

Gravitational Scattering of Solitonic Boson Stars: Analytics vs Numerics

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We study the scattering of two boson stars by comparing four sequences (at fixed energy and varying impact parameter) of numerical relativity simulations to an effective-one-body analytic description, taking into account both gravitational effects (point-mass and tidal) and short-range scalar-field interactions. We obtain excellent agreement between analytical and numerical results, exhibiting the importance of short-range scalar-field interactions for small impact parameters. Our Letter paves the way toward analytic waveform templates of exotic compact object binaries.

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Introduction—The first direct detection of a gravitational wave (GW) signal, GW150914, from a black-hole (BH) binary [1], followed by $\mathcal{O}(100)$ further detections to date [2–5], has established gravitational-wave astronomy as a new pathway for exploring the physics of our Universe. New tests of general relativity have added more evidence supporting the exceptional accuracy and range of validity of Einstein’s theory [6–8]. The multimessenger observation of the neutron-star (NS) binary system GW170817 [9] cemented a strong relation to short gamma ray bursts and efficient production of heavy elements.

While all events observed so far are compatible with compact binaries composed of NSs and BHs, the mass values inferred from some events raise intriguing questions. The analyses of GW190521 and GW231123 suggest constituent BHs with masses $\gtrsim 85M_\odot$ inside or above the pair-instability-induced mass gap [2,10], while the secondary of GW190814 with a mass $2.50\text{--}2.67M_\odot$ would either be the heaviest NS or the lightest BH observed. These observations have naturally triggered investigations into the possibility of alternative, more exotic origins of these events such as, for example, cosmic strings [11],

Proca stars [12], or primordial black holes [13]. Furthermore, injections of inspiral-merger-ringdown waveforms from boson stars (BSs) into simulated Laser Interferometer Gravitational-Wave Observatory noise have revealed significant degeneracy between signals from BS and BH binaries [14]. Such exotic GW sources have attracted a lot of attention in diverse contexts, including dark matter searches [15,16], structure formation [17,18], BH mimickers [19–22], and the validity of Thorne’s hoop conjecture [23].

Clearly, the capacity of GW observations to distinguish between BHs, NSs, and exotic compact objects will be crucial for identifying the nature of as yet unclassified sources, to understand their astrophysical origin, and to search for new physics beyond the standard model of particles. Systematic investigations are naturally drawn to two main characteristics of these diverse sources: (i) tidal interactions (e.g., Ref. [24]) and (ii) manifestations in either GW emission or the electromagnetic spectrum arising from the specific matter they are composed of (cf. Ref. [25] or the electromagnetic counterparts of GW170817).

In this Letter, we consider such effects (including the additional short-range scalar-field interaction) for the case of BS binary systems. A variety of stellar configurations composed of bosonic matter has been studied in the literature, such as, for example, Proca stars [26] or ℓ BSs [27]. Here, we focus on the class of BSs composed of a single complex scalar field first identified by Kaup [28]; see also Refs. [15,29,30] for reviews. We treat BSs as electromagnetically dark, so that direct effects of their scalar matter will manifest themselves exclusively through overlap in the regime where the two BSs are in close vicinity.

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Tidal effects, on the other hand, will be present at any stage of the binary evolution, albeit with decreasing magnitude at larger distance.

In this Letter, we compute for BS binaries the effective potential central for the construction of analytic GW templates using the effective-one-body (EOB) formalism [31], which in itself uses results of various perturbative approaches: post-Newtonian, post-Minkowskian (PM), Self-Force, Effective Field Theory (EFT), and Tutti Frutti. We furthermore employ numerical results that are necessary to determine the accuracy and validity of these approximants used to construct semianalytic banks of waveform templates essential for detection and parameter estimation of GW events.

While numerical relativity (NR) simulations of binary BSs have been performed for quasicircular orbits, in this Letter we present the first simulations of binary BS scattering along with analytically deriving for the first time the effect of scalar interactions for binary BSs. We then compare the scattering angle χ determined from NR to the analytical estimates from an EOB-potential model, including point-particle results (BH), gravitational tidal effects, and scalar-field interactions. We find that the scalar effect dominates for small impact parameters while the tidal effects are subdominant; nonetheless, the inclusion of tidal effects order by order improves the agreement with NR data.

We use a “mostly plus” signature for the spacetime metric and employ units where the speed of light $c = 1$.

Theory—The action describing the dynamics of a complex scalar field φ minimally coupled to gravity is

$$S = \int \frac{\sqrt{-g}}{2} \left\{ \frac{R}{8\pi G} - [g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + V(|\varphi|)] \right\} d^4x, \quad (1)$$

where R is the Ricci scalar, g the determinant of the spacetime metric $g_{\mu\nu}$, an overbar denotes the complex conjugate, and $V(|\varphi|)$ is the scalar-field potential. In this Letter, we consider solitonic potentials characterized by the scalar mass μ (with dimension of inverse length) and a parameter σ_0 ,

$$V(|\varphi|) = \mu^2 |\varphi|^2 (1 - 2|\varphi|^2/\sigma_0^2)^2. \quad (2)$$

The coupled Einstein-Klein-Gordon equations governing the BS solution are obtained by varying the action (1). Spherically symmetric solutions are obtained with the harmonic ansatz [32] $\varphi(t, r) = A(r)e^{i(\omega t + \Phi)}$, where $A(r)$ is the (real-valued) scalar-field amplitude profile, ω is the frequency, and Φ is an initial phase. We have also introduced a parameter $\epsilon = \pm 1$ determining the sense of rotation of the scalar field in the complex plane. By convention, $\epsilon = 1$ defines a BS while $\epsilon = -1$ is an “antiboson star” ($\overline{\text{BS}}$). Single BS and $\overline{\text{BS}}$ spacetimes are identical except for opposite Noether charge densities.

Our set of binary BS solutions depends on two dimensionful parameters (G and μ), and on two dimensionless parameters, which can be chosen to be $\sqrt{G}\sigma_0$ and $\sqrt{G}A_{\text{ctr}}$ (A_{ctr} denotes the central scalar-field amplitude of an isolated BS), which determine the frequency ω and compactness of ground-state solutions. We choose $\sqrt{G}\sigma_0 = 0.2$ and $\sqrt{G}A_{\text{ctr}} = 0.17$. These choices *uniquely* determine (modulo rescaling of mass and length units) our ground-state BS (or $\overline{\text{BS}}$) solutions, with frequency $\omega = 0.43916\mu$, mass $m = 0.713 G^{-1}\mu^{-1}$, and contain 99% of their total mass m within a radius $r_{99} = 5.58 Gm = 3.98 \mu^{-1}$. Our highly compact (intermediate between NSs and BHs) objects enable encounters at small impact parameters b where gravitational effects are strong and scalar interactions matter.

Numerical relativity simulations—We simulate our BS binaries numerically using the Lean code [33], which is based on the Cactus computational toolkit [34]. Lean discretizes the Einstein-Klein-Gordon equations in the conformal and covariant Z4 formulation [35] using fourth-order finite differencing. The code employs the moving puncture gauge and utilizes mesh refinement via CARPET [36]. Our computational domain extends to $448 GM$, where $M := m_1 + m_2 = 2m$ is the total BS rest mass, using eight nested refinement levels with standard resolution $\Delta x = 0.02 GM$ on the innermost level. We employ outgoing Sommerfeld boundary conditions.

We study the scattering of two equal-mass ($\nu := m_1 m_2 / M^2 = 1/4$), nonspinning BSs with scalar frequency $\omega = \omega_1 = \omega_2$. We investigate four distinct binary systems composed of a primary BS with fixed parameters ($\epsilon = 1, \Phi = 0$), and a secondary with parameters varying as follows: (i) ($\epsilon = 1, \Phi = 0$), denoted BSBS; (ii) ($\epsilon = 1, \Phi = \pi/2$), denoted BSBS $^{(\pi/2)}$; (iii) ($\epsilon = 1, \Phi = \pi$), denoted BSBS $^\pi$; and (iv) ($\epsilon = -1, \Phi = 0$), denoted BS $\overline{\text{BS}}$. The BSs start on the x axis with initial positions $\pm X$ and initial velocity parameters $\vec{v} = (\mp v_x, \pm v_y, 0)$, with corresponding kinematic momenta $\vec{p} := m\vec{v}/\sqrt{1 - \vec{v}^2}$. Unless specified otherwise, we set $|\vec{v}| = 0.2291$ and $X = 50.49 GM$. Defining the impact parameter $b := 2Xv_y/|\vec{v}| = 2Xp_y/|\vec{p}|$, the velocity vector of the two BSs is given by $(v_x, v_y) = \pm |\vec{v}|(-\sqrt{1 - [b/(2X)]^2}, b/(2X))$.

Initial data is constructed using the improved superposition of two boosted single-BS spacetimes as detailed in Ref. [37]. The constraint violations inherent to these initial data are reduced during the subsequent evolution by employing the conformal and covariant Z4 formulation with constraint damping parameter $\kappa_1 = 0.1/M$ following Ref. [35]. However, the constraint violations present in our initial data limit our ability to accurately compute the initial values of E and J from their standard integral expressions. We approximately remedied this issue in the following way.

The initial energy E is estimated as the sum of the relativistic (rest + kinetic) energy and the Newtonian binding energy, including a parameter c_V for relativistic corrections,

TABLE I. Impact parameter b , total angular momentum J_{NR} , and the scattering angles (together with their error bars) for the four BS binary sequences as extracted from the numerical simulations. For all configurations, the total energy is $E/M = 1.02394$.

(b/GM)	(J_{NR}/GM^2)	$\chi_{\text{NR}}^{\text{BSBS}} [^\circ]$	$\chi_{\text{NR}}^{\text{BSBS}} [^\circ]$	$\chi_{\text{NR}}^{\text{BSBS}(\pi/2)} [^\circ]$	$\chi_{\text{NR}}^{\text{BSBS}^\pi} [^\circ]$
9.9	1.15315	258.79(95)	216.41(1.41)	214.18(1.40)	198.41(1.26)
10.5	1.22360	170.51(89)	165.42(81)	165.28(81)	161.23(75)
11.0	1.28186	143.91(62)	142.32(60)	142.28(60)	140.83(58)
12.0	1.39839	114.43(64)	114.20(63)	114.33(53)	113.96(65)
13.0	1.51492	96.89(84)	96.89(84)	96.75(60)	96.81(82)
14.1	1.64982	83.39(1.14)	83.38(1.14)	83.32(1.10)	83.32(1.11)
15.0	1.74798	75.51(1.23)	75.73(1.37)	75.62(1.30)	75.62(1.30)
16.0	1.86450	68.16(1.27)	68.44(1.44)	67.97(1.14)	68.50(1.47)

$$E = 2 \frac{m}{\sqrt{1-v^2}} - c_v \frac{Gm^2}{2X}. \quad (3)$$

As discussed in Supplemental Material [38], the approximate analytical expression (3) with $c_v = 1.37$,

$$\frac{E}{M} = 1.02394 \quad \text{with} \quad \gamma_{\text{in}} = 1.09690, \quad (4)$$

defines a sufficiently accurate value of the initial energy; we adopt it in this Letter.

Starting from the formal initial-data value of J_{NR} (listed in Table I) obtained by extrapolating the Arnowitt-Deser-Misner like integral (7.63) of Ref. [40] to infinite extraction radius, and in absence of an analytic expression for J analogous to Eq. (3), we improved the estimate of the initial angular momenta by allowing for a rescaling $J = (1 + \epsilon_J)J_{\text{NR}}$, with an *a priori* unknown parameter ϵ_J expected to be $\sim (Gm/2X) = \mathcal{O}(10^{-2})$. As discussed in Supplemental Material [38] we determined the value $\epsilon_J \approx 0.006$ by using the subset of five BSBS simulations with $b/(GM) \leq 13.0$.

We adopt a method similar to that of Refs. [41,42] to determine the scattering angle. After converting the BS trajectories into standard polar coordinates (r, θ) and fitting the dependence of the angles on $1/r$ by polynomials $\theta_p := \sum_{n=0}^p \theta_{p,n}/r^n$ (with $\theta_{p,n}$ denoting, for each choice of maximal order p , the coefficient of $1/r^n$), the angles are obtained (for each resolution) from the following linear average of leading coefficients $n=0$: $\bar{\theta} = (\theta_{2,0} + \theta_{3,0} + \theta_{4,0} + \theta_{5,0})/4$. As a conservative estimate of the error due to polynomial fitting, we use half the range of $\theta_{p,0}$ with $p = 1, \dots, 6$. A convergence analysis using additional resolutions $\Delta x = 0.022$ and $0.017 GM$ and employing a fifth-order polynomial fit of the trajectory for the binary configuration $b = 9.9 GM$ yields second-order convergence with a discretization error $\sim 0.5^\circ$. The combined errors $\Delta\chi_{\text{NR}}$ obtained by adding in quadrature the discretization error to the polynomial fitting error are displayed in Table I. They range between 0.53° to 1.47° .

Analytical relativity (AR) computation of χ —Recently, there has been a significant effort on comparing analytic results to NR simulations of BH scattering [42–47]. Following Ref. [43], we analytically describe the scattering of BSs by transcribing the PM-expanded scattering angle $\chi(\gamma, j) = \sum_n 2\chi_n(\gamma)/j^n$ [where $j := J/(Gm_1m_2)$] into a corresponding energy-dependent EOB radial potential $w(\gamma, \bar{r}) = \sum_n w_n(\gamma)/\bar{r}^n$ (where $\bar{r} := \bar{R}/(GM)$ is a dimensionless isotropic EOB radial coordinate). Here, and in the following, γ denotes the EOB effective energy of the system, which is related to the total center-of-mass energy $E := Mh$ via $h = \sqrt{1 + 2\nu(\gamma - 1)}$ or, equivalently (for the equal-mass case $\nu = 1/4$), $\gamma = 2(E/M)^2 - 1$. (For scattering states, γ happens to be equal to the Lorentz factor between the two incoming world lines.) The dynamics of BS binaries involve two additional effects compared to those of BHs: tidal effects and scalar-field interactions, so that the PM-expansion of χ is naturally decomposed as

$$\chi(\gamma, j) = \chi^{\text{BH}}(\gamma, j) + \chi^{\text{tid}}(\gamma, j) + \chi^{\text{scalar}}(\gamma, j), \quad (5)$$

leading to a corresponding decomposition of the radial potential,

$$w(\gamma, \bar{r}) = w^{\text{BH}}(\gamma, \bar{r}) + w^{\text{tid}}(\gamma, \bar{r}) + w^{\text{scalar}}(\gamma, \bar{r}). \quad (6)$$

The PM expansion of w is uniquely determined by the PM expansion of χ via the EOB-derived map [48]

$$\pi + \chi(\gamma, j) = 2j \int_0^{\bar{u}_{\text{max}}(\gamma, j)} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w(\bar{u}, \gamma) - j^2 \bar{u}^2}}, \quad (7)$$

where $\bar{u} = 1/\bar{r}$, $\bar{u}_{\text{max}} \equiv 1/\bar{r}_{\text{min}}$, and $p_\infty^2 = \gamma^2 - 1$. Here, r_{min} is the radial turning point, defined as the largest root of the (total) radial potential, $p_r^2 = p_\infty^2 + w(1/\bar{r}, \gamma) - j^2/\bar{r}^2$.

Based on the 4PM knowledge of χ^{BH} [49], w^{BH} is fully known analytically up to 4PM order [43], and has been completed by a numerically fitted 5PM contribution, $w_5(\gamma)^{\text{BH,NR}}$ [44]—recall that w^{BH} is an effective potential that includes radiation-reaction effects [50]. The gravitational tidal potential w_n^{tid} starts at 6PM, and is computed

from the scattering angles derived in Refs. [49,52,53] using the following consequences of Eq. (7):

$$\chi_6^{\text{tid}} = \frac{15\pi}{32} p_\infty^4 w_6^{\text{tid}}, \quad \chi_7^{\text{tid}} = 4p_\infty^3 w_1 w_6^{\text{tid}} + \frac{8}{5} p_\infty^5 w_7^{\text{tid}}. \quad (8)$$

The last contribution to the potential, w^{scalar} , which describes the short-range scalar interaction between two BSs, has not been previously derived, nor its corresponding scattering-angle contribution part, χ^{scalar} . We have computed the leading-order (LO) PM analytical value of w^{scalar} describing the conservative part of the scattering for BS binaries, namely

$$w_{\text{LO}}^{\text{scalar}} = \frac{8\pi c_1 c_2}{G m_1 m_2} \frac{e^{-\bar{m} \bar{r}}}{\bar{r}} \cos(\Phi_{21}). \quad (9)$$

Here, c_1 and c_2 are real scalar charges (defined in the next section), $\bar{m} \equiv G M h \tilde{\mu}$ [with $\tilde{\mu}^2 \equiv \mu^2 - (2\omega_1 \omega_2 \gamma - \omega_1^2 - \omega_2^2)/(\gamma^2 - 1)$] is an inverse range, and $\Phi_{21} := \Phi_2 - \Phi_1$ is the dephasing between the two interacting BSs' scalar fields. This corresponds to an attractive potential for BSBS ($\Phi_{21} = 0$), a repulsive one for BSBS $^\pi$ ($\Phi_{21} = \pi$), and a vanishing one for BSBS $^{(\pi/2)}$ ($\Phi_{21} = \pi/2$). For the BSBS case we obtain a fast-oscillating potential that averages to zero. These results confirm in a quantitative way the qualitative considerations of Appendix B of Ref. [54].

We sketch the derivation of our results as follows. Our starting point is an EFT action describing the two BSs (labeled by $A = 1, 2$) as worldlines $\mathbf{z}_A(\tau_A)$ endowed with time-dependent complex sources $s_A(\tau_A)$,

$$S_{\text{EFT}} = \int \frac{\sqrt{-g}}{2} \left\{ \frac{R(g)}{8\pi G} - [g^{\mu\nu} \nabla_\mu \bar{\varphi} \nabla_\nu \varphi + \mu^2 \bar{\varphi} \varphi] \right\} d^4 x \\ - \sum_A \int \left\{ m_A - 2\pi \left[\varphi(\mathbf{z}_A) \bar{s}_A(\tau_A) + \bar{\varphi}(\mathbf{z}_A) s_A(\tau_A) \right] \right\} d\tau_A \\ + S_{\text{GF}}(g) + S_{\text{grav tidal}}. \quad (10)$$

Here, the gauge-fixing (GF) term $S_{\text{GF}}(g)$ fixes the harmonic gauge, τ_A is the proper time, and $S_{\text{grav tidal}}$ describes worldline gravitational tidal interactions [49,52,53]. We keep only the quadratic term $\mu^2 \bar{\varphi} \varphi$ in the scalar potential because the higher-order terms $\sim \varphi^4 + \varphi^6$ lead to exponentially faster decaying interactions. The complex sources $s_A(\tau_A)$ describing BSs are

$$s_A(\tau_A) = c_A e^{i(\epsilon_A \omega_A \tau_A + \Phi_A)}, \quad (11)$$

where each c_A is a constant (real) “scalar charge.” This harmonically varying complex scalar source arises from the time-dependent profile $\varphi(t, r) = A(r) e^{i(\epsilon \omega t + \Phi)}$ of an isolated BS in its rest frame. The nonlocal character of s_A (via

$\tau_A = \int^{z_A} \sqrt{-g_{\mu\nu}(z'_A) dz'_A{}^\mu dz'_A{}^\nu}$) can be described by using a Lagrange multiplier. The harmonically varying nature of the scalar sources $s_A(\tau_A)$ is a novel feature compared to existing EFT descriptions of pointlike objects interacting in scalar-tensor theories of gravity.

The perturbative analysis from this EFT leads to Feynman-type diagrams computing the impulse $\Delta p_A^\nu := p_{A\text{out}}^\nu - p_{A\text{in}}^\nu$ of each body under the exchange of the scalar field together with gravity (e.g., Ref. [49]). The LO impulse of body one is given by $\Delta p_1^\nu = \eta^{\mu\nu} \int d\tau_1 (\partial \mathcal{L}_1 / \partial z_1^\mu)$, where the effective Lagrangian corresponding to Fig. 1(a), describing scalar exchange between s_1 and s_2 , is

$$\mathcal{L}_1 = \frac{1}{\pi^2} \int d^4 k \int d\tau_2 (s_1 \bar{s}_2 + \bar{s}_1 s_2) \frac{e^{ik \cdot (z_1 - z_2)}}{k^2 + \mu^2}. \quad (12)$$

Inserting the scalar sources given in Eq. (11) and computing the integrals over k , τ_2 , and τ_1 yields the following LO impulse of body one:

$$\Delta p_{1\text{LO}}^{\nu \text{ scalar}} = -\frac{8\pi \tilde{\mu} c_1 c_2}{\sqrt{\gamma^2 - 1}} K_1(\tilde{\mu} |b|) \frac{b^\nu}{|b|} \cos(\Phi_{21}) \\ - \frac{8\pi c_1 c_2}{\sqrt{\gamma^2 - 1}} (\omega_1 \gamma - \omega_2) \tilde{y}_2^\nu \sin(\Phi_{21}) K_0(\tilde{\mu} |b|). \quad (13)$$

Here, $\tilde{y}_2^\nu = (u_2^\nu - u_1^\nu \gamma)/(\gamma^2 - 1)$, $b^\nu = b_1^\nu - b_2^\nu$ is the impact parameter, K_1 , K_0 are modified Bessel's functions, and $\tilde{\mu}$ is as above. The first (transverse) term on the right-hand side of (13) corresponds [using $j = p_\infty |b|/(GMh)$ and $\chi = \sqrt{\Delta p^2}/\sqrt{\gamma^2 - 1}$] to the scattering angle $\chi_{\text{LO}}^{\text{scalar}}$,

$$\frac{\chi_{\text{LO}}^{\text{scalar}}(j)}{h} = \frac{8\pi M \tilde{\mu}}{m_1 m_2} \frac{c_1 c_2}{\gamma^2 - 1} K_1(\tilde{\mu} |b|) \cos(\Phi_{21}). \quad (14)$$

The second (longitudinal) term on the right-hand side of (13) predicts that, due to (exponentially suppressed) scalar emission, the scattering is not conservative when $\Phi_2 \neq \Phi_1$. This nonconservative nature of the scattering can be analytically described by completing the conservative potential w^{scalar} by a dissipative force $\mathcal{F}_{\mu A}^{\text{scalar}}$ acting on each body. We checked on NR data, however, that the nonconservative nature of the scalar interaction introduces a negligible difference (for the cases we consider) between the scattering angle of particle 1, χ_1^{NR} , and that of particle 2, χ_2^{NR} , so that we can define $\chi^{\text{NR}} := \frac{1}{2}(\chi_1^{\text{NR}} + \chi_2^{\text{NR}})$. We henceforth discuss the construction of a scalar contribution, $w^{\text{scalar}}(\bar{r})$, in the radial potential (6) leading to χ^{NR} . Expanding Eq. (7) to linear order in w implies that the scalar contribution to χ is related to the scalar contribution to w via the following linear transformation:

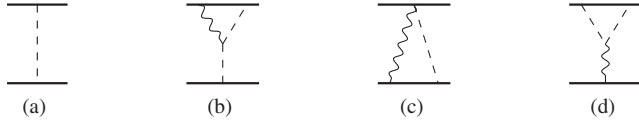


FIG. 1. Feynman diagrams needed for the computation of the scalar-field effective action up to $\mathcal{O}(Gc_Ac_B)$. The dashed (wavy) line represents the scalar (graviton) propagator. Diagram (a) represents the leading-order term in the scattering angle and diagrams (b)-(d) higher-order contributions as detailed in the text.

$$\chi^{\text{scalar}}(j) = -\frac{\partial}{\partial j} \int_{\bar{r}_{\min}}^{\infty} d\bar{r} \frac{w^{\text{scalar}}(\bar{r})}{\sqrt{p_\infty^2 - j^2/\bar{r}^2}}. \quad (15)$$

One readily checks that inserting in Eq. (15) the Yukawa-type potential $w_{\text{LO}}^{\text{scalar}}$, (9), gives rise to $\chi_{\text{LO}}^{\text{scalar}}$ (14).

To go beyond LO in the scattering angle, one needs to include both higher-order diagrams and iterated effects of the perturbed worldlines. The higher-order diagrams describing $\mathcal{O}(Gc_Ac_B)$ corrections to Fig. 1(a) are Figs. 1(b)–1(d) and their $A \leftrightarrow B$ counterparts. Figure 1(d), which is $\propto \mathcal{O}(Gc_Ac_A)$, decays twice faster with increasing impact parameter and can be neglected in our AR-NR comparison. As discussed in Supplemental Material [38], the leading fractional modifications of Eq. (14) are found to come from Figs. 1(b) and 1(c). We focus on the equal-mass case, implying $c_1 = c_2$, $m_1 = m_2$ and $\omega_1 = \omega_2 = \omega$. The fractional modification due to Fig. 1(c) is $\propto Gm_A/b$. By contrast, the fractional modification due to Fig. 1(b) is logarithmic and $\propto Gm_A[(2\omega^2 - \mu^2)/\bar{\mu}] \log(2e^{\gamma_E} \bar{\mu} b)$.

Motivated by the known *exponentiated* structure of the exact isolated, gravitationally dressed BS solution [see Eq. (19) below], we shall assume that the corresponding higher-order contribution to w can be approximated (when neglecting power law corrections $\propto GM/r$) by the factorized form

$$w_{\text{HO}}^{\text{scalar}} = w_{\text{LO}}^{\text{scalar}} w_{\text{g-dressing}}^{\text{scalar}}, \quad (16)$$

with the following exponentiated gravitational dressing factor:

$$w_{\text{g-dressing}}^{\text{scalar}} = e^{GM[(2\omega^2 - \mu^2)/\bar{\mu}] \log(2e^{\gamma_E} G\bar{\mu} \bar{r})}. \quad (17)$$

The length scale entering the logarithm agrees with the effective potential computations of [55].

We use the integral (7), with $w = w^{\text{BH}} + w^{\text{tid}} + w_{\text{HO}}^{\text{scalar}}$, to define the analytic prediction for the scattering angle.

Completing the AR description—The AR-predicted angle (5) depends on the knowledge of the following quantities: E (or equivalently $\gamma = 2(E/M)^2 - 1$), J , w_5^{BH} , w_6^{tid} , w_7^{tid} , the scalar-charge parameters $c_1 (= c_2)$, and the phase difference Φ_{21} . The determination of sufficiently accurate NR values of E and J was already discussed above (with details in Supplemental Material [38]). We now discuss in turn the derivation of w_5^{BH} , w_6^{tid} , w_7^{tid} , and $c_1 (= c_2)$.

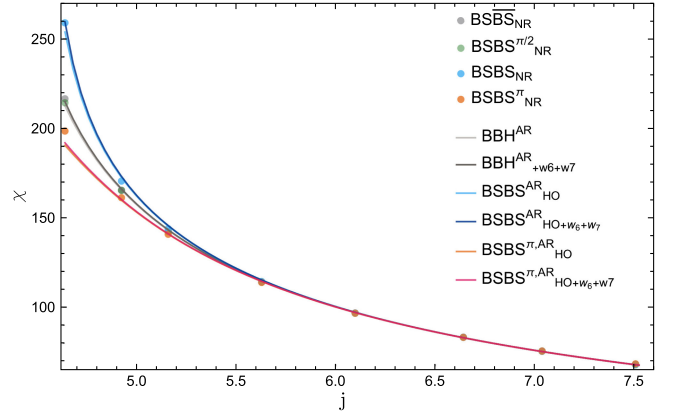


FIG. 2. Scattering angle comparison between the NR data (filled circles) and the EOB-resummed, PM-based analytical prediction (AR, solid lines) for equal-mass, nonspinning boson-star configurations for various rescaled initial angular momenta [$j = J/(Gm_1m_2)$].

Interpolating between the results of Refs. [41,45], we have determined the effective 5PM contribution to the BH potential $w^{\text{BH}}(\bar{r}, \gamma)$ corresponding to our initial energy (4),

$$w_5^{\text{BH}} = -1.03 \pm 0.03. \quad (18)$$

Next, we use the results of Ref. [56] to estimate the dimensionless even-parity quadrupolar tidal parameter $\Lambda_A := \lambda_A^{\text{tidal}}/(G^4 m_A^5)$, where λ_A^{tidal} denotes the tidal deformability of our solitonic BSs, and obtain $\Lambda_A \approx 7$ for each model. In the absence of corresponding results for the odd-parity quadrupolar and the even-parity octupolar tidal parameters, we insert $\Lambda_A \approx 7$ in Refs. [49,52,53] to determine χ_6^{tid} and χ_7^{tid} and then use Eq. (8) to obtain the corresponding tidal coefficients w_6^{tid} and w_7^{tid} entering the EOB w potential.

The scalar charges c_A are determined by matching the scalar profile numerically computed for isolated BSs to the (gravitationally dressed) EFT-predicted analytic expression of the asymptotic scalar profile generated by an isolated BS in harmonic coordinates, namely (recalling $\bar{\mu}_A := \sqrt{\mu^2 - \omega_A^2}$),

$$A(r) \approx \frac{c_A}{r} \exp \left[-\bar{\mu}_A r + Gm_A \frac{2\omega_A^2 - \mu^2}{\bar{\mu}_A} \log(2e^{\gamma_E} \bar{\mu}_A r) \right]. \quad (19)$$

The expression (19) is the massive ($\mu_A \neq 0$) generalization of the well-known massless wave asymptotics $\propto r^{-1} \exp\{\pm i\omega_A[r + 2Gm_A \log(r/s)]\}$, with a length scale s fixed by the EFT action. For our models with $G\mu m_A = 0.713$ and $\omega_A = 0.439\mu$ we obtain $\sqrt{G\mu} c_A = 1.608$.

Comparison between AR and NR results—We now compare our AR predictions χ_{AR} , defined by Eq. (7), with

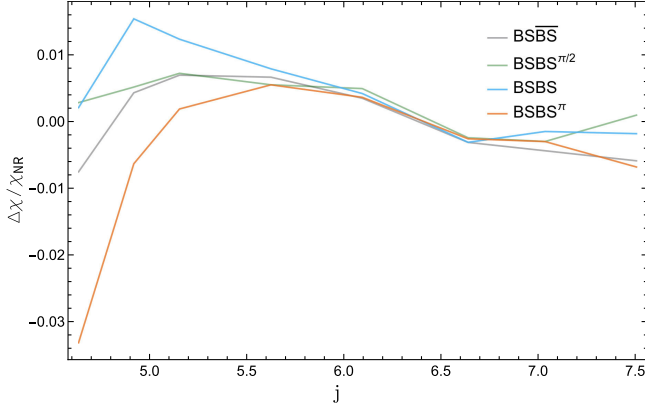


FIG. 3. Fractional differences $\Delta\chi/\chi_{\text{NR}} = (\chi_{\text{AR}} - \chi_{\text{NR}})/\chi_{\text{NR}}$ of analytical results (including both the tidal and scalar interactions) with respect to the numerical results.

our full set of 32 NR scattering data χ_{NR} (see Table I) composed of four types of BS binaries, each simulated for eight values of J (or, equivalently, b).

Figure 2 presents a global view of the comparison between our AR predictions (solid lines) and the corresponding NR data (circles). For all four sequences, the agreement is visually excellent for most impact parameters, and exhibits a small worsening only for the smallest impact parameters (leading to very large scattering angles, $\chi > 200^\circ$). Our AR predictions are presented both in their full version (using $w = w^{\text{BH}} + w^{\text{tid}} + w^{\text{scalar}}$) and a version using $w = w^{\text{BH}} + w^{\text{scalar}}$.

We quantify the (percent level) AR-NR agreement by displaying in Fig. 3 the fractional differences $(\chi_{\text{AR}} - \chi_{\text{NR}})/\chi_{\text{NR}}$. Further details about the NR-AR comparison are given in Table II, which explores the effect of various versions of the radial potential such as including only LO scalar interactions and/or various tidal terms. There, we define the quantity $\delta_\chi := (\chi_{\text{AR}} - \chi_{\text{NR}})/\Delta\chi_{\text{NR}}$ that measures the deviation of AR from NR values in units of the NR error estimate $\Delta\chi_{\text{NR}}$. Note that δ_χ is of order unity for our most accurate AR models.

The use of an analytic description based on $w^{\text{EOB-}}$ resummed angles is crucial to reach this level of agreement; PM-expanded angles would exhibit up to an order of magnitude larger differences from NR data for nearly all impact parameters. The best agreement is obtained for the BSBS sequence (black lines in Fig. 2) which, as expected, is very close to BH-binary scattering; that agreement was obtained by fitting only one parameter, namely ϵ_J . As indicated, tidal effects are small even for small impact parameters, $b = 9.90M, 10.5M$. The agreement for the BSBS($\pi/2$) sequence (green symbols in Fig. 2) is nearly as good, and also close to the BBH case. Its slightly larger disagreement for small b (leftmost data points in Fig. 3) likely arises because the vanishing of the w potential for BSBS($\pi/2$) is only valid at LO, i.e., only for the diagram Fig. 1(a). Excellent agreement with NR data is also obtained for the other two configurations: (i) BSBS (blue), and (ii) BSBS $^\pi$ (red). These cases also clearly demonstrate the attractive (repulsive) character of the scalar-mediated potential w^{scalar} , when $\Phi_{21} = 0$ ($\Phi_{21} = \pi$). In the attractive BSBS case, the two stars get very close to each other, $\bar{r}_{\text{min}} \approx 6.8$, and $w_{\text{HO}}^{\text{scalar}}$ [Eq. (16)] becomes quite significant; cf. Table II.

Our results also display the subdominant effect due to electric-type, quadrupolar tidal interactions. The inclusion, order by order, of the tidal potentials $w_6^{\text{tid}}, w_7^{\text{tid}}$, while nearly negligible for medium and large b , improves the agreement with NR data for all configurations with small impact parameters [38].

Conclusions—We have performed the first numerical computation of the scattering angle χ of nonspinning, equal-mass BS binary encounters with scattering angles between 68.2° and 259° . Our key results include (i) the derivation of the analytical LO scalar-exchange contribution to the scattering based on the EFT action (10) involving harmonically varying scalar sources, (ii) inclusion of the gravitational-dressing effects to complete the LO scalar interactions with HO effects, and (iii) determination of the effective scalar charges c_A by matching to NR computations of isolated BSs to EFT predictions.

TABLE II. Summary of the AR scattering angle for various EOB predictions with impact parameter b . Here, $\delta_\chi = (\chi_{\text{AR}} - \chi_{\text{NR}})/\Delta\chi_{\text{NR}}$ measures the deviation of AR from NR values in units of the NR error estimate.

(b/GM)	$\chi_{\text{AR}}^{\text{BBH}} [^\circ]$	$\delta_\chi^{\text{BSBS}}$	$\delta_\chi^{\pi/2}$	$\chi_{\text{LO,AR}}^{\text{scalar}} [^\circ]$				$\chi_{\text{HO,AR}}^{\text{scalar}} [^\circ]$				$\chi_{\text{HO,w}_6,\text{AR}}^{\text{scalar}} [^\circ]$				$\chi_{\text{HO,w}_6+w_7,\text{AR}}^{\text{scalar}} [^\circ]$			
				BSBS	δ_χ	BSBS $^\pi$	δ_χ^π	BSBS	δ_χ	BSBS $^\pi$	δ_χ^π	BSBS	δ_χ	BSBS $^\pi$	δ_χ^π	BSBS	δ_χ	BSBS $^\pi$	δ_χ^π
9.9	212.44	-2.81	-1.24	...	...	110.41	-69.84	253.68	-5.38	190.44	-6.33	256.24	-2.69	191.12	-5.79	259.32	0.56	191.89	-5.20
10.5	165.62	0.25	0.42	...	...	115.45	-61.04	172.51	2.25	159.78	-1.94	172.83	2.61	160.01	-1.63	173.11	2.93	160.21	-1.36
11.0	143.10	1.30	1.36	371.98	367.86	115.60	-43.50	145.47	2.50	140.90	0.12	145.58	2.69	141.00	0.30	145.67	2.84	141.09	0.45
12.0	114.90	0.12	1.08	124.70	16.06	107.50	-9.93	115.27	1.32	114.53	0.90	115.31	1.37	114.56	0.93	115.32	1.40	114.58	0.96
13.0	97.21	0.38	0.76	99.16	2.70	95.39	-1.73	97.28	0.46	97.14	0.40	97.29	0.47	97.15	0.42	97.30	0.48	97.15	0.42
14.1	83.12	-0.23	-0.19	83.46	0.07	82.76	-0.50	83.12	-0.23	83.10	-0.20	83.12	-0.23	83.10	-0.20	83.13	-0.23	83.10	-0.20
15.0	75.39	-0.24	-0.18	75.50	-0.01	75.28	-0.27	75.39	-0.10	75.39	-0.18	75.40	-0.10	75.39	-0.18	75.40	-0.10	75.39	-0.18
16.0	68.03	-0.28	-0.05	68.06	-0.08	68.01	-0.34	68.03	-0.10	68.03	-0.32	68.03	-0.10	68.03	-0.32	68.04	-0.10	68.03	-0.32

The comparison of the numerical and analytical results for BSs shows (i) the dominant effect due to scalar interactions, and (ii) the subdominant but real effect due to tidal interactions when the two stars get close. This indicates that the nature of the constituent matter of *exotic* compact objects can contribute more significantly to the dynamics than the tidal interactions.

Our simulations also revealed that for small impact parameters both BSs collapse into BHs after their close encounter and before separating to infinity. This phenomenon was not observed in quasicircular BS binaries with the same compactness, highlighting the potentially important role of time-dependent scalar perturbations in destabilizing BSs and triggering collapse. This opens a novel direction for probing smoking-gun signatures of exotic compact objects.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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