

Thermal Origin of the Attractor-to-General-Relativity in Scalar-Tensor Gravity

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The convergence of scalar-tensor gravity to general relativity, or the departure from it, is described in a new analogy with heat dissipation in a viscous fluid. This new thermal picture is applied to cosmology, shedding light on whether gravity deviates from general relativity early on and approaches it later in the cosmic history.

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Introduction—General relativity (GR) is quite successful but has its problems. First, it predicts spacetime singularities in cosmology and inside black holes, where physical quantities diverge, i.e., it predicts its own failure. It is believed that quantum mechanics, at or below the Planck energy scale, will cure these singularities. However, no quantum gravity model is satisfactory, and there is virtually no experiment to guide theory development. The lowest-order quantum corrections to GR inevitably modify it by introducing extra degrees of freedom or higher order field equations. Therefore, from the point of view of theoretical physics, GR cannot be the ultimate theory of gravity. Second, the standard cosmological model, the GR-based Λ cold dark matter model described by Friedmann-Lemaître-Robertson-Walker (FLRW) geometry, is plagued by various tensions. Moreover, in order to explain the present acceleration of the cosmic expansion discovered in 1998 with Type Ia supernovae, one must introduce an *ad hoc* dark energy of mysterious origin akin to a fudge factor. Hence, researchers have turned to the alternative of modifying gravity at the largest (cosmological) scales while giving up dark energy entirely. The most popular alternative to GR is $f(R)$ gravity (where R is the Ricci scalar curvature and f is a nonlinear function). A proof of principle was given that $f(R)$ gravity can explain the cosmic acceleration without dark energy (see Refs. [1–3] for reviews).

The simplest class of theories extending GR is scalar-tensor (ST) gravity, which only adds an extra scalar propagating degree of freedom ϕ to the GR degrees of

freedom (the two massless, spin-2 modes appearing as gravitational wave polarizations). This extra scalar field ϕ corresponds, roughly speaking, to the inverse of the effective gravitational coupling strength $G_{\text{eff}} \simeq \phi^{-1}$. That is, in ST gravity, Newton's constant G becomes a dynamical field. This was the original Brans-Dicke proposal [4], later extended to more general theories [5–8]. Notably, $f(R)$ gravity is a subclass of ST gravity. The quest for the most general ST theory with field equations of order not higher than 2 has led to the rediscovery of Horndeski gravity [9] and its degenerate higher-order scalar-tensor generalization, which are the subject of a large literature.

Solar System tests constrain severely ST gravity, and there are also stringent constraints from Big Bang nucleosynthesis. It was proposed long ago by Damour and Nordvedt that ST gravity was quite different from GR early in the history of the universe and that an *attractor-to-GR mechanism* brought it to GR during the radiation era, by making the scalar ϕ constant through an effective damping in its equation of motion [10,11]. This idea has generated a large literature with mixed conclusions about the presence and effectiveness of this attractor mechanism in the radiation, matter, or inflationary eras. The emerging picture is more involved than originally envisaged: the attractor-to-GR mechanism competes with a *repellor mechanism* [12], and it is clear that whether the final universe is a GR or a ST one depends heavily on the initial conditions, the scalar field potential $V(\phi)$ [13], and its matter content. The analysis of the approach of ST gravity to GR is complicated by the nonlinearity of the field equations. Unfortunately, one must make assumptions on the solutions themselves to draw conclusions on the dynamics of ST gravity. Essentially, no new result has been obtained for several years.

We revisit this subject in the light of a new “thermal” view consisting of an analogy between ST gravity and a dissipative fluid. The approach of ST gravity to GR is analogous to the relaxation of a viscous fluid to its “zero temperature” equilibrium state corresponding to GR. Configurations in which the extra degree of freedom ϕ

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is excited correspond to positive temperature states. This new formalism introduces a notion of “temperature of gravity” and an explicit equation describing the approach to the GR equilibrium or its departure from it, which makes it ideally suited to address the issue of the attractor-to-GR mechanism in cosmology. *This formalism, however, is not restricted to cosmology*, and we first discuss it in full generality.

Consider first-generation ST gravity [14] described by the Jordan frame action,

$$S_{\text{ST}} = \int \frac{d^4x}{16\pi} \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} \nabla^c \phi \nabla_c \phi - V \right] + S^{(\text{m})}, \quad (1)$$

where g is the determinant of the spacetime metric g_{ab} with Ricci scalar R , the “Brans-Dicke coupling” $\omega > -3/2$ to avoid ϕ being a phantom field, and $S^{(\text{m})}$ as the matter action. The Jordan frame field equations are

$$R_{ab} - \frac{R}{2} g_{ab} = \frac{8\pi}{\phi} T_{ab}^{(\text{m})} + T_{ab}^{(\phi)}, \quad (2)$$

$$T_{ab}^{(\phi)} := \frac{\omega}{\phi^2} \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi \right) + \frac{1}{\phi} (\nabla_a \nabla_b \phi - g_{ab} \square \phi) - \frac{V}{2\phi} g_{ab},$$

$$(2\omega + 3) \square \phi = 8\pi T^{(\text{m})} + \phi V' - 2V - \omega' \nabla^c \phi \nabla_c \phi, \quad (3)$$

where R_{ab} is the Ricci tensor, $T_{ab}^{(\text{m})}$ is the matter stress-energy tensor, $T^{(\text{m})} := g^{ab} T_{ab}^{(\text{m})}$, ∇ is the Levi-Civita connection, $\square := g^{ab} \nabla_a \nabla_b$, $T_{ab}^{(\phi)}$ is an effective stress-energy tensor of ϕ , and a prime denotes differentiation with respect to ϕ .

When the gradient $\nabla^c \phi$ is timelike and future-oriented, the effective stress-energy tensor of ϕ describes an effective dissipative fluid with 4-velocity $u^a := \nabla^a \phi / \sqrt{2X}$, with $2X = -\nabla^c \phi \nabla_c \phi$. The general form of a dissipative imperfect fluid of 4-velocity u^a reads

$$T_{ab} = \rho u_a u_b + P h_{ab} + \pi_{ab} + 2q_a u_b, \quad (4)$$

where ρ , P , π_{ab} , and q_a are, respectively, an effective energy density, effective isotropic pressure, effective anisotropic stress tensor, and effective heat flux density, $h_{ab} := g_{ab} + u_a u_b$, and the parentheses denote the symmetrized product of q_a and u_a . We shall denote the fluid quantities associated with $T_{ab}^{(\phi)}$ by $\rho^{(\phi)}$, $P^{(\phi)}$, $\pi_{ab}^{(\phi)}$, and $q_a^{(\phi)}$. Miraculously, the Eckart-Fourier constitutive law for dissipative fluids [16] $q_a = -\mathcal{K}(\nabla_a \mathcal{T} + \mathcal{T} \dot{u}_a)$ (where $\mathcal{K}$ is the thermal conductivity, $\mathcal{T}$ is the temperature, and $\dot{u}^a := u^c \nabla_c u^a$ is the fluid acceleration) is obeyed by this effective fluid. In fact, a direct computation yields $q_a^{(\phi)} = -\mathcal{K} T \dot{u}_a$ and [17–21]

$$\mathcal{K} T = \frac{\sqrt{-\nabla^c \phi \nabla_c \phi}}{8\pi \phi}. \quad (5)$$

ST gravity reduces to GR in the limit $\phi \rightarrow \text{const}$, in which $\mathcal{K} T \rightarrow 0$. The approach (departure) of ST gravity to (from) GR is described by [17–21]

$$\frac{d(\mathcal{K} T)}{d\tau} = 8\pi(\mathcal{K} T)^2 - \Theta \mathcal{K} T + \frac{\square \phi}{8\pi \phi}, \quad (6)$$

where τ is the proper time along the effective fluid lines and $\Theta := \nabla_c u^c$ is its expansion scalar. Equation (3) yields

$$\begin{aligned} \frac{d(\mathcal{K} T)}{d\tau} &= 8\pi(\mathcal{K} T)^2 - \Theta \mathcal{K} T + \frac{T^{(\text{m})}}{(2\omega + 3)\phi} + \frac{1}{8\pi(2\omega + 3)} \\ &\times \left(V' - \frac{2V}{\phi} - \frac{\omega'}{\phi} \nabla^c \phi \nabla_c \phi \right). \end{aligned} \quad (7)$$

Gravity is “heated” [$d(\mathcal{K} T)/d\tau > 0$] by positive terms in the right-hand side and is “cooled” [$d(\mathcal{K} T)/d\tau < 0$] by negative ones. We examine the combination $\mathcal{K} T(8\pi \mathcal{K} T - \Theta)$ for electrovacuum Brans-Dicke gravity later in this Letter. The sign of the matter contribution coincides with that of $T^{(\text{m})}$. Assuming matter to be an imperfect [i.e., with energy-momentum tensor of the form (4), see Refs. [16]] or a perfect (corresponding to $\pi_{ab}^{(\text{m})} = 0$, $q_a^{(\text{m})} = 0$ and zero viscous pressure) fluid, then the trace is $T^{(\text{m})} = -\rho^{(\text{m})} + 3P^{(\text{m})}$, which is negative if $P^{(\text{m})} < \rho^{(\text{m})}/3$, as guaranteed by the strong energy condition: then *matter “cools” gravity*. In particular, everything else equal, during the matter-dominated era with $T^{(\text{m})} = -\rho^{(\text{m})}$, it is easier for gravity to converge to GR.

Conformal matter has $T^{(\text{m})} = 0$ and includes pure electromagnetic fields, a radiation fluid with the equation of state $P^{(\text{m})} = \rho^{(\text{m})}/3$ (e.g., in the cosmic radiation era) and a scalar field ψ conformally coupled to the Ricci scalar R with quartic potential $U(\psi) = \lambda \psi^4$. Next, it is fortunate that one can draw general conclusions on the contribution of the potential: the combination $(V' - 2V/\phi)$ gives a positive contribution, “heating” gravity if $V' - 2V/\phi > 0$ or (since $V > 0$) $\frac{V'}{V} - \frac{2}{\phi} = \frac{d}{d\phi} \ln(V/\phi^2) > 0$. Since the logarithm is monotonically increasing, this condition means that $V(\phi)$ grows faster than ϕ^2 . Potentials slower than ϕ^2 “cool” ST gravity, a quadratic potential does not affect it, and potentials faster than quadratic “heat” gravity, which is particularly relevant during inflation driven by ϕ when matter is negligible. Since $\nabla^c \phi \nabla_c \phi < 0$ in our formalism, the last term in the right-hand side of Eq. (7) “heats” gravity when $\omega(\phi)$ is an increasing function and “cools” it when it is decreasing. Of course, for arbitrary functions $\omega(\phi)$, the sign of this contribution depends on the value of ϕ , but the scarce literature on ST theories with varying ω focuses on monotonic functions $\omega(\phi)$, and then

this contribution has a well-defined sign. Using the definition (5) of $\mathcal{KT}$, this term is seen to be proportional to $(\mathcal{KT})^2$, and Eq. (7) can be rewritten as

$$\frac{d(\mathcal{KT})}{d\tau} = 8\pi \left[1 + \frac{\phi}{2} \frac{d \ln(2\omega + 3)}{d\phi} \right] (\mathcal{KT})^2 - \Theta \mathcal{KT} + \frac{T^{(m)}}{(2\omega + 3)\phi} + \frac{V' - 2V/\phi}{8\pi(2\omega + 3)}. \quad (8)$$

Let us examine physical situations in which Eq. (8) simplifies substantially.

Electrovacuum Brans-Dicke gravity—In Brans-Dicke gravity, $\omega = \text{const}$, $V(\phi) = 0$, and we assume electrovacuum or conformal matter. Relevant cosmological situations are the early universe in which ϕ dominates the cosmic dynamics, and the radiation era, but the discussion applies to general geometries and is not restricted to cosmology. Then $\square\phi = 0$, and Eq. (7) reads

$$\frac{d}{d\tau}(\mathcal{KT}) = \mathcal{KT}(8\pi\mathcal{KT} - \Theta), \quad (9)$$

with $\mathcal{KT} \geq 0$. A partial analysis of this equation was given in [18,19], but here we provide a much more compact and intuitive analysis noting that the $(\Theta, \mathcal{KT})$ plane allows for a quick visualization of the dynamics of gravity (Fig. 1), especially in regard to the approach of ST gravity to GR. Although $\Theta = \nabla_c u^c = \nabla_c(\nabla^c \phi / \sqrt{2X})$ ultimately depends on $\mathcal{KT}$ through ϕ and its gradient, this plane is particularly convenient. It is not a phase plane, and trajectories of the system representing the evolution of gravity in this plane can, in principle, intersect. The Θ axis corresponds to thermal equilibrium at $\mathcal{KT} = 0$.

By the definition (5) of $\mathcal{KT}$, only the half-plane $\mathcal{KT} \geq 0$ is relevant. There are two, and only two, types of solutions with $\mathcal{KT} = \text{constant}$: $\mathcal{KT} = 0$ and $\mathcal{KT} = \Theta/8\pi$. In fact, since the left-hand side of Eq. (9) vanishes identically, it is $\mathcal{KT}(8\pi\mathcal{KT} - \Theta) = 0$, and they all lie on the $\mathcal{KT} = 0$ axis or on the half line $\mathcal{KT} = \Theta/8\pi$ through the origin. Since $\mathcal{KT} = \text{const}$ for these solutions, also $\Theta = 8\pi\mathcal{KT}$ is constant, and this entire half line is composed of points $(\Theta, \mathcal{KT}) = (\Theta_0, 8\pi\Theta_0)$ with constant Θ_0 . Typical solutions of this kind are de Sitter spaces with nonconstant scalar field ϕ , which are impossible in GR with a minimally coupled scalar field and are a signature of ST gravity.

In the quadrant $(\Theta > 0, \mathcal{KT} > 0)$, it is

$$\frac{d(\mathcal{KT})}{d\tau} > 0 \Leftrightarrow \mathcal{KT} > \frac{\Theta}{8\pi}; \quad (10)$$

if a solution of the Brans-Dicke field equations begins above the line $\mathcal{KT} = \Theta/8\pi$, then $\mathcal{KT}$ always increases (while the nearby solution on this line has constant $\mathcal{KT}$ and Θ instead) and $\mathcal{KT} \rightarrow +\infty$, diverging away from GR. If a solution instead begins below this line with $\mathcal{KT} < \Theta/8\pi$,

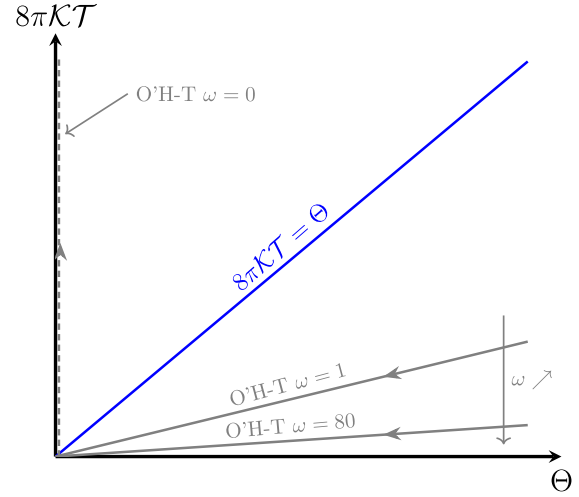


FIG. 1. The critical half line in the $(\Theta, \mathcal{KT})$ plane and lines representing the cosmological O’Hanlon-Tupper solutions for two values of ω . Solutions starting above this critical line deviate forever from GR; those starting below converge to GR. The critical line cannot be crossed.

then $\mathcal{KT}$ decreases, and the solution always remains below this line, converging to the Θ axis with $\mathcal{KT} = 0$ corresponding to the GR state of thermal equilibrium.

The half line $\mathcal{KT} = \Theta/8\pi$ (referred to as the *critical line*) is not the trajectory of a solution, it cannot be crossed dynamically by solutions, and it separates the $(\Theta > 0, \mathcal{KT} > 0)$ quadrant into two dynamically distinct regions. A *new notion of “thermal stability” of gravity emerges* from this picture: if the special solutions represented by points on the critical line are displaced slightly, their $\mathcal{KT}$ either diverges or goes to zero bringing them to GR. In this sense, all these special “point solutions” are *unstable* since any displacement from the critical line brings them away from it (more on this later).

The quadrant $(\Theta < 0, \mathcal{KT} > 0)$ is particularly simple (and for this reason it is not depicted in Fig. 1): here, $d(\mathcal{KT})/d\tau = \mathcal{KT}(8\pi\mathcal{KT} + |\Theta|) \geq 0$, and all solutions starting above the $\mathcal{KT} = 0$ axis move upward, diverging away from GR. This is the visualization of the sharp property of ST gravity, already enunciated in [18,19], that the contraction of the 3 space seen by observers comoving with the effective ϕ fluid “heats” gravity. Therefore, it is expected that, near spacetime singularities where the effective fluid lines converge, gravity deviates strongly from GR. This is the situation, for example, of contracting universes (which “heat” away from GR) and have $\mathcal{KT} \rightarrow +\infty$ near Big Crunch singularities.

Following Ref. [22], we introduce a typical length scale ℓ so that $\Theta = (3/\ell)(d\ell/d\tau)$, then ℓ^3 is the proper volume of a region of 3 space with unit comoving volume, and Eq. (9) reads

$$\frac{d}{d\tau} \ln(\ell^3 \mathcal{KT}) = 8\pi\mathcal{KT} > 0; \quad (11)$$

$\ell^3 \mathcal{K}T$ cannot decrease in time, both in τ time and in coordinate time t , because their directions coincide since the 4-velocity u^a of the effective ϕ fluid is future oriented, $u^0 = dt/d\tau > 0$. Hence, if ℓ decreases (i.e., in the $\Theta < 0$ quadrant), $\mathcal{K}T$ increases, and gravity departs from GR; $\mathcal{K}T$ can only decrease if the 3 space expands (i.e., $\Theta > 0$).

Let us study now special solutions of ST gravity that have already attracted attention in the thermal view of scalar-tensor gravity [23]: stealth solutions and de Sitter spaces with a nonconstant scalar field. Placing the analysis in the context of the $(\Theta, \mathcal{K}T)$ plane greatly elucidates their role in this formalism.

Stealth solutions—Stealth solutions of ST gravity are vacuum solutions with Minkowski geometry and non-constant scalar ϕ , for which terms in the effective stress-energy tensor $T_{ab}^{(\phi)}$ conspire to cancel out, but do not vanish identically. We restrict to stealth solutions for which $\nabla^a \phi$ is timelike and future oriented, and, in this section, we further require that $\square\phi = 0$ [24], which is satisfied if $\phi = \phi(t)$ and $\dot{\phi} < 0$.

In Minkowski space, $\Theta = 0$ and $\square\phi = 0$ imply that

$$\nabla^a \phi \nabla^b \phi \nabla_a \nabla_b \phi = \dot{\phi}^2 \ddot{\phi} = 0. \quad (12)$$

Thus, the Brans-Dicke scalar field must be linear [25], $\phi(t) = -|\phi_0|t + \phi_1$, with $\phi_{0,1}$ constants and $\phi_0 < 0$, $\phi_1 > 0$. To keep $\phi > 0$, only the semi-infinite time interval $t < \phi_1/|\phi_0| \equiv t_*$ is possible. As $t \rightarrow t_-^*$, the scalar field vanishes, and the effective gravitational coupling $G_{\text{eff}} = 1/\phi \rightarrow +\infty$. Even though the geometry is always exactly flat, a singularity of the gravitational coupling strength can rightly be regarded as a singularity of gravity.

In the $(\Theta, \mathcal{K}T)$ plane, stealth solutions live on the vertical $\mathcal{K}T$ axis and can only move upward in time because $d(\mathcal{K}T)/d\tau = 8\pi(\mathcal{K}T)^2 > 0$. This equation has the polelike solution,

$$\mathcal{K}T = \frac{1}{8\pi|\tau_0 - \tau|}, \quad (13)$$

exploding in a finite time τ_0 (only the increasing branch has physical meaning). All these stealth solutions run infinitely far away from GR in a finite time.

de Sitter solutions with nonstatic scalar field—A signature of ST gravity is the existence of vacuum de Sitter solutions with a nonconstant scalar field. By contrast, in GR sourced by a minimally coupled scalar field, de Sitter spaces necessarily have a constant scalar field.

In the $(\Theta, \mathcal{K}T)$ plane, these de Sitter solutions correspond to points on the critical line with $\Theta = 3H = \text{const}$, where $H \equiv \dot{a}/a$ is the Hubble function of the FLRW line element,

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \quad (14)$$

If $H = 0$, the de Sitter geometry degenerates into the Minkowski one, and we recover the $\Theta = 0$ stealth solutions (12) and (13). The equation $\square\phi = -(\ddot{\phi} + 3H\dot{\phi}) = 0$ admits the first integral $\dot{\phi} = C/a^3$, where C is a negative constant (for future orientation); therefore, $\phi = (|C|a_0/\Theta)e^{-\Theta t}$ and $\mathcal{K}T = |\dot{\phi}|/(8\pi\phi) = \Theta/8\pi$, so these de Sitter spaces are points lying on the critical line of the $(\Theta, \mathcal{K}T)$ plane (most of the de Sitter solutions with nonstatic ϕ in [23] have $\square\phi \neq 0$). No other de Sitter solutions with nonstatic ϕ are found under the restrictions discussed so far.

O'Hanlon and Tupper (O'H-T) solution—As an explicit example, let us consider the O'H-T solution of ST cosmology [26], corresponding to the theory with $V = 0$, $T_{ab}^{(m)} = 0$, $\omega > -3/2$, and $\omega \neq -4/3$. The solution is represented by power laws for the scale factor and scalar field, $a \sim t^{q_{\pm}}$ and $\phi \sim t^{s_{\pm}}$, with $q_{\pm} = \omega/[3(\omega + 1) \mp \sqrt{3(2\omega + 3)}]$ and $s_{\pm} = 1 - 3q_{\pm}$. Requiring $\phi > 0$ and $\nabla^a \phi$ future oriented, let us select the (+) solution, for which we have [27] $8\pi\mathcal{K}T = |s_+|/t$. Furthermore, since $\Theta = 3H = 3q_+/t$, the O'H-T solution corresponds to the straight line,

$$8\pi\mathcal{K}T = \frac{|s_+|}{3q_+} \Theta = \left| \frac{1}{3q_+} - 1 \right| \Theta,$$

which implies that $8\pi\mathcal{K}T < \Theta$ for all $\omega > -3/2$, $\omega \neq -4/3$, 0. Hence, based on our argument, gravity in the O'H-T solution, with $\omega > -3/2$, $\omega \neq -4/3$, 0, must relax to GR, which is exactly what happens at late times. Furthermore, the case $\omega = 0$ corresponds to $a \sim 1$ and $\phi \sim -t$, with $t < 0$. Then $\Theta = 0$, and $8\pi\mathcal{K}T = 1/|t|$ is above the critical line. Hence, as t increases, gravity diverges away from GR. These scenarios are represented in Fig. 1, where the arrows denote the behavior of $\mathcal{K}T$ as t increases.

Attractor-to-GR mechanism—With the assumptions made in this section, we are able to assess the attractor-to-GR mechanism found by Damour and Nordvedt [10,11] and discussed by many authors, including (i) an early epoch in which a free Brans-Dicke field ϕ that dominates the cosmic dynamics and matter is irrelevant; (ii) the radiation epoch in which $T^{(m)} = 0$ and ϕ is massless (which is the original situation of [10,11]).

In both cases we assume a spatially flat FLRW universe with line element (14) [28] and Brans-Dicke field $\phi(t)$. In both situations, $\ddot{\phi} + 3H\dot{\phi} = 0$, and the comoving time t coincides with the proper time τ of the effective ϕ fluid.

In the light of the previous discussion, the attractor-to-GR mechanism operates if and only if $\mathcal{K}T < \Theta/(8\pi)$ initially. In a FLRW universe, $\Theta = 3H = 3\dot{\ell}/\ell$ and $\ell = a$ for a unit comoving volume. Using the definition (5) of $\mathcal{K}T$, this criterion reads

$$\sqrt{-\nabla^e \phi \nabla_e \phi} < 3H\phi, \quad (15)$$

which is never satisfied in a contracting universe with $H < 0$. In an expanding universe, this criterion becomes $|\dot{\phi}|/\phi < 3H$, which reads as: there is an attractor-to-GR mechanism if and only if the scale of variation of the scalar field $\tau_\phi \equiv \phi/|\dot{\phi}|$ (equivalently, the scale of variation $\tau_G = \tau_\phi := G_{\text{eff}}/|\dot{G}_{\text{eff}}|$) satisfies

$$\tau_\phi > \frac{\tau_H}{3} := \frac{1}{3H}. \quad (16)$$

The physical interpretation is that if ϕ varies too fast ($\mathcal{KT}$ above the critical line), GR is never approached, while, if it varies sufficiently slowly, GR is always approached. The effective strength of gravity is precisely the extra degree of freedom added to the two massless spin-2 modes of GR. When the variation of ϕ is “fast,” the scalar mode dominates over the two GR modes, while the opposite is true when ϕ and G_{eff} vary slowly and the GR modes dominate, in a way made precise by Eq. (16) or by the thermal criterion $\mathcal{KT} < \Theta/8\pi$. Keep in mind that the scenario applies to the cosmic radiation era even though the scalar field is not dominant. Thus, the thermal view of scalar-tensor gravity provides insight on the attractor-to-GR problem in the restricted context of Brans-Dicke gravity with $\square\phi = 0$. The results can be extended immediately to the case $V(\phi) = m^2\phi^2/2$, the potential that does not contribute to $\square\phi$ (it does, however, contribute to sourcing g_{ab} ; therefore, the simple stealth and de Sitter solutions already discussed are no longer valid).

Conclusions—For an expanding 3 space (i.e., $\Theta > 0$) and $\square\phi = 0$, Brans-Dicke gravity “cools” and converges to GR if it starts sufficiently close to it (i.e., below the critical line), while it “heats up” and runs away from GR if it is sufficiently “hot” (i.e., above the critical line and, therefore, sufficiently far away from GR) initially. These conclusions hold, in particular, in FLRW cosmology and explain the attractor-to-GR mechanism and the repeller mechanism discovered in previous literature, without assuming anything on the solutions themselves, apart from the initial position in the $(\Theta, \mathcal{KT})$ plane. A clear physical interpretation of why the scalar degree of freedom dominates over the two tensor ones emerges naturally from this description.

Contrary to Refs. [10,11] and to most authors, who work in the Einstein conformal frame, we have performed our analysis entirely in the Jordan frame. However, the thermal analogy of scalar-tensor gravity has an equivalent description in the Einstein frame, in which one trades temperature (which is always zero) with chemical potential μ and an evolution equation for μ analogous to (6) is present [30].

Here we have covered the original scenario of [10,11]. More involved situations in which $\square\phi \neq 0$ and the extra

terms in the right-hand side of Eq. (8) play a role will be discussed in future work, in the context of scenarios proposed in the literature and involving specific potentials, matter fluids, and ϕ -dependent Brans-Dicke couplings. However, we can anticipate two results that highlight the generality of our approach compared to the current literature. Within the assumptions of the thermodynamics of scalar-tensor gravity, if $V > 0$, V grows faster than ϕ^2 , $2\omega + 3 > 0$, $\omega' \geq 0$, and assuming a matter perfect fluid with $w = p/\rho > 1/3$, then the last two terms in Eq. (7) are positive and

$$\frac{d(\mathcal{KT})}{d\tau} \geq 8\pi(\mathcal{KT})^2 - \Theta\mathcal{KT};$$

hence, if $\mathcal{KT} > \Theta/8\pi$, the theory will diverge away from GR also for more general models including matter and nontrivial potentials. Vice versa, for a perfect fluid $w < 1/3$, $2\omega + 3 > 0$, $\omega' < 0$, and V growing slower than ϕ^2 , then

$$\frac{d(\mathcal{KT})}{d\tau} < 8\pi(\mathcal{KT})^2 - \Theta\mathcal{KT},$$

which guarantees the existence of an attractor to GR if $\mathcal{KT} < \Theta/8\pi$. In other words, the first scenario pushes the critical curve separating the two regions in the $(\Theta, \mathcal{KT})$ plane down toward the Θ axis, whereas the second scenario pushes it up above the critical line $\mathcal{KT} = \Theta/8\pi$. More involved scenarios, to further extend the generality of this analysis, will be discussed elsewhere.

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- [1] T. P. Sotiriou and V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010).
 - [2] A. De Felice and S. Tsujikawa, *Living Rev. Relativity* **13**, 3 (2010).
 - [3] S. Nojiri and S. D. Odintsov, *Phys. Rep.* **505**, 59 (2011).
 - [4] C. Brans and R. H. Dicke, *Phys. Rev.* **124**, 925 (1961).
 - [5] P. G. Bergmann, *Int. J. Theor. Phys.* **1**, 25 (1968).
 - [6] K. Nordtvedt, *Phys. Rev.* **169**, 1017 (1968).
 - [7] R. V. Wagoner, *Phys. Rev. D* **1**, 3209 (1970).
 - [8] K. Nordtvedt, Jr., *Astrophys. J.* **161**, 1059 (1970).
 - [9] G. W. Horndeski, *Int. J. Theor. Phys.* **10**, 363 (1974).
 - [10] T. Damour and K. Nordtvedt, *Phys. Rev. Lett.* **70**, 2217 (1993).

- [11] T. Damour and K. Nordvedt, *Phys. Rev. D* **48**, 3436 (1993).
- [12] A. Serna, J. M. Alimi, and A. Navarro, *Classical Quantum Gravity* **19**, 857 (2002).
- [13] The original Damour-Nordvedt scenario contained a massless scalar field with no potential.
- [14] We follow the notation of Ref. [15].
- [15] R. M. Wald, *General Relativity* (Chicago University Press, Chicago, USA, 1984).
- [16] C. Eckart, *Phys. Rev.* **58**, 919 (1940).
- [17] V. Faraoni and J. Côté, *Phys. Rev. D* **98**, 084019 (2018).
- [18] V. Faraoni and A. Giusti, *Phys. Rev. D* **103**, L121501 (2021).
- [19] V. Faraoni, A. Giusti, and A. Mentrelli, *Phys. Rev. D* **104**, 124031 (2021).
- [20] A. Giusti, S. Zentarra, L. Heisenberg, and V. Faraoni, *Phys. Rev. D* **105**, 124011 (2022).
- [21] L. Gallerani, M. Miranda, A. Giusti, and A. Mentrelli, *Phys. Rev. D* **110**, 064087 (2024).
- [22] G. F. R. Ellis, *Proc. Int. Sch. Phys. Fermi* **47**, 104 (1971).
- [23] S. Giardino, A. Giusti, and V. Faraoni, *Eur. Phys. J. C* **83**, 621 (2023).
- [24] These requirements exclude several legitimate stealth solutions.
- [25] Ref. [23] discusses other stealth solutions with $\phi(t) = \phi_0 t^\beta$ or $\phi(t) = \phi_0 e^{\alpha t}$, but these are not solutions of Brans-Dicke gravity under the assumptions made in this section.
- [26] J. O' Hanlon and B. O. J. Tupper, *Nuovo Cimento B* **7**, 305 (1972).
- [27] S. Giardino, V. Faraoni, and A. Giusti, *J. Cosmol. Astropart. Phys.* **04** (2022) 053.
- [28] It is known that spatial curvature affects the attractor-to-GR mechanism [29]. Spatially curved universes will be discussed elsewhere.
- [29] D. I. Santiago, D. Kalligas, and R. V. Wagoner, *Phys. Rev. D* **58**, 124005 (1998).
- [30] V. Faraoni, S. Giardino, A. Giusti, and R. Vanderwee, *Eur. Phys. J. C* **83**, 24 (2023).