

Boiling After the Dust Settles: Constraining First-Order Phase Transitions During Dark Energy Domination

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A first-order phase transition could occur in the late Universe when vacuum energy begins dominating the energy density ($z \lesssim 0.3$) and convert some latent heat into other forms such as invisible radiation. This generic possibility also has concrete motivation in particle physics models, which invoke a multitude of vacua to address theoretical puzzles. The naïve constraint on such an event comes from measurements of the Hubble expansion rate, but this can only probe transitions involving $\mathcal{O}(10)\%$ of the dark energy. In this Letter, we show that significantly tighter constraints appear when accounting for phase transition fluctuations affecting CMB photon propagation anisotropically. For instance, if a completed phase transition has $\beta/H_\star \lesssim 25$, current CMB data limit the associated vacuum energy released to less than 1% of the dark energy. A transition to negative vacuum energy (quasi-anti-de Sitter) is allowed only for $\beta/H_\star \gtrsim 300$. For $\beta/H_\star \lesssim 500$, the Universe will not crunch for at least 14 Gyr.

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Introduction—Cosmology provides a powerful window into the dark sector, where new particles—including dark matter—can interact strongly with each other while remaining nearly invisible to the standard model (SM) particles. Decades of direct, indirect, and collider searches suggest that any interaction with SM particles must be extremely weak, if present at all. So far, gravity remains our most reliable tool for probing the dark sector. Fortunately, precision cosmological observations—sensitive to both the evolution and fluctuations of the Universe—allow us to study gravitational fluctuations induced by dark sector dynamics across a wide range of scales and epochs. In particular, they open the door to exploring dramatic events such as first-order phase transitions (FOPTs) that may occur entirely within the dark sector.

FOPTs have long been studied as efficient sources of gravitational waves (GWs). Transitions occurring at SM photon temperatures around 100 GeV to 1 TeV can produce signals detectable by future GW detectors like laser interferometer space antenna (LISA) and big bang observer (BBO) [1–9], with anisotropies that can reflect nontrivial inflationary or reheating dynamics [10–14]. FOPTs at MeV to GeV scales have been suggested as explanations for the stochastic GW background observed by pulsar timing arrays

[15–29]. In the eV range, they have been proposed to help resolve the Hubble tension [30–36] and may also produce observable CMB B-mode signals in future measurements [37]. Because of strong constraints from big bang nucleosynthesis (BBN) and the CMB, FOPTs below the 10 MeV scale are generally required to occur entirely within a dark sector to avoid excessive reheating of SM particles [38].

During FOPTs, bubble nucleation is stochastic and so occurs anisotropically even in the absence of primordial adiabatic perturbations. The resulting bubble wall collisions, sound waves, and magnetohydrodynamic motions of the plasma generate both scalar and tensor perturbations. While tensor modes, especially their role in producing GWs, have been extensively studied [39–41], scalar perturbations, including curvature and isocurvature modes, remain much less explored. Recent studies [42–48] have begun filling this gap, showing that precise measurements of the CMB anisotropies, energy spectrum, and matter power spectrum already provide tight constraints on FOPTs occurring between 1 eV and 10 keV temperature. Additional constraints arise from subhorizon density inhomogeneities generated during the PT, such as plasma sound waves [49] or curvature perturbations sourced by gravitational waves from bubble collisions [50].

In this Letter, we take a step further and explore extremely late-time FOPTs at redshifts $z \lesssim 0.3$, within the dark energy domination of the last few billion years. We derive upper bounds on the change in vacuum energy from a dark sector phase transition using constraints from CMB anisotropy measurements. We consider the effect of inhomogeneous redshifting of CMB photons due to

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variations in the time they enter true vacuum regions and show that this can impose much stronger constraints than those from the modification of cosmic expansion.

As to why such late FOPTs are well motivated to study and constrain, we recall the earlier periods in the conventional model of the early Universe when vacuum energy nears domination and note that, each time, this portends new vacuum dynamics, which turn this potential energy into dynamical degrees of freedom. This may occur by the rolling of an inflationary scalar field to its minimum, by a crossover as at the QCD phase transition, or by a violent FOPT as in proposals for electroweak baryogenesis. Quite recently in the history of the cosmos, we have seen vacuum energy reemerge as a dominant component, so it seems reasonable to ask whether this too may signal a recent or future phase transition.

Further beyond the SM, one large class of solutions to naturalness problems of particle physics uses large numbers of vacua and dynamical evolution between them. This strategy was famously invoked for the cosmological constant problem [51–62], and some mechanisms for the hierarchy problem [63–70] and the strong CP problem [71–75] work similarly. Generically, within such scenarios, it is possible to have a further vacuum transition at a recent time. This includes transitions to *negative* vacuum energy, which lead eventually to crunching Universes, on which we focus in the final section. This class of models, while seen as promising, has not been developed enough that late-time nucleation rates have been concretely calculated—this is where a model-independent analysis is crucial to broadly test these ideas.

Fluctuations of the phase transition time—An FOPT occurs through the nucleation and expansion of bubbles, the interiors of which are in the true vacuum. This process is inherently stochastic, so different regions transition at slightly different times. The bubble nucleation rate per unit time and volume is described by [76–78]

$$\Gamma = \Gamma_0 e^{-S(t)} \approx \Gamma_0 e^{-S(t_f)} e^{\beta(t-t_f)}, \quad (1)$$

where S is the bounce action for creating a critical bubble, and $\beta \approx -dS(t_f)/dt$. The timescale t_f serves as a reference time marking the PT's progression, defined as when an e^{-1} fraction of space remains in the false vacuum [79]. When $\beta \gg H_\star \equiv H(t_f)$, where $H(z)$ is the Hubble scale, the transition completes quickly, finishing in a small fraction of the Hubble time near t_f .

To estimate the redshift fluctuations of CMB photons entering true vacuum regions, we begin by evaluating the fluctuations in the PT time at different spatial points. We define the local transition time at position $\vec{x}$ as $t_c(\vec{x})$ and quantify its fluctuations using the two-point function $H_\star^2 \langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \rangle$, where $\delta t_c(\vec{x}) = t_c(\vec{x}) - \bar{t}_c$ and $\bar{t}_c$ is the average transition time. The corresponding dimensionless power spectrum is derived in [43] via its Fourier transform:

$$\mathcal{P}_{\delta t}(k) = \frac{k^3}{2\pi^2} \left(\frac{H_\star}{\beta} \right)^2 \int d^3r e^{i\vec{k}\cdot\vec{r}} \beta^2 \langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \rangle, \quad (2)$$

with $\vec{r} = \vec{x} - \vec{y}$. For perturbation modes with comoving wavelengths larger than the average comoving bubble separation, $k^{-1} \gg d_b \approx (8\pi)^{1/3} (1 + \bar{z}_{\text{pt}}) v_w \beta^{-1}$ [77,78], where $\bar{z}_{\text{pt}}$ is the redshift at which the PT completes, the power spectrum primarily reflects the standard deviation of the PT completion time averaged over a volume $\sim k^{-3}$. This leads to the dimensionless power spectrum [43]:

$$\mathcal{P}_{\delta t}(k) \approx 3(8\pi) v_w^3 \left(\frac{(1 + \bar{z}_{\text{pt}})k}{H_\star} \right)^3 \left(\frac{H_\star}{\beta} \right)^5, \quad (3)$$

assuming a bubble wall velocity v_w . For wavelengths shorter than the average bubble separation $k^{-1} \ll d_b$, the power spectrum is proportional to $\mathcal{P}_{\delta t}(k) \propto [(1 + \bar{z}_{\text{pt}})k/H_\star]^{-3} (H_\star/\beta)^{-1}$. In Supplemental Material, we show the full δt_c spectrum we use in our calculations [80], adapted from [43].

CMB redshift fluctuations from phase transition—We consider an FOPT in a dark-energy-dominated Universe, with the transition to the true vacuum completing at redshift $\bar{z}_{\text{pt}}$. Across the PT, a fraction $r > 0$ of the dark energy density is released. To isolate the minimal signatures, we consider a scenario where the latent heat is instantaneously converted into free-streaming, massless dark radiation, carrying an isocurvature perturbation. In a given complete model, energy conversion will occur over a finite timescale and the small-scale behavior depends on detailed bubble wall dynamics, but the existence and qualitative behavior of this large-scale perturbation signal will remain robust since the average energy density in the perturbed region always decreases following the phase transition.

We model the PT as leading to differential local Hubble expansion as follows:

$$H_{\text{pt}}(z, z_{\text{pt}}, r) \equiv H_0 \sqrt{\Omega_\Lambda (1-r) + r \Omega_\Lambda \left(\frac{1+z}{1+z_{\text{pt}}} \right)^4 + \Omega_m (1+z)^3}, \quad (4)$$

where z_{pt} (without a bar) varies anisotropically. Because the PT considered here only affects the CMB spectra at $\ell \lesssim 50$, while most Λ CDM parameter constraints are determined by higher ℓ modes, we adopt the Planck 2018 best-fit parameters [84]: $H_0 = 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_\Lambda = 0.69$, and $\Omega_m = 0.31$ in our analysis. Rather than performing a full fit of the Λ CDM + PT model to cosmological data, we estimate upper bounds on r from CMB anisotropy using these fixed parameters. As we will show, even when saturating the anisotropy bounds, the change in the Universe's average redshift due to the PT can be well below 1%, resulting in shifts in Λ CDM energy densities

smaller than the current precision. As the CMB fluctuations we consider come from random bubble distributions of PT, which are uncorrelated to the adiabatic perturbations, we turn off the adiabatic perturbations in the following discussion and consider the total CMB power spectrum to be the sum of PT and Λ CDM results.

A CMB photon traveling along the line of sight will enter the true vacuum region at $\bar{z}_{\text{pt}} + \delta z_{\text{pt}}(\theta, \phi)$, and we note that, as $\bar{z}_{\text{pt}}$ is defined at the redshift at which the PT finishes, $\delta z_{\text{pt}} \geq 0$. We focus on PTs with $\beta/H_* \approx 10 - 500$, and wall velocity $v_w = 1$, so the photon will then always be in the true vacuum region. As the Hubble is decreasing inside, the photons can be redshifted more or less before reaching us, depending on $\delta z_{\text{pt}}(\theta, \phi)$, which defines a “first encounter surface” (Fig. 1) that is similar to the last scattering surface of the CMB photon during recombination.

The spatial variations in the dark sector energy density could in general modify the local expansion at all later times, imprinting additional fluctuations in the photon spectrum. However, since we consider free-streaming dark radiation, the energy density becomes homogenized on the scale of the bubble size, which is where the dominant CMB anisotropy signal arises. For this reason, we take a conservative, simplifying assumption by only considering perturbations due to the inhomogeneous redshifting of dark radiation within $[\bar{z}_{\text{pt}} + \delta z_{\text{pt}}, \bar{z}_{\text{pt}}]$ (yellow region in Fig. 1). Beyond this time interval, we treat the energy density as homogeneous, and no further photon fluctuations are generated. (If the PT reheats into slow-moving fluids such as dark matter, the lack of free-streaming damping could preserve larger energy density fluctuations at low redshift, enhancing the CMB anisotropy. The perturbations we study are, thus, the minimal signatures, and we leave these cases for future work.)

We define the comoving distance as

$$\chi_{z_{\text{pt}}, r}(z_i, z_f) = \int_{z_f}^{z_i} \frac{dz}{(1+z)H_{\text{pt}}(z, z_{\text{pt}}, r)}. \quad (5)$$

In the no-PT case, $r = 0$, and we drop the z_{pt} dependence to write $\chi_{z_{\text{pt}}, r=0} \equiv \chi_0(z_i, z_f)$.

The first encounter surface is anisotropic and located at $\bar{z}_{\text{pt}} + \delta z_{\text{pt}}(\theta, \phi)$ (line BO in Fig. 1). The comoving distance from this entry point to us is

$$\chi_{BO} \Big|_{\delta z_0}^{\bar{z}_{\text{pt}} + \delta z_{\text{pt}}} = \chi_{\bar{z}_{\text{pt}}, r}(\bar{z}_{\text{pt}}, \delta z_0) + \chi_{\bar{z}_{\text{pt}} + \delta z_{\text{pt}}, r}(\bar{z}_{\text{pt}} + \delta z_{\text{pt}}, \bar{z}_{\text{pt}}), \quad (6)$$

where the final redshift δz_0 also depends on (θ, ϕ) . To determine this fluctuation, we can compare to the case where the PT occurs uniformly at $\bar{z}_{\text{pt}}$ without fluctuations. There are no fluctuations of the final redshift in this case, and $\delta z_0 = 0$ (line AO in Fig. 1). The comoving distance a

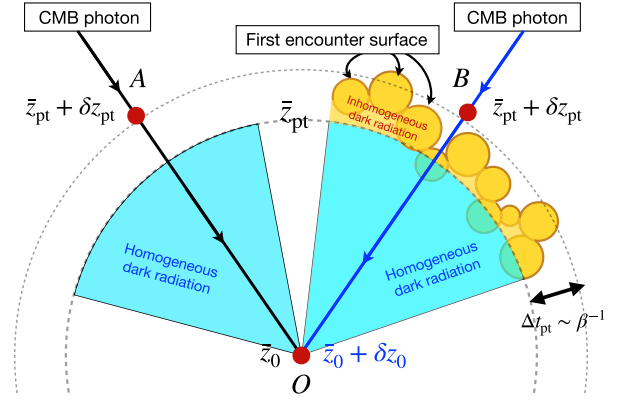


FIG. 1. Sketch of the origin of CMB fluctuations from a late-time FOPT, comparing photon trajectories before (left) and after (right) accounting for stochastic bubble nucleation. See text for details.

photon travels from redshift $\bar{z}_{\text{pt}} + \delta z_{\text{pt}}(\theta, \phi)$ to today is given by

$$\chi_{AO} \Big|_0^{\bar{z}_{\text{pt}} + \delta z_{\text{pt}}} = \chi_{\bar{z}_{\text{pt}}, r}(\bar{z}_{\text{pt}}, 0) + \chi_0(\bar{z}_{\text{pt}} + \delta z_{\text{pt}}, \bar{z}_{\text{pt}}). \quad (7)$$

Since the phase transition only affects the expansion rate of the Universe, the total comoving distance from the same initial redshift to us must remain the same. This gives the following condition:

$$\chi_{AO} \Big|_0^{\bar{z}_{\text{pt}} + \delta z_{\text{pt}}} = \chi_{BO} \Big|_{\delta z_0}^{\bar{z}_{\text{pt}} + \delta z_{\text{pt}}}. \quad (8)$$

Solving this equality gives the final redshift fluctuation δz_0 as a function of the PT-time fluctuation δz_{pt} . For small values $\bar{z}_{\text{pt}}, \delta z_0, \delta z_{\text{pt}} \ll 1$, the solution is well approximated by

$$\delta z_0 \approx \delta z_{\text{pt}}^2 \left[\frac{r \Omega_\Lambda}{(1 + \bar{z}_{\text{pt}})[\Omega_\Lambda + \Omega_m(1 + \bar{z}_{\text{pt}})^3]^{3/2}} \right]. \quad (9)$$

This expression agrees with the numerical solution at the $\mathcal{O}(1)\%$ level for the parameters we consider. The fluctuation scales with the PT energy r and with δz_{pt}^2 , where one power of δz_{pt} arises from its effect on photon propagation, and the other from having a significant redshift of dark radiation energy in the first δz_{pt} window. For $\delta z_{\text{pt}} > 0$, the CMB photon enters the true vacuum region earlier, and the Hubble expansion is slower at a later redshift due to a smaller total energy density compared to the uniform-PT case. This requires the expansion to end earlier with $\delta z_0 > 0$ to keep the same comoving distance.

We estimate the redshift power spectrum $\mathcal{P}_{\delta z_{\text{pt}}}$ using the power spectrum of the PT time at which each point transitions to the true vacuum,

$$\mathcal{P}_{\delta z_{\text{pt}}}(k) \approx (1 + \bar{z}_{\text{pt}})^2 \mathcal{P}_{\delta t}(k). \quad (10)$$

Combining this with Eq. (9), we compute the two-point function $\langle \delta z_{\text{pt}}^2(\theta_1, \phi_1) \delta z_{\text{pt}}^2(\theta_2, \phi_2) \rangle$ and obtain the power spectrum of the induced photon redshift:

$$\mathcal{P}_{\delta z_0}(k) \approx r^2 \Omega_{\Lambda}^2 [\Omega_{\Lambda} + \Omega_{\text{m}}(1 + \bar{z}_{\text{pt}})^3]^{-3} \mathcal{I}_{\delta t}(k), \quad (11)$$

where

$$\mathcal{I}_{\delta t}(k) \equiv k^3 \int_{k_{\min}}^{k_{\max}} \frac{dr}{r} \mathcal{P}_{\delta t}(r) \int_{-1}^1 d\mu s^{-3} \mathcal{P}_{\delta t}(s) \quad (12)$$

and $s = \sqrt{k^2 + r^2 - 2kr\mu}$. In the numerical integral, we set $k_{\min}$ ($k_{\max}$) to be 10 times smaller (larger) than the peak mode of $\mathcal{P}_{\delta t}$, which provides a good approximation. Further details of the derivation of Eqs. (9) and (11) are in Supplemental Material [80].

The anisotropy we obtain corresponds to a late-time integrated Sachs-Wolfe effect [85], sourced by metric perturbations from the PT around $\bar{z}_{\text{pt}}$. (See Ref. [86] for late-time ISW from a quintessence to matter transition.) The redshift perturbation δz_0 alters the CMB photon temperature, and its contribution to the CMB power spectrum is estimated by projecting $\mathcal{P}_{\delta z_0}(k)$ onto the first encounter surface using the standard method,

$$D_{\ell}^{\text{TT,pt}} = 2\ell(\ell+1) \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} \mathcal{P}_{\delta z_0}(k) j_{\ell}^2(k\Delta\tau). \quad (13)$$

We use $\Delta\tau = \chi_0(\bar{z}_{\text{pt}}, 0)$ as the no-PT case since the allowed PT only introduces a negligible ($< 1\%$) correction to the comoving distance. Since the PT signal we consider mainly peaks at $\ell < 20$, we omit the reionization suppression factor in the calculation. Including $e^{-2\tau} \approx 0.89$ [84] for

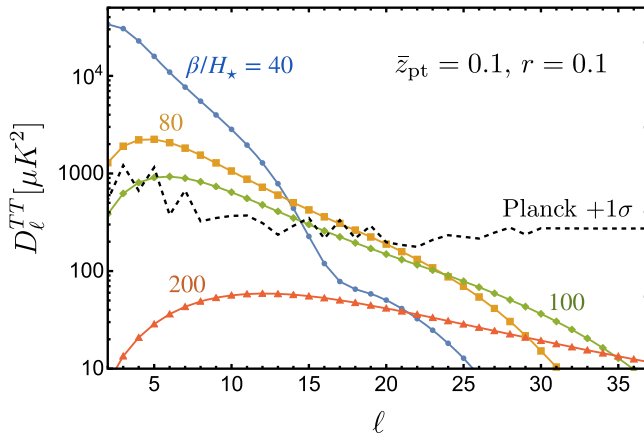


FIG. 2. CMB temperature power spectrum from a PT at $\bar{z}_{\text{pt}} = 0.1$ with $r = 0.1$. Discrete ℓ points are connected for visualization. The $+1\sigma$ error bar from Planck 2018 [87] is shown for comparison.

signal peaks at higher ℓ would only change the bound on r at the percent level. We show some examples of $D_{\ell}^{\text{TT,pt}}$ in Fig. 2.

We estimate the bounds on r using a standard χ^2 analysis, comparing $D_{\ell}^{\text{TT,pt}}$ with the 1σ error bars for each ℓ mode taken directly from the Planck 2018 archive [87] (see Supplemental Material for details [80]). While this method is not fully accurate—since Planck error bars are highly correlated and depend on the assumed baseline cosmology—it still gives a useful estimate of the allowed size of r without significantly modifying the observed D_{ℓ}^{TT} .

Figure 3 shows the upper bounds on r , assuming $v_w = 1$. Each point in the figure can be translated to a different $v_w \lesssim 1$ by rescaling $\beta/H_{\star} \rightarrow v_w(\beta/H_{\star})$ and the bound on r as $r \rightarrow r v_w^{-3/2}$. A useful analytic order-of-magnitude approximation is $r \lesssim 10^{-5}(\beta/H_{\star})^2$. Approximating the power spectrum with just its peak value around $\delta t_c \sim \beta^{-1}$, we get $\mathcal{P}_{\delta t} \sim (\beta/H_{\star})^{-2}$, and Eqs. (11) and (12) give $\mathcal{P}_{\delta z_0} \sim r^2(\beta/H_{\star})^{-4}$. The CMB bound is roughly $\mathcal{P}_{\delta z_0} \lesssim A_s \approx 2 \times 10^{-9}$, where A_s refers to the primordial scalar perturbation, leading to this estimate.

The r bounds also become weaker at lower $\bar{z}_{\text{pt}}$, as the peak of $\mathcal{P}_{\delta z_0}(k)$ shifts to such low k modes that it no longer contributes significantly to the power spectrum with $\ell \geq 2$. Note that a naïve estimate of the PT bound by requiring a

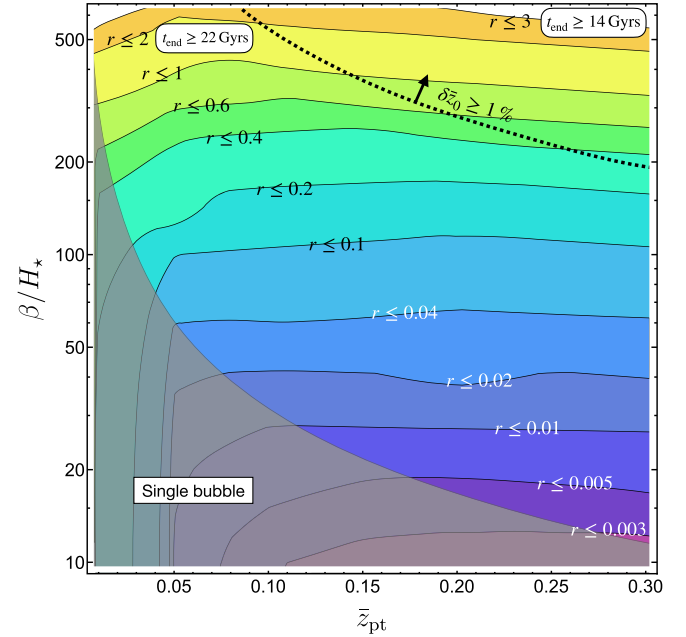


FIG. 3. Upper bounds on a FOPT converting a fraction r of dark energy into dark radiation, assuming wall velocity $v_w = 1$. The dashed curve indicates where the present-day redshift is shifted by 1% relative to the no PT, implying a $\approx 1\%$ rescaling of Λ CDM energy densities at earlier times if $z_0 = 0$ is fixed. The gray region indicates where the distance to $\bar{z}_{\text{pt}}$ falls within the average bubble size, where our anisotropy estimate does not apply. For $r > 1$, the square boxes show the minimal crunching time from Eq. (15).

$\lesssim 10\%$ suppression in the Hubble rate from Eq. (4) at $z \approx 0.2$, as motivated by the $h(z)/h^{\Lambda\text{CDM}}$ constraint from the DESI BAO analysis [88], allows a PT at $\bar{z}_{\text{pt}} = 0.3$ with $r \approx 0.65$. The CMB anisotropy bounds we obtained are significantly stronger for $\beta/H_\star \lesssim 200$.

Since our estimate assumes sufficient bubble statistics, the average comoving bubble separation must be smaller than the comoving distance from redshift $\bar{z}_{\text{pt}}$ to today:

$$d_b \approx (8\pi)^{1/3} v_w \beta^{-1} (1 + \bar{z}_{\text{pt}}) < \Delta\tau. \quad (14)$$

Although a single large bubble encompassing our region will strongly affect CMB photon propagation and may lead to stringent constraints on r , we leave a detailed analysis for future work.

Compared to standard ΛCDM with no PT, the change in H_{pt} also induces a homogeneous redshifting today, $\delta\bar{z}_0$, which we can determine by equating $\chi_{\bar{z}_{\text{pt}},r}(\bar{z}_{\text{pt}}, \delta\bar{z}_0) = \chi_0(\bar{z}_{\text{pt}}, 0)$. After rescaling today's redshift back to zero, the ΛCDM energy densities at earlier times could be modified at the level of $\delta\bar{z}_0$. For reference, Fig. 3 shows a dashed curve for $\delta\bar{z}_0 = 1\%$ when saturating the r bounds from the anisotropy estimate. This overall redshift shift is comparable to uncertainties in current ΛCDM density measurements, and the region above the curve may be further constrained by fitting ΛCDM energy densities to the full CMB data.

Note that PT perturbations can also generate CMB lensing and modify the matter power spectrum. In Supplemental Material, we provide order-of-magnitude estimates. We find that the matter power spectrum correction is well below current large-scale structure sensitivity due to the low redshift of the PT, while the lensing signal could approach current sensitivity [89] for a large $r = \mathcal{O}(1)$. A proper calculation requires a more careful treatment of the dark radiation-sourced perturbations, which we defer to future work.

The end times—For large $\beta/H_\star$, FOPTs with $r > 1$ become allowed. Here the released latent heat exceeds the dark energy density $\Lambda_0 = \rho_c \Omega_\Lambda$, where $\rho_c = (2.5 \text{ meV})^4$ [90] is the critical density, leading the Universe to *decelerate* and eventually *crunch*. This regime may be motivated from a UV perspective, as there is evidence from string theory that true vacua have $\Lambda_0 < 0$, and any positive Λ_0 must be metastable (see, e.g., Ref. [91] and references therein). This is often taken as motivation for quintessence [92–95], but a late-time FOPT would also satisfy these conjectures.

The second Friedmann equation after the PT is

$$\frac{\ddot{a}}{a} = -H_0^2 \Omega_\Lambda \left[r \left(\frac{1+z}{1+\bar{z}_{\text{pt}}} \right)^4 + (r-1) + \frac{\Omega_m}{2\Omega_\Lambda} (1+z)^3 \right]. \quad (15)$$

For $r > 1$ the right-hand side is strictly negative, so the expansion begins decelerating.

The continued expansion of the Universe further dilutes the radiation and matter densities until the Universe switches from expanding to contracting when $H(z_{\text{max}})^2 = 0$. Neglecting matter density ($\Omega_\Lambda \rightarrow 1$), this occurs at $1 + z_{\text{max}} = (1 + \bar{z}_{\text{pt}})(1 - 1/r)^{1/4}$. Following this point, the scale factor decreases, causing the energy density to grow and leading to increasingly rapid contraction. The Universe then collapses at a time $t_{\text{end}} \propto H_0^{-1}/\sqrt{r-1}$.

In the yellow and orange regions of Fig. 3 where $r > 1$ is allowed, we show the lower bounds on t_{end} corresponding to the upper bounds $r \leq 2$ and ≤ 3 , obtained by numerically solving Eq. (15). The anisotropy bound ensures that, for a PT with $\beta/H_\star \lesssim 500$ which has already completed, the Universe must live at least another $\gtrsim 14$ Gyr. As to humanity, we can offer no such guarantees.

Conclusion—In this Letter, we investigated constraints on late-time FOPTs from low- ℓ CMB temperature anisotropies and show that perturbations yield much stronger bounds than background evolution alone. It would be interesting to extend this to other CMB observables, including Sunyaev–Zeldovich (SZ) effects, a more precise lensing calculation, and the induced B -mode signals. Given the characteristic bubble scale of PT perturbations, the resulting CMB bispectrum could provide a distinctive signature of the transition. This work is one step toward a map of the constraints on FOPTs over all of cosmic time, which limits the most violent dark sector behaviors. Currently the constraints between recombination and dark energy domination are unknown and remain an obvious target for future exploration. It would also be interesting to consider phase transitions in sectors that are not entirely secluded but have some portal interaction with us. Overall, the signatures of phase transitions in the late Universe may be subtle enough that we will not see them if we do not look for them. Since this phenomenon is motivated both on general grounds and in specific theoretical frameworks, it would serve us well to learn how to characterize and search for signals from these drastic rearrangements of the degrees of freedom of the Universe.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [1] A. Kosowsky, M. S. Turner, and R. Watkins, Gravitational waves from first order cosmological phase transitions, *Phys. Rev. Lett.* **69**, 2026 (1992).
 - [2] A. Kosowsky, M. S. Turner, and R. Watkins, Gravitational radiation from colliding vacuum bubbles, *Phys. Rev. D* **45**, 4514 (1992).
 - [3] A. Kosowsky and M. S. Turner, Gravitational radiation from colliding vacuum bubbles: envelope approximation to many bubble collisions, *Phys. Rev. D* **47**, 4372 (1993).
 - [4] M. Kamionkowski, A. Kosowsky, and M. S. Turner, Gravitational radiation from first order phase transitions, *Phys. Rev. D* **49**, 2837 (1994).
 - [5] C. Caprini *et al.*, Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions, *J. Cosmol. Astropart. Phys.* **04** (2016) 001.
 - [6] C. Caprini *et al.*, Detecting gravitational waves from cosmological phase transitions with LISA: An update, *J. Cosmol. Astropart. Phys.* **03** (2020) 024.
 - [7] R. Caldwell *et al.*, Detection of early-universe gravitational-wave signatures and fundamental physics, *Gen. Relativ. Gravit.* **54**, 156 (2022).
 - [8] K. G. Arun *et al.* (LISA Collaboration), New horizons for fundamental physics with LISA, *Living Rev. Relativity* **25**, 4 (2022).
 - [9] P. Auclair *et al.* (LISA Cosmology Working Group), Cosmology with the laser interferometer space antenna, *Living Rev. Relativity* **26**, 5 (2023).
 - [10] M. Geller, A. Hook, R. Sundrum, and Y. Tsai, Primordial anisotropies in the gravitational wave background from cosmological phase transitions, *Phys. Rev. Lett.* **121**, 201303 (2018).
 - [11] S. Kumar, R. Sundrum, and Y. Tsai, Non-Gaussian stochastic gravitational waves from phase transitions, *J. High Energy Phys.* **11** (2021) 107.
 - [12] N. Bartolo *et al.* (LISA Cosmology Working Group), Probing anisotropies of the stochastic gravitational wave background with LISA, *J. Cosmol. Astropart. Phys.* **11** (2022) 009.
 - [13] A. Bodas and R. Sundrum, Large primordial fluctuations in gravitational waves from phase transitions, *J. High Energy Phys.* **06** (2023) 029.
 - [14] Y. Cui, S. Kumar, R. Sundrum, and Y. Tsai, Unraveling cosmological anisotropies within stochastic gravitational wave backgrounds, *J. Cosmol. Astropart. Phys.* **10** (2023) 064.
 - [15] Z. Arzoumanian *et al.* (NANOGrav Collaboration), The NANOGrav 12.5 yr Data Set: Search for an isotropic stochastic gravitational-wave background, *Astrophys. J. Lett.* **905**, L34 (2020).
 - [16] G. Agazie *et al.* (NANOGrav Collaboration), The NANOGrav 15 yr Data Set: Evidence for a gravitational-wave background, *Astrophys. J. Lett.* **951**, L8 (2023).
 - [17] S. Chen *et al.* (EPTA Collaboration), Common-red-signal analysis with 24-yr high-precision timing of the European Pulsar Timing Array: Inferences in the stochastic gravitational-wave background search, *Mon. Not. R. Astron. Soc.* **508**, 4970 (2021).
 - [18] J. Antoniadis *et al.* (EPTA and InPTA Collaborations), The second data release from the European Pulsar Timing Array—III. Search for gravitational wave signals, *Astron. Astrophys.* **678**, A50 (2023).
 - [19] J. Antoniadis *et al.* (EPTA and InPTA Collaborations), The second data release from the European Pulsar Timing Array—IV. Implications for massive black holes, dark matter, and the early Universe, *Astron. Astrophys.* **685**, A94 (2024).
 - [20] B. Goncharov *et al.*, On the evidence for a common-spectrum process in the search for the nanohertz gravitational-wave background with the Parkes Pulsar Timing Array, *Astrophys. J. Lett.* **917**, L19 (2021).
 - [21] D. J. Reardon *et al.*, Search for an isotropic gravitational-wave background with the Parkes Pulsar Timing Array, *Astrophys. J. Lett.* **951**, L6 (2023).
 - [22] H. Xu *et al.*, Searching for the nano-hertz stochastic gravitational wave background with the Chinese Pulsar Timing Array Data Release I, *Res. Astron. Astrophys.* **23**, 075024 (2023).
 - [23] T. Bringmann, P. F. Depta, T. Konstandin, K. Schmidt-Hoberg, and C. Tasillo, Does NANOGrav observe a dark sector phase transition?, *J. Cosmol. Astropart. Phys.* **11** (2023) 053.
 - [24] A. Addazi, Y.-F. Cai, A. Marciano, and L. Visinelli, Have pulsar timing array methods detected a cosmological phase transition?, *Phys. Rev. D* **109**, 015028 (2024).
 - [25] T. Ghosh, A. Ghoshal, H.-K. Guo, F. Hajkarim, S. F. King, K. Sinha, X. Wang, and G. White, Did we hear the sound of the Universe boiling? Analysis using the full fluid velocity profiles and NANOGrav 15-year data, *J. Cosmol. Astropart. Phys.* **05** (2024) 100.
 - [26] A. Salvio, Supercooling in radiative symmetry breaking: theory extensions, gravitational wave detection and primordial black holes, *J. Cosmol. Astropart. Phys.* **12** (2023) 046.
 - [27] A. Salvio, Pulsar timing arrays and primordial black holes from a supercooled phase transition, *Phys. Lett. B* **852**, 138639 (2024).
 - [28] M. W. Winkler and K. Freese, Origin of the stochastic gravitational wave background: First-order phase transition vs. black hole mergers, *Phys. Rev. D* **111**, 083509 (2025).
 - [29] A. Banik, Y. Cui, Y.-D. Tsai, and Y. Tsai, The sound of dark sectors in Pulsar Timing Arrays, *Phys. Dark Universe* **53**, 102398 (2026).
 - [30] A. G. Riess *et al.*, A comprehensive measurement of the local value of the Hubble constant with 1 km/s/Mpc uncertainty from the Hubble Space Telescope and the SH0ES Team, *Astrophys. J. Lett.* **934**, L7 (2022).

- [31] N. Schöneberg, G. Franco Abellán, A. Pérez Sánchez, S. J. Witte, V. Poulin, and J. Lesgourgues, The H0 Olympics: A fair ranking of proposed models, *Phys. Rep.* **984**, 1 (2022).
- [32] F. Niedermann and M. S. Sloth, New early dark energy, *Phys. Rev. D* **103**, L041303 (2021).
- [33] F. Niedermann and M. S. Sloth, Resolving the Hubble tension with new early dark energy, *Phys. Rev. D* **102**, 063527 (2020).
- [34] F. Niedermann and M. S. Sloth, New early dark energy as a solution to the H_0 and S_8 tensions, *The Hubble Constant Tension* (Springer, Singapore, 2024), 10.1007/978-981-99-0177-7_23.
- [35] M. Garny, F. Niedermann, H. Rubira, and M. S. Sloth, Hot new early dark energy bridging cosmic gaps: Supercooled phase transition reconciles stepped dark radiation solutions to the Hubble tension with BBN, *Phys. Rev. D* **110**, 023531 (2024).
- [36] A. Chatrchyan, F. Niedermann, V. Poulin, and M. S. Sloth, Confronting cold new early dark energy and its equation of state with updated CMB, supernovae, and BAO data, *Phys. Rev. D* **111**, 043536 (2025).
- [37] K. Greene, A. Ireland, G. Krnjaic, and Y. Tsai, Observable CMB B-modes from cosmological phase transitions, *Phys. Rev. D* **113**, 083527 (2026).
- [38] Y. Bai and M. Korwar, Cosmological constraints on first-order phase transitions, *Phys. Rev. D* **105**, 095015 (2022).
- [39] C. Caprini, R. Durrer, and G. Servant, Gravitational wave generation from bubble collisions in first-order phase transitions: An analytic approach, *Phys. Rev. D* **77**, 124015 (2008).
- [40] R. Jinno and M. Takimoto, Gravitational waves from bubble collisions: An analytic derivation, *Phys. Rev. D* **95**, 024009 (2017).
- [41] R. Jinno and M. Takimoto, Gravitational waves from bubble dynamics: Beyond the envelope, *J. Cosmol. Astropart. Phys.* **01** (2019) 060.
- [42] J. Liu, L. Bian, R.-G. Cai, Z.-K. Guo, and S.-J. Wang, Constraining first-order phase transitions with curvature perturbations, *Phys. Rev. Lett.* **130**, 051001 (2023).
- [43] G. Elor, R. Jinno, S. Kumar, R. McGehee, and Y. Tsai, Finite bubble statistics constrain late cosmological phase transitions, *Phys. Rev. Lett.* **133**, 211003 (2024).
- [44] M. Lewicki, P. Toczek, and V. Vaskonen, Black holes and gravitational waves from slow first-order phase transitions, *Phys. Rev. Lett.* **133**, 221003 (2024).
- [45] M. R. Buckley, P. Du, N. Fernandez, and M. J. Weikert, Dark radiation isocurvature from cosmological phase transitions, *J. Cosmol. Astropart. Phys.* **07** (2024) 031.
- [46] J. Zou, Z. Zhu, Z. Zhao, and L. Bian, Numerical simulations of density perturbation and gravitational wave production from cosmological first-order phase transition, *Sci. China Phys. Mech. Astron.* **69**, 270412 (2026).
- [47] G. Franciolini, Y. Gouttenoire, and R. Jinno, Curvature perturbations from first-order phase transitions: Implications to black holes and gravitational waves, *Phys. Rev. Lett.* **136**, 171404 (2026).
- [48] W. Yin, Direct detection of cosmic walls with paleodetectors, *Phys. Rev. D* **113**, 033003 (2026).
- [49] N. Ramberg, W. Ratzinger, and P. Schwaller, One μ to rule them all: CMB spectral distortions can probe domain walls, cosmic strings and low scale phase transitions, *J. Cosmol. Astropart. Phys.* **02** (2023) 039.
- [50] S. Bottaro, M. Geller, D. Redigolo, and M. Tsur, Post-recombination fluctuations from a sequestered dark sector, *arXiv:2605.03007*.
- [51] L. F. Abbott, A mechanism for reducing the value of the cosmological constant, *Phys. Lett. B* **150**, 427 (1985).
- [52] J. D. Brown and C. Teitelboim, Dynamical neutralization of the cosmological constant, *Phys. Lett. B* **195**, 177 (1987).
- [53] J. D. Brown and C. Teitelboim, Neutralization of the cosmological constant by membrane creation, *Nucl. Phys. B* **297**, 787 (1988).
- [54] M. J. Duncan and L. G. Jensen, Four forms and the vanishing of the cosmological constant, *Nucl. Phys. B* **336**, 100 (1990).
- [55] R. Bousso and J. Polchinski, Quantization of four form fluxes and dynamical neutralization of the cosmological constant, *J. High Energy Phys.* **06** (2000) 006.
- [56] J. L. Feng, J. March-Russell, S. Sethi, and F. Wilczek, Saltatory relaxation of the cosmological constant, *Nucl. Phys. B* **602**, 307 (2001).
- [57] G. R. Dvali and A. Vilenkin, Field theory models for variable cosmological constant, *Phys. Rev. D* **64**, 063509 (2001).
- [58] N. Kaloper and A. Westphal, Quantum-mechanical mechanism for reducing the cosmological constant, *Phys. Rev. D* **106**, L101701 (2022).
- [59] N. Kaloper, General relativity on the multiverse and nature's hierarchies, *Phys. Rev. D* **106**, 044023 (2022).
- [60] N. Kaloper, Axion flux monodromy discharges relax the cosmological constant, *J. Cosmol. Astropart. Phys.* **11** (2023) 032.
- [61] N. Kaloper, de Sitter space decay and cosmological constant relaxation in unimodular gravity with charged membranes, *Phys. Rev. D* **108**, 025005 (2023).
- [62] N. Kaloper, Discretely evanescent dark energy, *J. Cosmol. Astropart. Phys.* **11** (2025) 075.
- [63] G. Dvali and A. Vilenkin, Cosmic attractors and gauge hierarchy, *Phys. Rev. D* **70**, 063501 (2004).
- [64] G. Dvali, Large hierarchies from attractor vacua, *Phys. Rev. D* **74**, 025018 (2006).
- [65] P. W. Graham, D. E. Kaplan, and S. Rajendran, Cosmological relaxation of the electroweak scale, *Phys. Rev. Lett.* **115**, 221801 (2015).
- [66] D. E. Kaplan and R. Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, *Phys. Rev. D* **93**, 085007 (2016).
- [67] K. Choi and S. H. Im, Realizing the relaxation from multiple axions and its UV completion with high scale supersymmetry, *J. High Energy Phys.* **01** (2016) 149.
- [68] L. E. Ibanez, M. Montero, A. Uranga, and I. Valenzuela, Relaxion monodromy and the weak gravity conjecture, *J. High Energy Phys.* **04** (2016) 020.
- [69] R. S. Gupta, Z. Komargodski, G. Perez, and L. Ubaldi, Is the relaxation an axion?, *J. High Energy Phys.* **02** (2016) 166.
- [70] A. Hook and G. Marques-Tavares, Relaxation from particle production, *J. High Energy Phys.* **12** (2016) 101.
- [71] T. Banks, M. Dine, and N. Seiberg, Irrational axions as a solution of the strong CP problem in an eternal universe, *Phys. Lett. B* **273**, 105 (1991).

- [72] G. Dvali, A Vacuum accumulation solution to the strong CP problem, *Phys. Rev. D* **74**, 025019 (2006).
- [73] N. Kaloper and J. Terning, Landscaping the Strong CP Problem, *J. High Energy Phys.* **03** (2019) 032.
- [74] N. Kaloper, A quantal theory of restoration of strong CP symmetry, *Phys. Rev. D* **113**, 075020 (2026).
- [75] N. Kaloper, An alternative to axion, *Phys. Rev. D* **113**, L011701 (2026).
- [76] C. J. Hogan, Nucleation of cosmological phase transitions, *Phys. Lett.* **133B**, 172 (1983).
- [77] K. Enqvist, J. Ignatius, K. Kajantie, and K. Rummukainen, Nucleation and bubble growth in a first-order cosmological electroweak phase transition' *Phys. Rev. D* **45**, 3415 (1992).
- [78] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, Numerical simulations of acoustically generated gravitational waves at a first order phase transition, *Phys. Rev. D* **92**, 123009 (2015).
- [79] P. Athron, C. Balázs, A. Fowlie, L. Morris, and L. Wu, Cosmological phase transitions: From perturbative particle physics to gravitational waves, *Prog. Part. Nucl. Phys.* **135**, 104094 (2024).
- [80] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/1vyz-xdkb>, which includes Refs. [81–83].
- [81] J. Lesgourgues, The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview, [arXiv:1104.2932](https://arxiv.org/abs/1104.2932).
- [82] Z. Pan, L. Knox, and M. White, Dependence of the cosmic microwave background lensing power spectrum on the matter density, *Mon. Not. R. Astron. Soc.* **445**, 2941 (2014).
- [83] M. S. Madhavacheril *et al.* (ACT Collaboration), The Atacama Cosmology Telescope: DR6 gravitational lensing map and cosmological parameters, *Astrophys. J.* **962**, 113 (2024).
- [84] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020).
- [85] R. K. Sachs and A. M. Wolfe, Perturbations of a cosmological model and angular variations of the microwave background, *Astrophys. J.* **147**, 73 (1967).
- [86] M. A. Amin, P. Zukin, and E. Bertschinger, Scale-dependent growth from a transition in dark energy dynamics, *Phys. Rev. D* **85**, 103510 (2012).
- [87] <https://pla.esac.esa.int/#home>.
- [88] K. Lodha *et al.* (DESI Collaboration), Extended dark energy analysis using DESI DR2 BAO measurements, *Phys. Rev. D* **112**, 083511 (2025).
- [89] F. J. Qu *et al.* (ACT Collaboration), The Atacama Cosmology Telescope: A measurement of the DR6 CMB lensing power spectrum and its implications for structure growth, *Astrophys. J.* **962**, 112 (2024).
- [90] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [91] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, De Sitter space and the Swampland, [arXiv:1806.08362](https://arxiv.org/abs/1806.08362).
- [92] Y. Fujii, Origin of the gravitational constant and particle masses in scale invariant scalar—tensor theory, *Phys. Rev. D* **26**, 2580 (1982).
- [93] L. H. Ford, Cosmological constant damping by unstable scalar fields, *Phys. Rev. D* **35**, 2339 (1987).
- [94] B. Ratra and P. J. E. Peebles, Cosmological consequences of a rolling homogeneous scalar field, *Phys. Rev. D* **37**, 3406 (1988).
- [95] C. Wetterich, Cosmology and the fate of dilatation symmetry, *Nucl. Phys.* **B302**, 668 (1988).