

Dynamics of Superconducting Pairs in the Two-Dimensional Hubbard Model

G. Sordi^{1,*}, E. M. O’Callaghan¹, C. Walsh¹, M. Charlebois², P. Sémon³, and A.-M. S. Tremblay³

¹Department of Physics, Royal Holloway, University of London, Egham, Surrey, United Kingdom, TW20 0EX

²Département de Biochimie, Chimie, Physique et Science Forensique, Institut de Recherche sur l’Hydrogène, Université du Québec à Trois-Rivières, Trois-Rivières, Québec, Canada, G9A 5H7

³Département de physique, Institut quantique and RQMP, Université de Sherbrooke, Sherbrooke, Québec, Canada J1K 2R1



(Received 30 November 2025; revised 13 April 2026; accepted 18 May 2026; published 24 June 2026)

The frequency structure of the superconducting correlations in cuprates gives insights on the pairing mechanism. Here we present an exhaustive study of this problem in the two-dimensional Hubbard model with cellular dynamical mean-field theory. To this end, we systematically quantify the dependence on doping δ and interaction strength U of the superconducting gap, of the frequency scales where d -wave pairing occurs, and of their relative contribution to pairing. For all values of U and δ , we find pair-forming processes confined to frequencies set by the superexchange interaction and followed by pair-breaking processes, ruling out both pair-forming and pair-breaking processes on the scale of U . This suggests that at high frequencies, the effect of U is eliminated by the d -wave pairing, and that at small frequencies, U generates the superexchange interaction that leads to low-frequency pair-forming processes providing the net contribution to pairing.

DOI: [10.1103/22h2-jxh4](https://doi.org/10.1103/22h2-jxh4)

Introduction—The mechanism driving the superconducting pairing of electrons in cuprates is still under scrutiny [1–3]. It requires knowing how the electrons forming the Cooper pairs are correlated *both* in space and in time. On one hand, the d -wave symmetry of the superconducting gap and the short correlation length measured in cuprates suggest that the *spatial dependence* of the pair correlations is nonlocal and short ranged [3]. On the other hand, the limited experimental information about the *time dependence* of the pair correlations in cuprates [4–8] hinders a thorough characterization of the pairing mechanism [2,9].

While first-principle methods now allow material specific predictions [10–14], the detailed information of model calculations can provide valuable insights for clarifying the universal features of the pairing mechanism and for guiding experimental progress. These calculations are even more pressing in view of current proposals for measuring pair correlations with time-resolved [15] and coincidence [16–19] angle-resolved photoemission spectroscopy (ARPES). The strong electron-electron interaction in cuprates indicates that the two-dimensional (2D) Hubbard model, where electrons hop in a square lattice with an amplitude t and experience an onsite Coulomb repulsion U , is the point of

departure for modeling the superconducting pairing [2,20]. However, clarifying the origin of pairing in the Hubbard model is still a theoretical challenge, owing to the non-perturbative nature of the strong electronic correlations [21].

Regarding the *spatial dependence* of the superconducting correlations, the Hubbard model correctly captures their nonlocal character. Physically, this is because the repulsion U (i) disfavors the occurrence of two electrons on the same site and (ii) dynamically generates the antiferromagnetic superexchange interaction $J = 4t^2/U$, which favors antiparallel spins on neighboring sites, and thus their effective attraction [22,23]. Detailed studies confirm these effects brought about by U [2,21,24–26].

The issue of the *time dependence*, i.e., of the dynamics, of the superconducting correlations in the Hubbard model has been less explored and is controversial. It is the focus of this Letter. This is a difficult task, requiring a theory able to treat on an equal footing different and coexisting timescales, from short timescales associated with the interaction U to longer timescales associated with the superexchange J . Cluster extensions [24,25,27] of dynamical mean-field theory [28], which handle both spatial fluctuations (within the cluster) and all temporal fluctuations, provide such a theory.

The key questions concern the characteristic timescales that lead to pairing and their relative contribution to pairing. These features depend on U and doping. Previous studies [29–36] show that the main contribution to pairing arises from relatively long time (low frequency) scales ascribed to short-range spin singlet correlations generated by the superexchange J , with smaller contributions arising from shorter time (higher frequency) scales linked to U .

*Contact author: giovanni.sordi@rhul.ac.uk

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI.

However, these conclusions are drawn from studies on a selected range of parameters of the 2D Hubbard model and are based on different methodologies. For example, the pairing dynamics was studied in Ref. [29] with dynamical cluster approximation (DCA) at $U/t = 8, 10, 12$ and 20% doping at finite temperature T , in Ref. [30] with cellular dynamical mean-field theory (CDMFT) at $U/t = 8$ and few doping levels at $T = 0$, in Refs. [33,35,36] with DCA at $U/t = 5.5, 6$ and few doping levels at low T , and in Ref. [34] with CDMFT at $U/t = 9$ and several doping levels at low T .

Here we revisit the problem of the dynamics of the superconducting pairs in the 2D Hubbard model using state of the art calculations based on CDMFT [24,25,27] at finite temperature. Methodologically, the added value of our Letter is twofold. First, taking advantage of algorithmic improvements and large investment of computing time, we explore a dataset encompassing a wide range of interaction U and a comprehensive group of doping levels. Second, our findings are a direct computational result based solely on the Green's function obtained in CDMFT, with no other assumptions coming, e.g., from low-frequency theories. Using this approach, we systematically quantify the dependence on doping and U of the characteristic frequency scales where pairing occurs and their relative contribution to pairing. For all values of doping and U , we find a rich structure in the dynamics of the correlations of the paired electrons, with pair-forming processes mainly confined on the energy scale set by J and followed by pair-breaking processes. Hence, the net contribution to pairing comes only from the low frequency pair-forming processes.

Model and method—We study the 2D Hubbard model on the square lattice, $H = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i\sigma} n_{i\sigma}$. Here, $c_{i\sigma}$ ($c_{i\sigma}^\dagger$) is the operator that destroys (creates) an electron with spin $\sigma = \{\uparrow, \downarrow\}$ at site i , $n = c_{i\sigma}^\dagger c_{i\sigma}$ is the number operator, t_{ij} is the hopping amplitude between nearest neighbor sites, U is the onsite Coulomb repulsion, and μ is the chemical potential that changes the occupation n and thus the hole doping $\delta = 1 - n$. We set $t_{ij} = t = 1$ as our energy unit.

We solve this model in the $d_{x^2-y^2}$ -wave superconducting state and at finite temperature with CDMFT [24,25,27]. CDMFT maps the Hubbard model onto a cluster impurity model embedded in a self-consistent bath of noninteracting electrons. Here we consider the minimal cluster that describes d -wave superconductivity, i.e., a 2×2 plaquette. To solve the cluster impurity model, we use the hybridization expansion continuous-time quantum Monte Carlo method (CT-HYB) [37–39] with a LazySkip List algorithm [40] and with Monte Carlo updates of two pairs of creation and destruction operators to ensure ergodicity [41]. The CT-HYB method enables us to explore the superconducting state for a wide range of interaction strength U and to reach low temperatures. As shown by extensive studies, CDMFT on a 2×2 plaquette can describe several properties of the

superconducting state that are in qualitative agreement with experiments [21,25,26,42] and robust against the cluster size [26,43]. They include the domelike shape of the superconducting state [44–48] and its interplay with the pseudogap [44–47,49], the features of the density of states and of the spin susceptibility [45,50,51], and the changes in energy and entropy upon condensation [47,52,53].

This Letter focuses on the dynamics of the superconducting pairs, which is encoded in the Green's function. In the cluster momentum basis, the Green's function is

$$G_{\mathbf{K}}(\tau) = \begin{pmatrix} G_{\mathbf{K}\uparrow}(\tau) & F_{\mathbf{K}}(\tau) \\ F_{\mathbf{K}}^\dagger(\tau) & -G_{-\mathbf{K}\downarrow}(-\tau) \end{pmatrix}, \quad (1)$$

where, introducing the Matsubara frequencies, $G_{\mathbf{K}\sigma}(i\omega_n) = -\int_0^\beta d\tau e^{i\omega_n\tau} \langle T_\tau c_{\mathbf{K}\sigma}(\tau) c_{\mathbf{K}\sigma}^\dagger(0) \rangle$ is the Nambu diagonal (i.e., normal) Green's function and $F_{\mathbf{K}}(i\omega_n) = -\int_0^\beta d\tau e^{i\omega_n\tau} \langle T_\tau c_{\mathbf{K}\uparrow}(\tau) c_{-\mathbf{K}\downarrow}(0) \rangle$ is the Nambu off-diagonal (i.e., anomalous) Green's function. For d -wave superconductivity $F_{\mathbf{K}=(\pi,0)} = -F_{\mathbf{K}=(0,\pi)}$ is the only nonzero component. The superconducting order parameter is $\Phi = \langle F_{\mathbf{K}=(\pi,0)}(\tau = 0^+) \rangle$.

We perform the analytical continuation from imaginary to real frequencies using the maximum entropy software of Ref. [54]. However, for the anomalous Green's function, which shows both positive and negative spectral weight, the direct maximum entropy method is inapplicable. Instead, we use the MaxEntAux method of Ref. [55], which relies on an auxiliary Green's function with positive spectral weight, and exploit the symmetry properties of the anomalous Green's function (see Supplemental Material [56]).

Strategy—The strategy of this Letter is to systematically examine the dependence on doping and U of three key features of the superconducting state: (i) the superconducting gap, (ii) the characteristic frequency scales that contribute to superconducting pairing and (iii) their relative contribution to pairing.

To do that, we first take advantage of previous work that mapped out the superconducting state of the 2D Hubbard model with CDMFT on a 2×2 plaquette in the $U - \delta - T$ space [45–47,53]. This is condensed in Fig. 1(a), where we report the data of Ref. [53] showing the superconducting transition temperature T_c^{CDMFT} (defined as the temperature below which $|\Phi|$ is nonzero [57]) as a function of δ for different values of U straddling the normal state metal to Mott insulator transition at $U_{\text{MIT}} \approx 5.95$ [58]. Below U_{MIT} (squares), $T_c^{\text{CDMFT}}(\delta)|_U$ reaches its highest value at $\delta = 0$ and monotonically decreases with increasing doping. Above U_{MIT} (circles), $T_c^{\text{CDMFT}}(\delta)|_U$ acquires an asymmetric domelike shape versus δ , reaching its highest value at a finite doping, which is dependent on U . As a function of U , T_c^{CDMFT} is optimized just above U_{MIT} (green circles).

Next, we fix the temperature at $T = 1/50$, since it is below the optimum T_c^{CDMFT} for each value of U , and

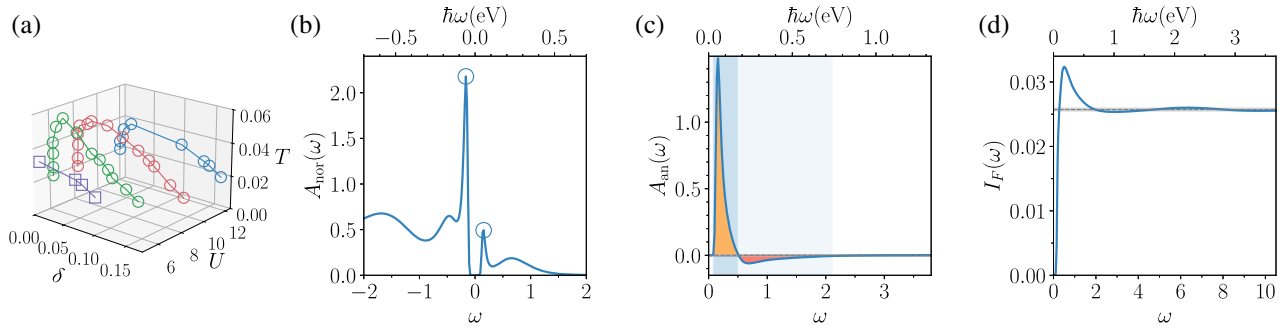


FIG. 1. (a) Superconducting transition temperature T_c^{CDMFT} versus δ for different values of U (data taken from Ref. [53]). (b) Density of states in the superconducting state $A_{\text{nor}}(\omega)$. The half distance between the position of the coherence peaks (marked by open circles) gives the superconducting gap Δ_{sc} and is analyzed in Fig. 2. (c) Anomalous spectral function $A_{\text{an}}(\omega)$. The frequency regions where $A_{\text{an}}(\omega)$ is positive (negative) are those that contribute to pairing (depairing): they are shaded with dark (pale) blue vertical bands and are examined in Fig. 3. The areas where $A_{\text{an}}(\omega)$ is positive or negative are colored and analyzed in Fig. 4. (d) Cumulative spectral weight of the order parameter $I_F(\omega)$. Horizontal line denotes the superconducting order parameter $|\Phi|$ computed independently. Data in panels (b)–(d) are for $U = 12$, $T = 1/50$, and $\delta \approx 0.02$. Frequencies are converted into physical units with $t = 350$ meV.

we explore the model for $U \in [5.2, 16]$ and several doping levels. For each value of U and δ we calculate the density of states in the superconducting state $A_{\text{nor}}(\omega) = -(1/\pi) \text{Im} G_{\mathbf{R}=(0,0)}(\omega)$, the anomalous spectral function $A_{\text{an}}(\omega) = -(1/\pi) \text{Im} F_{\mathbf{K}=(\pi,0)}(\omega)$, and the cumulative spectral weight of the order parameter [30,34] $I_F(\omega) = \int_{-\omega}^{\omega} (d\omega'/2\pi) A_{\text{an}}(\omega') f(-\omega')$. Figures 1(b)–1(d) show $A_{\text{nor}}(\omega)$, $A_{\text{an}}(\omega)$, and $I_F(\omega)$ for $U = 12$ and $\delta \approx 0.02$, as a sample of our calculations (see Supplemental Figs. S3–S7 [56] for spectra at other model parameters).

From $A_{\text{nor}}(\omega)$, we extract a key feature of the superconducting state, i.e., the superconducting gap $2\Delta_{\text{sc}}$. It is the minimum energy to break a Cooper pair. We estimate Δ_{sc} as the half distance between the position of the coherence peaks in $A_{\text{nor}}(\omega)$ [circles in Fig. 1(b)].

From $A_{\text{an}}(\omega)$, which for d -wave superconductivity is real and odd in frequency [55], we extract key features of the dynamics of the superconducting pairs, namely the frequency intervals where pairing occurs and their relative contribution to pairing. To do that, first we extract the frequency intervals where $A_{\text{an}}(\omega)$ is positive [shaded dark blue region in Fig. 1(c)]. These are the frequencies that contribute to pairing processes (i.e., pair-forming). Similarly, the frequencies over which $A_{\text{an}}(\omega)$ is negative contribute to depairing processes (i.e., pair-breaking) [shaded pale blue region in Fig. 1(c)]. Second, we calculate the corresponding areas between $A_{\text{an}}(\omega)$ and the frequency axis [colored areas in Fig. 1(c)] to identify the relative contribution to pairing for each frequency range, as explained in the following paragraph.

To show that positive and negative anomalous spectral weight determines the frequency range where pairing and depairing arise, we can turn to the behavior of the cumulative spectral weight of the order parameter $I_F(\omega)$ [Fig. 1(d)]. In the limit $\omega \rightarrow \infty$, $I_F(\omega)$ converges to the superconducting order parameter $|\Phi|$ (horizontal grey line)

(see also Supplemental Fig. S2 [56]). At low temperatures, $I_F(\omega)$ is approximately the integral of $A_{\text{an}}(\omega)$ over the positive frequencies, i.e., $I_F(\omega) \approx \int_0^{\omega} (d\omega'/2\pi) A_{\text{an}}(\omega')$. Hence, positive anomalous spectral weight $A_{\text{an}}(\omega)$ enhances $|\Phi|$ and thus is pair forming, whereas negative weight depletes $|\Phi|$ and thus is pair breaking. Hence, we can extract the pair-forming (pair-breaking) frequencies from the frequency ranges where $I_F(\omega)$ is increasing (decreasing). Similarly, we can extract the contribution to pairing (depairing) from the difference between a maximum (minimum) value of $I_F(\omega)$ and its preceding minimum (maximum) value.

This physical interpretation of $A_{\text{an}}(\omega)$ and $I_F(\omega)$ has been tested in the BCS and Eliashberg cases in Ref. [30].

Superconducting gap—Figure 2(a) shows the superconducting gap Δ_{sc} as a function of doping for different values of U . Δ_{sc} is defined as half of the energy distance between the coherence peaks in $A_{\text{nor}}(\omega)$. Physically, it reflects the pairing strength. Starting from high doping, as the doping decreases, Δ_{sc} increases. For $U > U_{\text{MIT}}$, and on decreasing doping further, Δ_{sc} flattens. Remarkably, for $U > U_{\text{MIT}}$, the doping dependence of Δ_{sc} contrasts with that of the superconducting order parameter $|\Phi|$, which first increases and then decreases as a function of doping (open squares in Fig. 4). The value of doping for which $|\Phi|$ is maximum, $\delta_{\Phi^{\text{max}}}$, is indicated by a filled symbol on each curve. Given that this doping is found [47] to be larger than the optimal doping [i.e. the doping that maximizes $T_c^{\text{CDMFT}}(\delta)$], the doping dependence of Δ_{sc} also contrasts with that of T_c^{CDMFT} . Hence, in the underdoped region for $U > U_{\text{MIT}}$, Δ_{sc} flattens while both T_c^{CDMFT} and $|\Phi|$ drop on approaching the Mott insulator at $\delta = 0$. This non-BCS behavior is compatible with previous calculations [34,44,45,50,51,59,60] and with ARPES experiments in hole-doped cuprates [61].

Figure 2(b) shows Δ_{sc} versus U at the fixed low doping $\delta \approx 0.02$. Remarkably, $\Delta_{\text{sc}}(U)$ shows a nonmonotonic

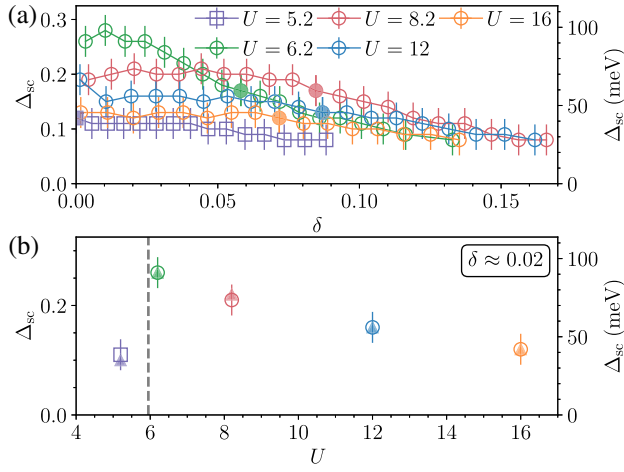


FIG. 2. (a) Superconducting gap Δ_{sc} versus δ for different values of U , as determined by the half of the frequency difference of the coherence peaks in $A_{nor}(\omega)$, see, e.g., Fig. 1(b). Error bars indicate the uncertainty in determining the peaks. For each value of U , the filled symbol denotes the doping at which the superconducting order parameter $|\Phi(\delta)|$ has a maximum. (b) Δ_{sc} at fixed doping $\delta \approx 0.02$ versus U . The dashed vertical line marks the critical threshold $U_{MIT} \approx 5.95$ [58] for opening the Mott gap at half filling. Filled up triangles denote the position of the low-frequency peak in $A_{an}(\omega)$, which tracks Δ_{sc} . Data are converted into physical units with $t = 350$ meV.

behavior, peaking around the underlying normal state Mott transition at U_{MIT} (dashed vertical line). In contrast, $|\Phi|$ at this doping level decreases monotonically with increasing U .

Frequency scales where pairing arises—Next, we analyze the frequency intervals where pairing occurs. These are given by the regions where $A_{an}(\omega)$ is positive. They are shown in Fig. 3 with vertical and dark colored bars, for several values of δ and U . Vertical and pale colored bars indicate the regions where $A_{an}(\omega)$ is negative, i.e., where depairing occurs. Up (down) triangles mark the maxima (minima) in each frequency region. Blank space between these bars indicates the regions where $A_{an}(\omega)$ is negligible [here, $|A_{an}(\omega)| < 0.005$]. This small threshold has been

introduced to filter out the noise in the spectra (see Supplemental Material [56]).

Although $A_{an}(\omega)$ for a given value of U and δ may show a complicated behavior due to the limitations of the analytical continuation, a few trends emerge. As a function of frequency, there is one main interval yielding pair-forming processes alternating with one main interval yielding pair-breaking processes, followed by higher frequency processes that are indistinguishable from noise.

First, at low frequencies $A_{an}(\omega)$ has a gap (blank space in Fig. 3) between $\omega = 0$ and $\omega \approx 0.1$ [see, e.g., Figs. 1(c) and 5 in End Matter for a low-frequency enlarged view of Fig. 3]. The behavior of this gap versus U and δ approximately follows that of Δ_{sc} , i.e., saturates at small doping levels and decreases with increasing doping and with increasing U above U_{MIT} .

Above this gap, $A_{an}(\omega)$ is positive on a narrow frequency range, which for all values of U and δ lies between $\omega \approx 0$ and $\omega \approx 0.6$ [see, e.g., dark shaded blue region in Fig. 1(c)]. (i) This frequency scale is of the order of $J/2$, suggesting that the long-lived pair-forming processes occurring in this frequency interval are associated with short-range spin fluctuations, as pointed out earlier [29,30,33]. (ii) This low-frequency region leading to pairing occurs for all doping levels and for all U , suggesting that it is unrelated to the underlying normal-state strongly correlated pseudogap, which only develops for $U > U_{MIT}$ and small dopings [62–64]. (iii) In this low-frequency region, $A_{an}(\omega)$ shows a prominent peak, whose position tracks Δ_{sc} , both as a function of U [filled up triangles vs open symbols in Fig. 2(b)] and δ (Fig. 5). As noted in Ref. [34], this suggests that the optimum spectral weight for pairing in $A_{an}(\omega)$ takes place on a frequency scale close to Δ_{sc} .

Above this low-frequency region leading to pairing, $A_{an}(\omega)$ becomes negative over a broad frequency range, which for all values of U and δ lies between $\omega \approx 0.6$ and $\omega \approx 3.0$ [see, e.g., pale shaded blue region in Fig. 1(c)]. This region leading to depairing is not restricted to low doping or large values of U .

For further higher frequencies, $A_{an}(\omega)$ remains overall small and indistinguishable from noise, for all doping levels

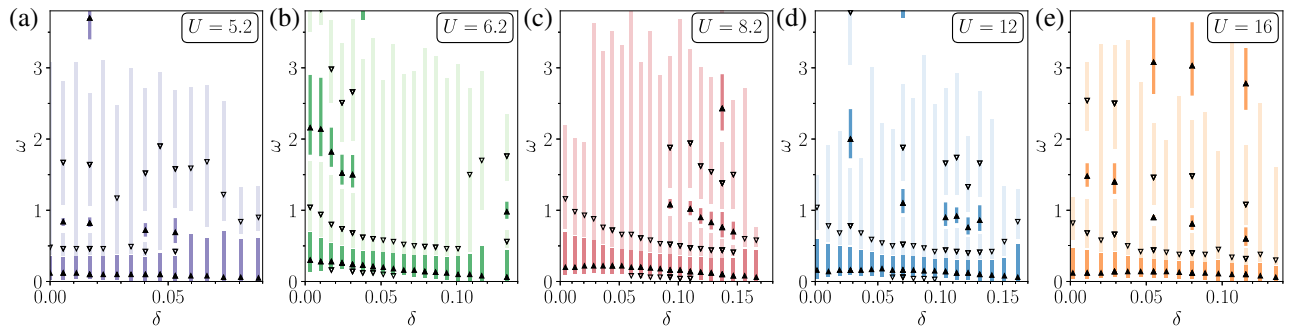


FIG. 3. (a),(b),(c),(d),(e) Vertical dark (pale) bars indicate the frequency regions where $A_{an}(\omega)$ is positive (negative), for several values of doping. Up (down) triangles denote the maxima (minima) in each frequency region. Blank space between these bars indicates the regions where $A_{an}(\omega)$ is negligible [here $|A_{an}(\omega)| < 0.005$]. Each panel shows data for a given interaction strength U .

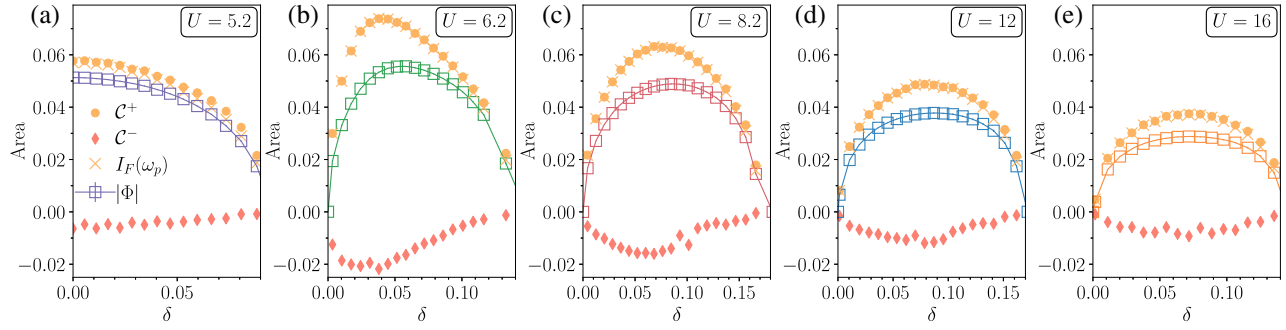


FIG. 4. (a),(b),(c),(d),(e) Largest area, divided by 2π , under the positive regions and above the negative regions of $A_{\text{an}}(\omega)$ versus doping δ , for different values of U . C^+ (positive area, yellow filled circles) and C^- (negative area, red filled diamonds). They are calculated as follows. With the composite trapezoidal method, we find the largest area under (above) $A_{\text{an}}(\omega)$ and above (under) the small threshold of ± 0.005 , to filter out noise. Yellow crosses indicate the value of the low-frequency peak of the cumulative spectral weight of the order parameter, $I_F(\omega_p)$. They follow C^+ . Open squares denote the superconducting order parameter $|\Phi|$.

and for all U [see e.g. blank region in Fig. 1(c) for $\omega \gtrsim 2.1$]. Although we cannot rule out that pair-forming and pair-breaking processes may also occur at higher frequency scales, and in particular on the scale of order U , our results show that their contribution to pairing is negligible, as we shall discuss in the next section.

Contribution to pairing—Next, we study the contribution to pairing or to depairing for each frequency interval where pairing or depairing occurs. This can be estimated by the area under the positive regions or above the negative regions of $A_{\text{an}}(\omega)$ [see, e.g., Fig. 1(c)]. Figure 4 shows the largest area, divided by 2π , under the positive regions and above the negative regions of $A_{\text{an}}(\omega)$ (C^+ , yellow filled circles for the largest positive area and C^- , red filled diamonds for the largest negative area) versus δ for different values of U . This largest positive (negative) area occurs for the main frequency interval where pairing (depairing) occurs that we have identified in Fig. 3, i.e., C^+ arises from the interval $0 \lesssim \omega \lesssim 0.6$ and C^- arises from the interval $0.6 \lesssim \omega \lesssim 3.0$. We verified that the positive and negative areas at other frequency scales are much smaller or negligible (see Supplemental Material [56]). This implies the key result that the contribution to pairing (or depairing) of higher frequencies is negligible, for all values of δ and U . Specifically, our results rule out an increased importance of higher frequency pair-forming processes with increasing U , a possibility left open in previous work [29,30].

For $U > U_{\text{MIT}}$, C^+ has a domelike shape versus δ and overall decreases with increasing U . Hence, a key finding is that on approaching the Mott insulator at $\delta = 0$, the drop of $|\Phi|$ and T_c^{CDMFT} is associated with the reduction of the area of the low frequency peak of $A_{\text{an}}(\omega)$ [30,33,34]. The behavior of C^- is similar to that of C^+ , although C^- is negative and is smaller in magnitude than C^+ .

Having analyzed the contribution to pairing and to depairing for each frequency interval, we examine their *net* contribution to pairing. Since we found that there is just one main frequency interval where pairing occurs, the

answer is clear: the net contribution to pairing comes only from the low-frequency pair-forming processes.

Specifically, the net contribution to pairing for a given frequency interval is the signed area of $A_{\text{an}}(\omega)$ in that frequency interval. Since we found that only C^+ and C^- are sizeable, $C^+ + C^-$ is approximately the order parameter $|\Phi|$. Physically, this means that only the low frequencies build up the order parameter. From Fig. 4, $C^+(\delta)$ is greater than $|\Phi(\delta)|$ (open squares), for all values of U and δ . It means C^+ is partly canceled out, so that only a fraction of C^+ contributes to $|\Phi|$ and thus to pairing. Physically, this means that the low frequency pair-forming processes outweigh the pair-breaking ones, so that only a fraction of the low-frequency pair-forming processes provides a net contribution to pairing. Put another way, the net contribution to pairing comes only from the low-frequency pair-forming processes.

The partial cancellation of the anomalous spectral weight can immediately be deduced from the behavior of the cumulative spectral weight of the order parameter $I_F(\omega)$ [see, e.g., Fig. 1(d)]: at low frequency, $I_F(\omega)$ overshoots $|\Phi|$. If pairing had a net contribution from higher frequencies, then $|\Phi|$ would be larger than the value of the low frequency peak, $I_F(\omega_p)$. Similarly, Fig. 4 shows that $I_F(\omega_p)$ (yellow crosses), which basically coincides with C^+ , is larger than $|\Phi|$ for all values of U and δ .

Summary—We studied the dynamics of the superconducting pairs in the 2D Hubbard model for a wide range of interaction U and doping. The superconducting gap Δ_{sc} extracted from $A_{\text{nor}}(\omega)$ does not scale with the superconducting order parameter, but instead saturates on approaching the Mott insulator. $A_{\text{an}}(\omega)$ has a rich structure as a function of frequency. (i) For all values of U and δ , the pair-forming processes, described by the positive weight of $A_{\text{an}}(\omega)$, are mainly confined on a single frequency interval set by the superexchange. (ii) They are followed by pair-breaking processes, described by the negative weight of $A_{\text{an}}(\omega)$, which are in turn followed by further higher

frequency processes that are indistinguishable from noise. The contribution to pairing and depairing can be extracted from the areas under the positive and negative regions of $A_{\text{an}}(\omega)$. (i) For all values of U and δ , the contribution to pairing of higher frequencies is negligible, ruling out pair-forming and pair-breaking processes on the scale of U . This suggests that d -wave pairing, which creates a node in the pair wave function when both electrons are on the same site, suffices to eliminate the effect of U at high frequency, which would be pair breaking in the s -wave channel. (ii) Since the low frequency pair-forming processes outweigh the pair-breaking ones, the net contribution to pairing comes only from the low-frequency pair-forming processes, which occurs on the scale of the superexchange J . This reflects that at small frequencies, U dynamically generates the interaction J , which plays a role analogous to the Debye frequency in the case of phonons, where pairing processes occur below that frequency and pair-breaking processes occur above that frequency. Our results provide predictions for experiments that plan to directly measure pair correlations with time-resolved [15] and coincidence [16–19] ARPES.

Acknowledgments—This work has been partially supported by the Canada First Research Excellence Fund. Simulations were performed on computers provided by the Canada Foundation for Innovation, Calcul Québec, and the Digital Research Alliance of Canada.

Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] M. R. Norman, The challenge of unconventional superconductivity, *Science* **332**, 196 (2011).
- [2] D. J. Scalapino, A common thread: The pairing interaction for unconventional superconductors, *Rev. Mod. Phys.* **84**, 1383 (2012).
- [3] B. Keimer, S. A. Kivelson, M. R. Norman, S. Uchida, and J. Zaanen, From quantum matter to high-temperature superconductivity in copper oxides, *Nature (London)* **518**, 179 (2015).
- [4] E. van Heumen, E. Muhlethaler, A. B. Kuzmenko, H. Eisaki, W. Meevasana, M. Greven, and D. van der Marel, Optical determination of the relation between the electron-boson coupling function and the critical temperature in high- T_c cuprates, *Phys. Rev. B* **79**, 184512 (2009).
- [5] J. P. Carbotte, T. Timusk, and J. Hwang, Bosons in high-temperature superconductors: An experimental survey, *Rep. Prog. Phys.* **74**, 066501 (2011).
- [6] S. D. Conte, C. Giannetti, G. Coslovich, F. Cilento, D. Bossini, T. Abebaw, F. Banfi, G. Ferrini, H. Eisaki, M. Greven, A. Damascelli, D. van der Marel, and F. Parmigiani,

- Disentangling the electronic and phononic glue in a high- T_c superconductor, *Science* **335**, 1600 (2012).
- [7] F. Cilento, S. D. Conte, G. Coslovich, F. Banfi, G. Ferrini, H. Eisaki, M. Greven, A. Damascelli, D. v. d. Marel, F. Parmigiani, and C. Giannetti, In search for the pairing glue in cuprates by non-equilibrium optical spectroscopy, *J. Phys. Conf. Ser.* **449**, 012003 (2013).
- [8] S. Dal Conte *et al.*, Snapshots of the retarded interaction of charge carriers with ultrafast fluctuations in cuprates, *Nat. Phys.* **11**, 421 (2015).
- [9] P. W. Anderson, Is there glue in cuprate superconductors?, *Science* **316**, 1705 (2007).
- [10] C. Weber, K. Haule, and G. Kotliar, Strength of correlations in electron- and hole-doped cuprates, *Nat. Phys.* **6**, 574 (2010).
- [11] C. Weber, C. Yee, K. Haule, and G. Kotliar, Scaling of the transition temperature of hole-doped cuprate superconductors with the charge-transfer energy, *Europhys. Lett.* **100**, 37001 (2012).
- [12] S. Acharya, C. Weber, E. Plekhanov, D. Pashov, A. Taraphder, and M. Van Schilfhaarde, Metal-insulator transition in copper oxides induced by apex displacements, *Phys. Rev. X* **8**, 021038 (2018).
- [13] B. Bacq-Labreuil, B. Lacasse, A.-M. S. Tremblay, D. Sénéchal, and K. Haule, Toward an *ab initio* theory of high-temperature superconductors: A study of multilayer cuprates, *Phys. Rev. X* **15**, 021071 (2025).
- [14] Z.-H. Cui, J. Yang, J. Tölle, H.-Z. Ye, S. Yuan, H. Zhai, G. Park, R. Kim, X. Zhang, L. Lin, T. C. Berkelbach, and G. K.-L. Chan, *Ab initio* quantum many-body description of superconducting trends in the cuprates, *Nat. Commun.* **16**, 1845 (2025).
- [15] F. Boschini, M. Zonno, and A. Damascelli, Time-resolved ARPES studies of quantum materials, *Rev. Mod. Phys.* **96**, 015003 (2024).
- [16] A. F. Kemper, F. Goto, H. A. Labib, N. Gauthier, E. H. da Silva Neto, and F. Boschini, Observing two-electron interactions with correlation-ARPES, *Phys. Rev. B* **112**, 035168 (2025).
- [17] T. P. Devereaux, M. Claassen, X.-X. Huang, M. Zaletel, J. E. Moore, D. Morr, F. Mahmood, P. Abbamonte, and Z.-X. Shen, Angle-resolved pair photoemission theory for correlated electrons, *Phys. Rev. B* **108**, 165134 (2023).
- [18] C. Stahl and M. Eckstein, Noise correlations in time- and angle-resolved photoemission spectroscopy, *Phys. Rev. B* **99**, 241111(R) (2019).
- [19] Y. Su and C. Zhang, Coincidence angle-resolved photoemission spectroscopy: Proposal for detection of two-particle correlations, *Phys. Rev. B* **101**, 205110 (2020).
- [20] P. W. Anderson, The resonating valence bond state in La_2CuO_4 and superconductivity, *Science* **235**, 1196 (1987).
- [21] A.-M. S. Tremblay, Strongly correlated superconductivity, in *Emergent Phenomena in Correlated Matter Modeling and Simulation*, edited by E. Pavarini, E. Koch, and U. Schollwöck (Verlag des Forschungszentrum, Jülich, 2013), Vol. 3, Chap. 10.
- [22] P. Morel and P. W. Anderson, Calculation of the superconducting state parameters with retarded electron-phonon interaction, *Phys. Rev.* **125**, 1263 (1962).

- [23] G. Kotliar and J. Liu, Superexchange mechanism and d -wave superconductivity, *Phys. Rev. B* **38**, 5142 (1988).
- [24] T. A. Maier, M. Jarrell, T. Pruschke, and M. H. Hettler, Quantum cluster theories, *Rev. Mod. Phys.* **77**, 1027 (2005).
- [25] A.-M. S. Tremblay, B. Kyung, and D. Sénéchal, Pseudogap and high-temperature superconductivity from weak to strong coupling. Towards a quantitative theory, *Low Temp. Phys.* **32**, 424 (2006).
- [26] M. Qin, T. Schäfer, S. Andergassen, P. Corboz, and E. Gull, The Hubbard model: A computational perspective, *Annu. Rev. Condens. Matter Phys.* **13**, 275 (2022).
- [27] G. Kotliar, S. Y. Savrasov, K. Haule, V. S. Oudovenko, O. Parcollet, and C. A. Marianetti, Electronic structure calculations with dynamical mean-field theory, *Rev. Mod. Phys.* **78**, 865 (2006).
- [28] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Dynamical mean-field theory of strongly correlated fermion systems and the limit of infinite dimensions, *Rev. Mod. Phys.* **68**, 13 (1996).
- [29] T. A. Maier, D. Poilblanc, and D. J. Scalapino, Dynamics of the pairing interaction in the Hubbard and $t - J$ models of high-temperature superconductors, *Phys. Rev. Lett.* **100**, 237001 (2008).
- [30] B. Kyung, D. Sénéchal, and A.-M. S. Tremblay, Pairing dynamics in strongly correlated superconductivity, *Phys. Rev. B* **80**, 205109 (2009).
- [31] M. Civelli, Evolution of the dynamical pairing across the phase diagram of a strongly correlated high-temperature superconductor, *Phys. Rev. Lett.* **103**, 136402 (2009).
- [32] D. Sénéchal, A. G. R. Day, V. Bouliane, and A.-M. S. Tremblay, Resilience of d -wave superconductivity to nearest-neighbor repulsion, *Phys. Rev. B* **87**, 075123 (2013).
- [33] E. Gull and A. J. Millis, Pairing glue in the two-dimensional Hubbard model, *Phys. Rev. B* **90**, 041110(R) (2014).
- [34] A. Reymbaut, M. Charlebois, M. F. Asiani, L. Fratino, P. Sémon, G. Sordi, and A.-M. S. Tremblay, Antagonistic effects of nearest-neighbor repulsion on the superconducting pairing dynamics in the doped Mott insulator regime, *Phys. Rev. B* **94**, 155146 (2016).
- [35] X. Dong, L. Del Re, A. Toschi, and E. Gull, Mechanism of superconductivity in the Hubbard model at intermediate interaction strength, *Proc. Natl. Acad. Sci. U.S.A.* **119**, e2205048119 (2022).
- [36] X. Dong, E. Gull, and A. J. Millis, Quantifying the role of antiferromagnetic fluctuations in the superconductivity of the doped Hubbard model, *Nat. Phys.* **18**, 1293 (2022).
- [37] K. Haule, Quantum Monte Carlo impurity solver for cluster dynamical mean-field theory and electronic structure calculations with adjustable cluster base, *Phys. Rev. B* **75**, 155113 (2007).
- [38] P. Werner, A. Comanac, L. de Medici, M. Troyer, and A. J. Millis, Continuous-time solver for quantum impurity models, *Phys. Rev. Lett.* **97**, 076405 (2006).
- [39] E. Gull, A. J. Millis, A. I. Lichtenstein, A. N. Rubtsov, M. Troyer, and P. Werner, Continuous-time Monte Carlo methods for quantum impurity models, *Rev. Mod. Phys.* **83**, 349 (2011).
- [40] P. Sémon, C.-H. Yee, K. Haule, and A.-M. S. Tremblay, Lazy skip-lists: An algorithm for fast hybridization-expansion quantum Monte Carlo, *Phys. Rev. B* **90**, 075149 (2014).
- [41] P. Sémon, G. Sordi, and A.-M. S. Tremblay, Ergodicity of the hybridization-expansion Monte Carlo algorithm for broken-symmetry states, *Phys. Rev. B* **89**, 165113 (2014).
- [42] S. Sakai, Nonperturbative calculations for spectroscopic properties of cuprate high-temperature superconductors, *J. Phys. Soc. Jpn.* **92**, 092001 (2023).
- [43] J. Liu, D.-X. Yao, and W. Wu, Interplay between the pseudogap and superconductivity in doped Mott insulators: A cluster dynamical mean-field theory study, *Chin. Phys. Lett.* **42**, 080711 (2025).
- [44] S. S. Kancharla, B. Kyung, D. Sénéchal, M. Civelli, M. Capone, G. Kotliar, and A.-M. S. Tremblay, Anomalous superconductivity and its competition with antiferromagnetism in doped Mott insulators, *Phys. Rev. B* **77**, 184516 (2008).
- [45] K. Haule and G. Kotliar, Strongly correlated superconductivity: A plaquette dynamical mean-field theory study, *Phys. Rev. B* **76**, 104509 (2007).
- [46] G. Sordi, P. Sémon, K. Haule, and A.-M. S. Tremblay, Strong coupling superconductivity, pseudogap, and Mott transition, *Phys. Rev. Lett.* **108**, 216401 (2012).
- [47] L. Fratino, P. Sémon, G. Sordi, and A.-M. S. Tremblay, An organizing principle for two-dimensional strongly correlated superconductivity, *Sci. Rep.* **6**, 22715 (2016).
- [48] C.-D. Hébert, P. Sémon, and A.-M. S. Tremblay, Superconducting dome in doped quasi-two-dimensional organic Mott insulators: A paradigm for strongly correlated superconductivity, *Phys. Rev. B* **92**, 195112 (2015).
- [49] D. Sénéchal, P.-L. Lavertu, M.-A. Marois, and A.-M. S. Tremblay, Competition between antiferromagnetism and superconductivity in high- T_c cuprates, *Phys. Rev. Lett.* **94**, 156404 (2005).
- [50] M. Civelli, M. Capone, A. Georges, K. Haule, O. Parcollet, T. D. Stanescu, and G. Kotliar, Nodal-antinodal dichotomy and the two gaps of a superconducting doped Mott insulator, *Phys. Rev. Lett.* **100**, 046402 (2008).
- [51] C. Walsh, M. Charlebois, P. Sémon, A.-M. S. Tremblay, and G. Sordi, Superconductivity in the two-dimensional Hubbard model with cellular dynamical mean-field theory: A quantum impurity model analysis, *Phys. Rev. B* **108**, 075163 (2023).
- [52] F. Carbone, A. B. Kuzmenko, H. J. A. Molegraaf, E. van Heumen, V. Lukovac, F. Marsiglio, D. van der Marel, K. Haule, G. Kotliar, H. Berger, S. Courjault, P. H. Kes, and M. Li, Doping dependence of the redistribution of optical spectral weight in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, *Phys. Rev. B* **74**, 064510 (2006).
- [53] C. Walsh, M. Charlebois, P. Sémon, G. Sordi, and A.-M. S. Tremblay, Information-theoretic measures of superconductivity in a two-dimensional doped Mott insulator, *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2104114118 (2021).
- [54] D. Bergeron and A.-M. S. Tremblay, Algorithms for optimized maximum entropy and diagnostic tools for analytic continuation, *Phys. Rev. E* **94**, 023303 (2016).
- [55] A. Reymbaut, D. Bergeron, and A.-M. S. Tremblay, Maximum entropy analytic continuation for spectral functions with nonpositive spectral weight, *Phys. Rev. B* **92**, 060509 (R) (2015).

- [56] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/22h2-jxh4> for the calculation of $A_{\text{an}}(\omega)$ by performing only a single analytical continuation, the consistency checks for $A_{\text{an}}(\omega)$, extended data for $A_{\text{nor}}(\omega)$, $A_{\text{an}}(\omega)$, $I_F(\omega)$, and the computation of $A_{\text{an}}(t)$ in the time domain.
- [57] This neglects Kosterlitz-Thouless physics.
- [58] C. Walsh, P. Sémon, D. Poulin, G. Sordi, and A.-M. S. Tremblay, Thermodynamic and information-theoretic description of the Mott transition in the two-dimensional Hubbard model, *Phys. Rev. B* **99**, 075122 (2019).
- [59] A. Paramekanti, M. Randeria, and N. Trivedi, High- T_c superconductors: A variational theory of the superconducting state, *Phys. Rev. B* **70**, 054504 (2004).
- [60] E. Gull, O. Parcollet, and A. J. Millis, Superconductivity and the pseudogap in the two-dimensional Hubbard model, *Phys. Rev. Lett.* **110**, 216405 (2013).
- [61] J. A. Sobota, Y. He, and Z.-X. Shen, Angle-resolved photoemission studies of quantum materials, *Rev. Mod. Phys.* **93**, 025006 (2021).
- [62] G. Sordi, K. Haule, and A.-M. S. Tremblay, Finite doping signatures of the Mott transition in the two-dimensional Hubbard model, *Phys. Rev. Lett.* **104**, 226402 (2010).
- [63] G. Sordi, K. Haule, and A.-M. S. Tremblay, Mott physics and first-order transition between two metals in the normal-state phase diagram of the two-dimensional Hubbard model, *Phys. Rev. B* **84**, 075161 (2011).
- [64] G. Sordi, P. Sémon, K. Haule, and A.-M. S. Tremblay, Pseudogap temperature as a Widom line in doped Mott insulators, *Sci. Rep.* **2**, 547 (2012).

End Matter

Figure 5 shows an enlarged view of Fig. 3 on the low frequencies.

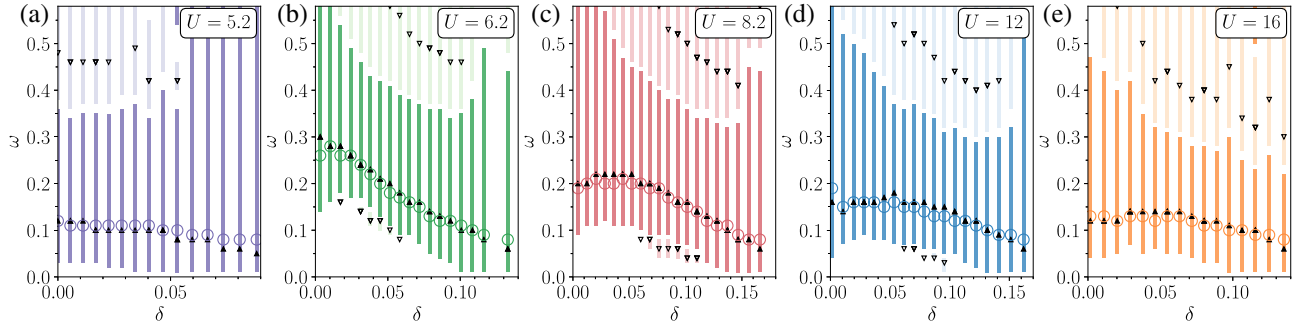


FIG. 5. (a),(b),(c),(d),(e) Low-frequency enlarged view of Fig. 3. For each value of δ and U , the frequency position of the low-frequency maximum of $A_{\text{an}}(\omega)$ (filled up triangles) tracks the size of the superconducting gap Δ_{sc} (open circles), suggesting that Δ_{sc} is the energy scale where pairing is maximum [34].