

Quantum Channels on Graphs: A Resonant Tunneling Perspective

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(Received 12 February 2026; accepted 8 September 2026; published 30 September 2026)

Quantum transport on structured networks is strongly influenced by interference effects, which can dramatically modify how information propagates through a system. We develop a quantum-information-theoretic framework for scattering on graphs in which a full network of connected scattering sites is treated as a quantum channel linking designated input and output ports. Using the Redheffer star product to construct global scattering matrices from local ones, we identify resonant concatenation, a nonlinear composition rule generated by internal back reflections. In contrast to ordinary channel concatenation, resonant concatenation can suppress noise and even produce superactivation of the quantum capacity, yielding positive capacity in configurations where each constituent channel individually has zero capacity. We illustrate these effects through models exhibiting resonant-tunneling-enhanced transport. Our approach provides a general methodology for analyzing coherent information flow in quantum graphs, with relevance for quantum communication, control, and simulation in structured environments.

DOI: [10.1103/21c5-cvnn](https://doi.org/10.1103/21c5-cvnn)

One of the most intriguing phenomena in quantum mechanics is resonant tunneling, where the transmission of a quantum particle through multiple potential barriers is enhanced due to constructive interference from multiple internal reflections [1]. Unlike classical wave propagation, where transmission typically decreases as barriers are added, resonant tunneling allows for perfect transmission under specific conditions. This phenomenon plays a crucial role in solid-state physics, where it underlies the operation of quantum well devices [2], and in photonics, where it governs light transmission in multilayer structures [3]. More broadly, resonant tunneling provides a fundamental example of how interference effects can be exploited to enhance the efficiency of quantum transport processes [4–7]. Beyond its relevance in one-dimensional scattering, resonant tunneling suggests a broader paradigm for improving quantum information transmission. The key principle is that when multiple scattering sites are arranged in a coherent structure, quantum interference can enhance the fidelity and efficiency of information transfer. This idea extends naturally to systems where a quantum particle scatters across complex networks, leading to the study of scattering processes on graphs [8–10]. A quantum graph consists of discrete scattering sites (vertices) connected by quantum pathways

(edges), where the propagation of quantum states is governed by local scattering matrices at each vertex. The interplay between local scattering matrices on each node and global interference patterns determines how information is processed and transmitted within the network. Moreover, the scattered particle often possesses internal degrees of freedom (d.o.f.), such as spin or polarization, which are modified as they pass through a given quantum graph. In this Letter, we develop a comprehensive framework for studying these effects in terms of the quantum channel formalism [11–13], situating our analysis at the intersection of quantum transport and quantum information theory. Inspired by quantum resonant tunneling, we introduce the concept of *resonant concatenation* of quantum operations—a genuine nonlinear process, which under certain conditions leads to noise suppression and enhanced information transmission efficiency in quantum networks that may result in superactivation (SA) effects analogous to those described in [14–16], where the composition is a tensor product of channels. Specifically it may happen that the quantum capacity Q [17–19] of the resonant concatenation of a collection of quantum channels is nonzero, while the individual constituents have explicitly zero Q values. More generally, by leveraging the Redheffer star product [20], extensively used in classical network theory and wave scattering, we study how the global quantum channel describing the information propagation on a network can be systematically constructed from the local scattering matrices of the individual nodes. Our formalism is closely related to the concept of scattering quantum walks introduced by Joye [21]. However, our perspective differs by explicitly focusing on the information-theoretic

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properties of the graph, treating the entire network as a quantum channel that maps a designated set of input ports to a set of output ports. Finally, we observe that the resonant effects in RC imply the absence of a well-defined causal order in the particle's traversal through multiple barriers—an effect reminiscent of behaviors found in models based on the quantum SWITCH construction [22,23]. However, while the RC mechanism analyzed here produces linear transformations on the Hilbert space of the system, it exhibits an intrinsically nonlinear dependence with respect to the individual constituents of the composition, marking a clear distinction from the linear composition rules that underlie the quantum SWITCH models. In contrast, the RC construction bears a closer resemblance to the phenomenology of quantum models introduced to analyze closed timelike curves [24–31], suggesting that it could serve as a useful tool for investigating such systems (see Supplemental Material [32] for details).

Resonant concatenation—The input-output relations which describe the (possibly noisy) evolution of signals transmitted through a quantum communication line can be formally represented by linear, completely positive, trace-preserving (CPTP) superoperators—also known as quantum channels—acting on the space of linear operators associated with the Hilbert space of the quantum system carrying the messages [11–13]. Analogously to classical information theory, the efficiency of a quantum channel Φ can be assessed through a set of figures of merit—its *quantum capacities*—which quantify the optimal ratio between the amount of information that can be transmitted reliably through the channel and the total amount of redundancy required to achieve such reliability [33,34]. In this Letter, we focus on one of these quantities, the *quantum capacity* $Q(\Phi)$, which measures the ability of the transmission line to faithfully convey quantum states [17–19]. Furthermore, we restrict our analysis to quantum channels models Φ_S schematically depicted in Fig. 1(a), where the sender of the message, Alice, encodes information in the internal d.o.f. of a flying qudit P (e.g., a spin), which in its route toward the receiver Bob undergoes (possibly spin-dependent) collisional events described by a scattering matrix (SM) S that may deflect it away. In the absence of back reflections, the overall transformation associated with two consecutive collisional events associated with the SMs S_1 and S_2 is provided by the direct concatenation of the individual channels Φ_{S_1} and Φ_{S_2} describing respectively the effects of the first and second scatterer encountered by the particle along its path to Bob, i.e., $\Phi_{S_2 \circ S_1}[\dots] := \Phi_{S_2} \circ \Phi_{S_1}[\dots] = \Phi_{S_2}[\Phi_{S_1}[\dots]]$ —see Fig. 1(b). A straightforward resource-counting argument shows that $\Phi_{S_2 \circ S_1}$ can never be less noisy than either of its components, leading to standard data-processing inequalities for the associated capacities [13],

$$Q(\Phi_{S_2 \circ S_1}) \leq \min\{Q(\Phi_{S_1}), Q(\Phi_{S_2})\}. \quad (1)$$

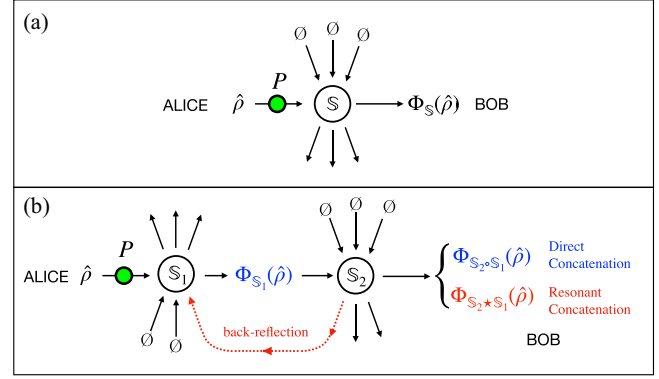


FIG. 1. (a) Schematic representation of a quantum channel Φ_S associated with the scattering of a quantum particle P (green element in the figure), prepared by Alice in the internal state $\hat{\rho}$ and received by Bob. The arrows pointing toward and away from the scattering center S indicate the possible spatial directions the particle can take. The symbol $\emptyset$ on an incoming line denotes that no particle enters from that particular direction. (b) Direct and resonant concatenation scenarios for two scattering events. In the absence of backscattering, the resulting quantum channel corresponds to the direct composition $\Phi_{S_2 \circ S_1}$ of the individual maps Φ_{S_1} and Φ_{S_2} . When backscattering from S_2 to S_1 is allowed (red dotted line), the resulting map is described by the RC channel $\Phi_{S_2 \star S_1}$.

This scenario, however, changes drastically once back-reflection events from the second scatterer to the first are allowed—a situation that naturally arises in the study of information propagation of quantum walkers over quantum graphs. We refer to the corresponding process as resonant concatenation (RC) of the transformations taking place at the scattering centers associated with S_1 and S_2 and denote with $\Phi_{S_2 \star S_1}$ the CPTP map associated with it. An intriguing property of RC is that the resulting quantum channel can exhibit less noise than either of the individual channels. This can lead to pronounced violations of the inequality (1), yielding strictly positive values of $Q(\Phi_{S_2 \star S_1})$ even in configurations where $Q(\Phi_{S_1}) = Q(\Phi_{S_2}) = 0$.

Information transfer on a graph—In our model, we assume that the propagation of the quantum particle P occurs on directed hypergraphs that include dangling edges—i.e., external edges connected to a single vertex—which serve as input and output ports for quantum states. We refer to this structured framework as a quantum graph. Formally, a quantum graph is defined as a triple $G = (V, E, I)$, where $V = \{v_1, v_2, \dots, v_l\}$ is a finite set of vertices, $E = \{e_1, e_2, \dots, e_m\}$ is a finite set of oriented edges. For each vertex $v_i \in V$ we define $E_i^{(\text{in})} := \{e_{i,1}^{(\text{in})}, e_{i,2}^{(\text{in})}, \dots, e_{i,k_i}^{(\text{in})}\}$ and $E_i^{(\text{out})} := \{e_{i,1}^{(\text{out})}, e_{i,2}^{(\text{out})}, \dots, e_{i,k_i}^{(\text{out})}\}$ the subsets of E formed, respectively, by incoming and outgoing edges at vertex v_i . In our analysis, we shall assume that both sets have the same cardinality k_i . Similarly, we shall also assume that the number n of dangling incoming edges of G equals the

number of outgoing edges of the graph. Each vertex v_i is associated with a local scattering matrix $\mathbb{S}_i$, which governs the local collisional process occurring at that site. Let $\vec{A}_i^{(j)} \in \mathbb{C}^d$ denote the vector of probability amplitudes describing the internal degree of freedom (d.o.f.) of the particle P as it approaches the vertex v_i along the incoming edge $e_{i,j}^{(\text{in})}$, and $\vec{B}_i^{(j)} \in \mathbb{C}^d$ the corresponding vector for the outgoing edge $e_{i,j}^{(\text{out})}$. Then $\mathbb{S}_i$ is a $dk_i \times dk_i$ matrix connecting these vectors through the mapping $\mathbf{u}_i^{(\text{out})} = \mathbb{S}_i \mathbf{u}_i^{(\text{in})}$, where $\mathbf{u}_i^{(\text{in})} := (\vec{A}_i^{(1)}, \vec{A}_i^{(2)}, \dots, \vec{A}_i^{(k_i)})^T$ and $\mathbf{u}_i^{(\text{out})} := (\vec{B}_i^{(1)}, \vec{B}_i^{(2)}, \dots, \vec{B}_i^{(k_i)})^T$. To extend this formalism to the entire graph G , we construct a global scattering matrix $\mathbb{S}_G$ that relates the quantum states from all the incoming dangling edges of G to those that are emerging from the graph through its outgoing dangling edges,

$$\mathbf{u}_G^{(\text{out})} = \mathbb{S}_G \mathbf{u}_G^{(\text{in})}, \quad (2)$$

where now $\mathbf{u}_G^{(\text{in})} := (\vec{A}_G^{(1)}, \vec{A}_G^{(2)}, \dots, \vec{A}_G^{(N)})^T$, $\mathbf{u}_G^{(\text{out})} := (\vec{B}_G^{(1)}, \vec{B}_G^{(2)}, \dots, \vec{B}_G^{(N)})^T$ are, respectively, the vectors of the incoming

and outgoing quantum amplitudes from the corresponding N dangling edges of the entire graph. The construction that connects the local SMs $\mathbb{S}_i$ to the global SM $\mathbb{S}_G$ is given by the Redheffer star product [20], which we review in [32]. Where the underlying graph contains loops, $\mathbb{S}_G$ exhibits a highly nonlinear dependence upon the $\mathbb{S}_i$'s, which gives rise to resonance effects in the propagation of the particle P . As an illustrative example, consider a graph G consisting of $l = 2$ vertices, the left vertex v_1 and right vertex v_2 , each associated with one incoming and one outgoing dangling edge: $e_{1,1}^{(\text{in})}, e_{1,1}^{(\text{out})}$ for v_1 on the left-hand side (l.h.s.), and $e_{2,2}^{(\text{in})}, e_{2,2}^{(\text{out})}$ for v_2 on the right-hand side (r.h.s.). The two vertices are connected by two internal (nondangling) edges, $e_{1,2}^{(\text{out})} = e_{2,1}^{(\text{in})}$ and $e_{2,1}^{(\text{out})} = e_{1,2}^{(\text{in})}$, linking the right-hand side of v_1 to the right-hand side of the v_2 . The local scattering matrix of the i th vertex can be written in block form as $\mathbb{S}_i = \begin{bmatrix} \mathbb{S}_i^{1,1} & \mathbb{S}_i^{1,2} \\ \mathbb{S}_i^{2,1} & \mathbb{S}_i^{2,2} \end{bmatrix}$, where each block $\mathbb{S}_i^{j,j'}$ is a $d \times d$ matrix mapping the incoming edges $e_{i,j'}^{(\text{in})}$ to outgoing edges $e_{i,j}^{(\text{out})}$. The global scattering matrix $\mathbb{S}_G = \mathbb{S}_2 \star \mathbb{S}_1$ then takes the form

$$\mathbb{S}_G = \begin{bmatrix} \mathbb{S}_1^{1,1} + \mathbb{S}_1^{1,2} \mathbb{L}_{\text{mp}}^{-1} \mathbb{S}_2^{1,1} \mathbb{S}_1^{2,1} & \mathbb{S}_1^{1,2} \mathbb{L}_{\text{mp}}^{-1} \mathbb{S}_2^{1,2} \\ \mathbb{S}_2^{2,1} \mathbb{S}_1^{2,1} + \mathbb{S}_2^{2,1} \mathbb{S}_1^{2,2} \mathbb{L}_{\text{mp}}^{-1} \mathbb{S}_2^{1,1} \mathbb{S}_1^{2,1} & \mathbb{S}_2^{2,2} + \mathbb{S}_2^{2,1} \mathbb{S}_1^{2,2} \mathbb{L}_{\text{mp}}^{-1} \mathbb{S}_2^{1,2} \end{bmatrix}, \quad (3)$$

where the loop matrix $\mathbb{L} := \mathbb{1} - \mathbb{S}_2^{1,1} \mathbb{S}_1^{2,2}$ encodes the effect of internal reflections and $\mathbb{L}_{\text{mp}}^{-1}$ its Moore-Penrose pseudoinverse, ensuring the expression [35] is well defined.

From $\mathbb{S}_G$ to Φ_G —Here we illustrate how to construct the CPTP map Φ_G that describes the propagation of the information through a quantum graph G . Let us define the d -dimensional Hilbert space of the internal d.o.f. of the quantum particle P , $\mathcal{H}_Q$, and the Hilbert spaces of its spatial degrees of freedom $\mathcal{H}_X^{(\text{in})}$ and $\mathcal{H}_X^{(\text{out})}$, spanned by the basis vectors $\{|j^{(\text{in})}\rangle_X\}_{j=1}^N$ and $\{|j^{(\text{out})}\rangle_X\}_{j=1}^N$, which label the spatial modes associated with the incoming and outgoing dangling edges of the graph, respectively. Accordingly, a generic pure state which describes the approach of the particle P toward the scattering center can be written as $|\psi^{(\text{in})}\rangle_{QX} = \sum_{j=1}^N \sum_{a=0}^{d-1} [\vec{A}_G^{(j)}]_a |a\rangle_Q |j^{(\text{in})}\rangle_X$, where $\{|a\rangle_Q\}_{a=0}^{d-1}$ is an orthonormal basis of $\mathcal{H}_Q$ and $\vec{A}_G^{(j)}$ the vector of the probability amplitudes associated with the j th incoming dangling edge of the graph—see Eq. (2). Analogously, a generic pure output state of P can be expressed as $|\psi^{(\text{out})}\rangle_{QX} = \sum_{j=1}^N \sum_{a=0}^{d-1} [\vec{B}_G^{(j)}]_a |a\rangle_Q |j^{(\text{out})}\rangle_X$, where now $\vec{B}_G^{(j)}$ is the probability amplitude vector associated with the j th outgoing dangling edge. The mapping

that connects $|\psi^{(\text{in})}\rangle_{QX}$ to $|\psi^{(\text{out})}\rangle_{QX}$ is hence provided by the isomorphism $\hat{\mathbb{S}}_G: \mathcal{H}_Q \otimes \mathcal{H}_X^{(\text{in})} \rightarrow \mathcal{H}_Q \otimes \mathcal{H}_X^{(\text{out})}$ represented by the matrix $\mathbb{S}_G$ of the graph in the computational basis of the model, i.e.,

$${}_Q\langle a|_X \langle j^{(\text{out})} | \hat{\mathbb{S}}_G | b \rangle_Q | k^{(\text{in})} \rangle_X = [\mathbb{S}_G^{j,k}]_{a,b}, \quad (4)$$

where $a, b = 0, \dots, d-1$ label the internal basis states and $\mathbb{S}_G^{j,k}$ is the $d \times d$ block of $\mathbb{S}_G$ that connects $\vec{A}_G^{(k)}$ to $\vec{B}_G^{(j)}$ in Eq. (2). We next focus on the special (yet nontrivial) case where Alice is encoding messages on the internal d.o.f. of the particle P using states that only occupy a single incoming dangling edge of the graph—say, the one associated with the vector $|j = 1^{(\text{in})}\rangle_X$ —so that the input states $\rho_{QX}^{(\text{in};1)}$ of the model are now restricted to configuration spanned by vectors $|\psi^{(\text{in})}\rangle_{QX}$ where $\vec{A}_G^{(j)} = 0$ for all $j = 2, \dots, n$, i.e., $|\psi^{(\text{in};1)}\rangle_{QX} = \sum_{a=0}^{d-1} [\vec{A}_G^{(1)}]_a |a\rangle_Q |1^{(\text{in})}\rangle_X$. Similarly, we assume that Bob collects data only from the n th outgoing dangling edge. Under these conditions, the state Bob gets can be represented by the density matrix

$$\begin{aligned}\hat{\rho}_{QX}^{(\text{out};n)} &= \Phi_G(\hat{\rho}_{QX}^{(\text{in};1)}) \\ &:= \hat{P}_X^{(n)} \hat{S}_G \hat{\rho}_{QX}^{(\text{in};1)} \hat{S}_G^\dagger \hat{P}_X^{(n)} + \text{Tr}[\hat{Q}_X^{(n)} \hat{S}_G \hat{\rho}_{QX}^{(\text{in};1)} \hat{S}_G^\dagger] |\emptyset\rangle\langle\emptyset|,\end{aligned}\quad (5)$$

where $\hat{P}_X^{(n)} := |n^{(\text{out})}\rangle_X \langle n^{(\text{out})}|$ and $\hat{Q}_X^{(n)} := \hat{1}_X - \hat{P}_X^{(n)}$ are respectively the projector on the n th outgoing dangling edge of the graph and its complement to the identity, while $|\emptyset\rangle$ represents the vacuum state (no particle). Equation (5) indicates the map Φ_G is a state-dependent erasure channel (EC) [36] with flagged state $|\emptyset\rangle$, where Bob receives a (possibly deteriorated) version of Alice's input state with probability $p := \text{Tr}[\hat{P}_X^{(n)} \hat{S}_G \hat{\rho}_{QX}^{(\text{in};1)} \hat{S}_G^\dagger]$, and no signal (i.e., $|\emptyset\rangle$) with probability $1 - p = \text{Tr}[\hat{Q}_X^{(n)} \hat{S}_G \hat{\rho}_{QX}^{(\text{in};1)} \hat{S}_G^\dagger]$. Notice if $\mathbb{S}_G$ acts uniformly on the internal d.o.f. of P , then p is a constant parameter that does not depend on the transmitted message. In this special case Φ_G reduces to a standard EC $\mathcal{E}_p$ whose quantum capacity is expressed by the formula [13,37]

$$Q(\mathcal{E}_p) = \max\{0, (2p - 1) \log_2 d\}, \quad (6)$$

which is zero for p smaller than the critical value $1/2$. For state-dependent EC, no such formula exists; however, as rigorously shown in Sec. II. A of [32] via singular value decomposition and unitary equivalence arguments, the capacity of the channel Φ_G only depends on the singular eigenvalues $\{\sqrt{p_1}, \sqrt{p_2}, \dots, \sqrt{p_d}\}$ of the operator $\hat{P}_X^{(n)} \hat{S}_G$. Furthermore, by exploiting Eq. (1) one can establish the following bounds:

$$Q_{\text{low}}(\Phi_G) := Q(\mathcal{E}_{p_{\min}}) \leq Q(\Phi_G) \leq Q(\mathcal{E}_{p_{\max}}) =: Q_{\text{up}}(\Phi_G), \quad (7)$$

with $p_{\max/\min} := \max/\min\{p_1, p_2, \dots, p_d\}$ —see Supplemental Material [32].

SA by RC—We now present examples of the channel Φ_G which display SA effects. In particular, we consider the configuration illustrated in Fig. 2, where a spin-1/2 particle P ($d = 2$), with kinetic energy E propagates along a 1D line in the presence of two (possibly spin-dependent) potential barriers associated with the SMs $\mathbb{S}_1$ and $\mathbb{S}_2$. The barriers are separated by a distance w , across which the particle interacts with a pointlike, spin-insensitive, and energy-independent scattering matrix $\mathbb{S}_\eta$ emulating transmission losses. The first barrier, $\mathbb{S}_1$, corresponds to a square potential step $\hat{V}_1(x)$ which is zero for $|x| > a$ while for $|x| \leq a$ is equal to $V_0(\hat{1} + \epsilon \hat{\sigma}_z)$ with $V_0 > 0$ and $\epsilon \in [0, 1]$; the second barrier, $\mathbb{S}_2$, is obtained by translating $\hat{V}_1(x)$ by a distance w , i.e., $\hat{V}_2(x) = \hat{V}_1(x - w)$. These potentials act as barriers of energy $(1 + \epsilon)V_0$ for the spin-up component of P ($|\uparrow\rangle_r$) and $(1 - \epsilon)V_0$ for the spin-down component ($|\downarrow\rangle_r$). Solving the scattering problem for $\mathbb{S}_i$ yields a 4×4 matrix

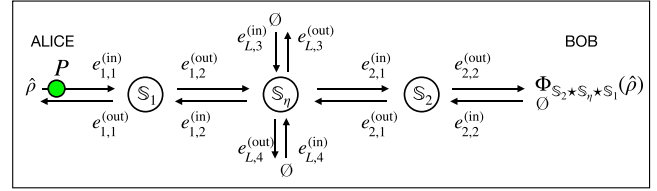


FIG. 2. Superactivation induced by RC. Here, $\mathbb{S}_1$ and $\mathbb{S}_2$ represent potential barriers that can reflect P , while $\mathbb{S}_\eta$ serves as a scatterer that deflects P from the horizontal paths into the vertical ones, thereby modeling losses.

composed by 2×2 diagonal blocks, $\mathbb{S}_i^{j,j'} = \begin{bmatrix} S_i^{j,j'} & 0 \\ 0 & S_i^{j,j'}(\downarrow) \end{bmatrix}$,

where for $j, j' = 1, 2$, $S_i^{j,j'}(\uparrow)$ and $S_i^{j,j'}(\downarrow)$ denote spin-resolved scattering coefficients describing the input-output relations of P [for the spin-independent case $\epsilon = 0$, $S_i^{j,j'}(\uparrow) = S_i^{j,j'}(\downarrow)$]. Analytical expressions for such coefficients are provided in [32]: they depend explicitly on the ratio E/V_0 and on the parameters a , w , and ϵ . The lossy scatterer is modeled by assuming that P has spin- and energy-independent probability $1 - \eta$ to reach the opposite barrier and probability η to be deflected into the outgoing vertical edges $e_{L,3}^{\text{out}}$ and $e_{L,4}^{\text{out}}$ of Fig. 2. This process can be described by the 8×8 block matrix

$$\mathbb{S}_\eta := \begin{bmatrix} 0 & 0 & \sqrt{\eta} \mathbb{1} & \sqrt{1-\eta} \mathbb{1} \\ 0 & 0 & -\sqrt{1-\eta} \mathbb{1} & \sqrt{\eta} \mathbb{1} \\ \sqrt{\eta} \mathbb{1} & -\sqrt{1-\eta} \mathbb{1} & 0 & 0 \\ \sqrt{1-\eta} \mathbb{1} & \sqrt{\eta} \mathbb{1} & 0 & 0 \end{bmatrix}, \quad (8)$$

where $\mathbb{1}$ is the 2×2 identity matrix acting on the spin degree of freedom. The global scattering process of the model is given by the resonant concatenation of $\mathbb{S}_2$, $\mathbb{S}_1$, and $\mathbb{S}_\eta$, i.e., by the matrix $\mathbb{S}_2 \star \mathbb{S}_\eta \star \mathbb{S}_1 = \mathbb{S}_2 \star (\mathbb{S}_\eta \circ \mathbb{S}_1)$, where the replacement of the second star product with a direct concatenation follows from the absence of back reflection between the first barrier and the loss element. Assuming that Alice encodes her signals into the port $e_{1,1}^{(\text{in})}$ and Bob retrieves them from port $e_{2,2}^{(\text{in})}$, as shown in the figure, the corresponding quantum channel $\Phi_{\mathbb{S}_2 \star (\mathbb{S}_\eta \circ \mathbb{S}_1)}$ is given by Eqs. (4) and (5), setting $\mathbb{S}_G = \mathbb{S}_2 \star (\mathbb{S}_\eta \circ \mathbb{S}_1)$ and $n = 2$. To highlight the superactivation effect, we compare its performance with that of a configuration without back reflection from $\mathbb{S}_2$, whose SM—up to an irrelevant global unitary—is given by the direct concatenation of $\mathbb{S}_1$ and $\mathbb{S}_\eta$, yielding the channel $\Phi_{\mathbb{S}_\eta \circ \mathbb{S}_1}$. In the $\epsilon = 0$ case, $\mathbb{S}_1$ and $\mathbb{S}_2$ are spin independent, both channels $\Phi_{\mathbb{S}_2 \star (\mathbb{S}_\eta \circ \mathbb{S}_1)}$ and $\Phi_{\mathbb{S}_\eta \circ \mathbb{S}_1}$ reduce to regular ECs, whose quantum capacities can be computed from Eq. (6). Figure 3 shows these capacities as a function of the input energy E of P . While both channels achieve the same capacity for large energies,

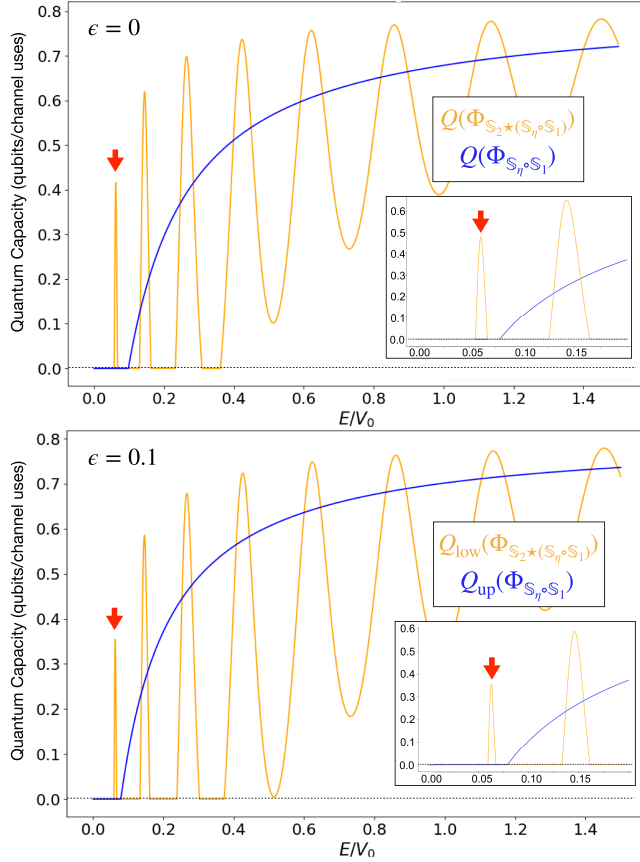


FIG. 3. Top: quantum capacities $Q(\Phi_{S_2 \star (S_\eta \circ S_1)})$ (orange) and $Q(\Phi_{S_\eta \circ S_1})$ (blue) for spin-independent potentials ($\epsilon = 0$) as a function of the input energy E , for fixed loss. Bottom: lower bound $Q_{\text{low}}(\Phi_{S_2 \star (S_\eta \circ S_1)})$ of Eq. (7) for the quantum capacity of $\Phi_{S_2 \star (S_\eta \circ S_1)}$ (orange), and upper bound $Q_{\text{up}}(\Phi_{S_\eta \circ S_1})$ for the quantum capacity of $\Phi_{S_\eta \circ S_1}$ (blue) in the spin-dependent case with $\epsilon = 0.1$. In all plots $d = 2$, $\eta = 0.1$, $a = 0.06\sqrt{20}$, and $w = 10\sqrt{20}$ where a and w are intended in units of $k_0^{-1} = \frac{\hbar}{\sqrt{2m}V_0}$. Red arrows indicate the energy intervals where the model exhibits SA [i.e., $Q(\Phi_{S_2 \star (S_\eta \circ S_1)}) > 0$ while $Q(\Phi_{S_\eta \circ S_1}) = 0$].

the data reveal specific energy values where $Q(\Phi_{S_2 \star (S_\eta \circ S_1)})$ is strictly larger than $Q(\Phi_{S_\eta \circ S_1})$ including regimes where the latter vanishes entirely. The case of spin-dependent potentials is analyzed in the bottom panel of the figure, where using Eq. (7) we compare the lower bound of $Q(\Phi_{S_2 \star (S_\eta \circ S_1)})$ with the upper of $Q(\Phi_{S_\eta \circ S_1})$: also in this case one notices that the star product does not obey the inequality (1) enforcing SA effects in some regimes. The robustness and generality of these phenomena are further demonstrated in [32], confirming that the quantum capacity enhancement persists over extended regions of the parameter space, both in the lossless and lossy regimes.

Conclusions—We developed a quantum-channel framework for scattering on graphs, showing that resonant concatenation, arising from internal back reflections, acts as a nonlinear composition rule capable of suppressing

noise and enabling SA of the quantum capacity. Our approach provides a direct route from local scattering matrices to global channel properties, offering a versatile tool for analyzing coherent transport in structured environments. Future extensions will address continuous-variable settings and multiparticle scattering effects, where additional interference mechanisms may further enrich the phenomenology uncovered here.

Acknowledgments—We acknowledge F. Taddei for useful discussion. We acknowledge financial support by MUR (Ministero dell’Università e della Ricerca) through the PNRR MUR Project No. PE0000023-NQSTI.

Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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