

Time Correlations from Steady-State Expectation Values

Wojciech Górecki^{1,*}, Simone Felicetti^{2,3}, Lorenzo Maccone^{1,4}, and Roberto Di Candia^{5,2,4}

¹*INFN Sezione di Pavia, via Bassi 6, I-27100 Pavia, Italy*

²*Institute for Complex Systems, National Research Council (ISC-CNR), Via dei Taurini 19, 00185 Rome, Italy*

³*Physics Department, Sapienza University, Piazzale Aldo Moro 2, 00185 Rome, Italy*

⁴*Dipartimento di Fisica, University of Pavia, via Bassi 6, I-27100 Pavia, Italy*

⁵*Department of Information and Communications Engineering, Aalto University, Espoo, 02150 Finland*



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Recovering properties of correlation functions is typically challenging. On the one hand, experimentally, it requires measurements with a temporal resolution finer than the system's dynamics. On the other hand, analytical or numerical analysis requires solving the system evolution. Here, we use recent results of quantum metrology with continuous measurements to derive general lower bounds on the relaxation and second-order correlation times that are both easy to calculate and measure. These bounds are based solely on *steady-state* expectation values and their derivatives with respect to a system parameter, and can be readily extended to the autocorrelation of arbitrary observables. We validate our method on two examples of critical quantum systems: a critical driven-dissipative resonator, where the bound matches analytical results for the dynamics, and the infinite-range Ising model, where only the steady state is solvable, and thus the bound provides information beyond the reach of existing analytical approaches. Our results can be applied to the experimental characterization of ultrafast systems and to the theoretical analysis of many-body models whose dynamics are hard to compute.

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Analyzing the correlation functions of emitted signals is a standard tool across different physical disciplines to characterize the properties of atomic, molecular, and solid-state systems [1–6]. The characteristic timescales of correlation functions are particularly relevant for classical [7] and quantum [8] systems undergoing phase transitions, as dynamic correlations of order parameters exhibit universal scalings near the critical point. However, fundamental and practical constraints limit our capacity to infer correlation functions, both experimentally and theoretically.

Experimentally, the measurement of, e.g., the second-order intensity autocorrelation function of light $g^{(2)}(\tau)$ [Eq. (6)] is limited by the resolution time of photodetectors. Time-resolved measurement of $g^{(2)}(\tau)$ is routinely performed when its width is in the nanosecond scale, to characterize emitters such as quantum dots [9], single molecules [10], and diamond color centers [11]. Substantial efforts [12–14] are

dedicated to resolving correlations below the 50 ps resolution of photodetectors. Theoretically, the analysis of quantum many-body models is often an intractable task: even in a steady state and when the quantum regression theorem can be applied [15], extracting correlation times requires the analytical solution or numerical evaluation of the system's dynamics. Alternative approaches tend to lack universality and often rely on specialized analytical methods. For example, the fluctuation-dissipation theorem [16] relates symmetrized correlation functions to the response function, which can be evaluated using the Kubo formula [17]. However, this approach assumes thermalization, a condition frequently violated in, e.g., driven-dissipative quantum systems [18]. The Keldysh formalism [19,20] may be used to compute correlation functions, such as $g^{(1)}(\tau)$ or $g^{(2)}(\tau)$, and their timescales [21]. Obtaining these results involves a highly nontrivial analytical treatment grounded in quantum field theory and relies on approximations that are generally not controllable.

In this Letter, we establish a universal relationship between a system's steady-state sensitivity to small changes of a system parameter and the second-order correlation function $g^{(2)}(\tau)$, using information-theoretic bounds from quantum parameter estimation [22–33]. In particular, our result provides rigorous lower bounds on the correlation and relaxation times. The main idea is that, in continuous steady-state measurements, the sensitivity of an observable with respect to the estimated parameter can be quantified

*Present address: Freie Universität Berlin, Fachbereich Physik und Dahlem Center for Complex Quantum Systems, Arnimallee 14, 14195 Berlin.

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via the signal-to-noise ratio (SNR), which is bounded from above by quantum Fisher information (QFI). If the measurements were uncorrelated in time, the total SNR would accumulate additively, resulting in a precision consistent with the statistics of independent samples. Instead, if the precision seems to exceed the fundamental bounds for quantum parameter estimation obtained under this assumption, we must conclude that the measurement outcomes are temporally correlated. These correlations are captured by the two-time function $g^{(2)}(\tau)$, and we show that such metrological tools can be used to lower bound its characteristic time.

Our bound has both theoretical and experimental implications: it serves both as a conceptual framework for the theoretical analysis of many-body quantum systems and as a practical method for experimentally probing the correlation times of ultrafast dynamics that cannot be resolved by available detectors. Indeed, it enables the estimation of the correlation time based solely on the single-time expectation values of the system's steady state, without requiring resolution of the dynamics. This method can be readily extended to the autocorrelation of arbitrary observables, and it particularly fits systems showing strong bunching and large susceptibility, such as critical systems [34–36].

We benchmark our method on two complementary examples. First, we consider a parametrically driven resonator, where the bound can be saturated (up to the leading order). This validates the approach and emphasizes its relevance for real-world scenarios, as confirmed by recent experiments [37,38]. Second, we study the infinite-range dissipative transverse-field Ising model, for which a direct solution of the equations of motion is not available. In this case, our method provides new insight into the system's correlation behavior that would otherwise be hard to access.

Setting and fundamental bounds—As sketched in Fig. 1, we consider a system governed by the Markovian master equation

$$\frac{d\rho}{dt} = -i[H, \rho] + \gamma \mathcal{D}[\rho], \quad \text{with } H = \omega G_0 + G_1, \quad G_0 \geq 0, \quad (1)$$

where ωG_0 is the free Hamiltonian of the system and G_1 is a driving part. The G_0 has integer spectrum and is positive [39]. The Lindbladian is $\mathcal{D}[\rho] = \sum_k D_k \rho D_k^\dagger - \frac{1}{2} \{D_k^\dagger D_k, \rho\}$, where D_k are the lowering operators, mapping between eigenstates of G_0 corresponding to neighboring eigenvalues, such that $G_0 = \sum_k D_k^\dagger D_k$. There is no assumption about G_1 . This situation arises in many open-driven systems [40–42] (see also the Examples section). Note that the steady state of such a system ρ_{ss} is mixed but is not thermal, and the eigenbasis of ρ_{ss} does not correspond to the eigenbasis of the full Hamiltonian. Quantum jumps correspond to photon emission into the environment [2]. The mean emission rate is $\gamma \langle G_0 \rangle$, and we denote the emitted mode by b .

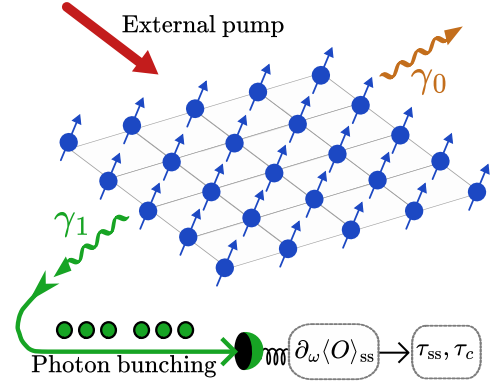


FIG. 1. We consider a general open quantum system coupled to a Markovian environment, possibly subject to an external pump. The total emission rate is $\gamma = \gamma_0 + \gamma_1$, where γ_0 corresponds to dissipation into unmonitored modes, while γ_1 represents the fraction of output photons that are collected and measured. We focus on the steady-state expectation value $\langle O \rangle_{ss}$ of an observable O , which depends on the system parameter ω . Using recent theoretical results for quantum parameter estimation, we demonstrate that the sensitivity $\partial_\omega \langle O \rangle_{ss}$ to small variations in ω provides a lower bound to the relaxation time (τ_{ss}) and, if O is proportional to the number of emitted photons, also to the second-order correlation time (τ_c). Our method enables the estimation of the time correlations based solely on the single-time expectation values of the system's steady state, as a function of ω .

In what follows, we relate characteristic dynamical times to the steady-state sensitivity to small changes of ω using quantum-metrological arguments. We consider an estimation protocol in which the system is initially prepared in a state independent of the actual value of ω , and information about ω is encoded in the system state (and in the emitted light) during the subsequent evolution. Let O denote a measurable scalar quantity used to estimate ω —for instance, an observable measured on the system at time t , or a time-integrated observable of the emitted field. The usefulness of O is quantified by its signal-to-noise ratio, $\text{SNR} = (|\partial_\omega \langle O \rangle|^2 / \Delta^2 O)$. The maximal SNR is bounded by the quantum Fisher information, $\text{SNR} \leq \text{QFI}$, both defined with respect to ω .

Let us decompose the total photon-loss (emission) rate as $\gamma = \gamma_0 + \gamma_1$, where $\gamma_1 \in [0, \gamma]$ denotes the rate of the detected (accessible) output and γ_0 the rate of light lost to the environment (inaccessible). The maximal QFI achievable over a time t is fundamentally bounded by [31–33] (see Supplemental Material Sec. I [43]):

$$\text{QFI} \leq \frac{4}{\gamma_0} \int_0^t \langle G_0 \rangle_{t'} dt', \quad (2)$$

where $\langle G_0 \rangle_{t'} = \text{Tr}(\rho(t') G_0)$. Notably, this bound depends only on the rate of photons lost to the environment γ_0 , not on the total emission rate γ .

Trivially, for the best precision in estimating ω , one should measure all emitted photons, i.e., setting $\gamma_1 = \gamma$ and $\gamma_0 = 0$. However, in that case, the bound Eq. (2) diverges and becomes non-informative [44]. Since our aim is not to find an optimal strategy for estimating ω , but rather to establish a relation between metrological bounds and characteristic dynamical times, γ_1 is only a theoretical tool, and it is treated as a free parameter.

Main results—In the following, we derive bounds on the relaxation time τ_{ss} , which is the time required to reach the steady state, and on the second-order correlation time τ_c [Eq. (7)]. First, consider the case in which, after reaching a steady state, one performs a direct (destructive) measurement in the eigenbasis of an observable O and attempts to estimate ω from its average value. We introduce $\langle G_0 \rangle_{\max} = \max_{t' \in [0, \tau_{ss}]} \langle G_0 \rangle_{t'}$, so, from Eq. (2), $\text{QFI} \leq 4\tau_{ss} \langle G_0 \rangle_{\max} / \gamma_0$. We set $\gamma_0 = \gamma, \gamma_1 = 0$ (since here no information is extracted from emitted photons), use the inequality $\text{SNR} \leq \text{QFI}$, and rearrange the terms to get

$$\tau_{ss} \geq \frac{\gamma}{4\langle G_0 \rangle_{\max}} \frac{|\partial_\omega \langle O \rangle_{ss}|^2}{\Delta_{ss}^2 O}. \quad (3)$$

Note that, for finite-dimensional systems, $\langle G_0 \rangle_{\max}$ is bounded by the operator norm $\langle G_0 \rangle_{\max} \leq \|G_0\|$ [45].

Next, consider the case in which, after obtaining the steady state, one wants to estimate ω from observing emitted photons. If we consider a very long time $t \gg \tau_{ss}$, the integral in Eq. (2) approximates $t \langle G_0 \rangle_{ss}$, so

$$\text{QFI}/t \leq \frac{4\langle G_0 \rangle_{ss}}{\gamma_0}. \quad (4)$$

To properly describe the time correlation, we switch now to the Heisenberg picture, choosing as $t = 0$ an arbitrary time after attaining the steady state; i.e., $\rho(0) = \rho_{ss}$. For any observables A, B , we have $\langle A(t)B(t') \rangle_{ss} = \text{Tr}(A(t)B(t')\rho_{ss})$, where $A(t), B(t')$ are obtained via the action of the adjoint Liouvillian superoperator [46] associated with Eq. (1). Single-time quantities like $\langle A(t) \rangle_{ss}$ do not depend on the actual value of t and are therefore denoted simply as $\langle A \rangle_{ss}$. One aims to estimate ω from the average number of photons counted in the time interval t . For this particular quantity, the SNR formula per unit time is given by (see Supplemental Material Sec. II [43])

$$\text{SNR}/t = \frac{|\partial_\omega \gamma_1 \langle G_0 \rangle_{ss}|^2}{\gamma_1 \langle G_0 \rangle_{ss} + \gamma_1^2 \langle G_0 \rangle_{ss}^2 \int_{-\infty}^{+\infty} (g^{(2)}(\tau) - 1) d\tau}, \quad (5)$$

where $g^{(2)}(\tau)$ is the normalized, normally ordered second-order correlation function in the steady state:

$$g^{(2)}(\tau) = \frac{\langle b^\dagger(0)b^\dagger(\tau)b(\tau)b(0) \rangle_{ss}}{\langle b^\dagger b \rangle_{ss}^2}, \quad (6)$$

where $b(t)$ is the output mode [47,48]. We define the second-order correlation time as

$$\tau_c := \int_{-\infty}^{+\infty} [g^{(2)}(\tau) - 1] d\tau. \quad (7)$$

This definition generalizes the standard integral correlation time of classical stationary stochastic processes [49,50] to the quantum domain, measuring the typical timescale over which photon-number fluctuations are correlated.

Using inequality $\text{SNR}/t \leq \text{QFI}/t$, the upper bound for QFI equation (4) and $\gamma_0 = \gamma - \gamma_1$ we therefore obtain

$$\frac{|\partial_\omega \langle G_0 \rangle_{ss}|^2}{\frac{\langle G_0 \rangle_{ss}}{\gamma_1} + \langle G_0 \rangle_{ss}^2 \tau_c} \leq \frac{4\langle G_0 \rangle_{ss}}{\gamma - \gamma_1}. \quad (8)$$

Rearranging the terms yields a family of bounds for τ_c :

$$\tau_c \geq \frac{1}{\langle G_0 \rangle_{ss}} \left[\frac{(\gamma - \gamma_1) |\partial_\omega \langle G_0 \rangle_{ss}|^2}{4\langle G_0 \rangle_{ss}^2} - \frac{1}{\gamma_1} \right]. \quad (9)$$

The correlation time τ_c depends only on the system dynamics Eq. (1) and not on γ_1 ; therefore, the tightest version of the bound may be obtained by maximizing right-hand side over $\gamma_1 \in [0, \gamma]$, what results in $\gamma_1 = \min\{2\langle G_0 \rangle_{ss}/|\partial_\omega \langle G_0 \rangle_{ss}|, \gamma\}$. Assuming $2\langle G_0 \rangle_{ss}/|\partial_\omega \langle G_0 \rangle_{ss}| \leq \gamma$ (which is typically satisfied at, e.g., the critical point of a quantum system undergoing a phase transition), this yields the following bound expressed solely in terms of system parameters:

$$\tau_c \geq \frac{1}{\langle G_0 \rangle_{ss}} \left[\frac{\gamma |\partial_\omega \langle G_0 \rangle_{ss}|^2}{4\langle G_0 \rangle_{ss}^2} - \frac{|\partial_\omega \langle G_0 \rangle_{ss}|}{\langle G_0 \rangle_{ss}} \right]. \quad (10)$$

Even though a suitable choice of G_1 in Eq. (1) may induce critical behavior, it enters the bound only indirectly through its effect on $\langle G_0 \rangle_{ss}$. Near criticality, the first term on the right-hand side of Eq. (10) typically dominates and becomes arbitrarily large, signaling strong bunching in the output light (recall that τ_c reflects both the bunching strength and the correlation decay time).

Although derived within a metrological context, the bounds remain valid in any framework. In particular, we can consider a situation where ω is a known, controllable parameter. The bound can then be evaluated directly from experimental data, without explicitly measuring time correlations. Notice that the decomposition $\gamma = \gamma_0 + \gamma_1$ is a purely theoretical construct used to derive the bound in Eq. (10), which depends only on γ . Therefore, no experimental tuning of γ is required, but only the knowledge of its value. Both $\langle G_0 \rangle_{ss}$ and $|\partial_\omega \langle G_0 \rangle_{ss}|$ can be easily estimated by varying ω , and therefore also the bound in Eq. (10).

The bound is particularly advantageous when $g^{(2)}(\tau)$ cannot be measured directly, e.g., due to detector dead time exceeding the relaxation time, or in the weak-coupling and

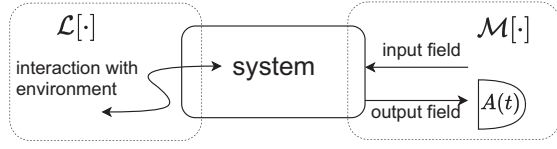


FIG. 2. Schematic representation of an open quantum system driven by an input field, with continuous measurement performed on the output field, described by Eq. (11).

limited-acquisition regime (see Supplemental Material Sec. III [43]).

Arbitrary parameter and observable of the output field—The above discussion can be generalized to an arbitrary system Hamiltonian parameter ν and arbitrary observable A . Consider the master equation

$$\frac{d\rho}{dt} = -i[H(\nu), \rho] + \mathcal{L}[\rho] + \mathcal{M}[\rho], \quad (11)$$

where $H(\nu)$ is the full Hamiltonian of the system, $\mathcal{L}[\rho] := \sum_k L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\}$ describes the interaction with an environment, and $\mathcal{M}[\rho]$ accounts for all effects related to the interaction with a controlled field (see Fig. 2).

If the operator $\partial_\nu H$ can be expressed as a linear combination of identity, Lindblad operators, and their products—i.e., $\partial_\nu H \in \text{span}\{\mathbb{1}, L_i, L_i^\dagger, L_i^\dagger L_j\}$ —then the SNR per unit time can be bounded by [31–33]

$$\text{SNR}/t \leq C[\partial_\nu H, \{L_i\}_i], \quad (12)$$

where C is a scalar constant that depends only on $\partial_\nu H$ and $\{L_i\}_i$. Then, for the time-integrated observable $O = \int_0^t A(t') dt'$ measured on the output field in the steady state, for large t , the SNR per unit time ($|\partial_\nu O|^2 / \Delta^2 O$)/ t may be expanded as

$$\frac{|\partial_\nu \langle A \rangle_{ss}|^2}{\int_{-\infty}^{+\infty} [\langle A(0)A(t) \rangle_{ss} - \langle A \rangle_{ss}^2] dt} \leq C[\partial_\nu H, \{L_i\}_i], \quad (13)$$

so if $\partial_\nu \langle A \rangle_{ss}$ and $C[\partial_\nu H, \{L_i\}_i]$ are known or measurable directly, it constitutes the lower bound for autocorrelation of $A(t)$. Further expanding Eq. (13) into an explicit bound on the correlation time requires choosing a specific model; once the model is specified, $C[\partial_\nu H, \{L_i\}_i]$ can be written explicitly. E.g., the reasoning used to derive Eq. (10) can be seen as a special case of the above with $\nu = \omega$, $A(t) = b_1^\dagger(t)b_1(t)$ (see Supplemental Material Sec. II [43]), $\mathcal{L}[\cdot] = \gamma_0 \mathcal{D}[\cdot]$ and $\mathcal{M}[\cdot] = \gamma_1 \mathcal{D}[\cdot]$. Equations (3), (10), and (13) are the main results of our Letter.

Examples—We illustrate the validity and applicability of our approach through two representative examples. Example (i) is a single-mode degenerate parametric amplifier, described by a squeezing Hamiltonian with losses [48].

This quantum optical system admits analytical solutions for both the steady state and the full dynamics, allowing a verification of our bounds against exact results. Example (ii) is the dissipative infinite-range Ising model with a transverse field, for which an analytical steady-state solution has been recently derived [41]; however, its dynamics is not analytically tractable. This case illustrates the effectiveness of our method in exploring regimes that are beyond the reach of conventional analytical techniques.

(i) Lossy cavity with parametrically induced squeezing: Consider a single-mode cavity governed by a squeezing Hamiltonian and coupled to the vacuum bath, i.e.,

$$\begin{aligned} H &= \omega a^\dagger a + \frac{\epsilon}{2} (a^{\dagger 2} + a^2), \\ \mathcal{D}[\rho] &= \rho a^\dagger a - \frac{1}{2} \{a^\dagger a, \rho\}. \end{aligned} \quad (14)$$

This Hamiltonian can be effectively realized by driving a nonlinear resonator with a pump at approximately twice the cavity frequency [51]. The required nonlinearity can be engineered using suitable Josephson-junction circuits [52–54]. Far from the critical threshold, the static nonlinearity is negligible, yielding the quadratic Hamiltonian equation (14). The detuning ω can be controlled via magnetic tuning of SQUID elements [55].

For $\epsilon < \sqrt{\omega^2 + (\gamma/2)^2}$, the system steady state is a Gaussian state with zero mean and an average number of photons given by $N_{ss} = \epsilon^2 / [2(\omega^2 + (\gamma/2)^2 - \epsilon^2)]$. As it has been recently observed in experiment [37,38], the system exhibits critical behavior when ϵ approaches the critical value $\epsilon_c = \sqrt{\omega^2 + (\gamma/2)^2}$, and in that regime we have $\partial_\omega N_{ss} \approx -(4\omega/\epsilon_c^2)N_{ss}^2$, and $\Delta^2 N_{ss} \approx 2N_{ss}^2$. Here $G_0 = a^\dagger a$ and $\langle G_0 \rangle_{ss} = \langle a^\dagger a \rangle_{ss} = N_{ss}$. Assuming $\langle G_0 \rangle_{\max} = \langle G_0 \rangle_{ss}$, which is always true for $\epsilon > \omega$ [42], from Eq. (3) with $O = G_0$ we obtain

$$\tau_{ss} \geq \frac{2\omega^2 \gamma}{\epsilon_c^4} N_{ss}, \quad (15)$$

which is consistent with the analytical solution, as the inverse of the lowest nonzero real part of the Liouvillian eigenvalue also scales linearly with N_{ss} [42].

For the correlation function, by substituting the exact formulas for $\langle G_0 \rangle_{ss}$ and $\partial_\omega \langle G_0 \rangle_{ss}$ into Eq. (10), we obtain

$$\tau_c \geq \frac{2\omega^2 \gamma}{\epsilon^2} \frac{1}{\omega^2 + (\gamma/2)^2 - \epsilon^2} - \frac{4\omega}{\epsilon^2} \approx 4 \frac{\omega^2 \gamma}{\epsilon_c^4} N_{ss}, \quad (16)$$

so bunching of the emitted photons scales linearly with the average photon number [56]. Since the model is analytically solvable, one can compute τ_c directly and obtain $\tau_c \approx (4\gamma/\epsilon_c^2)N_{ss}$ (see Supplemental Material Sec. V [43]). We observe that our bound is tight in the regime $\omega \gg \gamma$, whereas it becomes ineffective for $\omega \ll \gamma$. In intermediate

regimes, the bound provides an accurate estimate of the order of magnitude.

(ii) Dissipative transverse-field Ising model: Consider a system of N spin-1/2 particles with the Hamiltonian

$$H = \omega \sum_{i=1}^N \sigma_i^z + \sum_{i=1}^N h_i^x \sigma_i^x + \sum_{i \neq j} J_{ij} \sigma_i^z \sigma_j^z, \quad (17)$$

where the terms $h_i^x \sigma_i^x$ represent an external pump, and $J_{ij} \sigma_i^z \sigma_j^z$ accounts for interactions between spins. For now, we make no further assumption of the range of interaction. The dissipative part

$$\mathcal{D}[\rho] = \gamma \sum_{i=1}^N \left[\sigma_i^- \rho \sigma_i^{-\dagger} - \frac{1}{2} \{ \sigma_i^- \sigma_i^-, \rho \} \right] \quad (18)$$

describes spontaneous emission in the z basis, with each decay corresponding to the emission of a single photon.

To use the bounds, we first define $G_0 = \sum_{i=1}^N \sigma_i^z + (N/2)\mathbb{1}$ (the shift ensures the lowest eigenvalue of G_0 is zero). We define the normalized magnetization operator $m = G_0/N$, so that $\langle m \rangle$ ranges between $[0, 1]$. The total flux of emitted photons is $\gamma N \langle m \rangle$. Without any further assumptions, we can bound $\langle G_0 \rangle_{\max} \leq \|G_0\| = N$, so that Eq. (3) with $O = G_0$ yields for the relaxation time

$$\tau_{ss} \geq \frac{\gamma}{4N} \frac{|\partial_\omega \langle m \rangle_{ss}|^2}{\Delta_{ss}^2 m}. \quad (19)$$

From Eq. (10), we obtain for the correlation time

$$\tau_c \geq \frac{1}{N \langle m \rangle_{ss}} \left[\frac{\gamma |\partial_\omega \langle m \rangle_{ss}|^2}{4 \langle m \rangle_{ss}^2} - \frac{|\partial_\omega \langle m \rangle_{ss}|}{\langle m \rangle_{ss}} \right]. \quad (20)$$

These two formulas are valid for arbitrary values of h_i^x, J_{ij} —for finite or infinite range of interaction—with their influence encapsulated in the quantity $\partial_\omega \langle m \rangle_{ss}$.

We now focus on the special case of the infinite-range Ising model with uniform couplings $\forall_{ij} J_{ij} = J$ and transverse fields $\forall_i h_i^x = h^x$. While its steady state is known analytically [41], the full dynamics remains unsolved. This makes our constraints particularly valuable, as they allow temporal characteristics of the system to be inferred from stationary features alone. The thermodynamic limit is obtained by taking $N \rightarrow \infty$ keeping $\bar{J} := NJ$ constant, so we adopt $1/\bar{J}$ as the natural unit of time. The system exhibits critical behavior and a sharp change of magnetization for a certain value of ω , which we call $\omega_c := \arg\max_\omega |\partial_\omega \langle m \rangle_{ss}|$.

An exact expression for the magnetization m is known [41], allowing both bounds to be written analytically (although the resulting formulas are too lengthy to display). In Fig. 3, we show Eq. (20) over a broad range of ω and h^x for fixed $\gamma = 0.1[\bar{J}]$ and $N = 128$, computed using

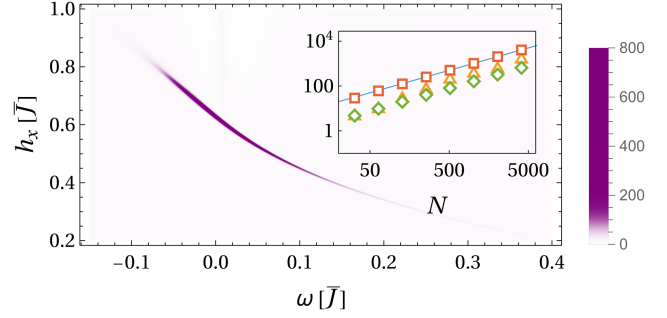


FIG. 3. Density plot of the value of the lower bound for output photon correlation τ_c for the infinite-range dissipative transverse-field Ising model [Eq. (20)] for fixed loss rate $\gamma = 0.1[\bar{J}]$ and number of spins $N = 128$. The bound takes a significantly large value on the line corresponding to the phase transition. Inset: the bounds for time correlations τ_c (yellow triangles) and relaxation time τ_{ss} (green diamonds), and value of $|\partial_\omega \langle m \rangle_{ss}|$ (red squares) as a function of N , calculated for fixed values $\gamma = 0.1[\bar{J}]$, $h^x = 0.5[\bar{J}]$, at critical point $\omega_c \approx 0.064[\bar{J}]$. The blue solid line represents the linear function $f(N) = N$ for the reference.

Eqs. (14) and (S39) of [41]. The high-value ridge in the density plot coincides with the phase transition [cf. Fig. 3(a) of [41]].

In the inset of Fig. 3, for fixed $\gamma = 0.1[\bar{J}]$ and $h^x = 0.5[\bar{J}]$, we analyze the N dependence of the bounds at the critical point $\omega_c \approx 0.064[\bar{J}]$, using the approximation formula [Eq. (16) of [41]]. The steady-state magnetization is $\langle m \rangle_{ss} \approx 0.4$ [57]. At criticality, the distribution of m becomes bimodal, so its variance approaches a finite value in the thermodynamic limit, $\Delta_{ss}^2 m \approx 0.16$. Moreover, $|\partial_\omega \langle m \rangle_{ss}| \approx N[1/\bar{J}]$, implying linear scaling of both τ_{ss} and τ_c with system size, in line with the behavior observed in the previous example.

This strong bunching contrasts with the single-spin case, where the emitted light is necessarily antibunched, since emission projects the system to its ground state [2]. Applying our bound thus reveals a fundamental dynamical property of an open quantum many-body system.

Conclusions—In this Letter, we established a connection between the correlation time of out-of-equilibrium open quantum systems and their sensitivity to parameter changes, providing lower bounds on the correlation and relaxation times. For a broad class of critical models, we show that the enhanced sensitivity of the mean excitation is intrinsically connected to long relaxation and second-order correlation times. This relation provides a powerful tool for analyzing the second-order correlation function $g^{(2)}(\tau)$ in scenarios where only steady-state properties are accessible, and the full time dynamics are analytically or numerically intractable. In fact, recent results allow one to efficiently estimate steady-state observables numerically, with certified error bounds provided by semidefinite programming methods, even when the steady state itself is not accessible [58]. Our method allows one to lower bound the correlation

time using only the average number of photons emitted within a given time interval, measured for two closely spaced values of the system parameter, which makes it particularly useful from an experimental point of view. In this sense, this Letter extends the theoretical toolbox [59–62] for characterizing quantum materials derived in the context of quantum information. The framework we developed can be extended to relate temporal autocorrelations of arbitrary observables to the system’s sensitivity to general Hamiltonian perturbations, opening new avenues for probing quantum criticality in dissipative systems. Extending these results to non-Markovian environments would be an interesting but nontrivial direction for future research [63].

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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