
SXS:BBH:3994 [93]	1	100	8000	5600
SXS:BBH:3995 [94]	1	200	8000	5600
SXS:BBH:3996 [95]	1	400	8000	5600

in Fig. 2. In SM [53], Table II, we report the details of additional *SpEC* simulations in which there is causal contact between the boundary and the extraction spheres, clearly showing a strong impact on the behavior of the tail.

GW extraction—The *RWZHyP* software uses a numerical domain that is decomposed into two regions, a compact inner region and an outer hyperboloidal layer [96–98]. The inner region contains the particle trajectories and has a uniform grid in r_* , ranging from $r_* = -100M$ to a large positive r_* value. The hyperboloidal layer, compactified over r_* , is attached at the positive end of the inner region in order to bring $\mathcal{I}^+$ into the computational domain. As a result, perturbative waveforms can be computed both at finite distances, in terms of the coordinate time t , as well as at $\mathcal{I}^+$ in terms of the retarded time u .

In *SpEC* the GW information is extracted on spheres, whose radii R_{ext} are distributed in the interval $(300M, 1200M)$ and are held fixed across all simulations in this Letter, as opposed to standard *SpEC* runs in which their radii scale with R_{out} . The extracted finite-radius

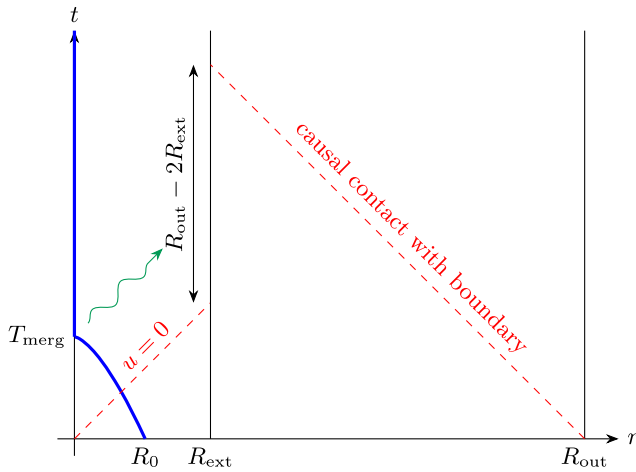


FIG. 2. Schematic spacetime diagram demonstrating the geometry of the head-on merger and remnant, retarded time u , extraction spheres, and outer boundary.

waveform data are extrapolated to $\mathcal{I}^+$ by a standard polynomial fit using the method of [99] as implemented in the *SCRI* package [100–104].

We give additional details of the simulations in SM [53], where we present convergence tests in terms of resolution, extrapolation methods, and initial separation, validating the robustness of our results. Furthermore, there we also discuss results from waveform extraction by Cauchy-characteristic evolution.

Results—In Table I we report the parameters of the simulations carried out with the above methods. Figure 1 shows the resulting news function, normalized by the symmetric mass ratio, with respect to the retarded time u . A first striking result from Fig. 1 concerns the dependence on the mass ratio. In fact, after mass rescaling the waveforms, the comparable-mass waveforms with different mass ratios are very similar (differences around the percent level), suggesting that finite mass ratio corrections do not play a significant role in the waveform generation, including the tail part.

We also compute the tail exponent $p(t)$ [Eq. (1)], reported in the inset of Fig. 1. Its magnitude is much smaller than the asymptotic Price-law value ($\bar{p} = -4$, for this multipole), toward which we expect it to slowly converge [40]. Such decrease in exponent magnitude significantly boosts the tail amplitude at intermediate times. This result agrees with the analytical perturbative picture of [40], according to which tail emission is maximized for motion at large distances from the central BH, with small angular velocity. The additional numerical time derivative required for the exponent computation, combined with finite resolution, tends to introduce high-frequency noise. To compute the tail exponent at late times we therefore apply a Savitzky-Golay filter [105] on the waveform to suppress high-frequency oscillations. In SM [53] we compare the unfiltered tail exponent with the one computed after applying the filter, showcasing that our conclusions are not impacted by the filtering.

The test-mass perturbative case (similarly rescaled, and aligned minimizing the postpeak mismatch) shows a remarkable agreement with the nonlinear evolutions, all the way from the peak until the transition to a tail-dominated behavior. At later times ($\sim 150M$), a gradually growing relative mismatch is present, albeit the waveforms exhibit a similar (slowly decaying) morphology, with a consistent initial tail amplitude. Such a mismatch is expected given the hereditary nature of the tail signal, as differences in the inspiral trajectory accumulate in time, giving rise to an increasing difference among the two signals [40]. The initial agreement, together with the *slowly* growing nature of the later times relative mismatch displayed in Fig. 1, suggest that the same linear mechanism behind QNMs and tail excitation is present in the nonlinear case, and dominates at intermediate times. These results confirm that the tail is primarily generated by the source term, aligning with

previous findings, which showcased how test-particle perturbative evolutions proved to be a remarkably accurate tool for the modeling of inspiral-merger-ringdown waveforms [46,47,106]. Somewhat surprisingly though, this framework provides *quantitatively accurate* predictions even for comparable-mass systems [107,108], as we confirm here. This is true even in the (*a priori* strong-field) merger stage, as notably depicted also in Fig. 2 of [109].

The relative mismatch at later times could also arise from nonlinear effects; its slow growth suggests it could be due to a subleading component in the tail generation. For instance, two tail signals could be at play in Fig. 1: Price’s law and the nonlinear tail component of [52] (see also Refs. [110,111]). The nonlinear tail decays slower than Price’s tail, and, if initially less excited, the strain could first be dominated by a hereditary tail transient, later transitioning to the nonlinear tail. The impact of other nonlinear effects, such as finite mass-ratio corrections [112] or time-dependent background effects [113–117] could accumulate in time, progressively amplified by the hereditary nature of Price’s tail propagator. However, we do not expect time-dependent background effects to strongly affect the tail, since they become relevant during merger, while the hereditary tail observed at late times is mainly emitted before merger [40].

Future avenues—We have uncovered late-time gravitational-wave tails in $3 + 1$ nonlinear simulations of black hole collisions through the highly accurate `spEC` code, robustly validating this result through a number of numerical tests. The waveforms display a remarkable agreement with perturbative results until the tail-dominated phase, a fact that deserves further scrutiny in future works.

Our results raise several interesting questions. On the modeling side, these include the following. What is the nonlinear content of late-time tails? Can second-order tails explain the observed differences between linear and nonlinear evolutions? Or can these instead be accounted for by higher-order (self-force type) corrections to the trajectory or the dynamical background? A one-power-slower decay is predicted by mathematical relativity results of Ref. [118] when accounting for nonlinearities. Further studies are needed to understand if this could be the cause of our deviations from perturbation theory predictions. Precision studies on this subject will benefit from longer simulations and the development of Cauchy-characteristic matching techniques [119–121] or nonlinear codes in hyperboloidal coordinates [122–125] to uncover the subtle role of nonlinearities.

On the observational side, the tail sensitivity to the long-range spacetime structure implies it might be used as a tool to detect “environmental” effects around binaries, such as those induced by dark matter halos, accretion disks, boson clouds, as well as a tertiary object. This could open up an entire new way of characterizing nonvacuum spacetimes, as displayed, e.g., in Fig. 1 of [126] and discussed in [52].

The impact of these configurations on tails will be the subject of future investigation. Observability studies will greatly benefit from the construction of analytical tail models for spinning remnants (akin to [40]). We do not expect spin to significantly impact the intrinsic tail-generation mechanism, as the propagator governing this effect is insensitive to deviations from spherical symmetry [30]. However, the dynamics of eccentric orbits in the spinning case might enhance tails even further [43]. Finally, we note that tails bear significant potential as fundamental physics probes, e.g., they can be used to investigate the number of spacetime dimensions [127]. The source-driven enhancement of the tail brings it much closer to the reach of upcoming GW observatories; hence our results open the possibility of observational tests of general relativistic predictions on the long-range structure of highly curved spacetimes. A detailed tail detectability study is the subject of ongoing investigations.

The version of the `RWZHyp` code used bears the tag “tails” on the “`rwzhyp_eccentric`” branch. The Letter content has been derived using publicly available software: Matplotlib [128], NumPy [129], SciPy [130], and SXS [131].

Note added—We recently became aware of an independent work studying late-time tails in fully nonlinear numerical evolutions of head-on collisions [132]. The two investigations complement each other. The present Letter focus is on the tails’ phenomenology in comparable-mass mergers, studying the mass ratio dependence and comparing the behavior with perturbative results. In [132] the main focus is on applying an advanced numerical method, the Cauchy-characteristic matching integration scheme, to tail extraction at $\mathcal{I}^+$ for nonlinear simulations. A phenomenological investigation is then performed by filtering out the QNMs from the strain in order to isolate the tail. In our case, the tail extraction was achieved through a robust extrapolation scheme, leveraging a perturbative understanding. The two studies yield consistent results for the equal-mass case, the only overlapping dataset.

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Data availability—SpEC simulation data are available from Caltech Data [90–95,133–140] and can also be accessed through the SXS Python package [89,131].

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