Mendonça, Jachymski, and Wang Reply In the previous Comment [1], it is pointed out that the Dicke-Ising phase diagram of our Letter [2] omits an intermediate phase with coexisting antiferromagnetic and superradiant order, and a change in the order of the superradiant transition for ferromagnetic Ising couplings. As the Comment notes, these lie at the resolution and beyond the range of our scans, respectively. We use this Reply to place both features within the framework of our Letter, and to show that our results and conclusions are unaffected.

We first address the order of the transition for $J > J_c = -\varepsilon/4$. The continuous transition we reported holds for $J < \varepsilon/2$, and after extending the parameter range, our method resolves the change from a continuous to a discontinuous superradiant transition, which occurs at a tricritical point at $J = \varepsilon/2$ in agreement with Refs. [1,3] and [4], as shown in Fig. 1. Rohn *et al.* [5] found a discontinuous transition at vanishing longitudinal field, where an additional spin-flip symmetry is present. We misread the scope of that work, and the corresponding remark in our Letter is incorrect.

We turn to the AFM-SP. The mean-field ansatz of Ref. [8] predicts a broad region in which antiferromagnetic order coexists with superradiance. As argued in our Letter, symmetry allows either a direct discontinuous transition or two continuous transitions through a phase breaking both translation and parity. Fine scans and finite-size analysis prompted by the Comment show that the coexistence region survives in a window narrower than the resolution of our Letter. The resulting boundaries, shown in Fig. 1, agree well with the quantum Monte Carlo (QMC) results of Refs. [1] and [3]. The reduction of the region relative to the mean-field prediction originates in the spin correlations, while the photon remains close to a coherent state.

The boundaries in Fig. 1 are drawn from the $\lambda = 0$ limit of the variational family of our Letter, in which the photon is a Gaussian state and the spins remain correlated and are solved by DMRG. For the Dicke-Ising chain, the full ansatz and DMRG on the whole spin-photon chain show transitions that approach these boundaries as N grows [6], allowing a direct comparison with the large-N QMC of Ref. [3].

The central result of our Letter concerns the Dicke-XXZ model. In the range of J_z where the matter subsystem develops XY order, the residual symmetry enforces $\langle S^x \rangle = 0$, so in a factorized spin-photon state, the light-matter coupling reduces to a self-consistent field $\propto \langle S^x \rangle S^x$ that vanishes. The full ansatz supplies instead the photon-mediated term $\propto (S^x)^2$, which does not require $\langle S^x \rangle \neq 0$ and produces a finite photon occupation throughout this parameter range. Whether the photon can be eliminated for $N \to \infty$ for the full dressing $\lambda = 1$, where these correlations are retained, remains open.

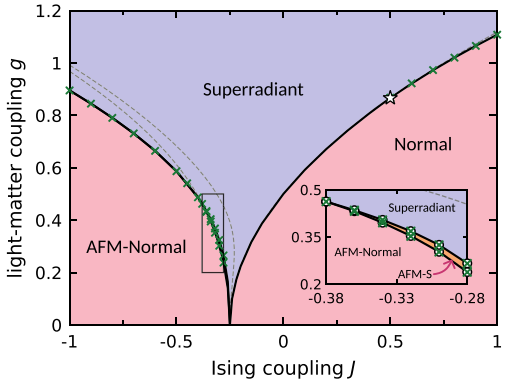


FIG. 1. Dicke-Ising phase diagram comparing NGS-DMRG [2,6] (solid lines; circle markers in the inset), QMC [1,3,7] (crosses), and MFT [8] (dashed lines). Tricritical point (star) at $J = \varepsilon/2$. NGS-DMRG data are available in Ref. [9].

In conclusion, the two features identified in the Comment refine the Dicke-Ising phase diagram of our Letter without altering its results and conclusions. A detailed presentation of the method, together with an extended analysis of the Dicke-Ising model as a benchmark, is given in Ref. [6], with code and data publicly available.

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Data availability—The datasets generated and analyzed during the current study are publicly available [9]. The open-source code package, NonGaussianDMRG.jl, is available at [10].

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