Exact Solution of Two-Dimensional Palatini Gauss-Bonnet Theory on a Strip

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We analyze the two-dimensional Palatini Gauss-Bonnet theory on an infinite strip (product of a finite interval with the infinite line, corresponding to "time"). The theory has only boundary degrees of freedom. Its phase space is the cotangent bundle to the group manifold of $SL(2,\mathbf{R})$, subject to a (first-class) constraint quadratic in the momenta. With the simplest choice of boundary Hamiltonian, namely $H=0$, the theory is shown to describe geodesics on the group manifold of $SL(2,\mathbf{R})$, with a "mass" determined by the Palatini Gauss-Bonnet coupling constant. Other choices of boundary Hamiltonians compatible with gauge invariance are also possible. The symmetry group contains (left and right) group translations on $SL(2,\mathbf{R})$. These are "boundary symmetries" from the bulk point of view, with one copy acting on one end of the interval and the other copy acting on the other end. Comments on the quantum theory are also given.

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Introduction—The complexity of four-dimensional gravity, at both the classical and quantum levels, has motivated the study of lower-dimensional models ($d=3$ and $d=2$). These theories have provided an invaluable testing ground for developing and exploring ideas relevant to four-dimensional gravity.

Somehow puzzling, the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence remained an open problem for many years. This picture changed dramatically after the works of Refs. [1–3] and others, who showed that the breaking of conformal symmetry gives rise to a Schwarzian action describing the boundary dynamics of nearly AdS_2 Jackiw-Teitelboim (JT) gravity [4,5] and the boundary effective theory obtained after integrating the microscopic fermionic degrees of freedom in the Sachdev–Ye–Kitaev model model [6,7]. These ideas have been under intense scrutiny is recent years.

A salient feature of JT gravity, as of most two-dimensional gravitational models [8], is the introduction of a scalar field ("dilaton") multiplying the Einstein-Hilbert density, which otherwise defines by itself a trivial theory.

A different mechanism that also leads to a nontrivial two-dimensional gravitational action while dispensing with the scalar field was introduced in a recent work [9], where we considered the "Palatini Gauss-Bonnet theories." These theories exist in all even spacetime dimensions $D=2n$ and involve as dynamical fields a $GL(2n,\mathbf{R})$-connection $\Gamma^{\alpha}_{\beta\mu}$ and an internal metric $g_{\alpha\beta}$.

The purpose of this Letter is to study in detail the Palatini Gauss-Bonnet theory in two spacetime dimensions. As JT gravity, this theory has only boundary degrees of freedom (see Ref. [9] for details) and is therefore an interesting holographic laboratory. We prove that Palatini Gauss-Bonnet theory formulated on a manifold with two disconnected boundaries (an infinite strip $[r_1,r_2]\times\mathbf{R}$) is equivalent to a one-dimensional relativistic particle propagating on a three-dimensional anti–de Sitter space. We leave for the future a direct comparison with JT gravity and its relation to $\mathrm{AdS}_2/\mathrm{CFT}_1$. This is an important aspect but beyond the scope of this Letter.

In two dimensions, the action that we consider is

$$I[g,\Gamma]=k\int\sqrt{|g|}\epsilon^{\alpha}_{\beta}R^{\beta}_{\alpha}(\Gamma)-\int dtH\Big|_{r_1}^{r_2}, \tag{1}$$

where $\epsilon^{\alpha}_{\beta}$ is defined by $\epsilon^{\alpha}_{\beta}=g^{\alpha\gamma}\epsilon_{\gamma\beta}$ and $k\neq 0$ is a dimensionless coupling constant [10]. We have allowed for the possibility of a boundary term $-\int dtH|_{r_1}^{r_2}$ at the boundary of the strip. This term will play the role of boundary Hamiltonian (integrated over time) and is necessary to make the variational principle well defined when some fields, in our specific case Γ_0, do not go to zero at the boundary. Giving H completes the definition of the theory. We come back to its admissible forms below and its connection with the boundary behavior of Γ_0.

As shown in [9], this action can be rewritten as a constrained *B-F* system [11,12], which, in Hamiltonian form [13], reads as

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$$I[B,\Gamma,\Gamma_0,\lambda_1,\lambda_2]=\int dt\int_{r_1}^{r_2}dr\,\mathcal{L}-\int dtH\Big|_{r_1}^{r_2}$$
$$\mathcal{L}=\mathrm{Tr}(B\dot{\Gamma}+\Gamma_0\nabla B-\lambda_1 B-\lambda_2(B^2+k^2\eta I_{2\times 2})), \tag{2}$$

where the matrix B belongs to $gl(2)$. Full equivalence of (2) with (1) was established in Ref. [9]. (We split the $1+1$ spacetime connection as $\Gamma_\mu=\{\Gamma_t,\Gamma\}$ with $\Gamma\equiv\Gamma_r$; ∇ is the radial covariant derivative; $\nabla B\equiv\partial_r B+[\Gamma,B]$; and $\eta=[(|\det g|)/(\det g)]=\pm 1$ [9]. More on our conventions is given in Ref. [16].)

The kinetic term in the action has the standard "$p\dot{q}$" canonical form; therefore the spatial connection Γ and the B-field are canonically conjugate,

$$\{\Gamma^\alpha_{\beta 1}(r),B^\gamma_\delta(r')\}=\delta^\alpha_\delta\delta^\gamma_\beta\delta(r-r'). \tag{3}$$

The other bulk terms in the action are constraint terms, with respective Lagrange multipliers Γ_0, λ_1, and λ_2.

Gauge generators in the presence of boundaries—The Gauss constraint, $\nabla B=0$, involves spatial gradients of the fields leading to "improper gauge symmetries" [17,18], which provide useful insight on the boundary theory. The gauge transformations associated with this constraint are the standard $GL(2)$ gauge transformations,

$$\delta\Gamma^\alpha_{\beta 1}=\nabla_1\varepsilon^\alpha_\beta,\qquad \delta B^\alpha_\beta=[B,\varepsilon]^\alpha_\beta. \tag{4}$$

The canonical generator for these symmetries is

$$G[\varepsilon]=-\int dr\,\mathrm{Tr}(\varepsilon\nabla B)+\mathrm{Tr}(\varepsilon B)\Big|_{r_1}^{r_2}. \tag{5}$$

The boundary terms at r_1 and r_2 are necessary if $\epsilon(r_1)\neq 0$ [respectively, $\epsilon(r_2)\neq 0$]; otherwise $G[\varepsilon]$ does not have well-defined variations, and the Poisson bracket does not exist [17].

If the matrix ε is a multiple of the identity at r_1 (respectively, r_2), the boundary term at r_1 vanishes because of the linear constraint $\mathrm{Tr}(B)=0$: the transformation is then a proper gauge transformation. Thus, the center $\mathbf{R}I_{2\times 2}$ defines a proper gauge symmetry.

For arbitrary values of B compatible with the constraints and arbitrary traceless gauge parameters $\epsilon(r_1)$ and $\epsilon(r_2)$, the surface term does not vanish, even when the constraints hold. The gauge transformations are in that case improper and correspond to transformations having a nontrivial physical action [18]. The improper gauge symmetries are parametrized at each boundary by the quotient algebra, which is $sl(2)$:

$$sl(2)\simeq\frac{gl(2)}{\mathbf{R}I_{2\times 2}}. \tag{6}$$

One might fear that, because of the Gauss constraint $\nabla B=0$, which is a first order equation relating the values of $B(r_2)$ to the value of $B(r_1)$, the boundary terms at the two boundaries are not independent. This would be the case if the group were Abelian since then the covariant derivative would coincide with the ordinary derivative and the constraint would imply $B(r_2)=B(r_1)$, leading to the equality of the boundary terms when $\varepsilon(r_1)=\varepsilon(r_2)$. But in the present case, the relation between $B(r_2)$ and $B(r_1)$ involves the phase space variables so that the boundary terms are functionally independent, as the explicit expressions given below manifestly indicate.

$G[\varepsilon]$ generates gauge transformations on all fields, including itself. This implies $\{G[\varepsilon_1],G[\varepsilon_2]\}=\delta_2 G[\varepsilon_1]$ for any two parameters ε_1 and ε_2. One can use this relation for a quick evaluation of the algebra. One finds

$$\{G[\varepsilon_1],G[\varepsilon_2]\}=-\int dr[\varepsilon_1,\varepsilon_2]\nabla B+[\varepsilon_1,\varepsilon_2]B\Big|_{r_1}^{r_2},$$
$$=G\{[\varepsilon_1,\varepsilon_2]\}, \tag{7}$$

as expected (Tr symbol omitted). This is an example of the general theorem proved in Ref. [19], showing that the boundary-term issue arises only once.

Since the values of the gauge parameters at the endpoints are independent, one gets two commuting copies of $sl(2)$, one at each endpoint. The algebra of improper gauge symmetries is thus $sl(2)\times sl(2)$. This can be seen explicitly by decomposing the gauge parameter as

$$\varepsilon=\epsilon+\tilde{\epsilon}, \tag{8}$$

where $\epsilon(r_1)=0$ and $\tilde{\epsilon}(r_2)=0$. The gauge generator decomposes accordingly as $G[\varepsilon]=G[\epsilon]+G[\tilde{\epsilon}]$. One finds from (7) that, up to purely bulk terms that vanish with the Gauss constraints, the generators $G[\epsilon]$ and $G[\tilde{\epsilon}]$ have vanishing Poisson brackets and separately fulfill the $sl(2)$ algebra.

The $sl(2)$ canonical charges at each endpoint are subject to the same quadratic relation $\mathrm{Tr}(B^2+k^2\eta I_{2\times 2})=0$, which is a restriction on the value of the (quadratic) Casimir in terms of the coupling constant and the signature of the metric $g_{\alpha\beta}$.

Boundary action—One can reformulate the theory as a $1+0$-dimensional boundary theory by integrating the Gauss constraint "*à la* Moore-Seiberg" [20]. Since the Hamiltonian plays no role in the reduction to the boundary theory, we first take $H=0$. This means that the improper gauge transformation that accompanies the time evolution is equal to zero, i.e., that the Lagrange multipliers Γ_0 vanish at the boundaries, $\Gamma_0(r_1)=\Gamma_0(r_2)=0$. We comment later on the case $H\neq 0$.

The trace constraint being algebraic is easily dealt with. It just expresses that the gauge algebra is $sl(2)$. We can thus use the equivalent description of the system as a $SL(2)$ BF system and assume from now on that Γ is an $SL(2)$ connection and that B is identically traceless. In that description, the sole constraints are the quadratic constraint and the Gauss constraint.

In order to solve the Gauss constraint, we make the phase space change of variables $[\Gamma(r), B(r)] \to [U(r), b(r)]$, with $U \in SL(2)$ and $b \in sl(2)$ defined by

$$\Gamma(r) = U(r)^{-1}U(r)', \qquad B(r) = U(r)^{-1}b(r)U(r). \tag{9}$$

This is a noncanonical but invertible transformation if we impose $U(r_1) = I_{2\times 2}$, with the inverse being given by

$$U(r) = \mathrm{P}e^{\int_{r_1}^{r}\Gamma(r)dr}, \qquad b(r) = U(r)B(r)U(r)^{-1}. \tag{10}$$

Under gauge transformations $\Gamma \to S^{-1}dS + S^{-1}\Gamma S$, the Wilson line U and the new variable b transform as

$$U(r) \to S(r_1)^{-1}U(r)S(r), \qquad b(r) \to S(r)^{-1}b(r)S(r). \tag{11}$$

In the new variables, the Gauss constraint becomes simply $U(r)^{-1}b'(r)U(r) = 0$, the solution of which is $b = b(t)$. Plugging this solution into the bulk action reduces it to a surface term at r_2, which is the searched-for action in $1+0$ dimension:

$$I[V, b; \lambda] = \int dt\, \mathrm{Tr}\left[b\dot{V}V^{-1} - \lambda(b^2 + k^2\eta I_{2\times 2})\right]. \tag{12}$$

Here $\lambda(t)$ is related to λ_2 as $\lambda(t) \coloneqq \int_{r_1}^{r_2} dr\, \lambda_2(r,t)$ and $V(t) \equiv U(t, r_2)$. To reach (12), we used the relation $\dot{\Gamma} = \nabla(U^{-1}\dot{U})$ that follows from the expression of Γ, the constraint $\nabla(U^{-1}bU) = 0$, and the boundary condition $U(r_1) = I_{2\times 2} \Rightarrow \dot{U}(r_1) = 0$.

Varying (12) with respect to b, V, and λ we obtain the equations of motion, $\dot{V}V^{-1} = 2\lambda b$, $\dot{b} = [\dot{V}V^{-1}, b]$, and $0 = \mathrm{Tr}(b^2 + \eta k^2)$, respectively.

All of the information extracted via the analysis of charges in bulk functionals can be reobtained from the boundary action. The improper gauge transformations of the original action discussed above appear now as Noether symmetries of the boundary action (12), which has indeed an explicit $SL(2) \times SL(2)$ symmetry. The left $SL(2)$ symmetry [improper gauge transformation at r_1, $S(r_1) = h$, $S(r_2) = I_{2\times 2}$] is given by the transformations

$$V \to h^{-1}V, \qquad b \to h^{-1}bh \qquad [h \in SL(2)]. \tag{13}$$

The right $SL(2)$ symmetry [improper gauge transformation at r_2, $S(r_1) = I_{2\times 2}$, $S(r_2) = g$] reads as

$$V \to Vg, \qquad b \to b \qquad [g \in SL(2)], \tag{14}$$

with h and g being time independent. Both sets of transformations are clearly symmetries of (12).

The corresponding Noether charges coincide exactly with the surface terms associated with the bulk improper gauge symmetries,

$$Q_\zeta = -\mathrm{Tr}[\zeta B(r_1)] = -\mathrm{Tr}(\zeta b), \tag{15}$$

$$Q_\epsilon = \mathrm{Tr}[\zeta B(r_2)] = \mathrm{Tr}(\epsilon V^{-1}bV), \tag{16}$$

where we have expanded h and g near the identity as $h = I_{2\times 2} + \zeta$, $g = I_{2\times 2} + \epsilon$. The conservation of these charges, $(db/dt) = 0$ and $(d/dt)(V^{-1}bV) = 0$, can in fact be verified directly from the equations of motion of the boundary theory.

It is tempting to think of the boundary theory as living at r_2. However, the set of asymptotic symmetries defined at the other boundary r_1 does act nontrivially at r_2. This is because Gauss' law relates the two boundaries, the field $V(r_2)$ itself being nonlocal.

Anti–de Sitter formulation—We now show that the action (12) can be seen as a massive particle propagating on AdS_3. This is not too surprising given the isomorphism $SL(2) \simeq \mathrm{AdS}_3$ [21]. Indeed, the unit determinant condition of $SL(2)$ matrices,

$$V = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL(2), \qquad (\alpha\delta - \beta\gamma = 1), \tag{17}$$

has a one-to-one map with the $\mathbf{R}^{2,2}$ AdS_3 embedding $-T_0^2 + X_1^2 + X_2^2 - T_3^2 = -1$ via $\alpha = T_0 + X_1$, $\delta = T_0 - X_1$, $\beta = T_3 + X_2$, $\gamma = -T_3 + X_2$. A quick way to see the relation between (12) and a free particle on AdS_3 is to note that b, and then λ, can be eliminated from their own equations arriving to the group manifold action $\int dt\sqrt{V^{-1}\dot{V}V^{-1}\dot{V}}$. Parametrizing V gives the Lagrangian of a free relativistic particle on AdS_3. We choose a route that takes us directly to the Hamiltonian version appropriated to study the quantum theory.

Let us parametrize V in terms of the familiar AdS global coordinates $\{T, \rho, \Psi\} \equiv q^A$ via the explicit formulas $\alpha = \cosh\rho\cos T + \sinh\rho\cos\Psi$, $\beta = \cosh\rho\sin T - \sinh\rho\sin\Psi$, $\gamma = -\cosh\rho\,\sin T - \sinh\rho\,\sin\Psi$, and $\delta = \cosh\rho\,\cos T - \sinh\rho\,\cos\Psi$.

The matrix b lives in $sl(2)$. We parametrize it in terms of three momenta $\{p_0, p_1, p_2\} \equiv \{p_T, p_\rho, p_\Psi\} \equiv \{p_A\}$,

$$b = \frac{1}{2}RP - \frac{1}{2}(p_0 + p_2)i\sigma_2, \tag{18}$$

where σ_2 is the Pauli matrix, with R and P given by

$$R = \begin{pmatrix} \cos S & \sin S \\ -\sin S & \cos S \end{pmatrix}, \qquad P = \begin{pmatrix} p_1 & -P_{12} \\ -P_{12} & -p_1 \end{pmatrix}. \tag{19}$$

Here, $S = T + \Psi$ and $P_{12} \equiv p_0 \tanh\rho + p_2 \coth\rho$.

By direct replacement into the action (12) one finds (with $\frac{1}{2}\lambda \to \lambda$)

$$I = \int dt \Big[p_A \dot{q}^A - \lambda(\gamma^{AB} p_A p_B + 4\eta k^2) \Big], \tag{20}$$

where

$$\gamma_{AB} dq^A dq^A = -\cosh^2(\rho) dT^2 + d\rho^2 + \sinh^2(\rho) d\Psi^2 \tag{21}$$

is the AdS_3 metric [22]. Thus, as announced, the action (12) is equivalent to a particle propagating on AdS_3 with mass-shell condition $\gamma^{AB} p_A p_B + m^2 = 0$, where the mass is given by

$$m^2 = 4k^2\eta. \tag{22}$$

This is negative for the Minkowskian signature of $g_{\alpha\beta}$ ($\eta = -1$) and positive for the Euclidean signature ($\eta = 1$). Note that the signature of the two-dimensional metric $g_{\alpha\beta}(x)$ only affects the sign of m^2, not the signature of the metric γ_{AB}, which is always AdS_3.

The boundary symmetries identified before are now just the isometries of AdS_3, three left translations L_a^A and three right translations R_a^A on the group [23]. Both sets of Killing vectors satisfy the $sl(2)$ algebra in the Lie bracket, $[\xi_a, \xi_b]^A = \epsilon_{ab}^c \xi_c^A$. In the anti–de Sitter parametrization, the boundary Noether charges read simply as $Q_a = -\xi_a^A p_A$.

In particular, the AdS energy is the conserved Noether charge associated with the Killing vector $\partial/\partial T$, $E = Q_2^R + Q_2^L$, and can be rewritten as

$$E_{\mathrm{AdS}} = -m\boldsymbol{\gamma}\left(\frac{\partial}{\partial T}, \frac{dq^M}{d\tau}\right) = -m\gamma_{00}\frac{dT}{d\tau}. \tag{23}$$

It is positive for future-oriented timelike motions.

The equivalence of the theory with geodesic motion in anti–de Sitter space, and the resulting physical interpretation, naturally leads to the consideration of the covering space $CAdS_3$ as configuration space for the "particle," in order to avoid closed timelike curves.

Nonzero Hamiltonian—Including a nonzero boundary Hamiltonian in (1) is permissible but changes the interpretation of the theory. The choice of the boundary Hamiltonian goes hand in hand with the choice of the boundary values of the Lagrange multipliers $\Gamma_0(r_i)$ associated with the Gauss constraint as most clearly seen in the form (2) of the action. Indeed, if $\Gamma_0(r_i) \neq 0$, one must add a boundary term to cancel the boundary term picked up upon integration by parts in the variation of the action, which is the time integral of $-[\mathrm{Tr}[\Gamma_0(r)\delta B(r)]]_{r_1}^{r_2}$. This is necessary in order to have a well-defined action principle [17]. Such a boundary term will exist only if $\mathrm{Tr}[\Gamma_0(r)\delta B(r)]$ is integrable at the boundaries, i.e., if $\Gamma_0(r_i) = \{[\partial H^{(i)}(B_i)]/\partial B_i\}$, $(i = 1, 2)$, for some $H^{(i)}(B_i)$ with $B_i \equiv B(r_i)$. This yields the Hamiltonian $H = H^{(2)}(B_2) - H^{(1)}(B_1)$, or

$$H = H^{(2)}(Q_i^R) - H^{(1)}(Q_i^L). \tag{24}$$

The Hamiltonian $\mathcal{H}$ is invariant under proper gauge symmetries, as it should. Because the algebra is not Abelian, it is not invariant under improper ones unless one takes $H^{(i)}(B_i) = \mathrm{Tr}(B_i^2)$, but given the constraint enforced by λ_2, this is equivalent to $H^{(i)}(B_i) = \text{constant}$ [and $\Gamma_0(r_i) = 0$]. Since invariance under improper gauge transformations is not a condition to be imposed on the theory, arbitrary choices of $H^{(i)}(B_i)$ are definitely allowed. For generic choices of H, one gets in general $\{H, Q_i^{L,R}\} \neq 0$ except for the unbroken "subalgebra" of transformations that commute with H. The charges associated with the symmetries that do not commute with H will be explicitly time dependent. With the Hamiltonian being a function of the generators of the algebra, the $sl(2) \times sl(2)$ algebra is nevertheless a symmetry [24].

Quantum theory—We follow Dirac prescription for constructing the quantum theory, in which one imposes the constraints on physical states. In the representation where the group coordinates q^A are diagonal and $\hat{p}_A = i(\partial/\partial q^A)$, this constraint on $\phi(q^A)$ becomes the Klein-Gordon equation on AdS_3,

$$-\Box_q \phi + m^2 \phi = 0, \tag{25}$$

where $\Box_q$ is defined with the metric (21). Since $\Box_q$ commutes with the AdS generators, the states will transform into representations of the AdS algebra.

If the boundary Hamiltonian H is taken to be zero, the problem reduces to the much studied question of solving the Klein-Gordon equation in anti–de Sitter space [26–31]. It is then natural to impose for stability that the AdS energy should be bounded from below. This means that only the representations from the lowest weight discrete series are allowed [for a review of representations of $SL(2)$ and the AdS groups, see Refs. [32–34]]. The lowest energy state [35] is $\phi_h = c(e^{-2ihT}/\cosh^{2h}\rho)$ (c is a normalization constant) and has energy $E_{\mathrm{AdS}} = 2h$. It satisfies $Q_-^L \phi_h = Q_-^R \phi_h = 0$ (lowering operators) together with $Q^{L_0}\phi_h = h\phi_h$, $Q^{R_0}\phi_h = h\phi_h$. Since the eigenvalues of L_0 and R_0 are equal this state has zero angular momentum (eigenvalue of $Q^{R_0} - Q^{L_0} = i\partial_\Psi$). The wave equation implies the relation $m^2 = 4h(h-1)$, where we recall that the mass is related to

the coupling k by (22). The descendants have quantized energy levels $E = 2h + n$ (with n being a positive integer).

The quantum number h must be positive for positivity of the Klein-Gordon norm. From the expression for the mass, we find $h = 1 + \sqrt{1 + \eta k^2}$; thus, for $\eta = -1$, k must fulfill the Breitenlhoner-Freedman bound [29,30],

$$k^2 < 1, \qquad (\eta = -1). \tag{26}$$

This is the only condition in the case of $C\mathrm{AdS}_3$ and $\overline{SL(2)}$.

If one does not unwrap the closed timelike curves and sticks to the original AdS_3, the weight h is restricted by single valuedness of the wave function on the time circle by the quantization condition $2h =$ integer, that is, h can be an integer or semi-integer. This implies that the Palatini Gauss-Bonnet coupling k must also be quantized since the mass, and hence k, is related to h through $m^2 = 4h(h-1)$.

If $H \neq 0$, the quantum states $\phi(q^A, t)$ also depend on the boundary time t through the Schrödinger equation $i\hbar(\partial\phi/\partial t) = H\phi$ (in addition to the Klein-Gordon equation). Because the Hamiltonian does not in general commute with all of the generators of $sl(2) \oplus sl(2)$, the energy (defined now as the eigenvalue of H) takes different values within an irreducible representation. The splitting can be easily computed since H is a function of the $sl(2) \oplus sl(2)$ charges.

If H is a positive definite function of the AdS charges, such as $H = (E_{\mathrm{AdS}})^2$ (say), then it would seem admissible to consider all of the solutions of the Klein-Gordon equation, including those corresponding to other (normalizable) representations, such as the continuous principal series, or the discrete highest weight representations (see, e.g., Ref. [25]).

To conclude, we have established in this Letter the equivalence of the two-dimensional Palatini Gauss-Bonnet theory with the dynamics of a point moving on AdS_3 and subject to the Klein-Gordon constraint, with a mass that depends on the signature of the internal metric $g_{\alpha\beta}$ and the coupling constant k. This constraint fully characterizes the system when the boundary Hamiltonian H vanishes, but different choices of H are permissible and define different theories. Nonvanishing Hs remain fully tractable because they depend on the generators of the algebra.

It would be interesting to extend these results to include spin, by starting with a supersymmetric version of the two-dimensional Palatini Gauss-Bonnet. We leave this for future work.

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[10] $R^{\beta}_{\alpha}(\Gamma)$ is the curvature 2-form of the $GL(2)$ connection $\Gamma^{\alpha}_{\beta\mu}$, and g is the determinant of the internal metric $g_{\alpha\beta}$. From now on, $GL(2) \equiv GL(2, \mathbf{R})$, $SL(2) \equiv SL(2, \mathbf{R})$ since we deal only with the real linear groups. Furthermore, we take for definiteness the explicit realization of the group elements in terms of 2×2 real matrices (and not any other representation).
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[13] With respect to the standard B-F system, the theory has an extra (quadratic) constraint that would not arise by dimensional reduction from three-dimensional Chern-Simons theory. This constraint implies that the theory has no degree of freedom in the absence of boundaries, even if the spatial topology is nontrivial (circle). This is because the holonomies are then fixed up to conjugation [9]. The presence of this extra constraint makes the analysis of the Palatini Gauss-Bonnet theory different from that pursued in the papers [14,15]. This constraint leads to the Klein-Gordon equation in the quantum theory. Note also that we do not implement any $sl(2)$ Hamiltonian reduction through boundary conditions at the edges of the strip. In fact, we impose no boundary condition at r_1 and r_2 on the canonical variables Γ_1 and B.
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[16] Our notations are as follows. (i) The matrix $B \equiv B^\alpha_\beta T^\beta_\alpha$ is the B-field transforming as $B \to S^{-1}BS$ under $GL(2)$ transformations S, where T^b_a is a basis of generators of the Lie algebra $gl(2)$. A definite choice is not needed at this stage. (ii) $\Gamma \equiv \Gamma^\alpha_{\beta 1} T^\beta_\alpha$ is the spatial component of the $GL(2)$-valued connection 1-form $\Gamma^\alpha_{\beta\mu} T^\beta_\alpha dx^\mu$, transforming as $\Gamma \to S^{-1}dS + S^{-1}\Gamma S$. (iii) $\Gamma_0 \equiv \Gamma^\alpha_{\beta 0} T^\beta_\alpha$ is its temporal component, which acts as a Lagrange multiplier for the Gauss constraint $\nabla B = 0$. (iv) $\nabla B \equiv \nabla_1 B = \partial_1 B + [\Gamma_1, B]$ is the spatial covariant derivative of B. (v) λ_1 and λ_2 are Lagrange multipliers for the constraints $\mathrm{Tr}B = 0$ and $\mathrm{Tr}(B^2 - k^2 \eta I_{2\times 2}) = 0$. (vi) $I_{2\times 2}$ is the identity in two dimensions. In the following we not systematically write it. (vii) $\eta = \pm 1$ is the sign of the determinant of the metric $g_{\alpha\beta}$.

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[21] Note that AdS_3 means here "wrapped" AdS space (with closed timelike lines) and not its covering space $C\,AdS_3$. The isometry group acts by left and right multiplication as $A \to LAR$, $A, L, R \in SL(2)$ and is isomorphic to the quotient $\{[SL(2) \times SL(2)]/\mathbf{Z}_2\}$, where $\mathbf{Z}_2$ is the center of $SL(2)$ (obtained by simultaneously replacing L by $-L$ and R by $-R$). Since the original action involved only Lie algebra elements, the previous considerations extend straightforwardly to the covering group $\overline{SL(2)}$ and the unwrapped covering space $CAdS_3$. Going in the opposite direction (taking a quotient rather than unwrapping), one can consider $PSL(2) \simeq (SL(2)/\mathbf{Z}_2)$, i.e., AdS_3 quotientized by the isometry $T_0 \to -T_0$, $T_3 \to -T_3$, $X_1 \to -X_1$, $X_2 \to -X_2$. All these groups share the same Lie algebra.

[22] Note that the group coordinates are dimensionless, and AdS space can be taken to have unit radius of curvature.

[23] Left translations, in terms of AdS coordinates, read as

$$L_0 = \frac{1}{2}\left(\frac{\partial}{\partial T} - \frac{\partial}{\partial \Psi}\right) \tag{27}$$

$$L_1 = \frac{1}{2}\sin(T-\Psi)\left(-\frac{\sinh\rho}{\cosh\rho}\frac{\partial}{\partial T} + \frac{\cosh\rho}{\sinh\rho}\frac{\partial}{\partial \Psi}\right) + \frac{1}{2}\cos(T-\Psi)\frac{\partial}{\partial \rho} \tag{28}$$

$$L_2 = \frac{1}{2}\cos(T-\Psi)\left(-\frac{\sinh\rho}{\cosh\rho}\frac{\partial}{\partial T} + \frac{\cosh\rho}{\sinh\rho}\frac{\partial}{\partial \Psi}\right) - \frac{1}{2}\sin(T-\Psi)\frac{\partial}{\partial \rho}, \tag{29}$$

while right translations R^A_a become

$$R_0 = \frac{1}{2}\left(\frac{\partial}{\partial T} + \frac{\partial}{\partial \Psi}\right) \tag{30}$$

$$R_1 = \frac{1}{2}\sin(T+\Psi)\left(-\frac{\sinh\rho}{\cosh\rho}\frac{\partial}{\partial T} - \frac{\cosh\rho}{\sinh\rho}\frac{\partial}{\partial \Psi}\right) + \frac{1}{2}\cos(T+\Psi)\frac{\partial}{\partial \rho} \tag{31}$$

$$R_2 = \frac{1}{2}\cos(T+\Psi)\left(-\frac{\sinh\rho}{\cosh\rho}\frac{\partial}{\partial T} - \frac{\cosh\rho}{\sinh\rho}\frac{\partial}{\partial \Psi}\right) - \frac{1}{2}\sin(T+\Psi)\frac{\partial}{\partial \rho}. \tag{32}$$

[24] In Ref. [9], we pointed out that the diffeomorphism invariance of the action (1) was not an independent symmetry but could be expressed in terms of the gauge symmetries generated by the first class constraints. Of these, the Gauss constraint generates internal gauge transformations, while the trace constraint generates projective transformations. The transformation generated by the quadratic constraint was called "new symmetry" as it was not in the original list of original symmetries. Time diffeomorphisms could be expressed in terms of it, but we were reluctant to identify the new symmetry with time diffeorphisms because this involved dividing by a variable (here, λ) that was not always different from zero. It is interesting to note, however, that with $H = 0$ and the understanding of the dynamics as that of a free particle in AdS space, the quadratic constraint, which is the mass shell condition, does generate reparametrizations in the time t. In that case, identifying the two symmetries is therefore meaningful (the condition $\lambda > 0$ on the "lapse" λ then makes sense). But if the boundary Hamiltonian H is not zero, reparametrizations of the boundary time is not a gauge symmetry, yet the transformations generated by $\mathcal{K}$ are proper gauge symmetries. In fact, translations in t at the boundary are, as we have seen, improper gauge symmetries generated by the Gauss constraint with, in general, a B-dependent parameter, but arbitrary reparametrizations are not symmetries at the boundary. It follows that there is no hope to express, at the boundary, the proper gauge symmetries generated by $\mathcal{K}$ in terms of the time diffeomorphisms.

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